CKM Matrix from Lattice QCD and the z Expansion

Andreas S. Kronfeld
Fermilab & IAS TU München

GENIE z-expansion Tutorial
September 1, 2016
Fermilab

Outline

- Basics of Lattice QCD
- z Expansions Used in B Physics
- z Fits: Lattice QCD, Experiment, Combined
- Compare and Contrast ν and B Physics

LATTICE GAUGE THEORY
FOR BEGINNERS

The QCD Lagrangian

- SU(3) gauge symmetry and 1 + n_f + 1 parameters:

$$\mathcal{L}_{\rm QCD} = \frac{1}{g_0^2}\,{\rm tr}[F_{\mu\nu}F^{\mu\nu}] \qquad m_\Omega,\ \Upsilon(2S\text{-}1S),\ \text{or}\ r_1,\ w_0,\ldots$$

$$- \sum_f \bar\psi_f(\not{D} + m_f)\psi_f \qquad m_\pi,\ m_K,\ m_{J/\psi},\ m_\Upsilon,\ \ldots$$

$$+ \frac{i\theta}{32\pi^2}\varepsilon^{\mu\nu\rho\sigma}\,{\rm tr}[F_{\mu\nu}F_{\rho\sigma}] \qquad \theta = 0.$$

- Observable CP violation $\propto \theta = \vartheta - \arg\det y_f$ (if all Yukawas nonvanishing):
  - neutron electric-dipole moment sets limit $\theta \lesssim 10^{-11}$;
  - bafflingly implausible cancellation called the strong CP problem.

Lattice Gauge Theory

K. Wilson, PRD 10 (1974) 2445

- Invented to understand asymptotic freedom without the need for gauge-fixing and ghosts [Wilson, hep-lat/0412043].

- Gauge symmetry on a spacetime lattice:

  - mathematically rigorous definition of QCD functional integrals;

$$\langle \bullet \rangle = \frac{1}{Z} \int \mathcal{D}U \mathcal{D}\psi \mathcal{D}\bar{\psi} \exp\left(-S\right) [\bullet]$$

  - enables theoretical tools of statistical mechanics in quantum field theory and provides a basis for constructive field theory.

- Discrete space (a la Heisenberg & Pauli) and discrete time (a la Feynman).

Numerical Lattice QCD

- Nowadays “lattice QCD” usually implies a numerical technique, in which the functional integral is integrated numerically on a computer.

- Big computers:

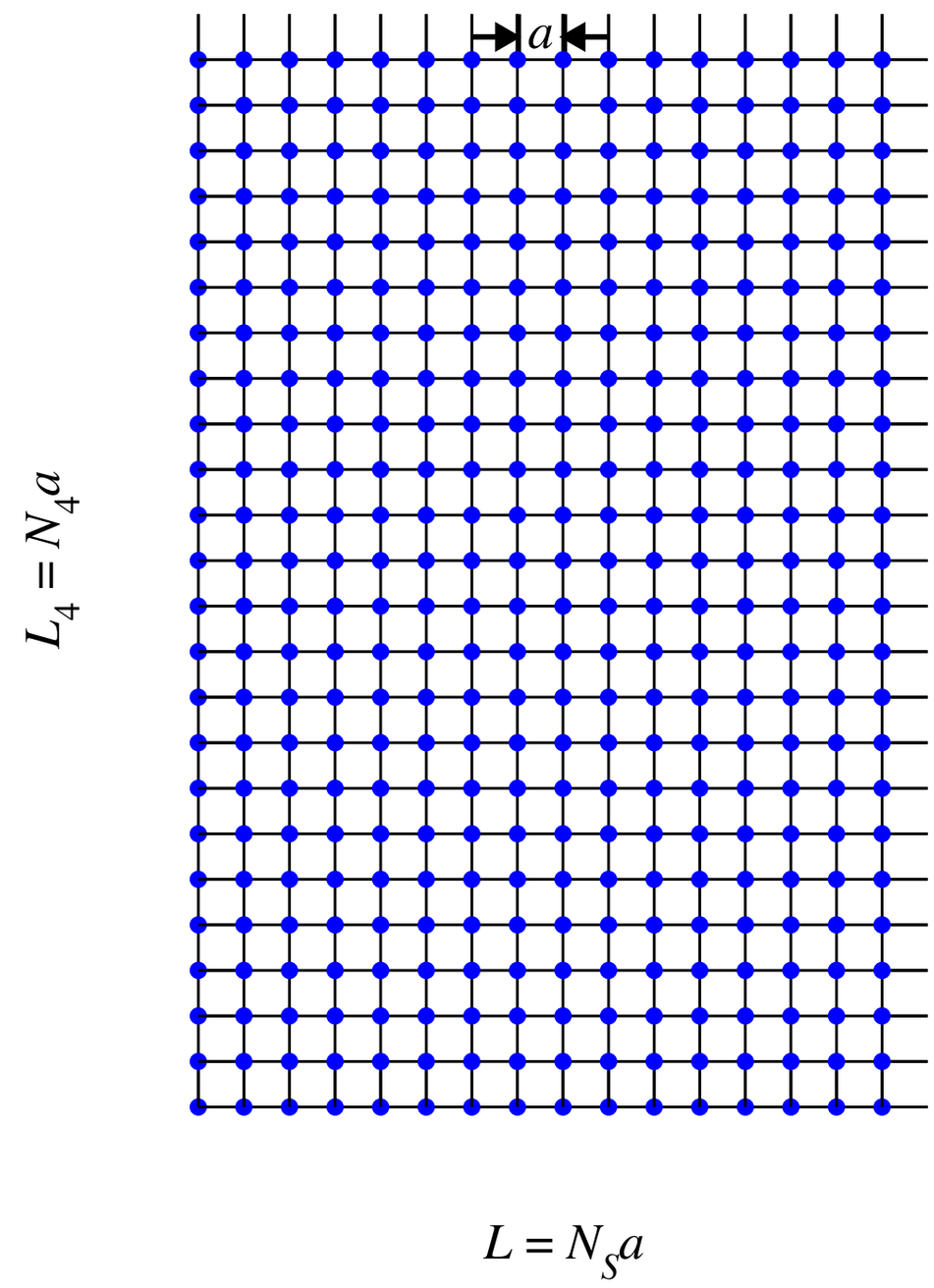
- Some compromises:
  - finite human lifetime ⇒ Wick rotate to Euclidean time: $x_4 = ix_0$;
  - finite memory ⇒ finite space volume & finite time extent;
  - finite CPU power ⇒ light quarks until recently heavier than up and down.

Lattice Gauge Theory

$$\langle \bullet \rangle = \frac{1}{Z} \int \underbrace{\mathcal{D}U}_{\text{MC}} \underbrace{\mathcal{D}\psi \mathcal{D}\bar{\psi}}_{\text{hand}} \exp\left(-S\right) [\bullet]$$

$$= \frac{1}{Z} \int \mathcal{D}U \ \mathrm{Det}(\not{D} + m) \exp\left(-S_{\text{gauge}}\right) [\bullet']$$

- Infinite continuum: uncountably many d.o.f. (⇒ UV divergences);
- Infinite lattice: countably many; used to define QFT;
- Finite lattice: finite dimension ~ 10^8, so compute integrals numerically.



Correlators Yield Masses & Matrix Elements

- Two-point functions for masses, $\pi(t) = \bar{\psi}_u \gamma_5 S \psi_d(t)$, $N(t) = {}^c\psi_u \gamma_5 \psi_d \psi_d(t)$:

$$\langle \pi(t)\pi^\dagger(0)\rangle = \sum_n |\langle 0|\hat{\pi}|\pi_n\rangle|^2 \exp(-m_{\pi_n} t)$$

- Two-point functions for decay constants:

$$\langle J(t)\pi^\dagger(0)\rangle = \sum_n \langle 0|\hat{J}|\pi_n\rangle \langle \pi_n|\hat{\pi}^\dagger|0\rangle \exp(-m_{\pi_n} t)$$

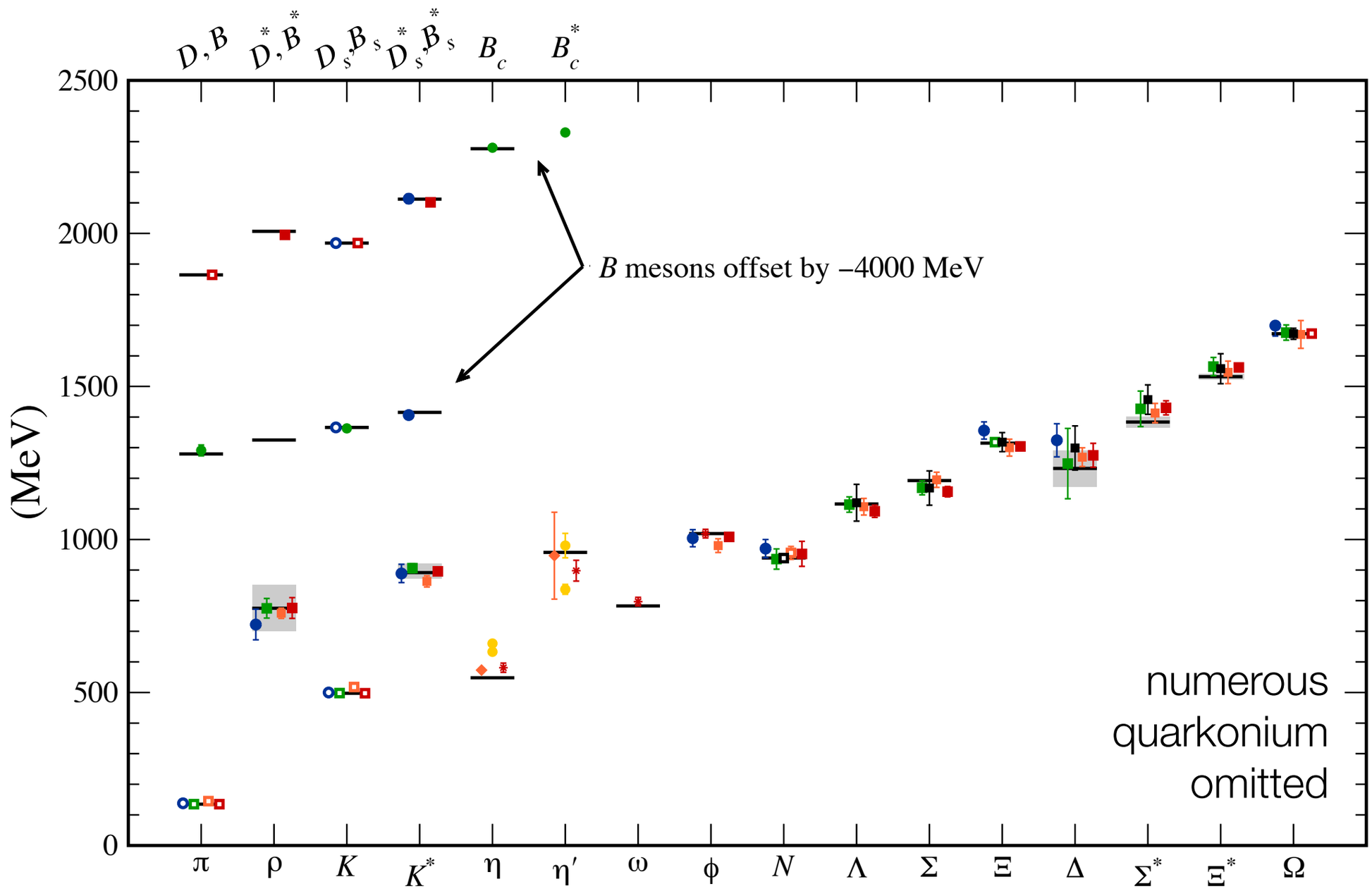
- Three-point functions for form factors, mixing:

$$\langle \pi(t)J(u)B^\dagger(0)\rangle = \sum_{mn} \langle 0|\hat{\pi}|\pi_m\rangle \langle \pi_n|\hat{J}|B_m\rangle \langle B_m|\hat{B}^\dagger|0\rangle \times \exp[-m_{\pi_n}(t-u) - m_{B_m} u]$$

- LHS needs supercomputers; RHS needs students, postdocs, junior faculty.

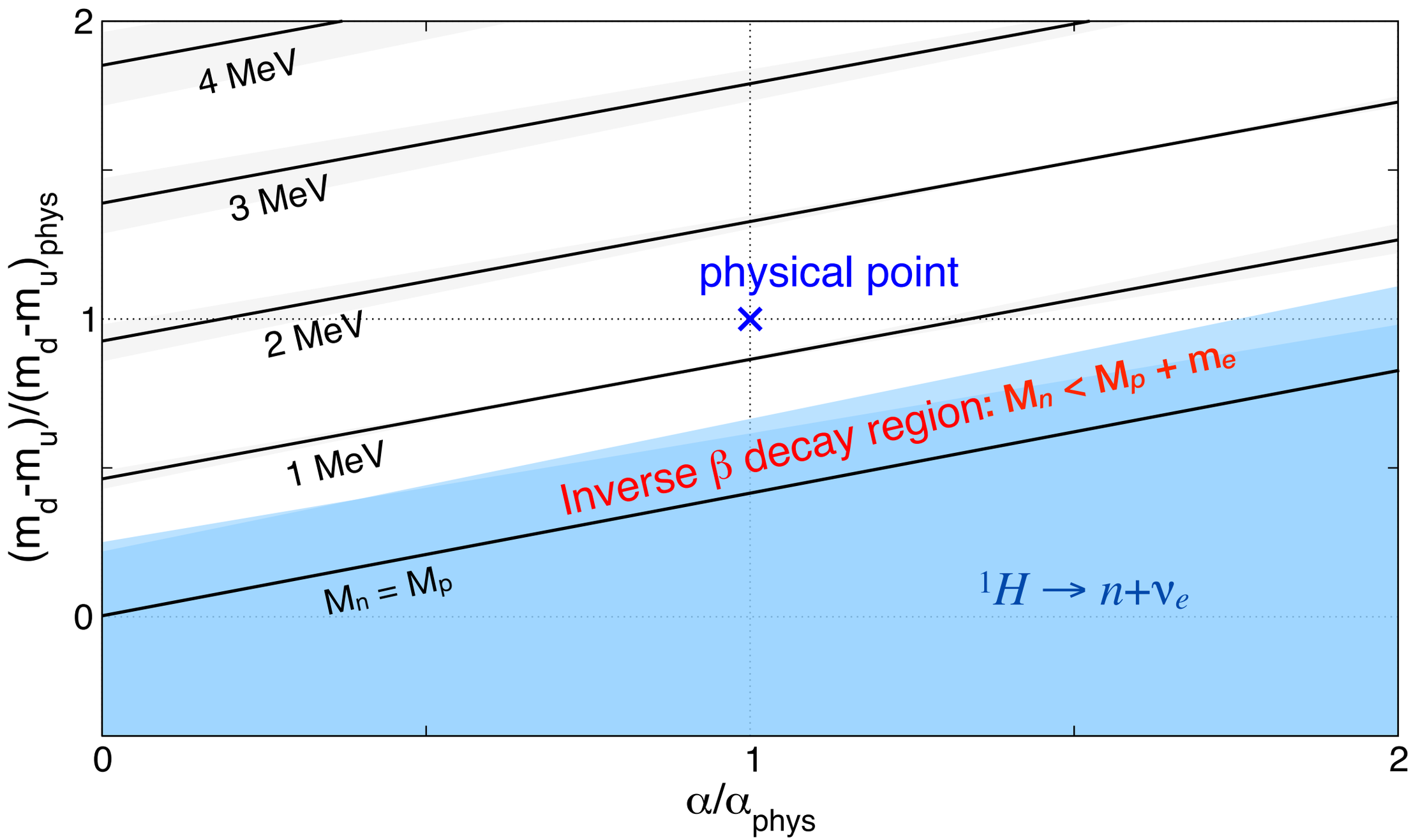
QCD Hadron Spectrum

$\pi \ldots \Omega$: BMW, MILC, PACS-CS, QCDSF; ETM (2+1+1);
η-η': RBC, UKQCD, Hadron Spectrum (ω);
D, B: Fermilab, HPQCD, Mohler&Woloshyn



Neutron-Proton Mass Difference

BMW Collab., arXiv:1406.4088 (see also Horsley *et al*. arXiv:1508.06401)



Form-Factor Calculations I

- Matrix elements decomposed into Lorentz-covariant forms, multiplied by “form factors”.
- Compute them from three-point functions, with several
  - daughter momenta;
  - lattice spacing;
  - quark masses, physical volumes, etc.
- Fit lattice data to EFT formulas to get continuum limit.

Form-Factor Calculations II

- Output of these fits in continuum and at physical quark masses is a set of fit parameters, errors, and their correlations for the EFT formula.

- Fit this information to the z expansion:

  - "synthetic data": evaluate fit function at # of points less than # of fit parameters;

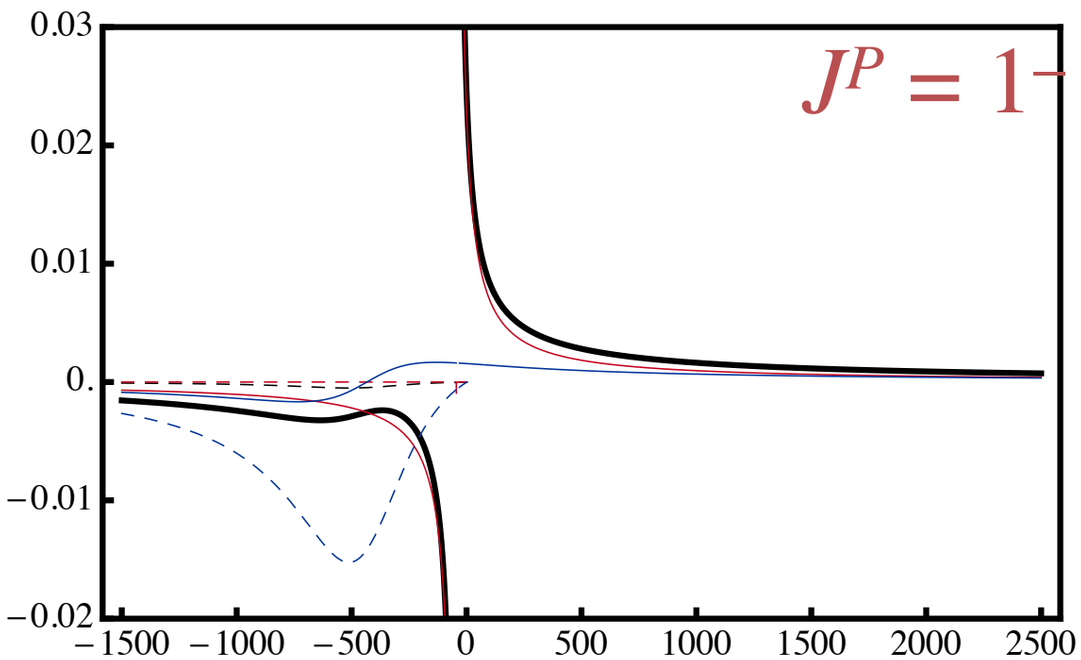
  - functional z expansion (here): finds best fit of second polynomial, $a_k z^k$, to an initial one (EFT expression).

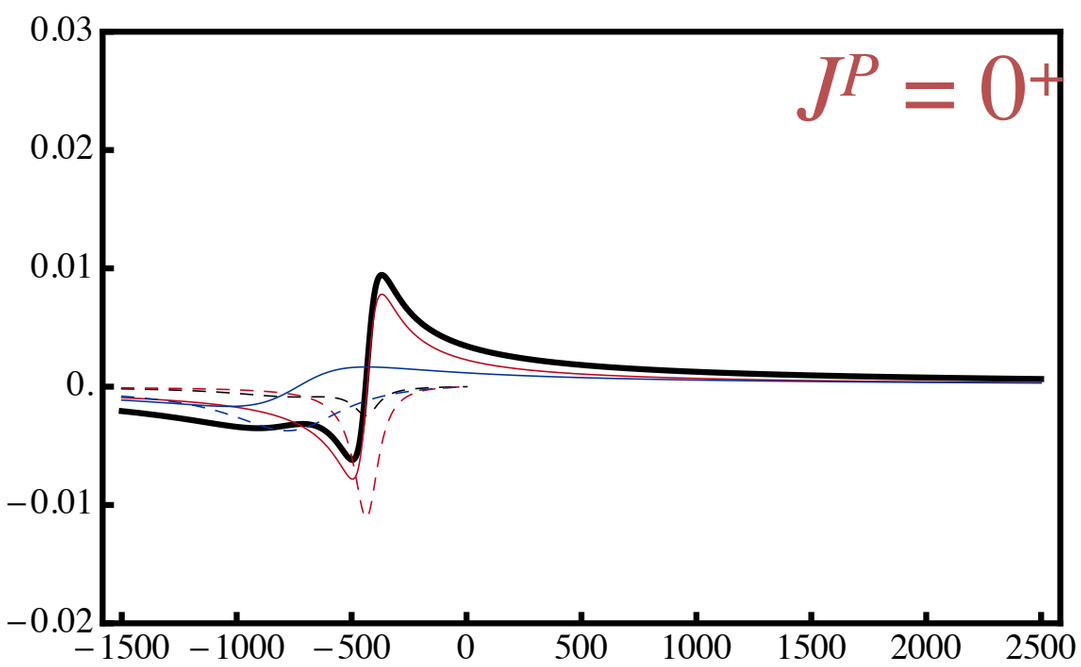
z Expansions Used in B Physics

Three Channels

- scattering $\nu n \to pl$, $\nu B \to \pi l$, …: $q^2 < 0$.
- decay $n \to pl\nu$, $B \to \pi l\nu$, …: $0 = q^2 \leq (M_{\text{parent}} - M_{\text{daughter}})^2$.
- s-channel, $l\nu$ annihilation into pn, $B\pi$, etc.: $s = q^2 \geq t_{\text{cut}}$, but in general also subthreshold poles:
  - flavor-tag in $B\pi$ implies $t_{\text{cut}} = (M_B + M_\pi)^2$, but note also pole at $s = M^2{}_{B^*}$ when $J = 1$;
  - radius of convergence of q^2 expansion is t_{pole} or t_{cut}.

Pology





- Physics looks exciting for annihilation kinematics,
- but dull for decay and scattering:
  - all connected by analyticity.

$$E_\pi = (M_B^2 + M_\pi^2 - q^2)/2M_B$$

General z Expansion

- In general, write $$f(t) = \frac{1}{P(t)\phi(t)} \sum_{k=0}^{\infty} a_k z(t, t_0)^k$$

poles expressed as "inner function" aka "Blaschke factor"

unitarity constraints managed with "outer function"

- Unitarity constrains $|f(t)|^2$:

BGL:

$$1 \geq \sum_{k=0}^{\infty} |a_k|^2$$

$\phi =$ mess

BCL:

$$1 \geq \sum_{k=0}^{\infty} B_{jk} a_j^* a_k, \quad B_{jk} = \text{mess}$$

$\phi = 1$

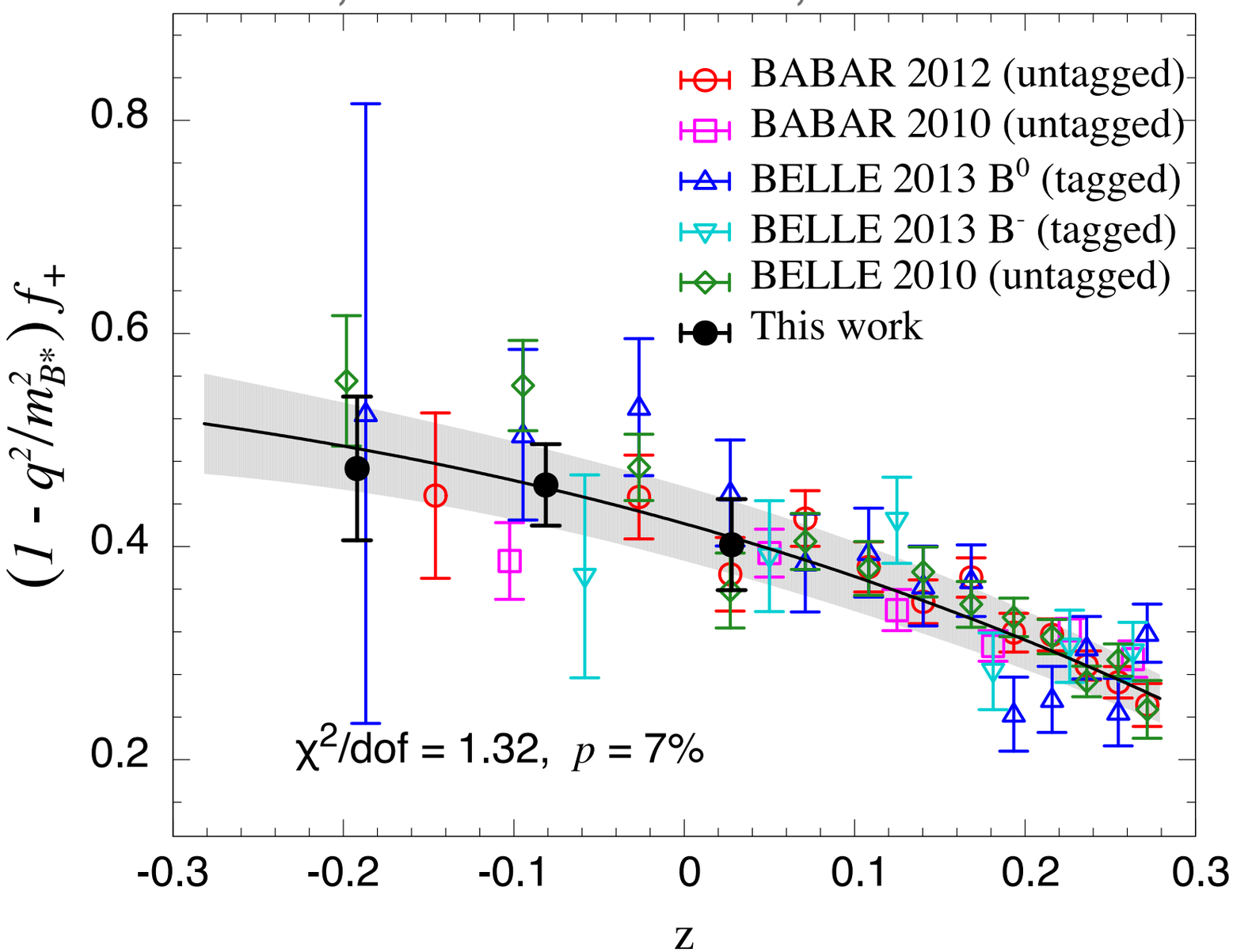
References

- Meiman, Sov. Phys. JETP 17 (1963) 830.
- Okubo and Shih, PRD 4 (1971) 2020.
- Singh and Raina, Fort. Phys. 27 (1979) 561.
- Boyd, Grinstein, and Lebed (BGL), hep-ph/9412324.
- Boyd and Savage, hep-ph/9702300.
- Bourrely, Caprini, and Lellouch (BCL), arXiv:0807.2722.

z Fits

Combining Lattice QCD with Experiment

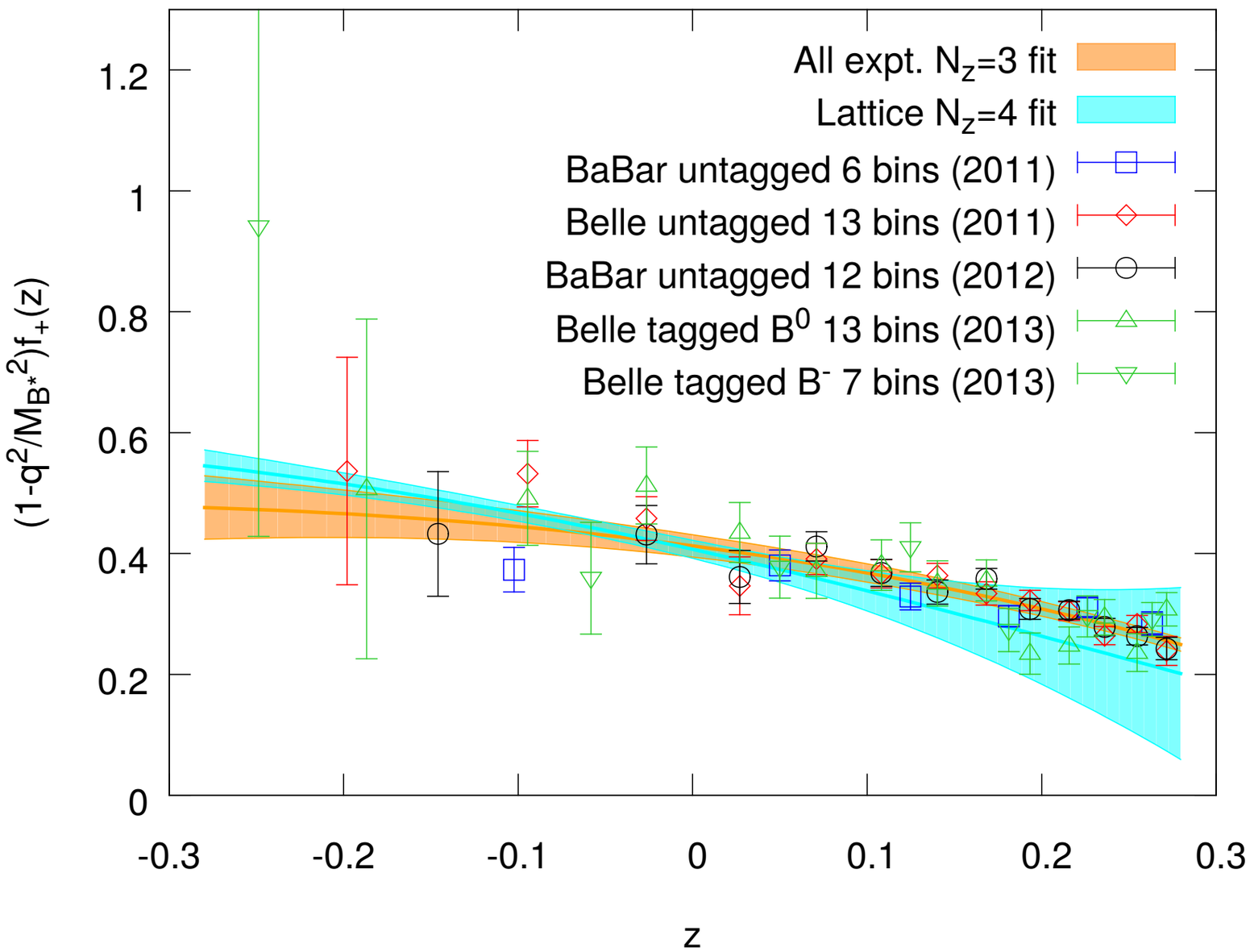
- $B \to \pi l \nu$, RBC/UKQCD, arXiv:1501.05373.



$$10^3|V_{ub}| = 3.61(32)$$

Combining Lattice QCD with Experiment

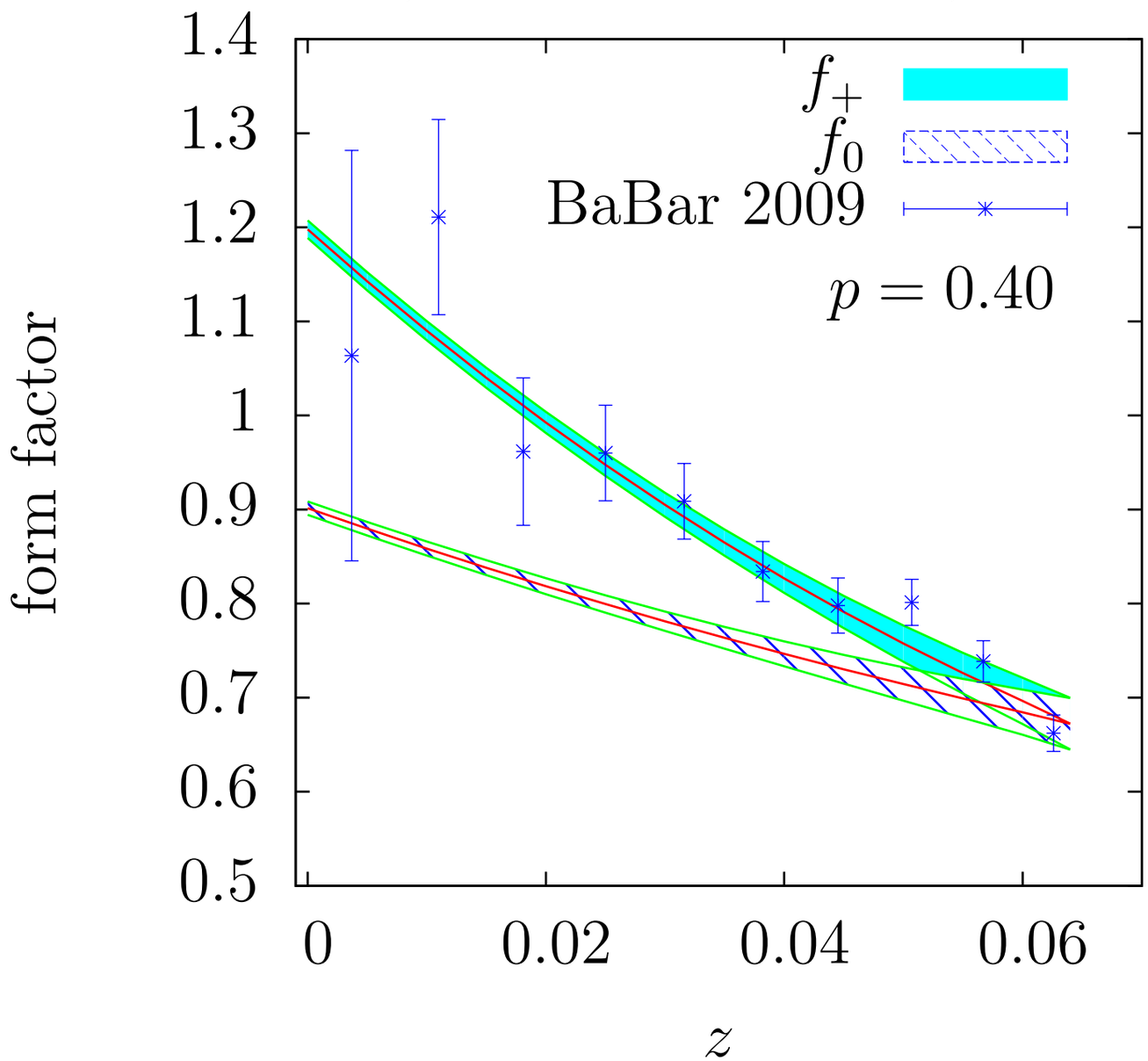
- $B \to \pi l \nu$, Fermilab/MILC, arXiv:1503.07839.



$$10^3|V_{ub}| = 3.72(16)$$

Combining Lattice QCD with Experiment

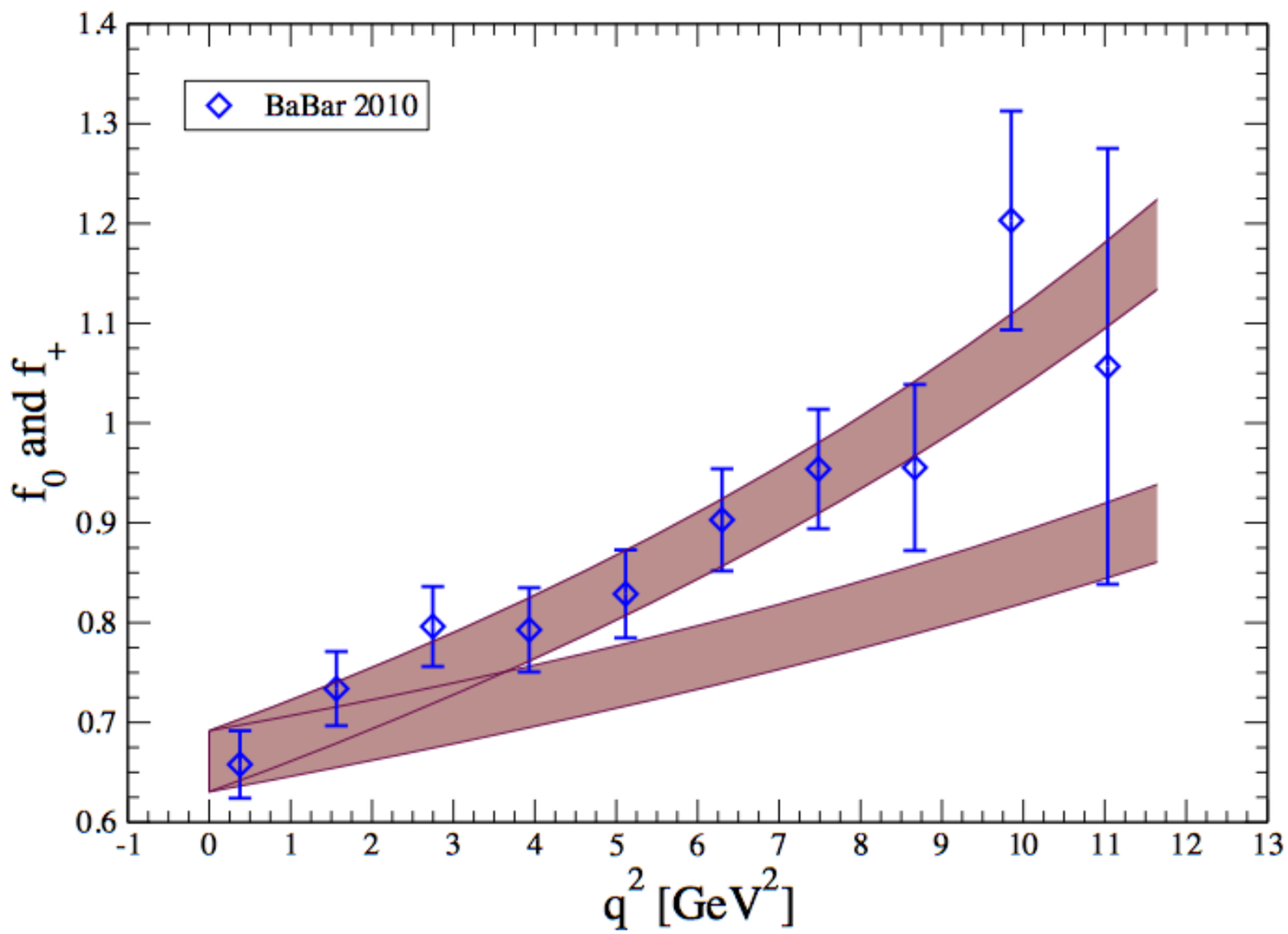
- $B \to Dl\nu$, Fermilab/MILC, arXiv:1503.07237.



$10^3|V_{cb}| = 39.6(1.8)$

Combining Lattice QCD with Experiment

- $B \to Dl\nu$, HPQCD, arXiv:1505.03925.



$$10^3|V_{cb}| = 40.2(2.1)$$

Combining Lattice QCD with Experiment

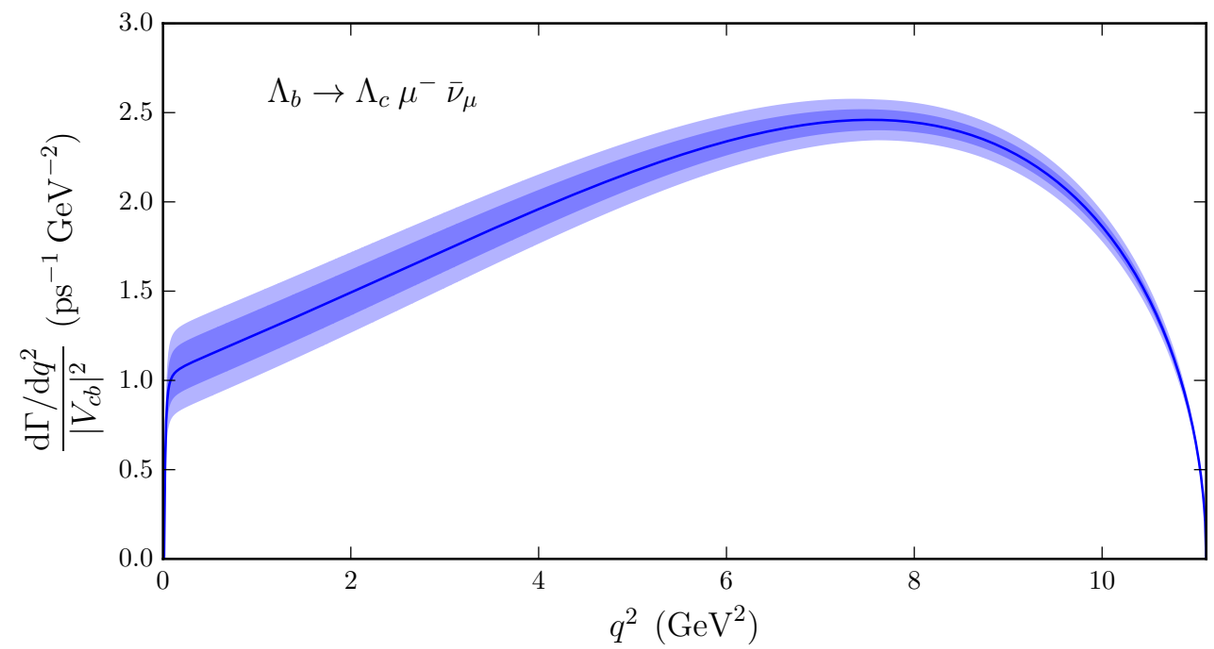
- $\Lambda_b \to p l \nu / \Lambda_b \to \Lambda_c l \nu$, Detmold, Lehner, Meinel, arXiv:1503.01421.

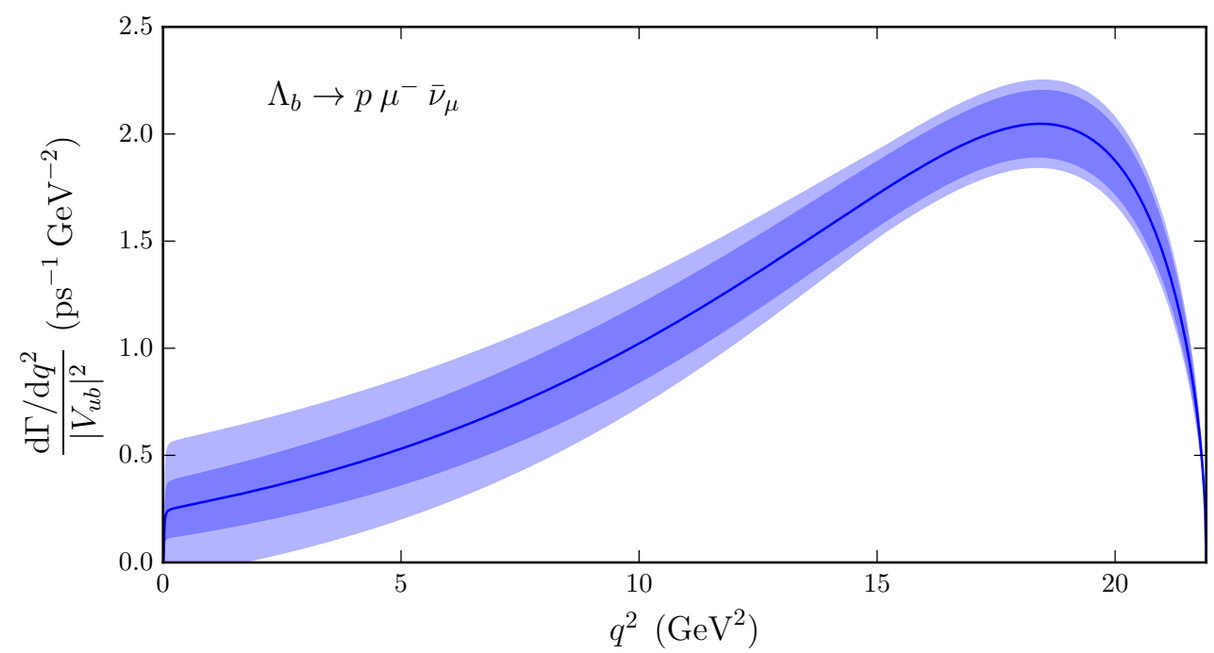
- LHCb measures these rates (over some range):

  $$|V_{ub}|/|V_{cb}| = 0.083(4)(4)$$

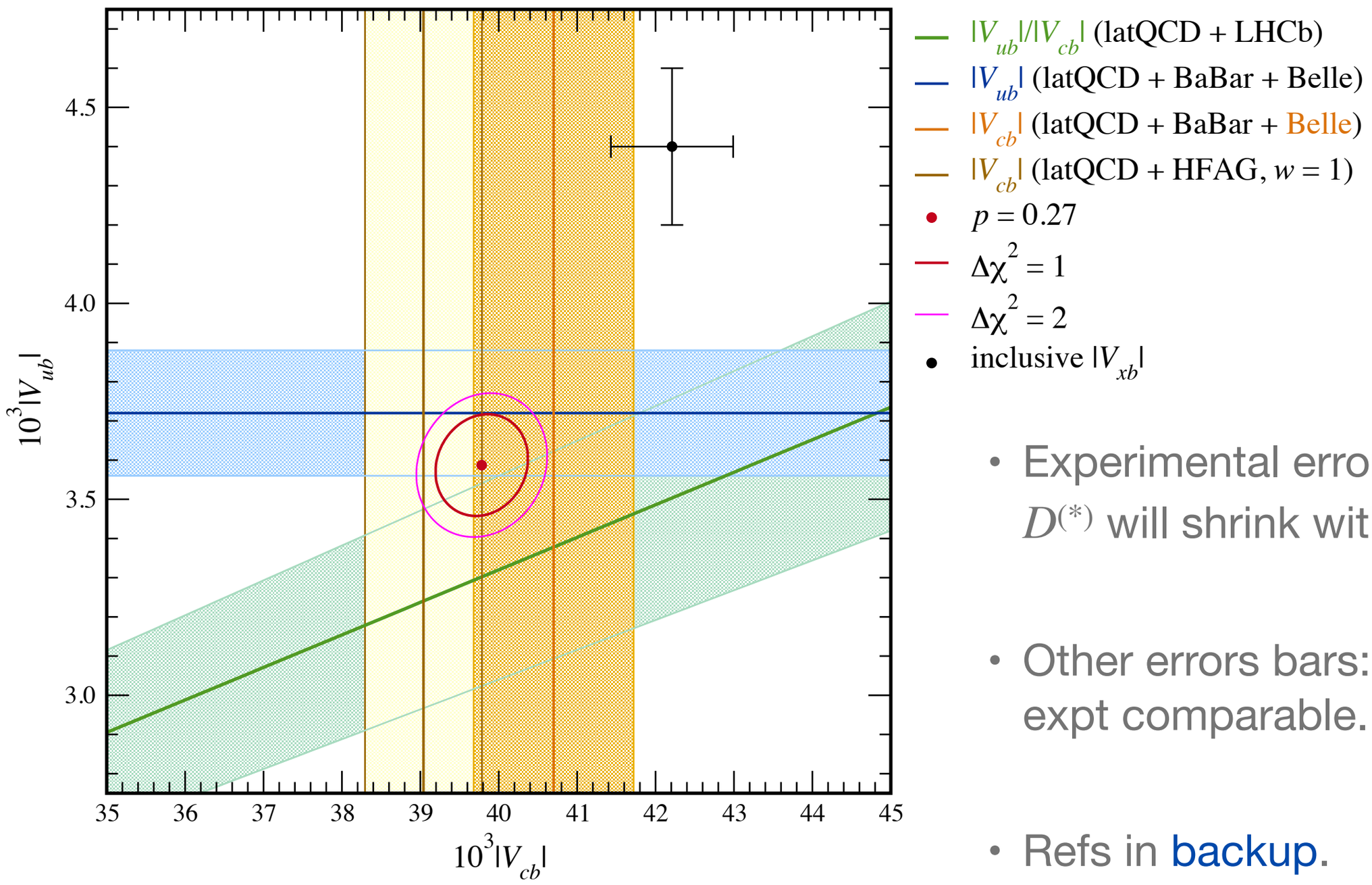
  arXiv:1504.01568.

- Quiz: what's $t_{\rm cut}$ here?





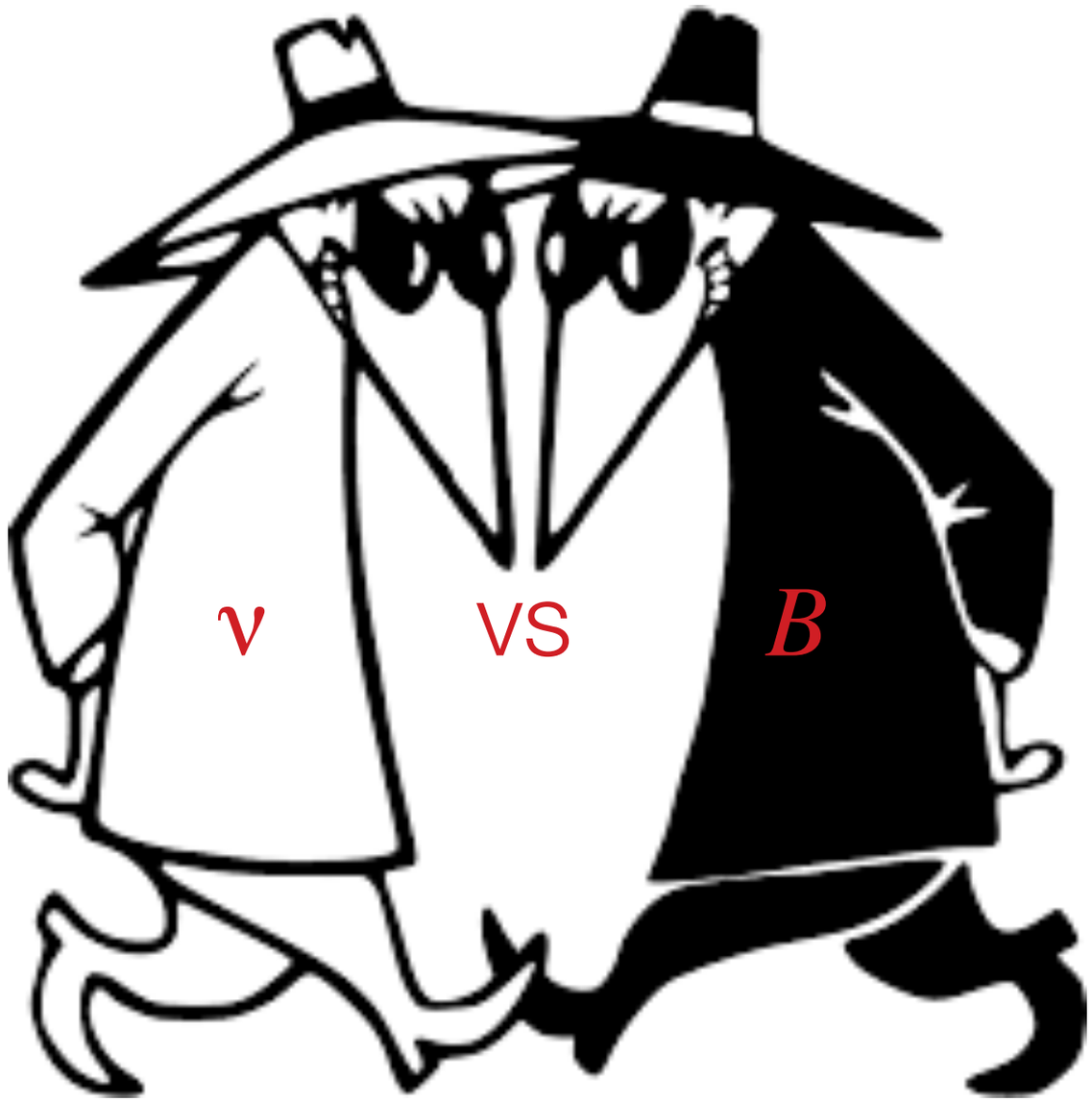
Synthesis of $|V_{ub}|$ & $|V_{cb}|$ Calculations



- Experimental errors for $B \to D^{(*)}$ will shrink with Belle 2.
- Other errors bars: QCD and expt comparable.
- Refs in backup.

How Are Lattice QCD Results Reported?

- Curve and error band, described by
  - the z formulas (t_0, $t_{\rm cut}$, inner and outer functions);
  - coefficients a_k, their errors, and their correlation matrix;
  - cross correlations, *e.g.*, between F_V and F_A.
- Lattice-QCD community working on ways to easily and robustly combine calculations with each other, and with experimental data.


ν
VS
B

Similarities

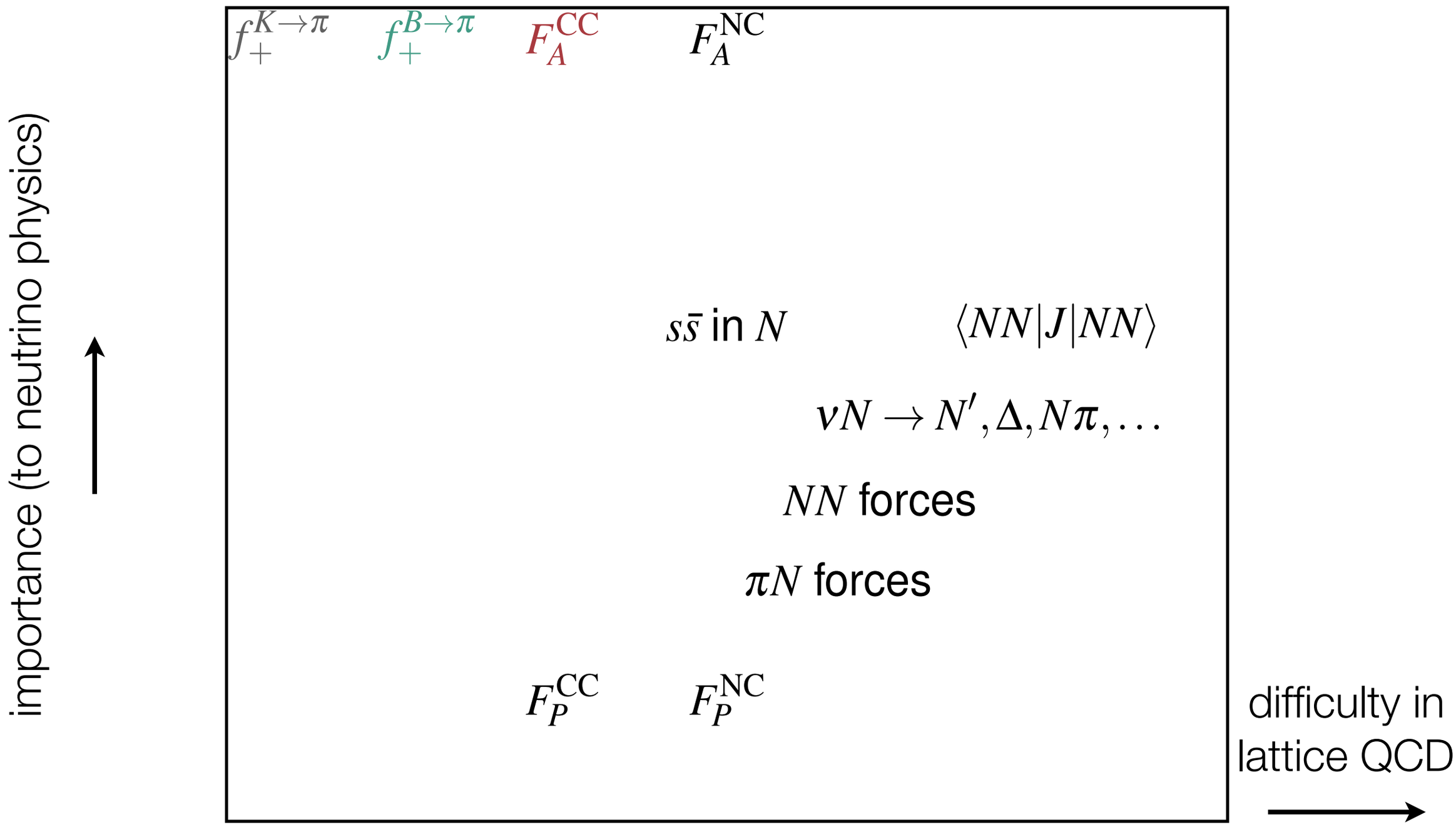
- Same theoretical underpinnings:
  - analyticity constraints on shape ⇒ z expansion with (more than) adequate radius of convergence;
  - lattice gauge theory = first-principles tool for QCD.
- Similar interval in z (for optimal t_0) for D decay and ν scattering—
  - larger (smaller) range for B decay (K decay).

Differences

- Kinematic region—unimportant in practice (I think).
- Experiments measure
  - form factor shape (essentially) directly in B physics;
  - complicated nuclear process in ν physics.
- Meson form factors may have poles, a mild complication;
  - nucleons don't have this (thresholds at $3m_\pi$, $2m_\pi$).

Outlook

Further Calculations of Interest



The Advocated Paradigm

- Replace *Ansätze* for nucleon-level physics with *ab-initio* QCD (*i.e.*, continuum limit of lattice QCD).
- Use success of lattice QCD for meson form factors to bolster confidence in nucleon form factors.
  - (With further checks from F_V and g_A, of course.)