The Microstrip SQUID Amplifier for the Axion Dark Matter eXperiment (ADMX)

12 January 2017

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Outline

- Motivations from the Axion search
- Principle of SQUIDs as microwave amplifiers
- Practical MSA design and optimization
- Planned work

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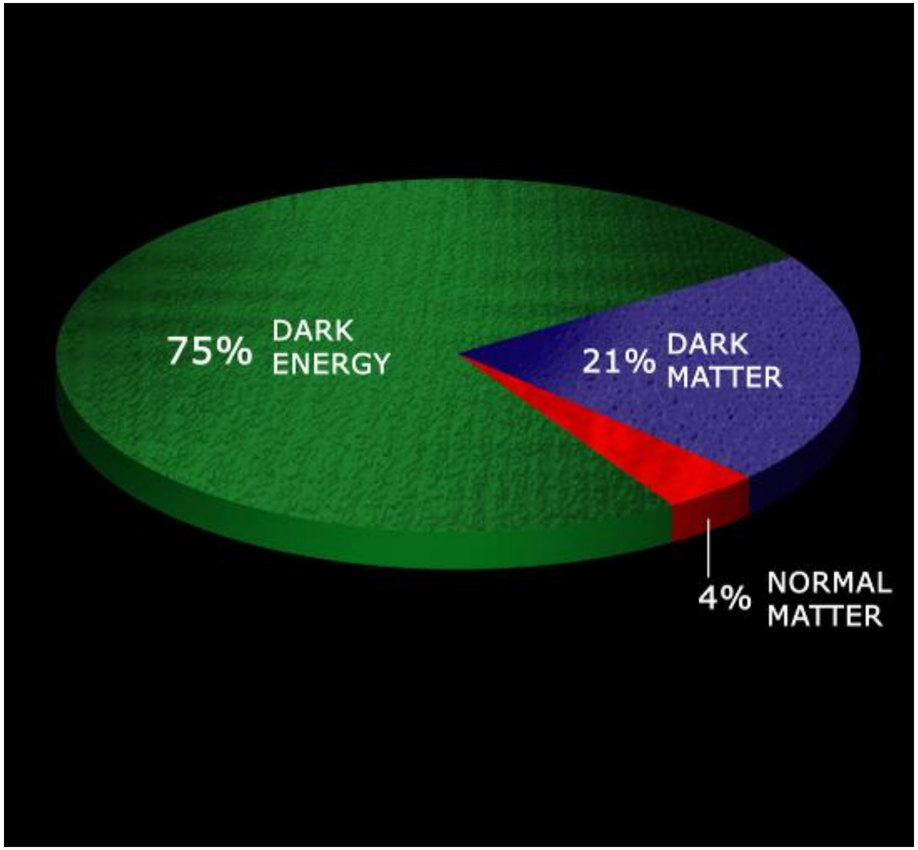
Our Bizarre Universe



- **Ordinary Matter**

Astronomical observations indicate that baryonic matter accounts for only 4% of the mass-energy of the universe.

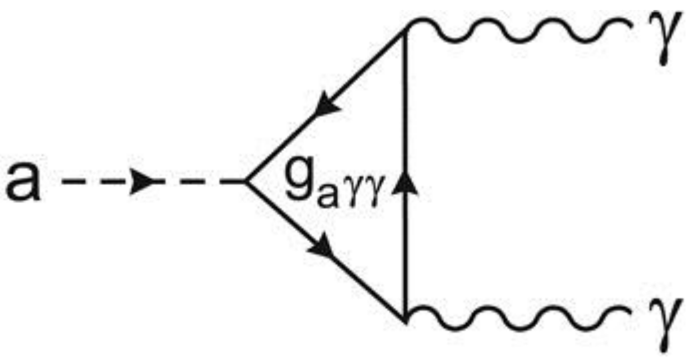
- **Dark Matter**

Orbital kinematics of starts in galaxies, galaxies in clusters, and observations of gravitational lensing all point towards the presence of about 5 times more mass than can be accounted for by stars, gas, and other ordinary matter.

- **Dark Energy**

The observation that our universe is not just expanding, but accelerating indicates that the universe’s total mass-energy is dominated by the cosmological constant, quintessence, or other dark energy.

The Axion: a Candidate for DM



- The axion was originally proposed in 1977 by Peccei and Quinn (before the idea of dark matter) as a solution that "cleans up" the problem of extremely high symmetry observed in the strong force.

- If axions exist, they would have been produced in the big bang, and are an excellent dark matter candidate because they are cold (non-relativistic) and interact with ordinary light and matter very weakly.

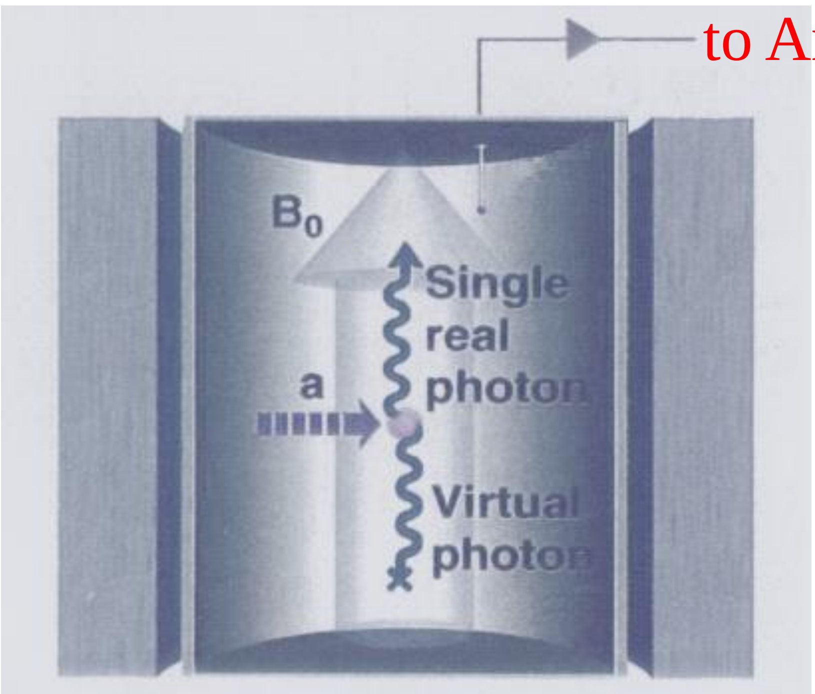
The Axion: a Candidate for DM

- The Axion has recently been observed at UC Berkeley, among a disused lab sink deep in the second basement of Birge hall!
- Initial data suggests a non-virialized velocity distribution and highly non-homogenous density, so universal abundance remains an open question and no competing DM candidates have yet been excluded.
- Even 10 years after the expiration date, Axion remains an excellent degreaser.

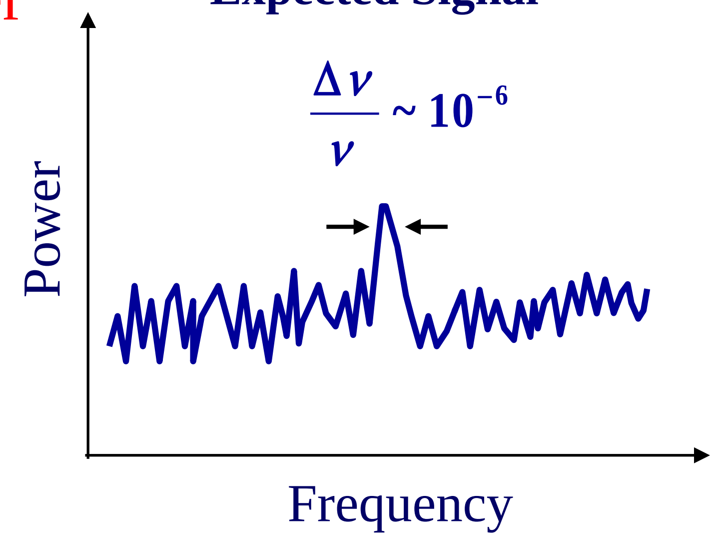
How to Find an Axion

Pierre Sikivie (1983)

Primakoff Conversion

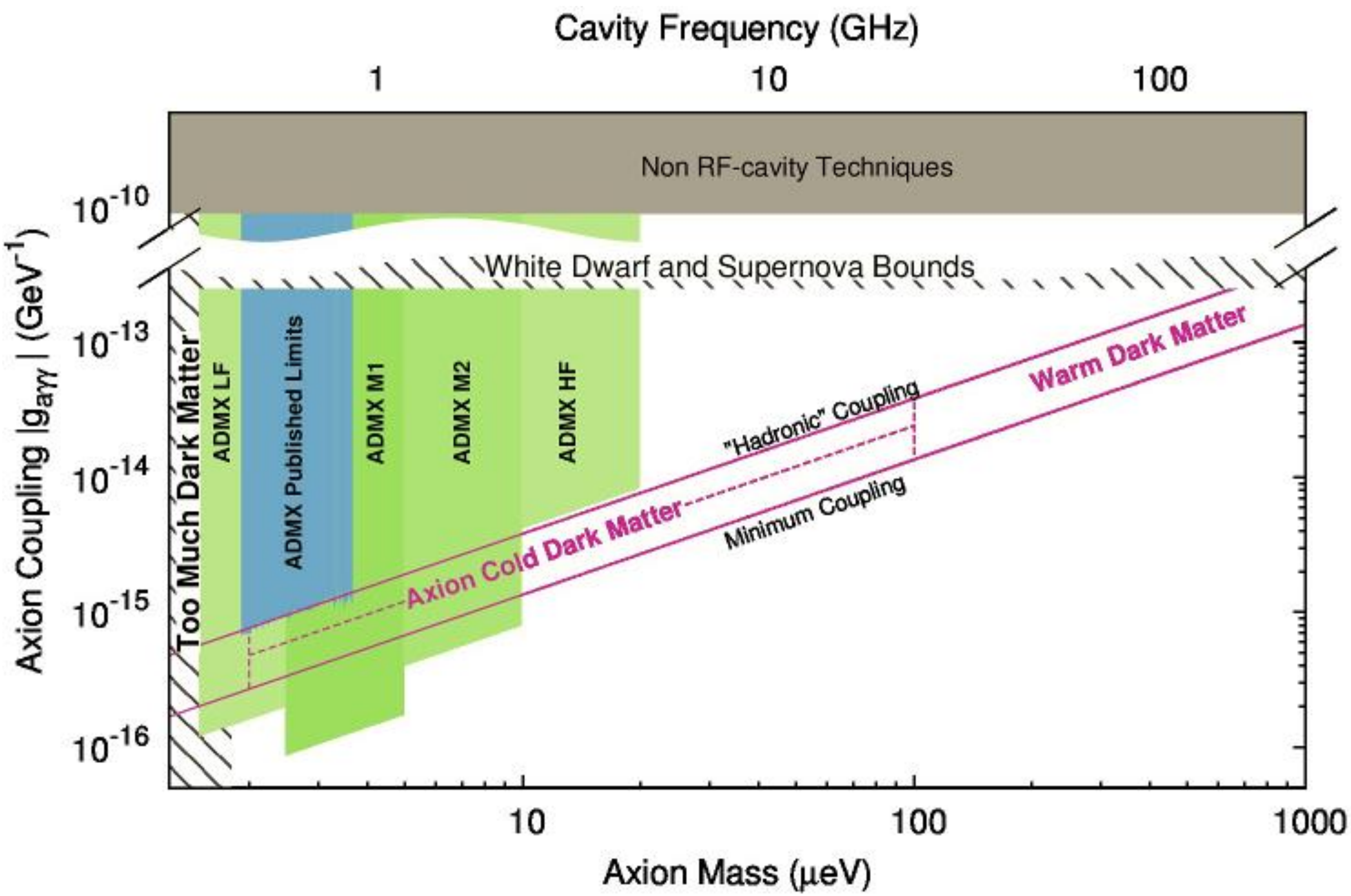


Expected Signal



Need to scan frequency
Need low noise floor

The Axion Search Space



3 orders of magnitude in mass/frequency to search

The Importance of Noise Temperature

- Original system noise temperature: $T_S = T + T_N = 3.2$ K

  Cavity temperature: $T = 1.5$ K (pumped He_4)

  Amplifier noise temperature: $T_N = 1.7$ K (HEMT)

- Time* to scan the frequency range from $f_1 = 0.24$ to $f_2 = 0.48$ GHz:

$$\tau(f_1, f_2) = 4 \times 10^{17} (3.2\text{K}/1\ \text{K})^2 (1/f_1 - 1/f_2)\ \text{sec} \approx \mathbf{270\ years}$$

*Dine-Fischler-Srednicki-Zhitnitsky (DFSZ) theory

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- Next generation:

  Cavity temperature: $T = 50$ mK (He_3 dilution unit)

  Amplifier noise temperature: $T_N = 50$ mK (MSA)

- Time* to scan the frequency range from $f_1 = 0.24$ to $f_2 = 0.48$ GHz:

$$\tau(f_1, f_2) = 4 \text{ x } 10^{17}(0.1\text{K}/1\text{ K})^2(1/f_1 - 1/f_2) \text{ sec} \approx \mathbf{100\ days}$$

*Dine-Fischler-Srednicki-Zhitnitsky (DFSZ) theory

ADMX at UW

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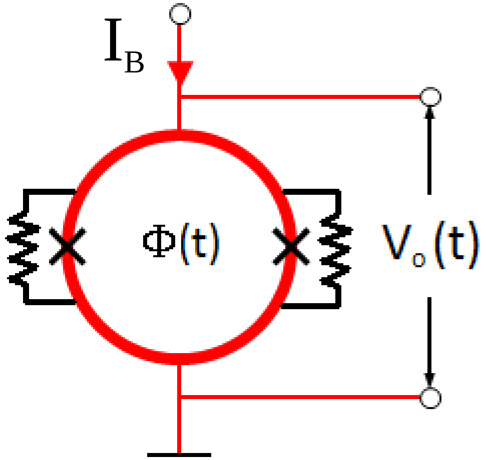
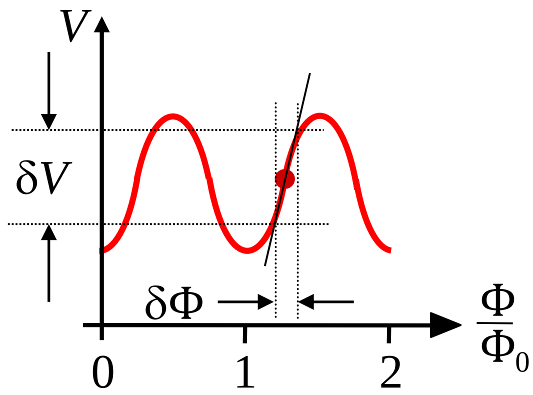
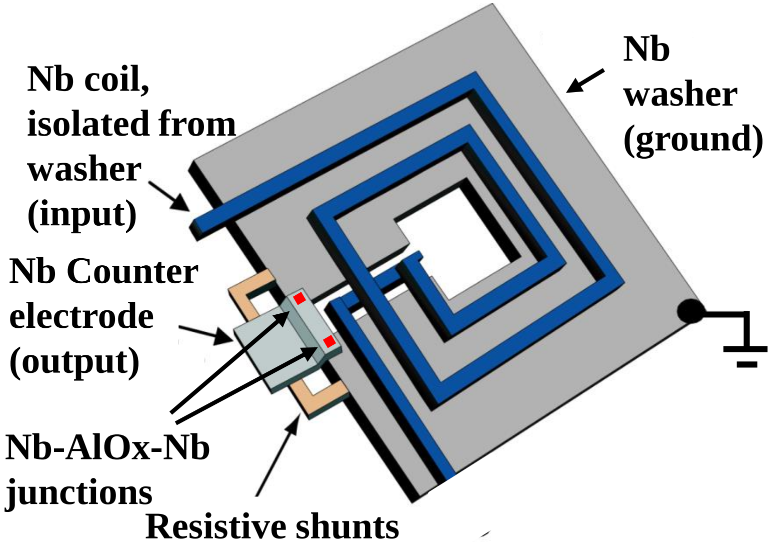
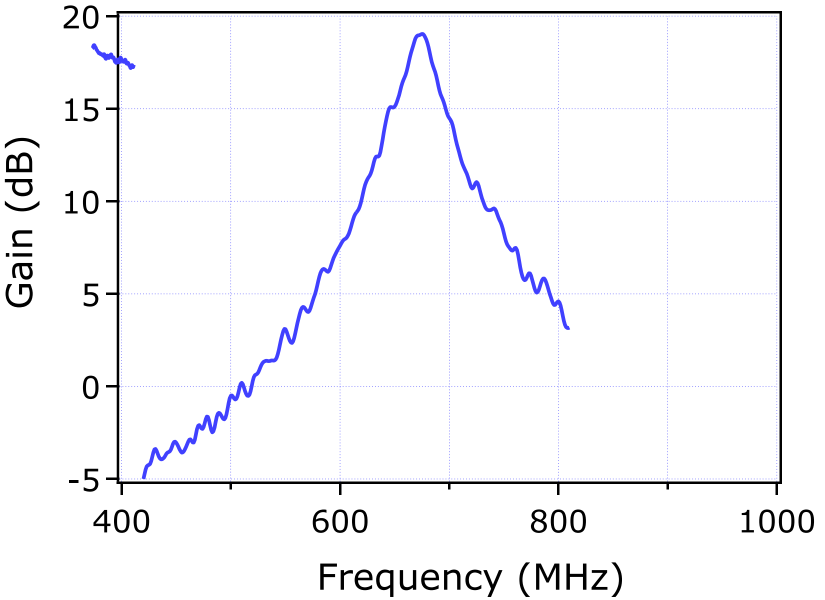
The Microstrip SQUID Amplifier





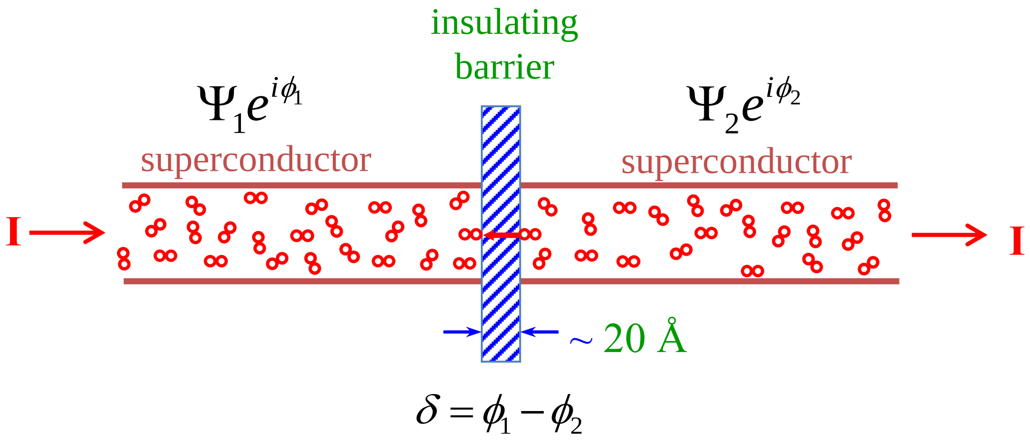
Microstrip SQUID Amplifier (MSA):





Superconductivity

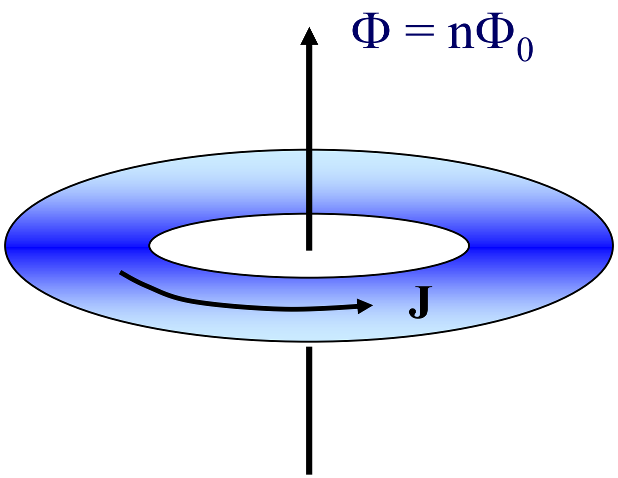
Josephson Tunneling



Superconducting state has macroscopic wavefunction.
I and V across the junction are given by the Josephson relations:

$I = I_0 \sin \delta$ $\qquad$ $V = \dot{\delta}\Phi_0/2\pi$

Flux Quantization



$\Phi = n\Phi_0$ (n = 0, ±1, ±2, ...)
$\Phi_0 = h/2e$

In presence of Josephson element the quantization condition becomes:

$\Phi - (\delta/2\pi)\, \Phi_0 = n\Phi_0$

The RCSJ Model

From Kirchhoff's laws:

$$I = I_0 \sin\delta + \frac{V}{R} + C\dot{V}$$

substituting the 2nd Josephson relation:

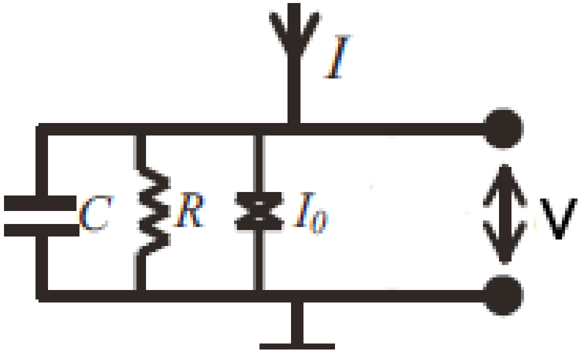
$$I - I_0 \sin\delta = \frac{\Phi_0}{2\pi}\frac{1}{R}\dot{\delta} + \frac{\Phi_0}{2\pi}C\ddot{\delta}$$

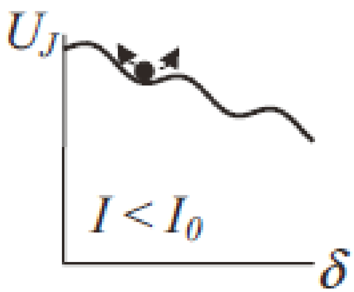
or

$$-\frac{2\pi}{\Phi_0}\frac{\partial U}{\partial\delta} - \frac{\Phi_0}{2\pi}\frac{1}{R}\dot{\delta} = \frac{\Phi_0}{2\pi}C\ddot{\delta}$$

with

$$U = \frac{\Phi_0}{2\pi}\left[I_0(1-\cos\delta) - I\delta\right]$$





"phase" particle on a tilted washboard:

tilt ←→ I

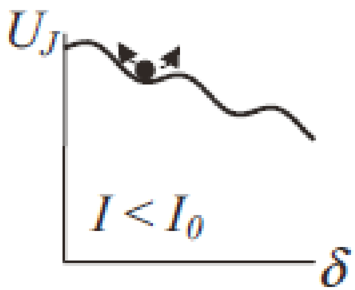
position ←→ δ

velocity ←→ V

mass ←→C

damping ←→1/R

The RCSJ Model



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tilt ←→ I
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mass ←→C
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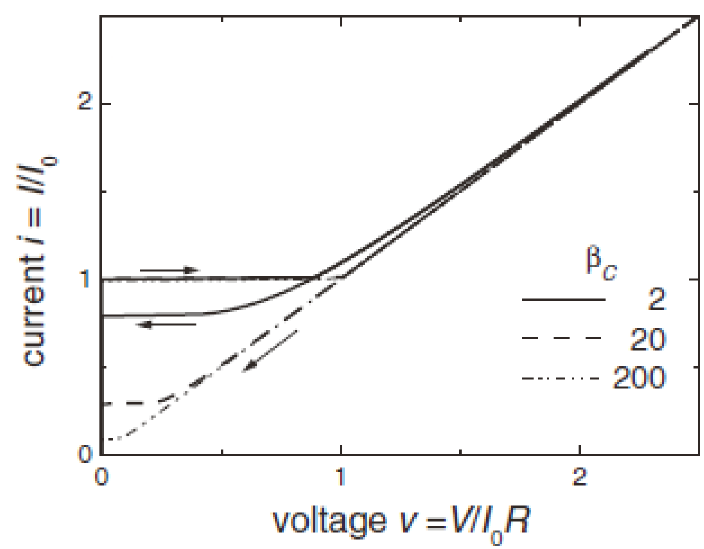
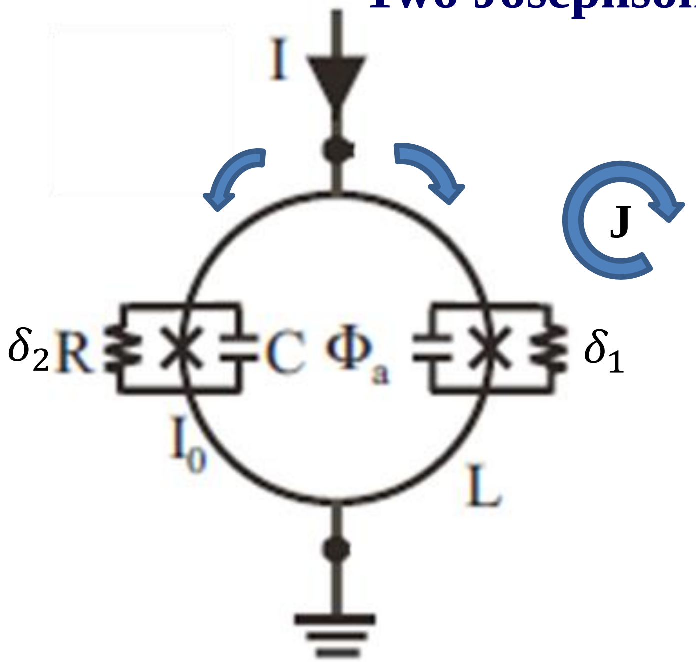
Insight from tilted washboard potential:

- V=0 for any $I < I_0$ (starting flat, at rest)
- As soon as $I > I_0$, V > 0 (particle rolls downhill)
- For small damping terms, V may remain non-zero, even if $I < I_0$
- Critical damping parameter $\beta_c = \frac{2\pi}{\Phi_0} I_0 R^2 C$ determines if V→0 for $I < I_0$ regardless of tilt



The DC SQUID

Two Josephson junctions on a superconducting ring



$$\frac{I}{2} + \mathrm{J} = I_0 \sin\delta_1 + \frac{\Phi_0}{2\pi R}\dot{\delta}_1 + \frac{\Phi_0}{2\pi} C_1 \ddot{\delta}_1 + I_{N,1}$$

$$\frac{I}{2} - \mathrm{J} = I_0 \sin\delta_2 + \frac{\Phi_0}{2\pi R}\dot{\delta}_2 + \frac{\Phi_0}{2\pi} C \ddot{\delta}_2 + I_{N,2}$$

$$\delta_1 - \delta_2 = \frac{2\pi}{\Phi_0}(\Phi_a + LJ)$$

$$i = I/I_0$$

$$j = J/I_0$$

$$\varphi_a = \Phi_a/\Phi_0$$

$$\tau = \Phi_0/2\pi I_0 R$$

$$\beta_C = \frac{2\pi}{\Phi_0} \mathrm{I}_0 R^2 C$$

$$\beta_L = \frac{2\mathrm{L}\mathrm{I}_0}{\Phi_0}$$

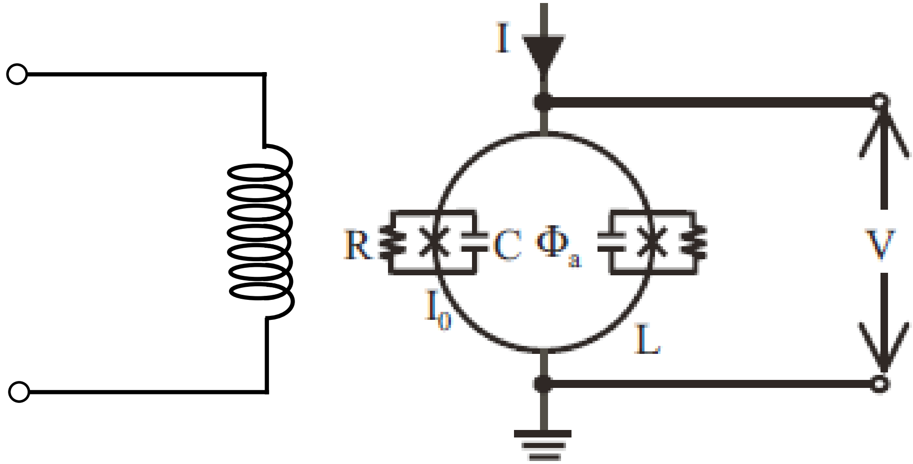
$$\frac{i}{2} + \mathrm{j} = \sin\delta_1 + \dot{\delta}_1 + \beta_C \ddot{\delta}_1 + i_{N,1}$$

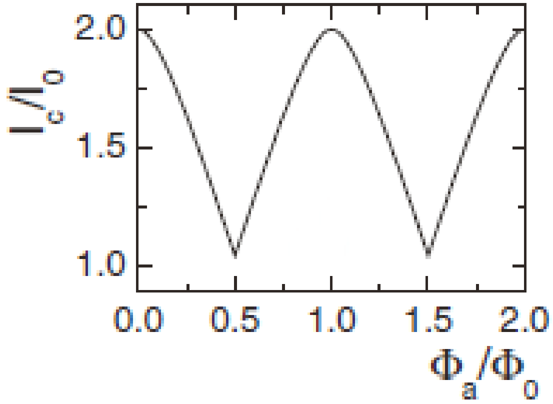
$$\frac{i}{2} - \mathrm{j} = \sin\delta_2 + \dot{\delta}_2 + \beta_C \ddot{\delta}_2 + i_{N,2}$$

$$\delta_1 - \delta_2 = 2\pi\left(\varphi_a + \frac{1}{2}\beta_L j\right)$$

The DC SQUID

Two Josephson junctions on a superconducting ring





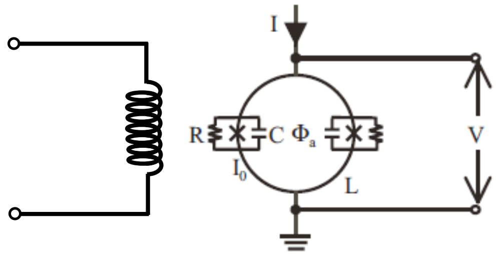
Critical Current I_c is modulated by magnetic flux

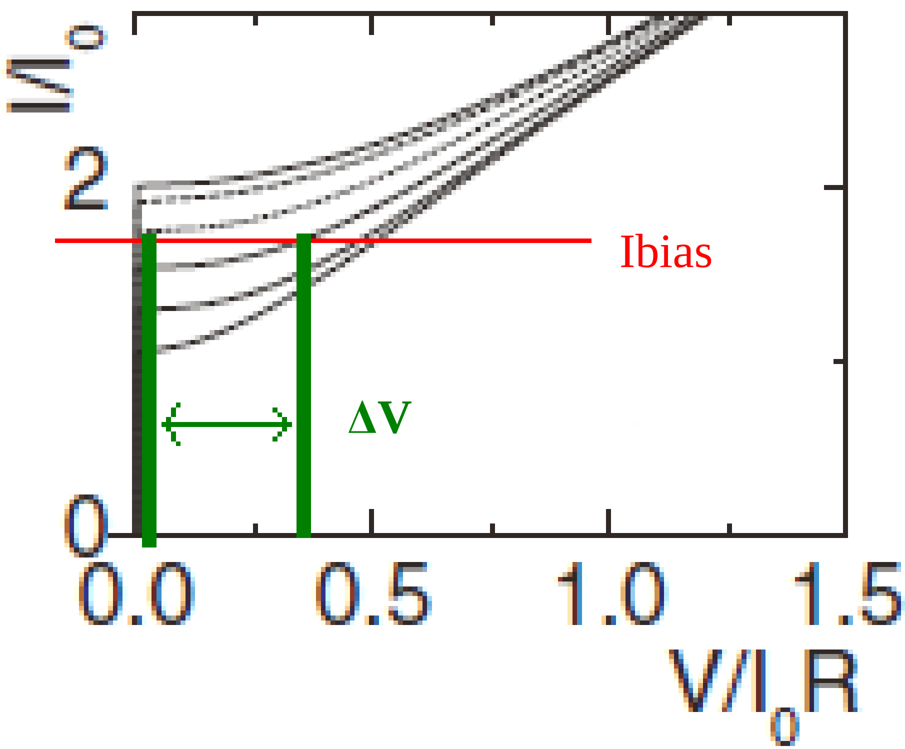
A flux through the SQUID loop (Φ_a) induces a circulating current to satisfy the flux quanitzation condition, adding to the current through one junction, subtracting from the other, and inducing a difference in the phases across the junctions.

Interference of the superconducting wave functions in the two SQUID arms sets the maximum current Ic that can flow at V = 0

With some simplifying assumptions (like symmetric junctions) the **DC SQUID can be treated as a single, flux-modulated Josephson junction**

DC SQUID as Flux-to-Voltage Transducer

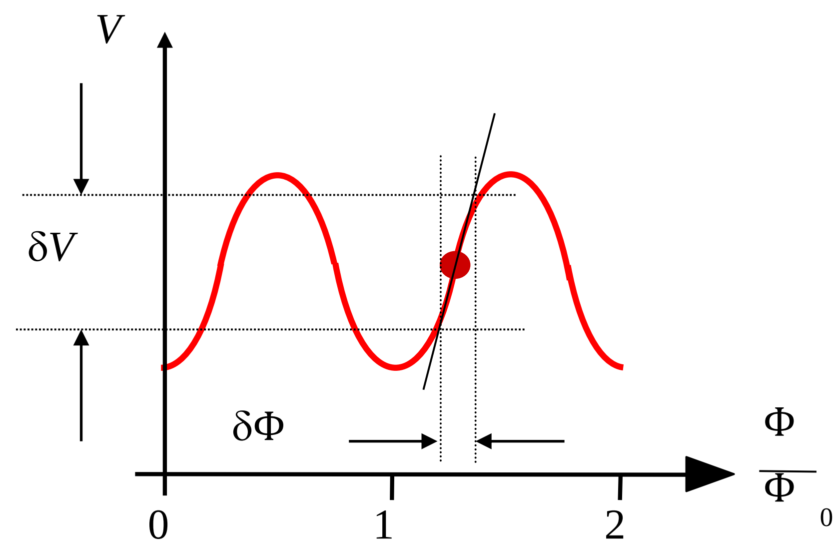




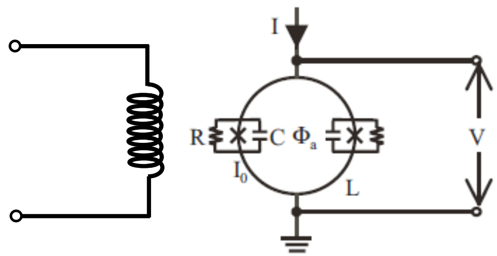
Normalized I-V plot for various DC flux biases from 0 to $0.5\Phi_0$

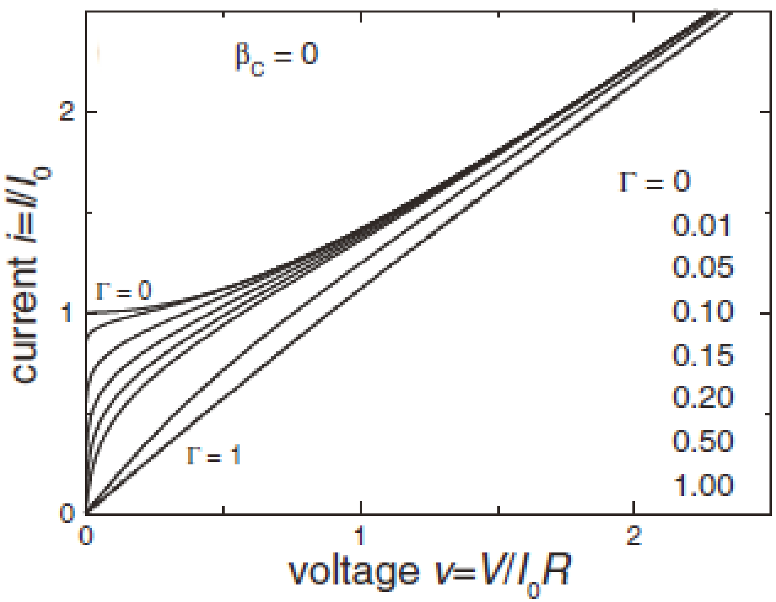
For use as a flux transducer:

- Bias flux around $\Phi_0/4$ for max $dI_c/d\Phi$
- Apply a DC bias current slightly above Ic to select a high dynamic impedance part of the I-V curve
- Small variations in Φ yield large swings in V

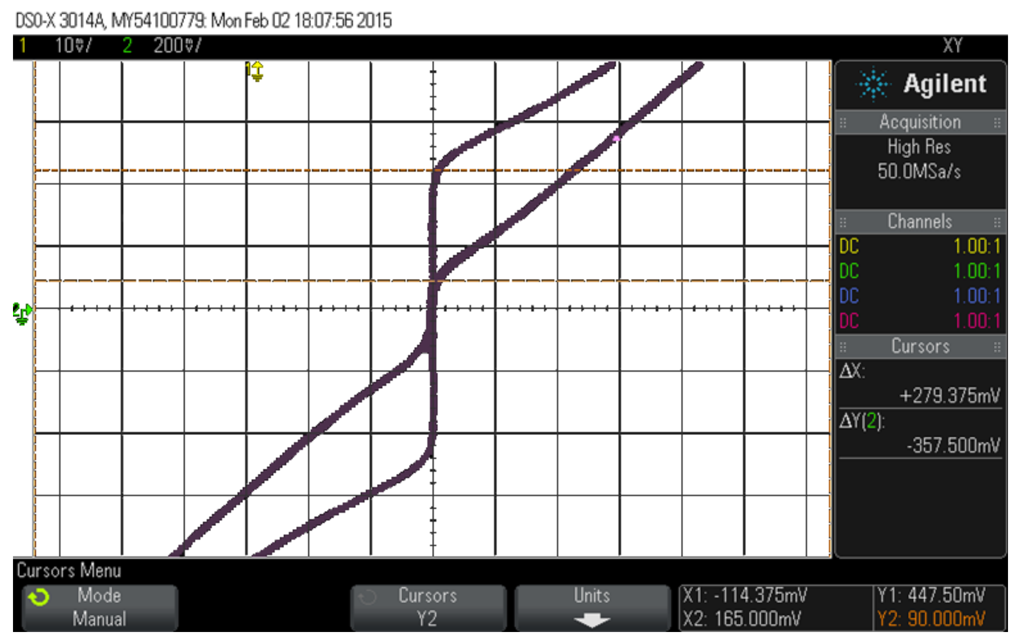


DC SQUID Thermal Effects





$$\Gamma \equiv \frac{2\pi kBT}{I_0\Phi_0}$$

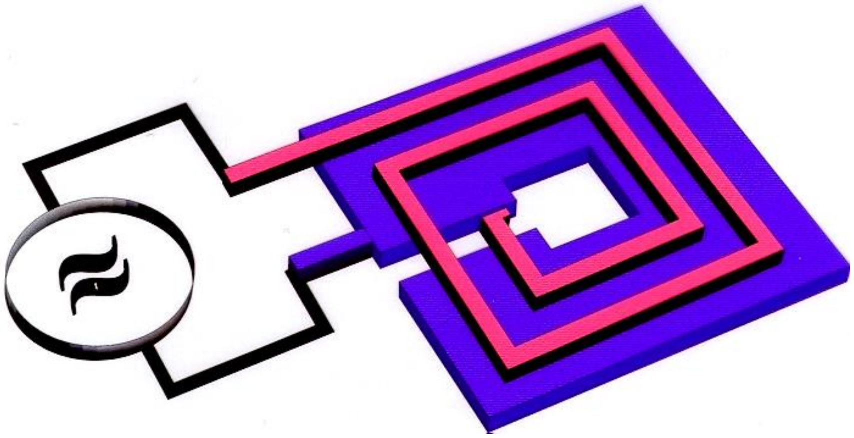


X: 10 μA/div Y: 2 μA/div
T = 4.2K
Max Ic = 4.47 μA
Min Ic = 0.9 μA
Γ @ Max I_c= 0.04
Γ @ Min I_c= 0.20

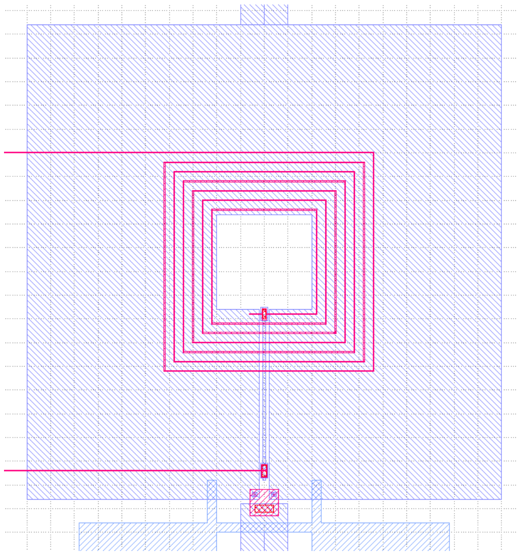
DC SQUID as an RF amplifier (MSA)

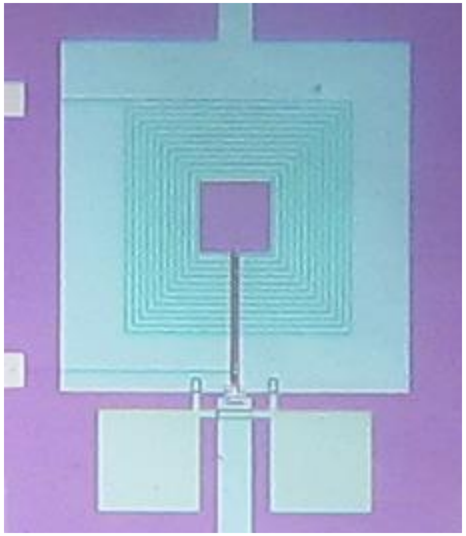
To couple a microwave signal into the SQUID:

- Cover the washer with an insulating layer (350nm of SiO_2)
- Add a spiral path of conductor around the central hole



This creates a resonant **microstrip** transmission line between the input coil and SQUID washer





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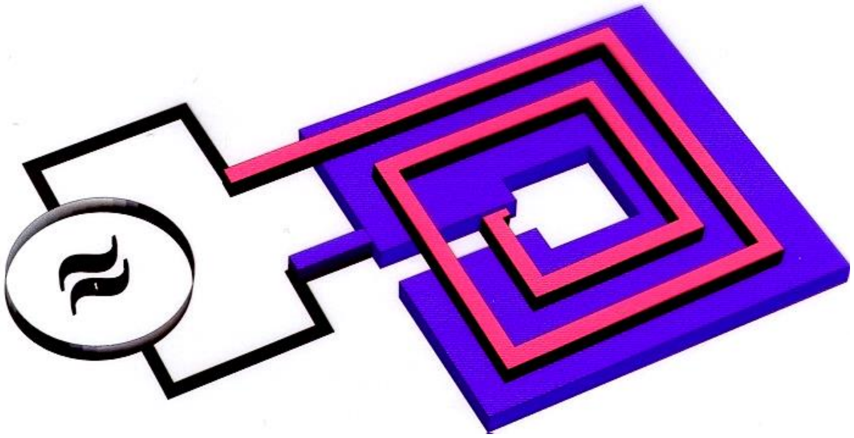
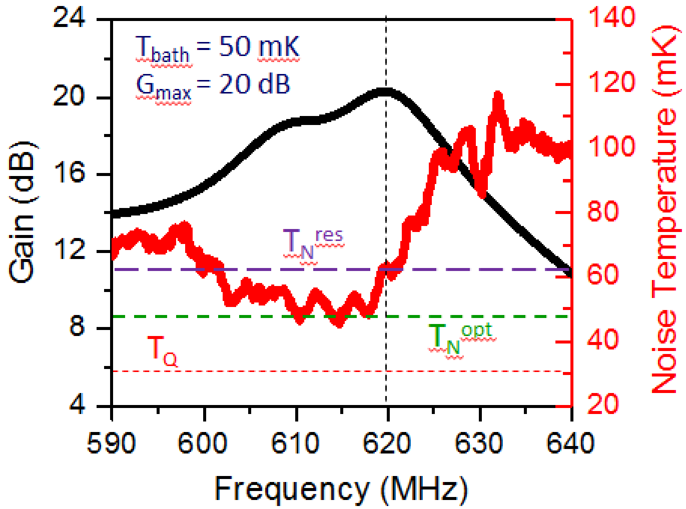
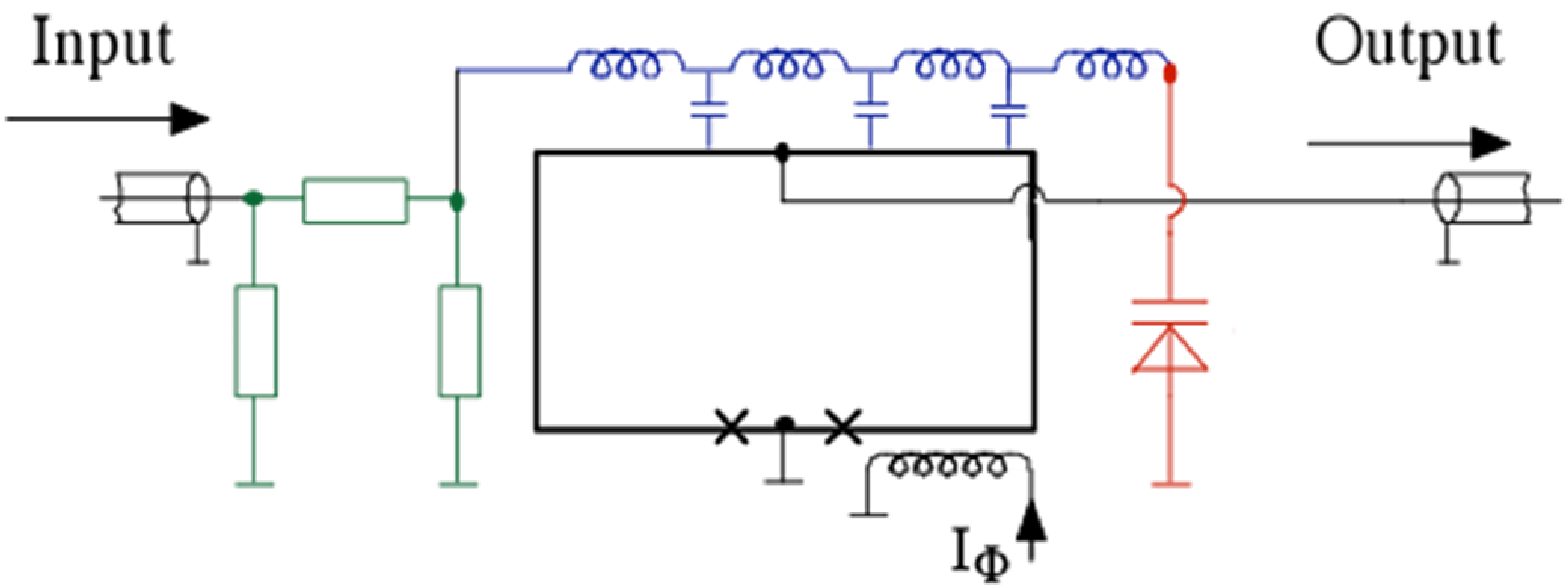


This creates a resonant **microstrip** transmission line between the input coil and SQUID washer

- Best historical MSAs have a $T_N \approx T/2$
- Prior work has demonstrated T_N of 48 ± 5 mK at 600 MHz, 1.7 times the quantum limit

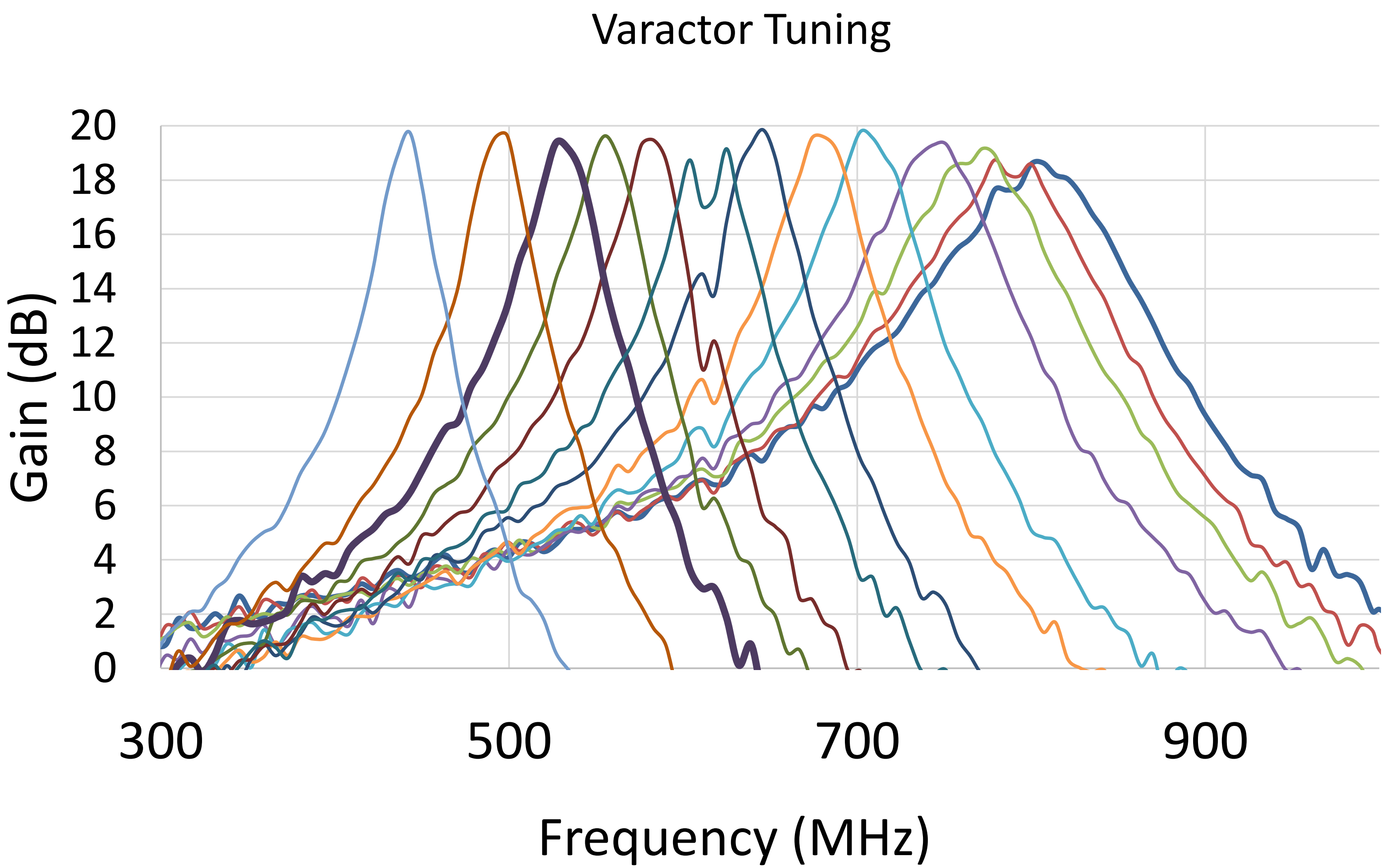


Varactor tuning an MSA



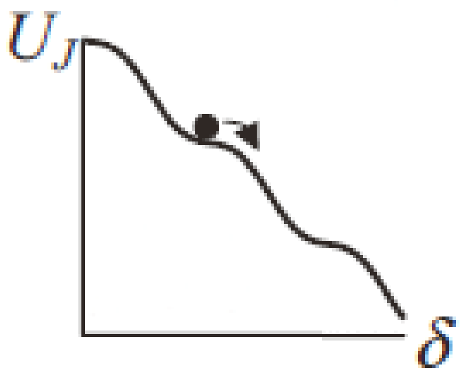
- Varying the capacitance modifies the phase change on reflection, effectively changing the length of the microstrip
- As the phase changes from a node to anti-node, the standing wave changes from λ/2 to λ/4, and the resonant frequency varies by a factor of 2
- Varactors must be GaAs (Si freezes out), high Q, very low inductance

Varactor tuning an MSA



How high in frequency is "DC"?

At finite voltage the phase will evolve with both a DC and AC component as the phase particle "rolls down a bumpy hill". The frequency of oscillation is ω_j.



$$\omega_j = \frac{2\pi V_j}{\Phi_0}$$

For typical a typical value of V = 10 uV
$f_j \approx 30$GHz

The "DC" SQUID can operate reliably only for $f < f_j$
"DC" operation becomes problematic around $10f > f_j$, around 3GHz in this example.

RF frequency limits are currently constrained by microwave engineering, not Josephson junction physics

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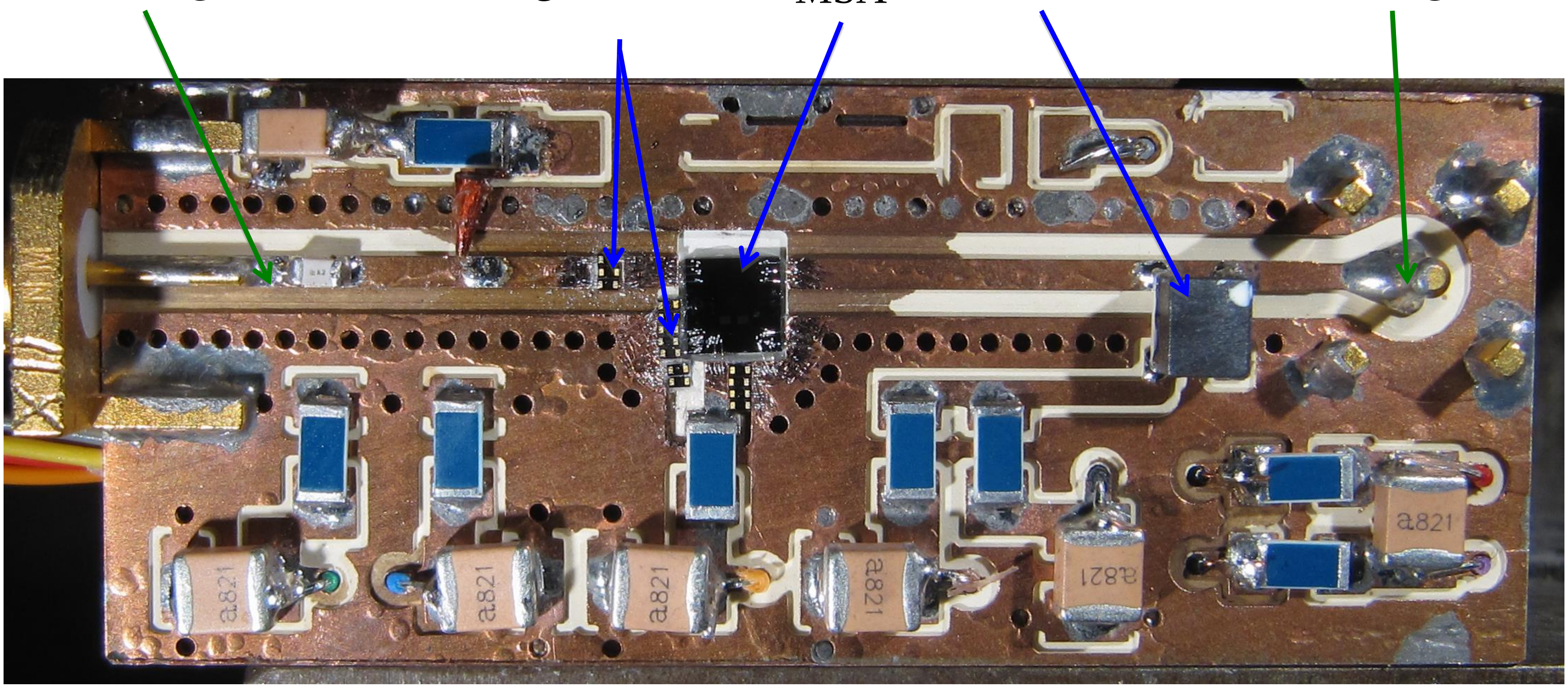
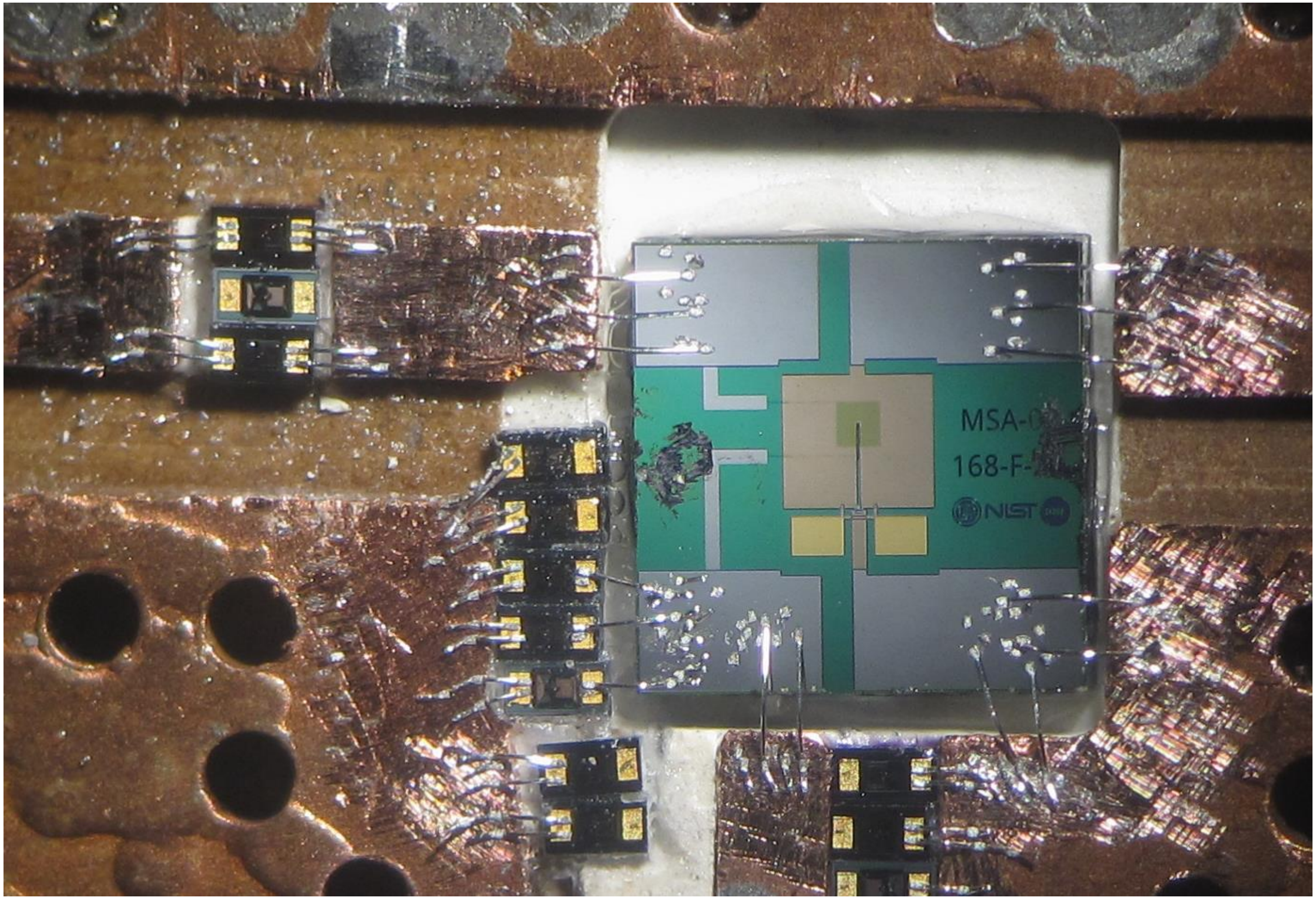
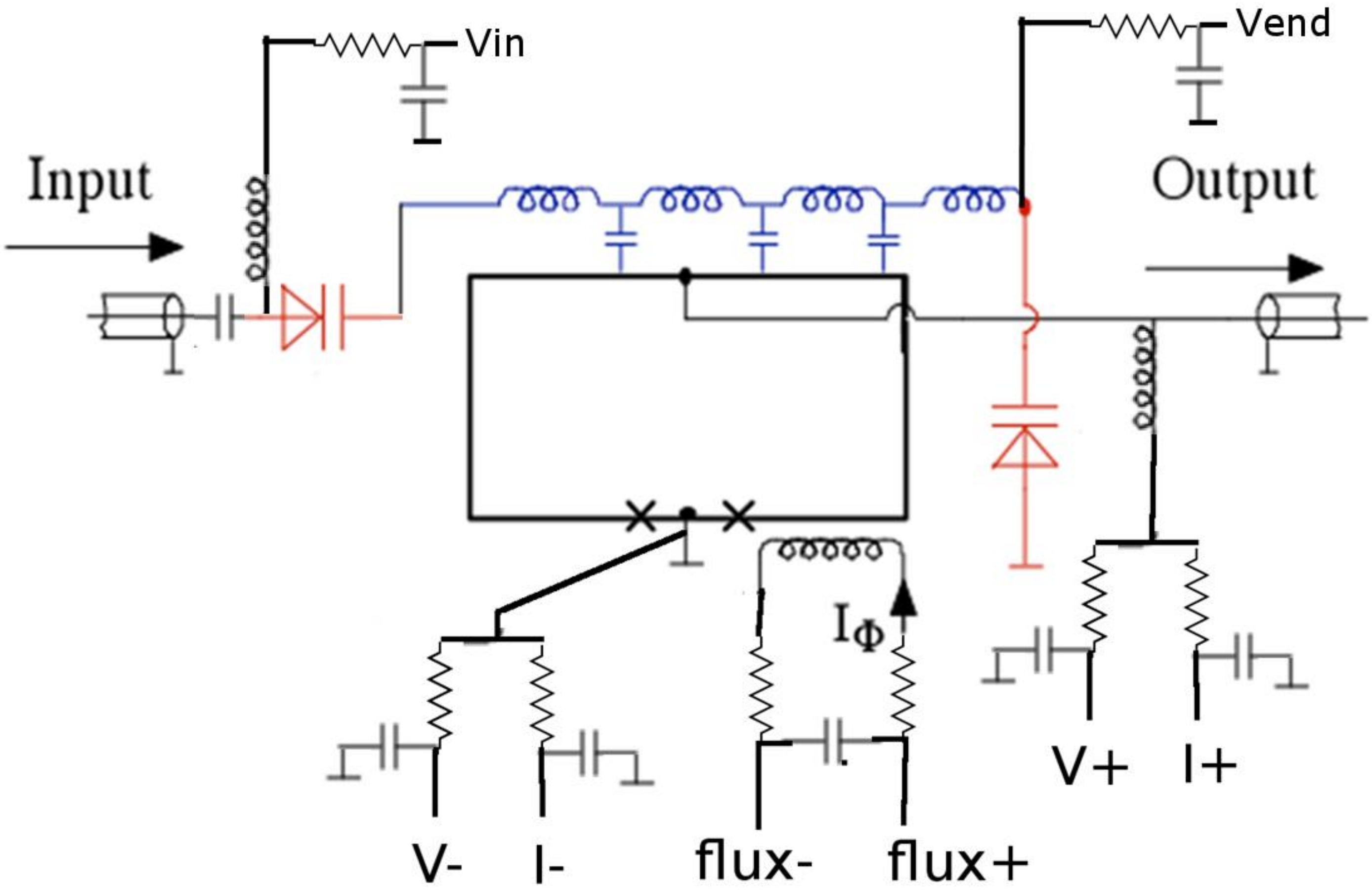
Practical Circuit Realization



Practical Circuit Realization

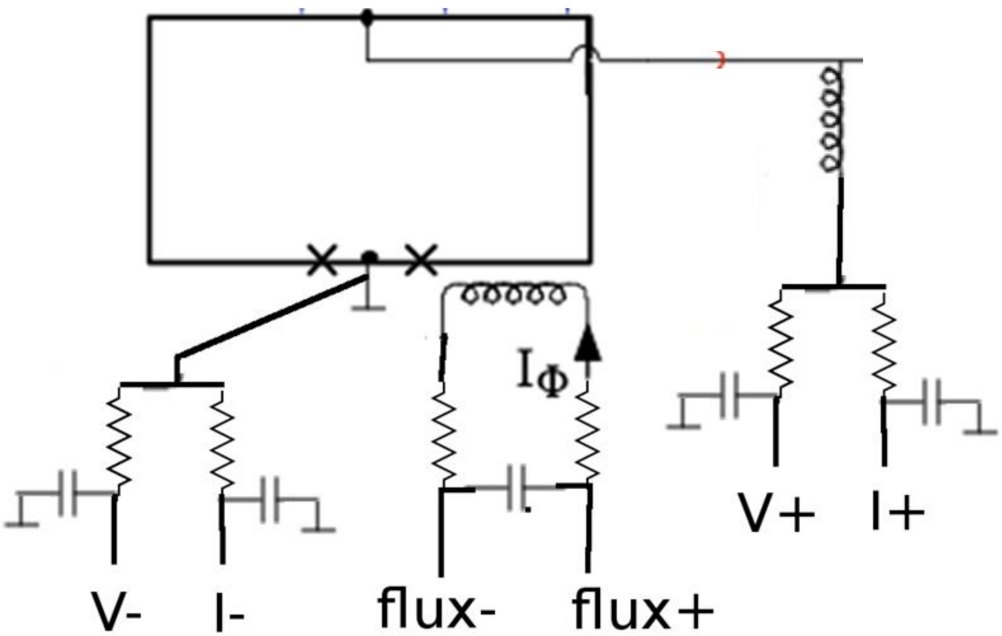


MSA Circuit Schematic



- Floating 4-wire, RC filtered DC bias network
- Floating flux bias
- Two varactor tuning voltages

MSA DC Schematic



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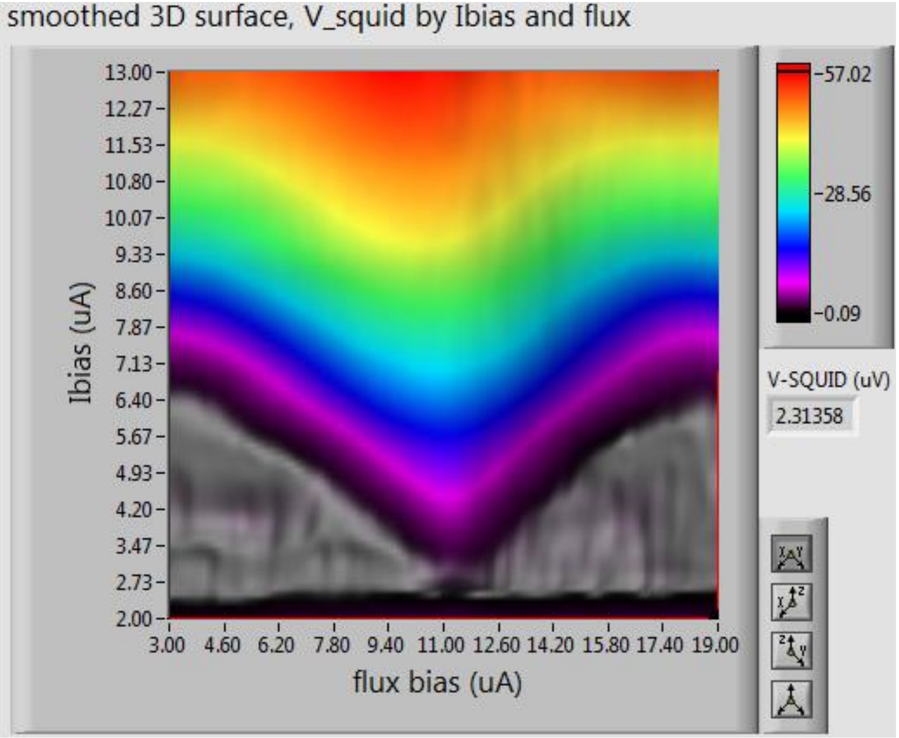
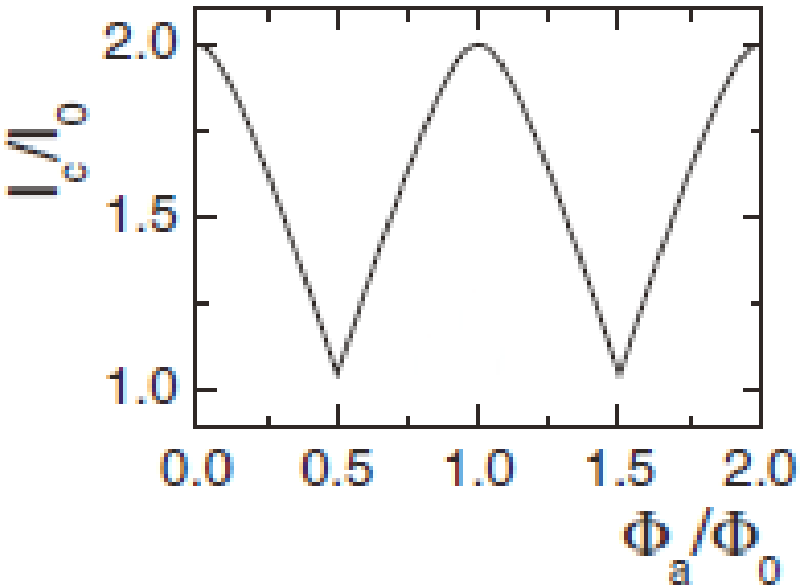
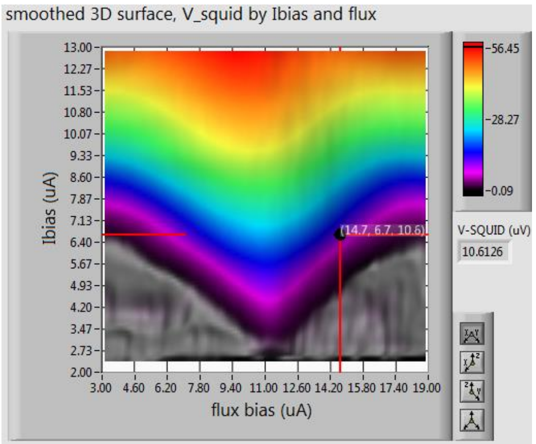
MSA DC Characteristics



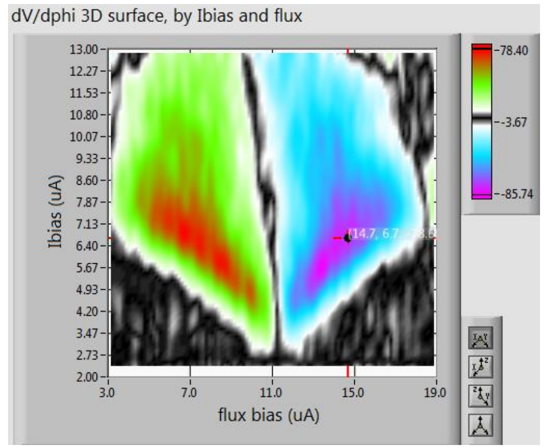


MSA DC Characteristics

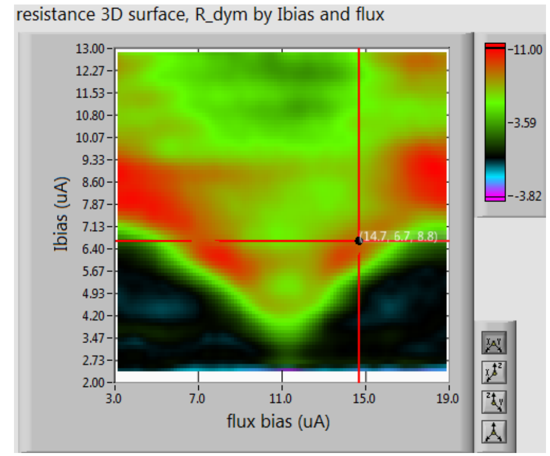
SQUID voltage



$dV/d\phi$



dV/dI_{bias}

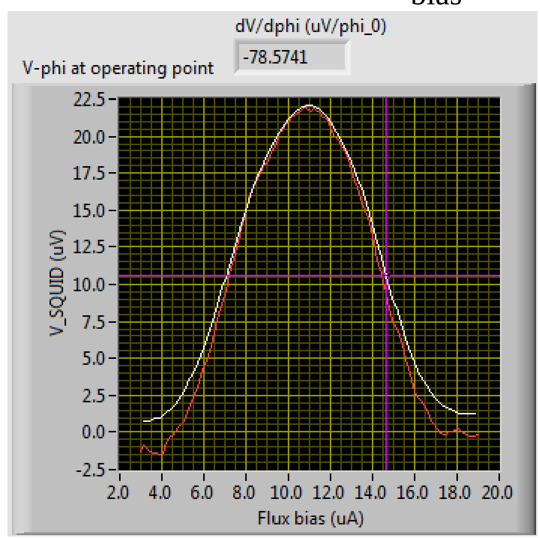


Typical DC bias point is around:

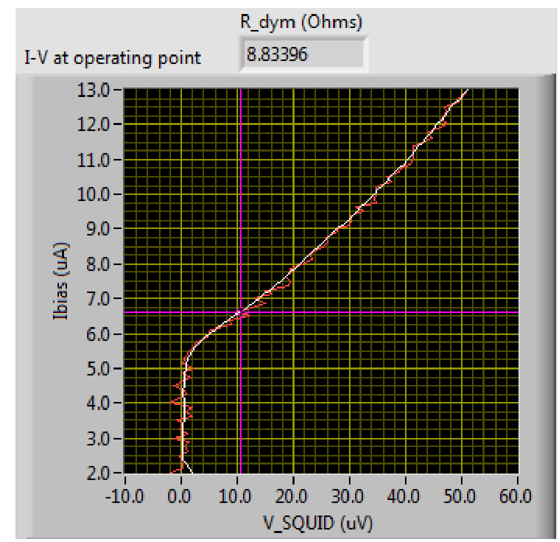
Current $\approx I_c$

Flux $\approx$ ¼ or ¾ ϕ_0

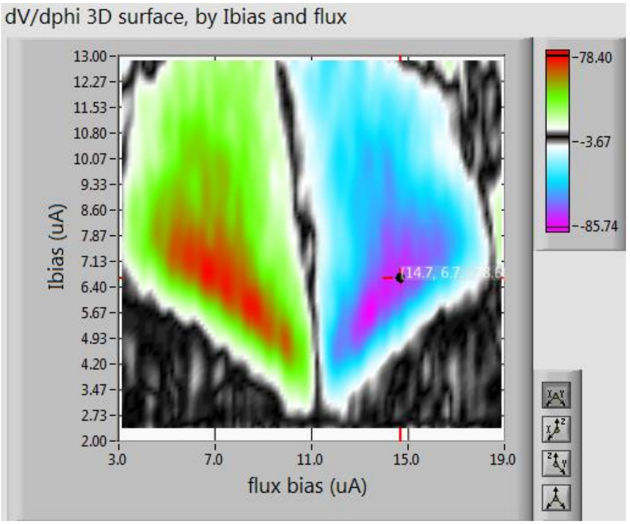
V vs flux, fixed I_{bias}

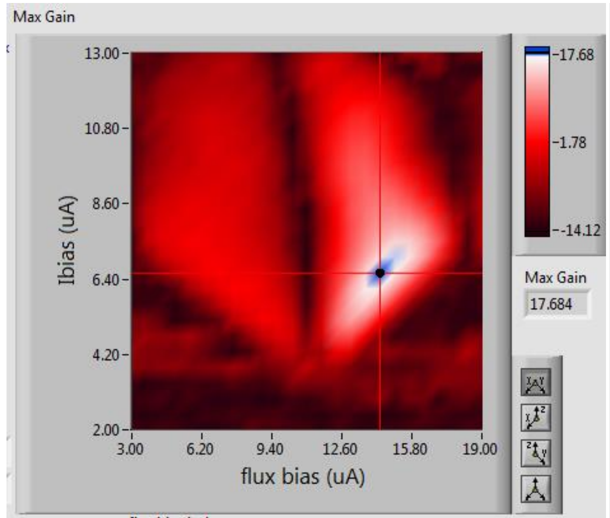


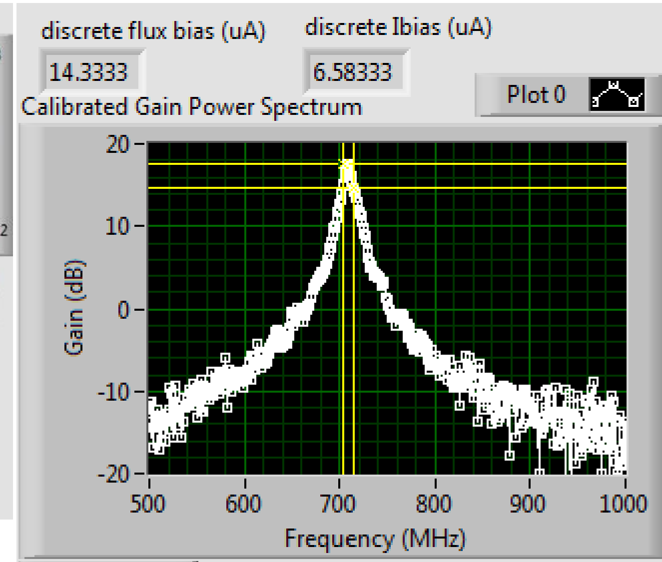
V vs I_{bias}, fixed flux

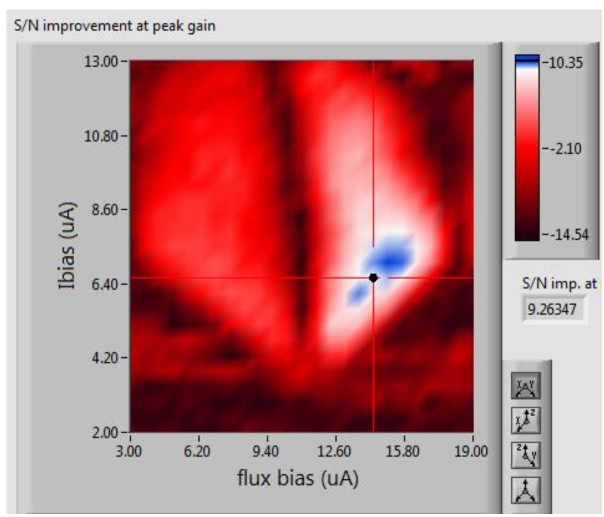


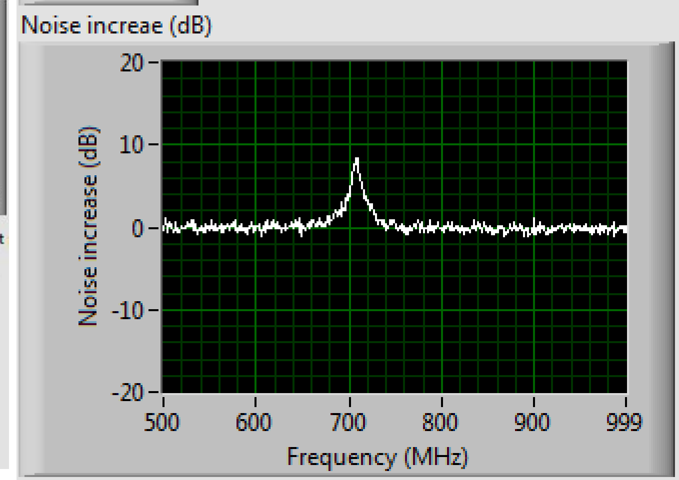
MSA RF Characteristics







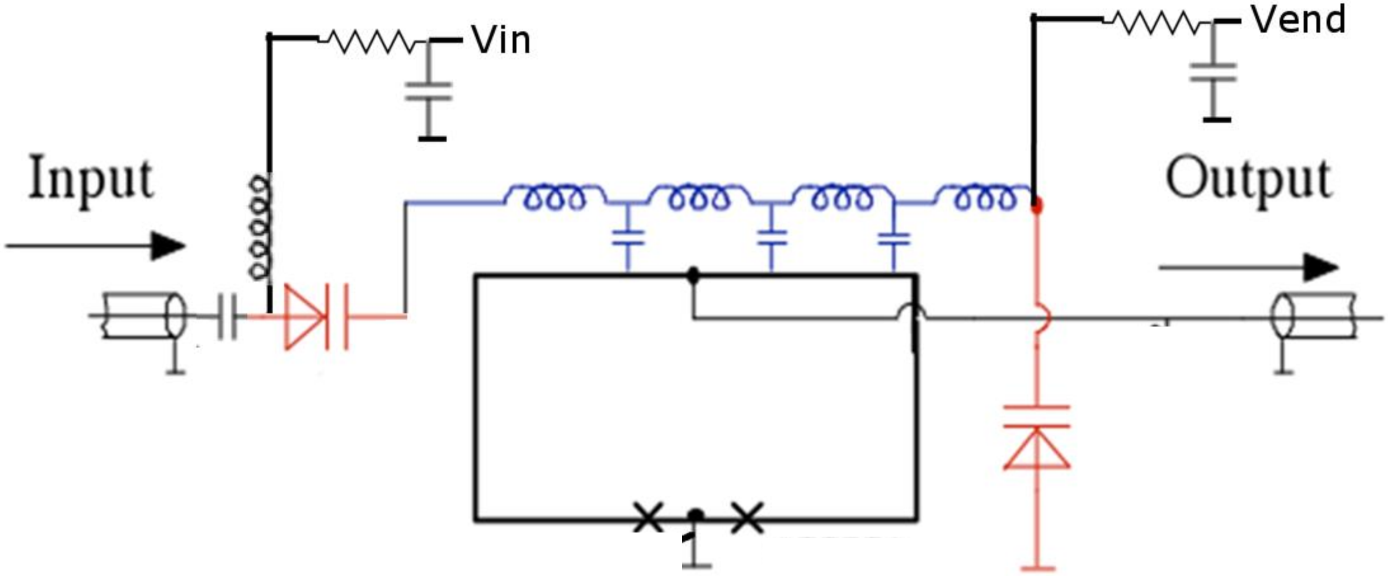




Note asymmetry between (+) and (-) $dV/d\phi$

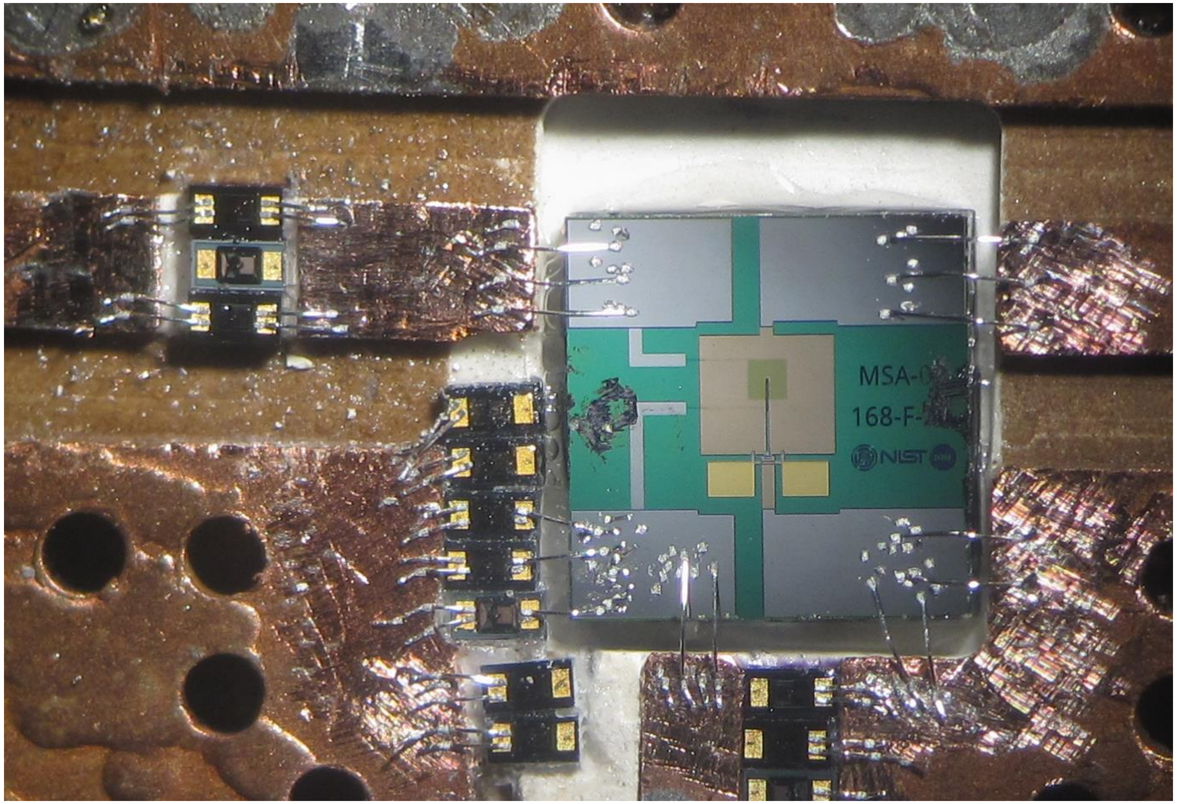
The explanation lies in feedback

MSA RF Schematic



- Input microstrip is referenced to the *active* SQUID washer, not to ground.
- This results in capacitive feedback from the SQUID output voltage to the input coil

MSA RF Connections



- Input microstrip is referenced to the *active* SQUID washer, not to ground.
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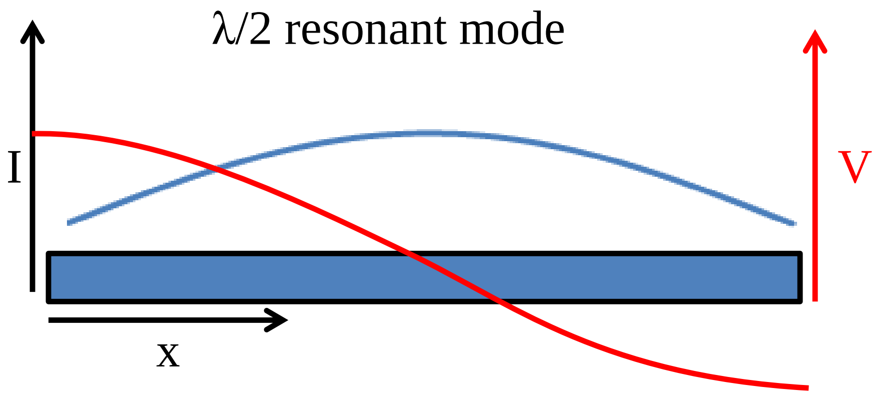
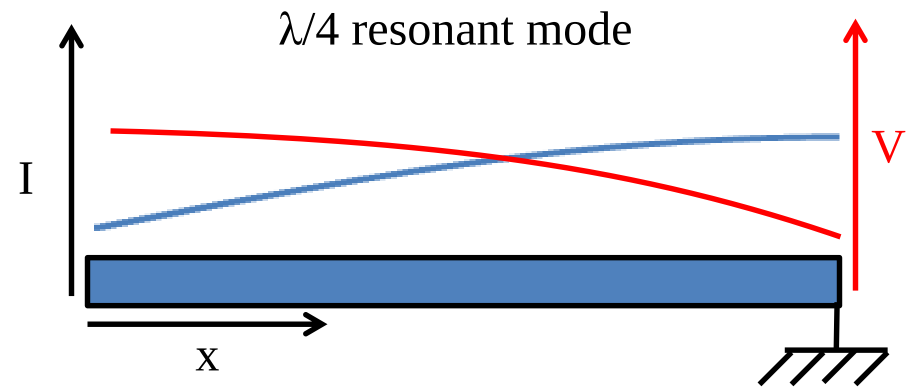
MSA feedback concept





Sign of feedback:

+ 0 -

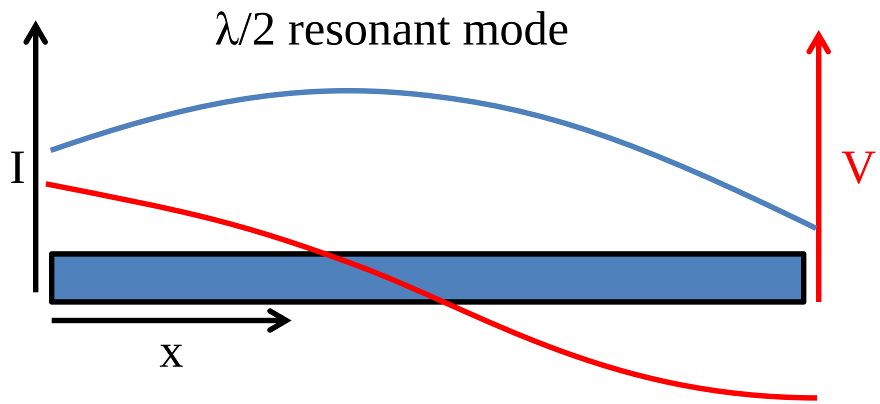
Capacitive feedback canceled

Sign of feedback:

+ 0

Capacitive feedback positive

MSA feedback concept

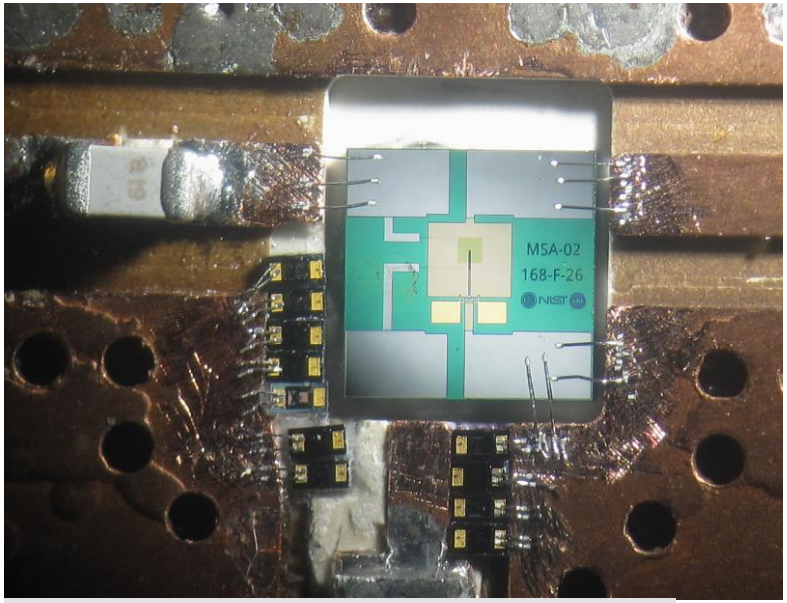


Sign of feedback:

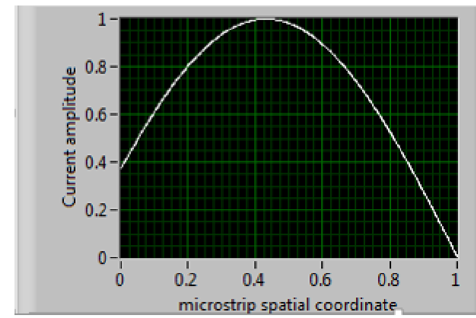
+ 0 −

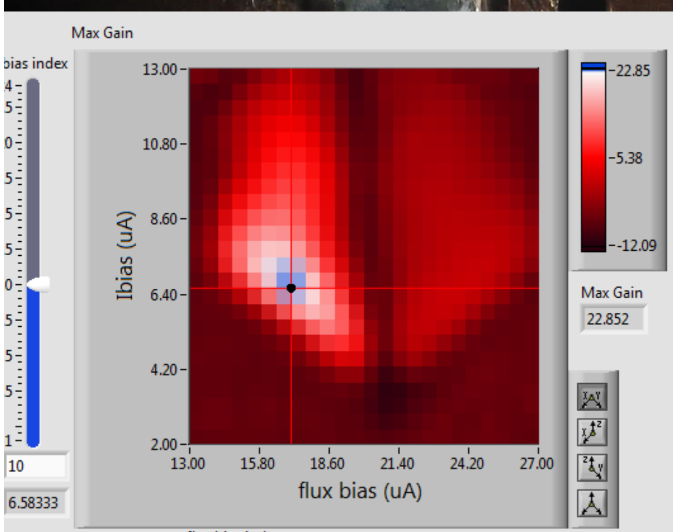
Capacitive feedback *negative*

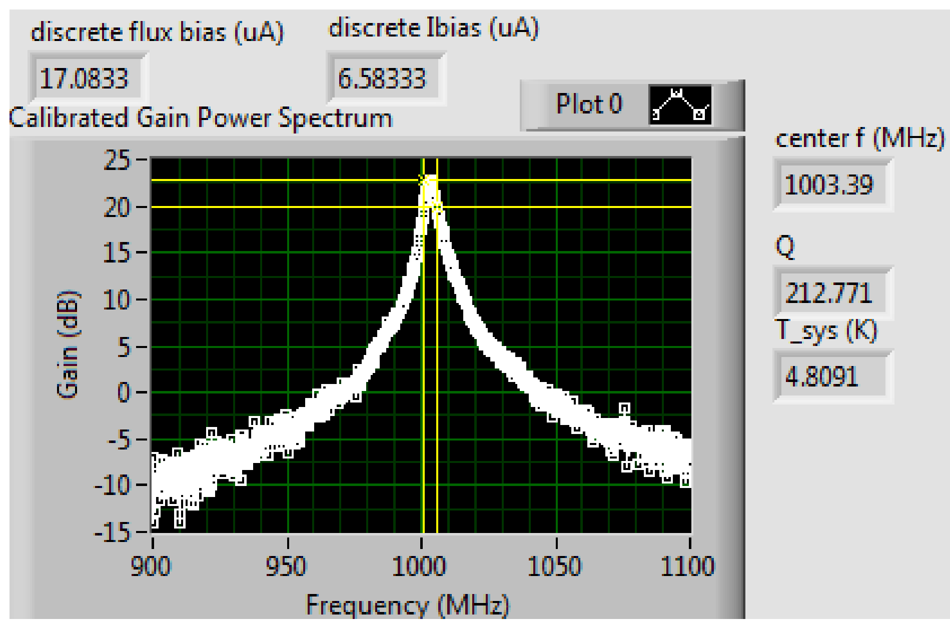
MSA feedback demonstration



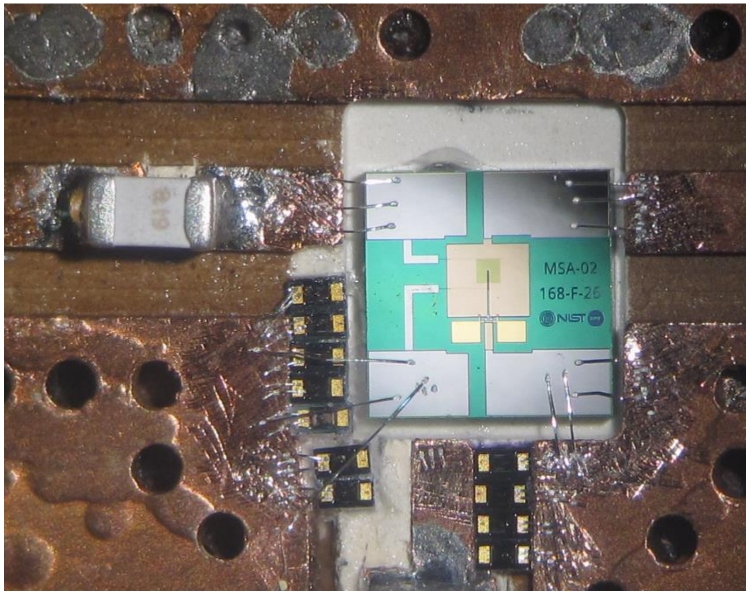
- Fixed input capacitor
- Open coil end
- High frequency
- Moderate (+) feedback
- Moderate Gain
- Low T_{SYS}



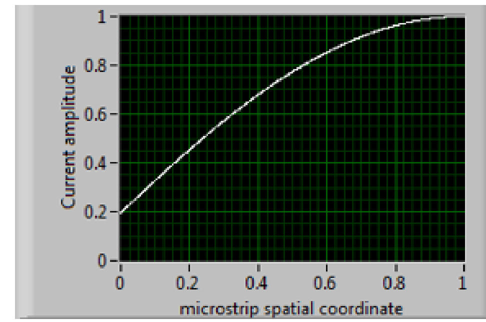


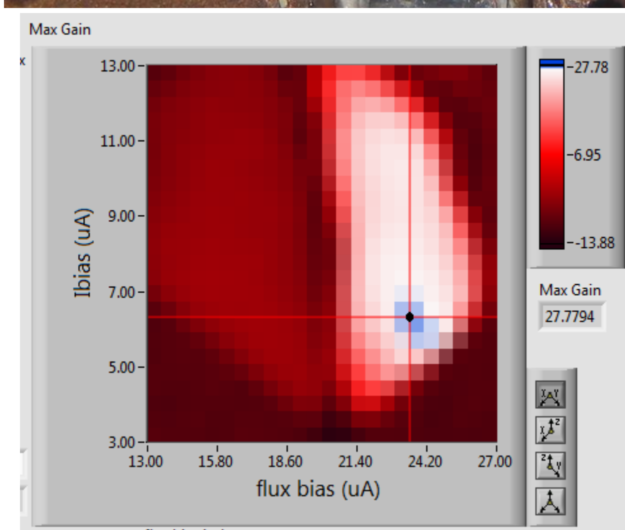


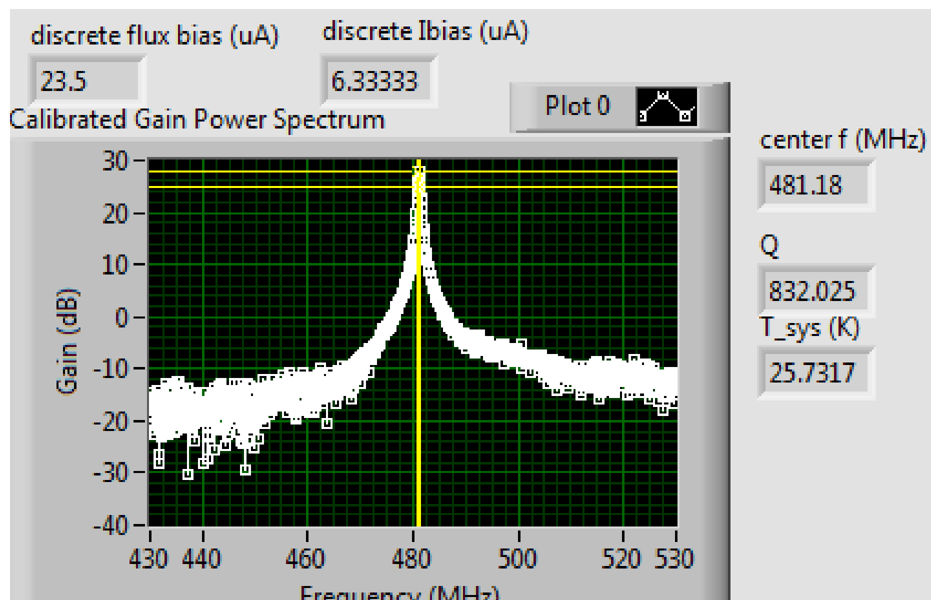
MSA feedback demonstration



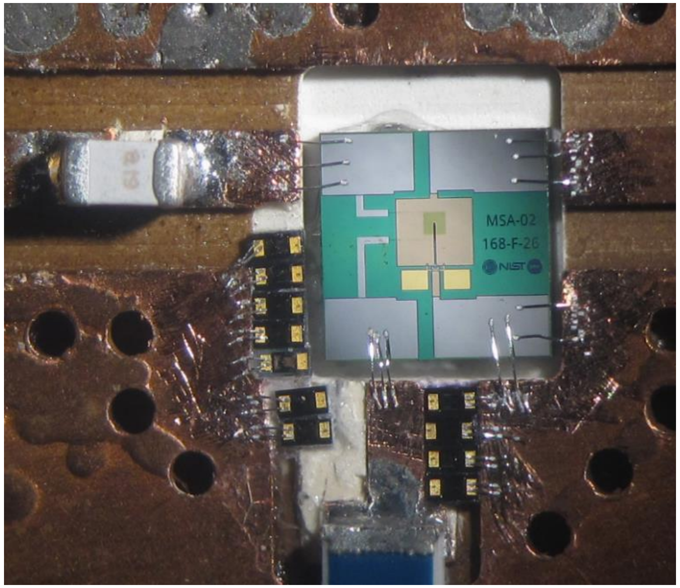
- Fixed input capacitor
- Coil end short to ground
- Low frequency
- High (-) feedback
- High Gain
- High T_{SYS}



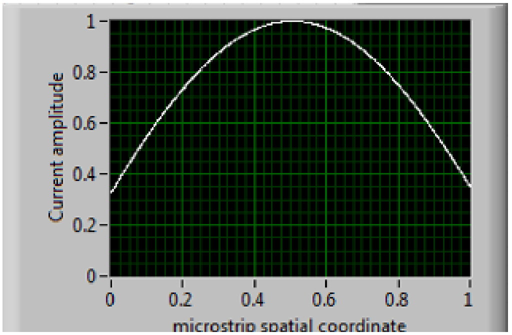


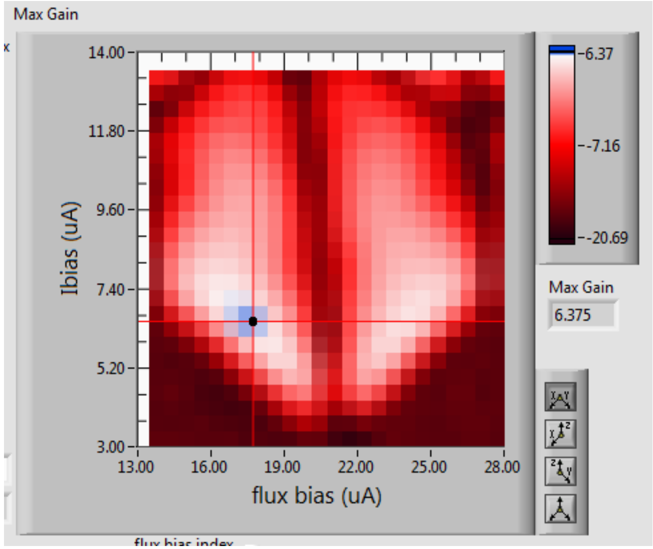


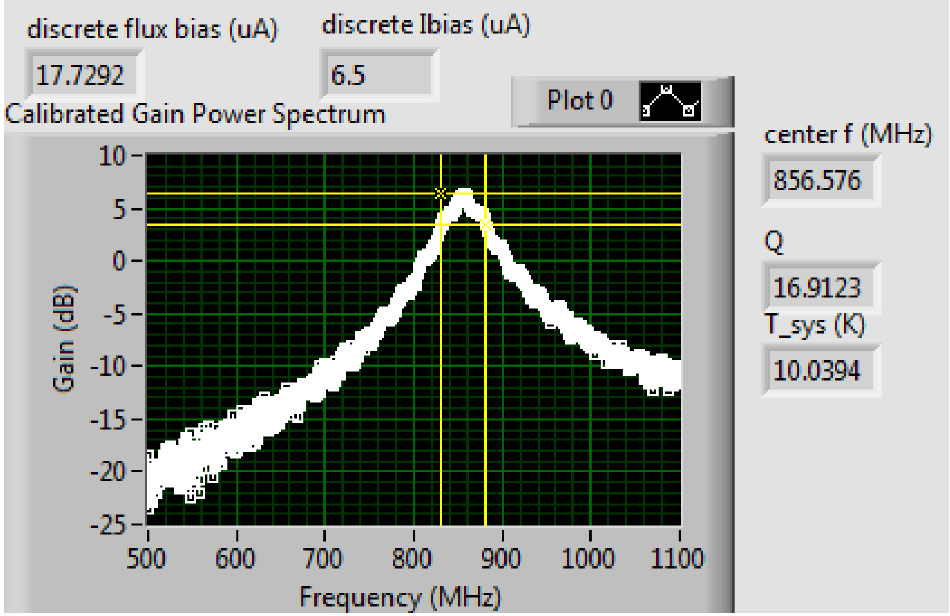
MSA feedback demonstration



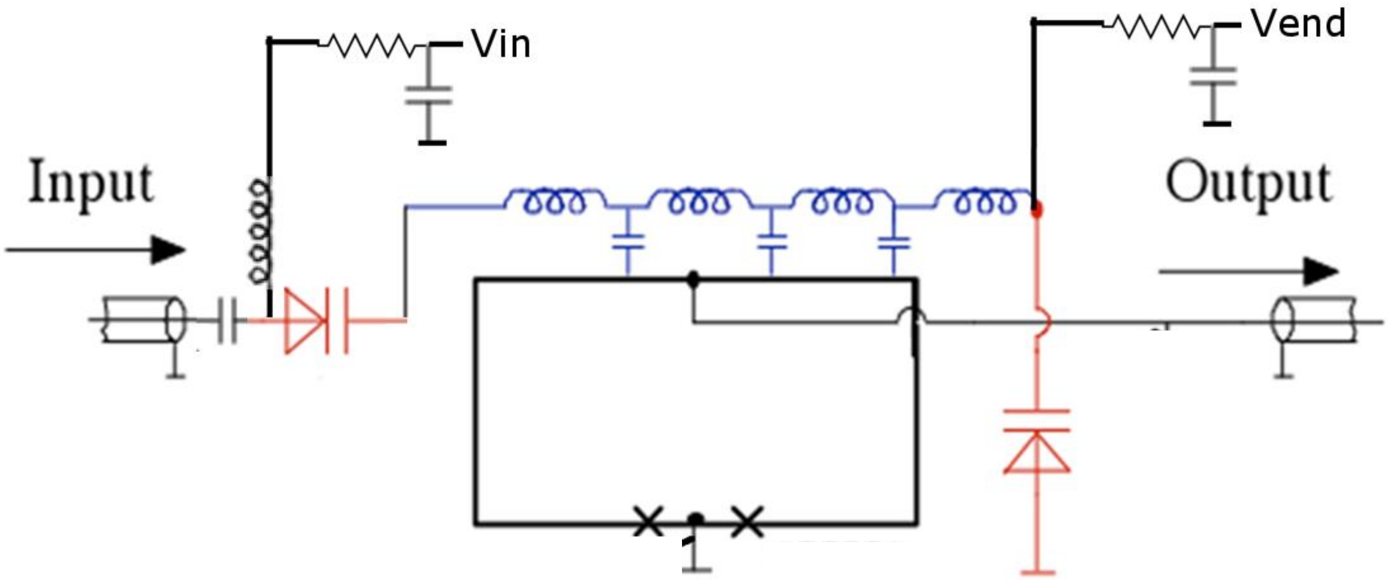
- Fixed input capacitor
- Fixed end capacitor
- Moderate frequency
- Zero (0) feedback
- Low Gain
- High T_{SYS}





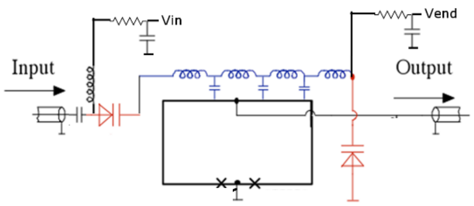


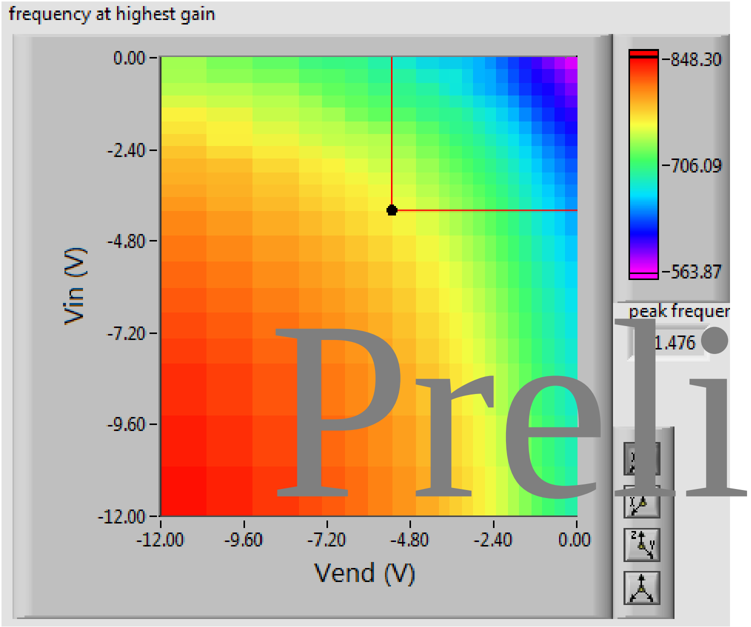
MSA RF Schematic

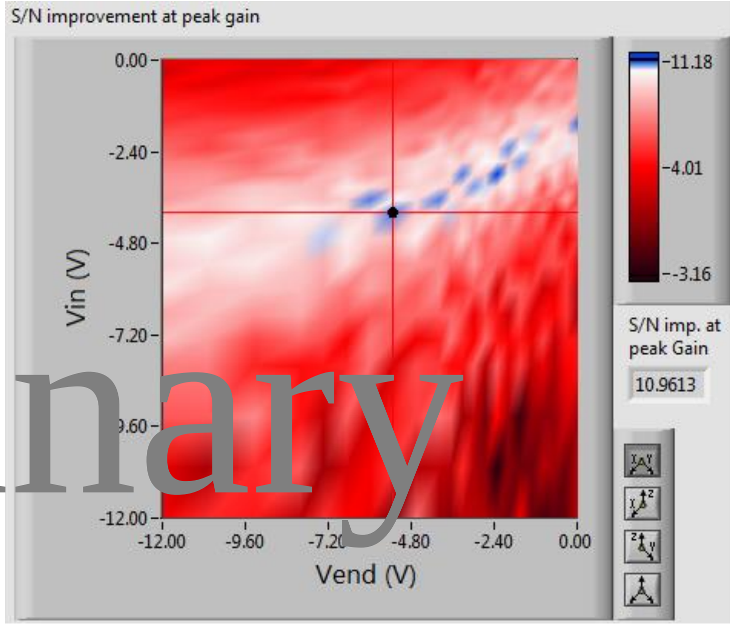


• Independent varactor control allows simultaneous tuning of frequency and feedback

MSA RF 2-end varactor tuning







- Independent varactor control allows simultaneous tuning of frequency and feedback
- Early data shows that the “best S/N ridge” spans the frequency space

SQUID design parameters

Adjustable parameters:

- Junction critical current density j_0
- Junction area
- Shunt resistor design
- SQUID geometric inductance
- Input coil # of turns
- Input coil width
- Dielectric thickness (between washer and input coil)
- **Input coupling**
- Output coupling
- **End tuning**
- DC filtering

Effects:

- Reliability/repeatability
- Input coil Impedance Z_0
- Native frequency f_0
- Output impedance
- Stray inductance
- $dV/d\Phi$
- Feedback

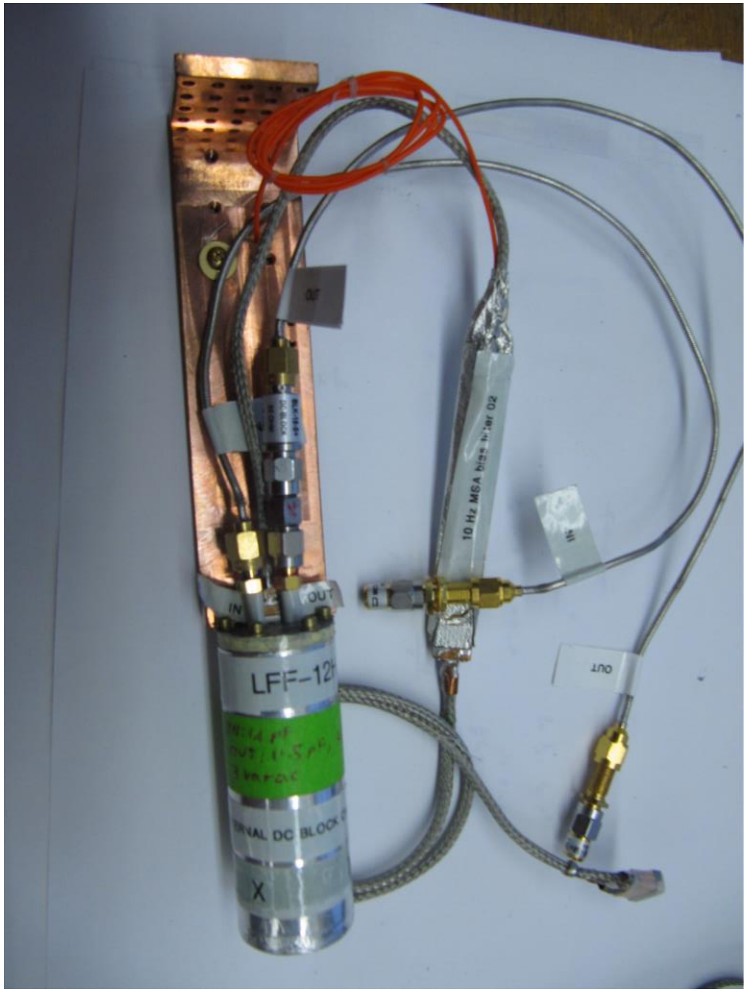
Ultimate performance concerns:

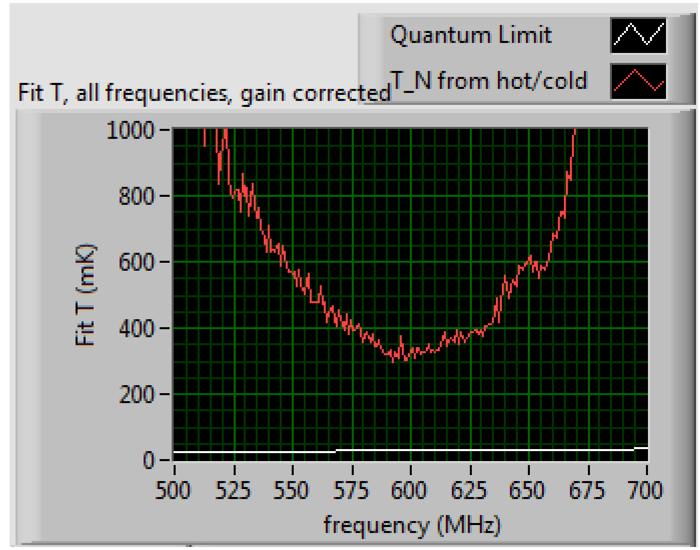
- Noise Temperature
- Gain
- Tunability

Outline

- Motivations from the Axion search
- Principle of SQUIDs as microwave amplifiers
- Practical MSA design and optimization
- Planned work

MSA RF 2-end varactor tuning





- Best (single-varactor) T_{SYS} measured with a hot/cold load is 300mK, estimated MSA T_N = 200mK
- Can the historic 50mK T_N or be matched or beaten with active tuning and input coupling?
- Tests at T=60mK incorporating 2-varactor tuning and other improvements soon to come!

Further planned work

- Deliver tunable low T_N MSAs to cover ADMX frequency span
- Deliver backup MSA’s for ADMX
- Test “stretch” devices (already fabricated) designed for frequencies from 1 to 3GHz and 250 to 500MHz

Acknowledgments

This work was made possible through the combined efforts of many skilled and competent collaborators who variously contributed guidance, insight, hard work, devices, and fabrication.

UC Berkeley
John Clarke
Jørn Hansen (Technical University of Denmark)

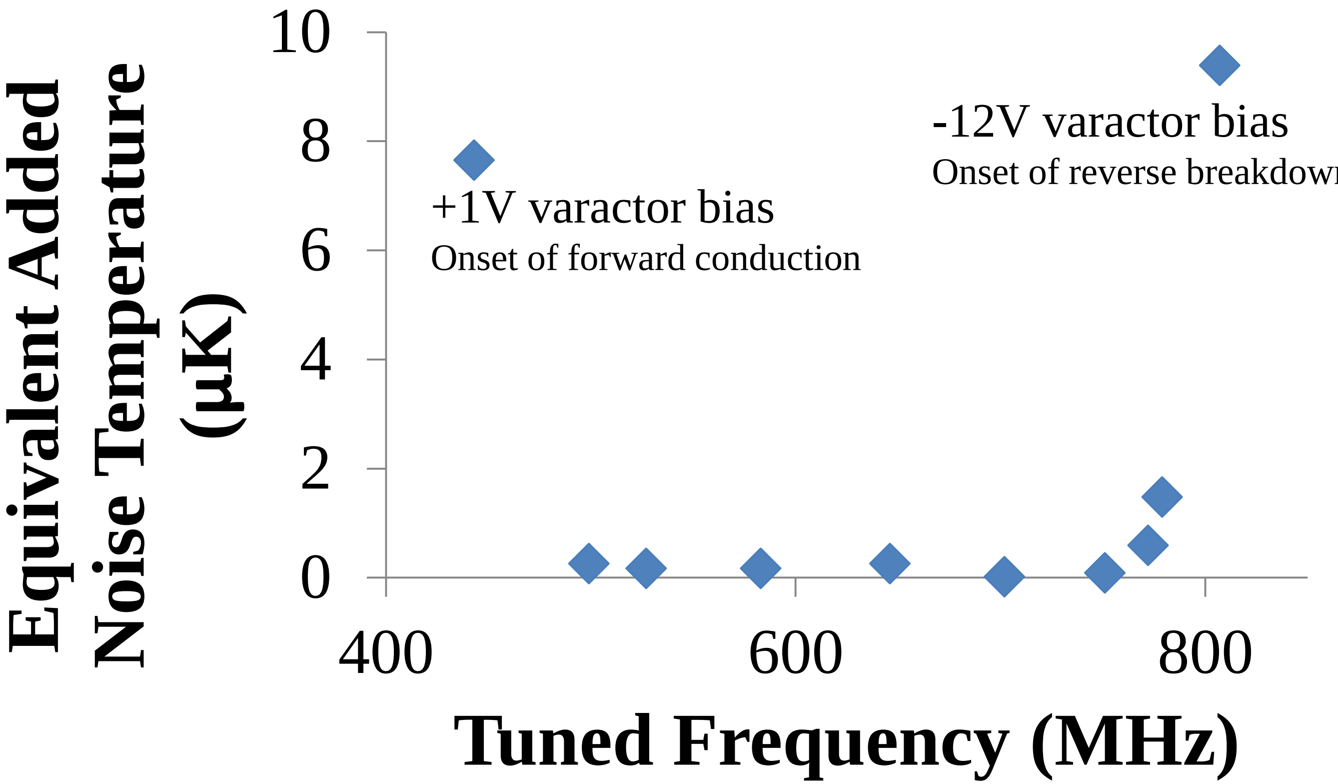
ADMX Collaboration
including collaborators at
U Washington
U Florida
LLNL

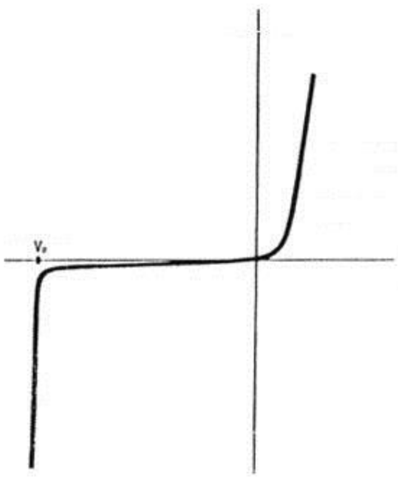
Device Fabrication
Gene Hilton (NIST Boulder)

U.S. DEPARTMENT OF ENERGY | Office of Science

Noise Added by Varactors

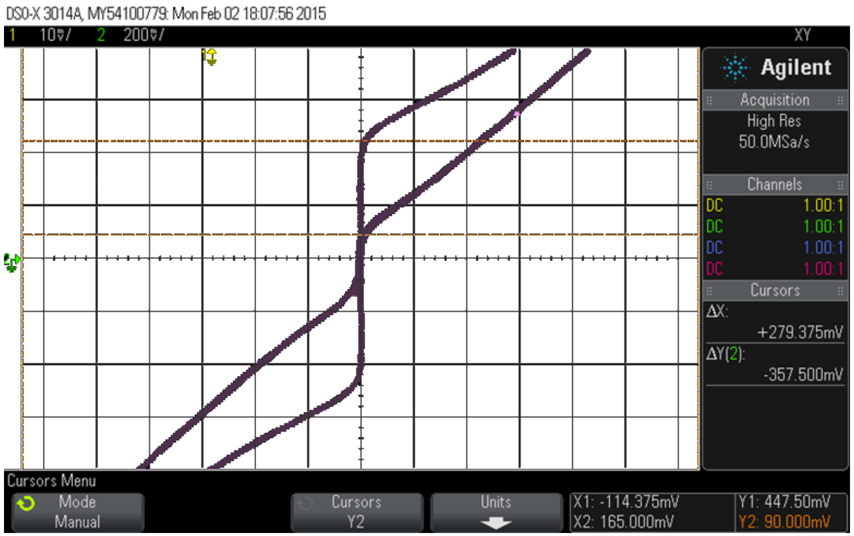
$$T_N = (eI_{leakage}Z_0)/2k_B$$



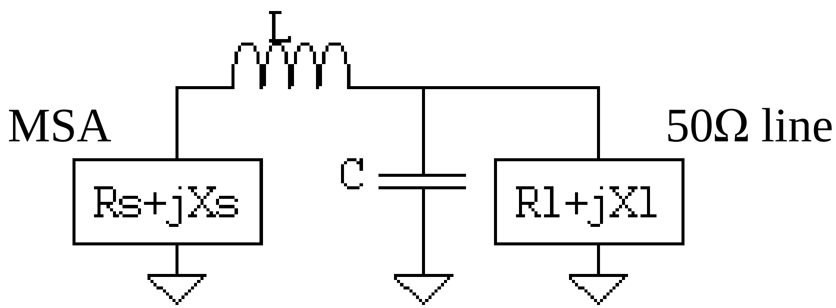


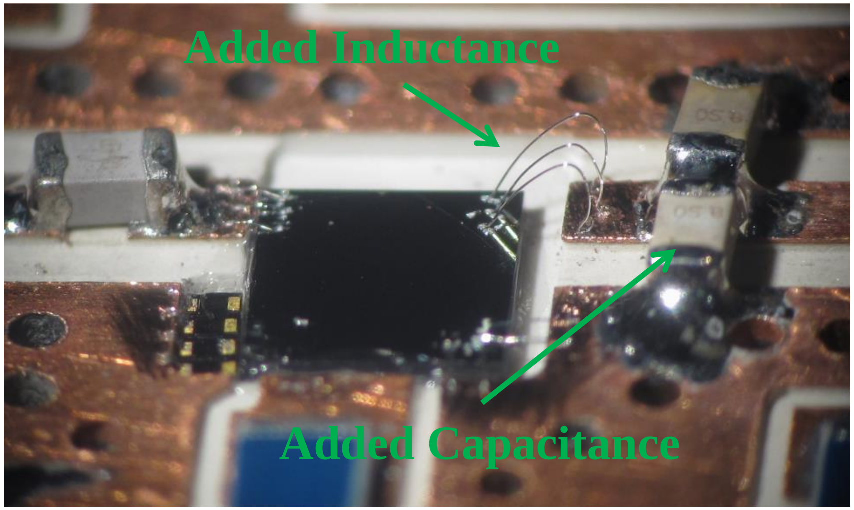
Assumes Z_0 = 50 Ω, leakage current measured at 4.2 K

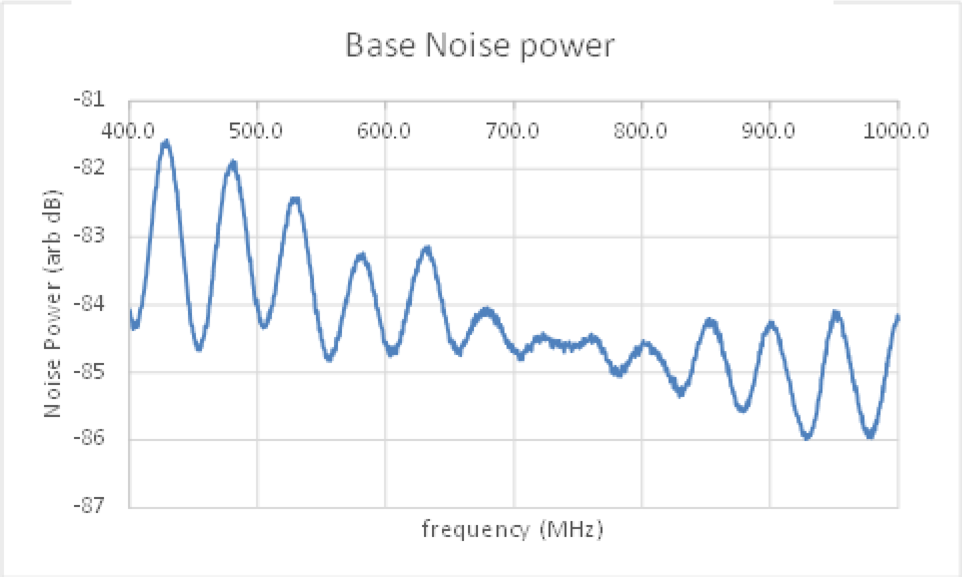
Output Coupling Optimization



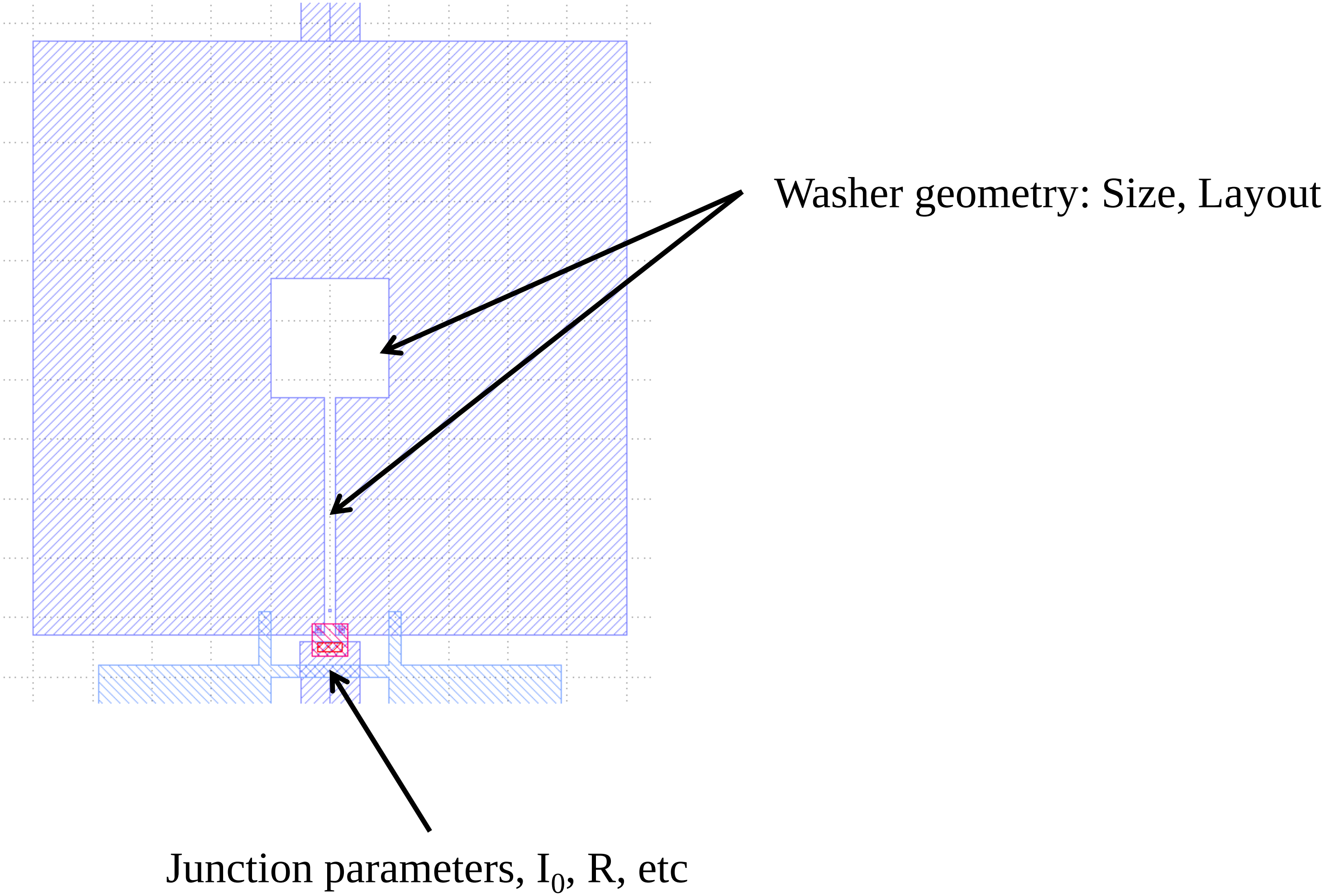
MSA output impedance ≈ 10 Ω
Transmission line = 50 Ω





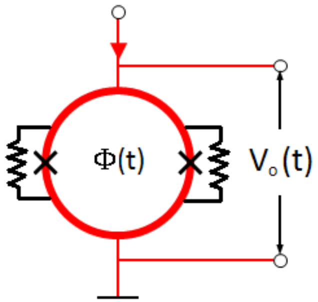


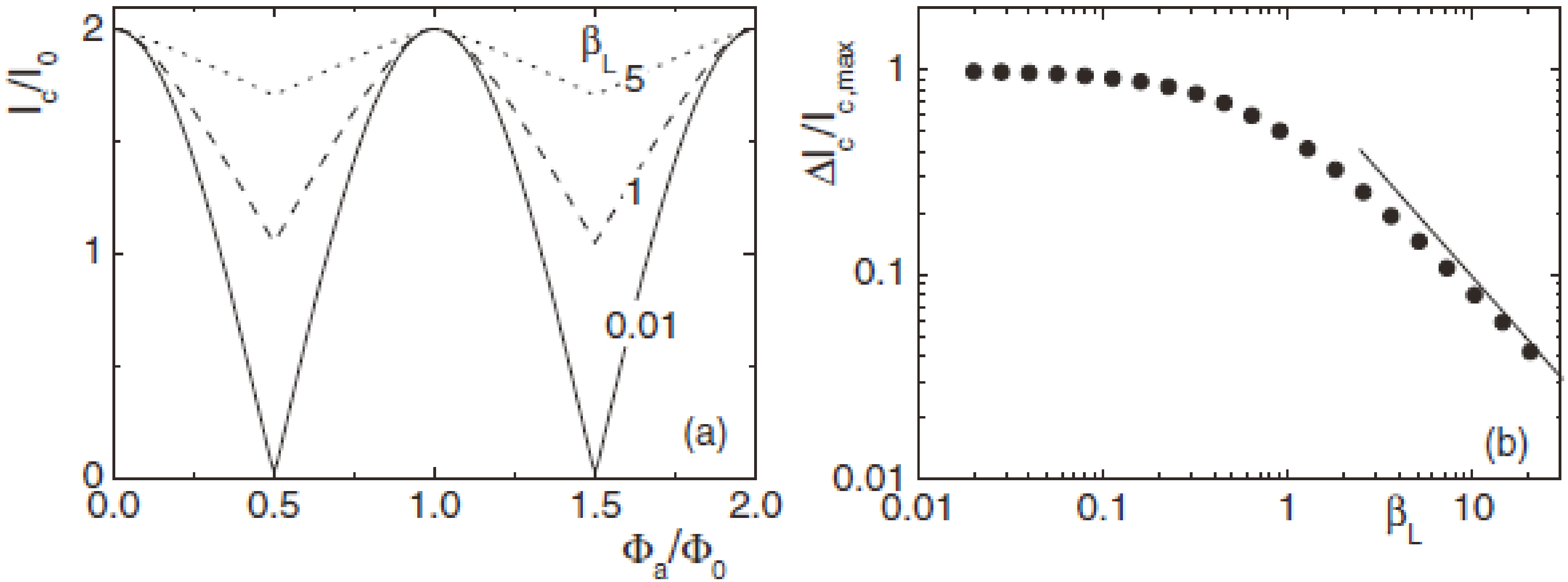
SQUID Layout



The screening parameter β_L

$$\beta_L = 2LI_0/\Phi_0$$

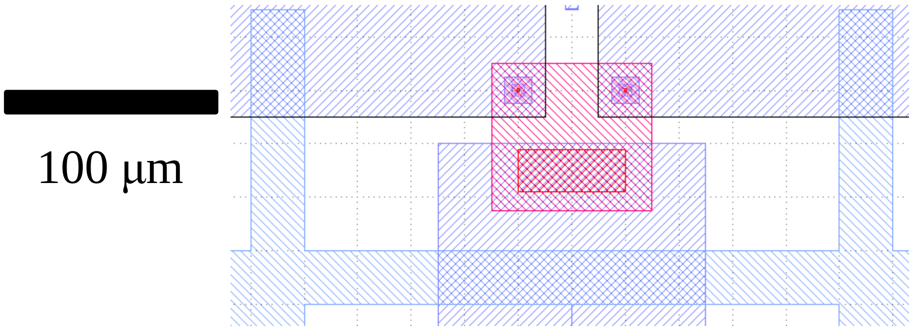


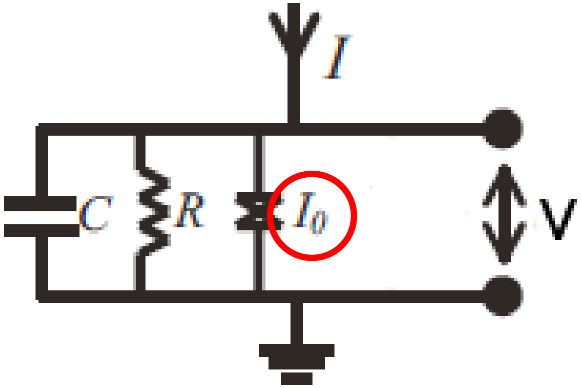


- β_L is essentially the ratio of geometric inductance to Josephson inductance.
- Smaller β_L yields greater modulation depth and thus greater potential amplification.
- Thermal effects limit the practicality of $\beta_L << 1$
- Design to $\beta_L \approx 1$ or slightly below as a rule of thumb.

Choosing Junction Parameters: I_0

Our MSA's are made by Gene Hilton at NIST, who has a set of very reliable recipes for junction fabrication, which constrain our choice of parameters.





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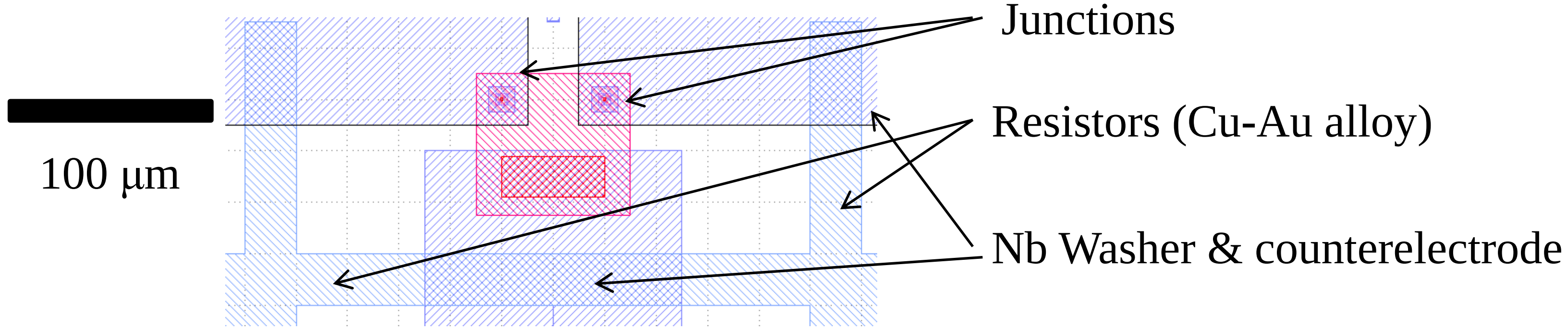

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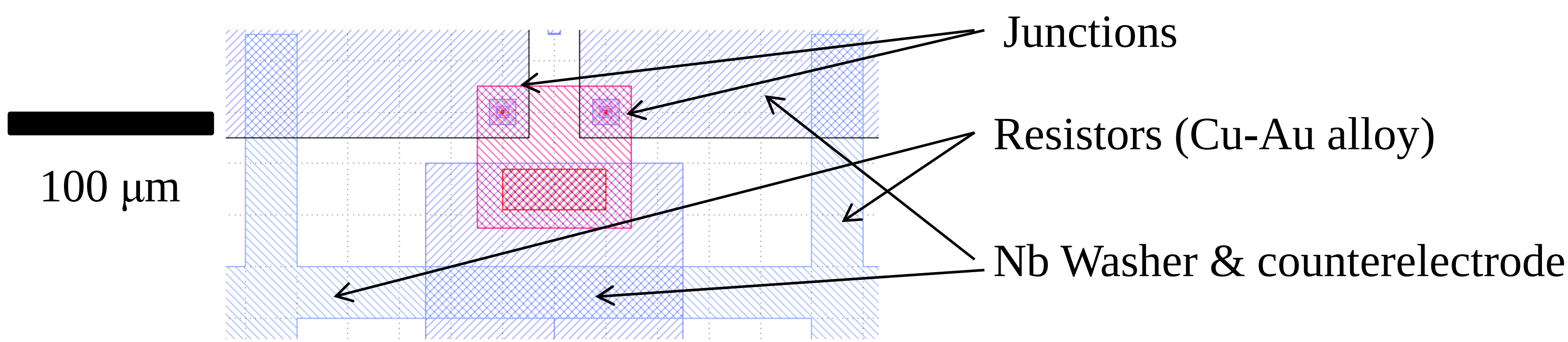
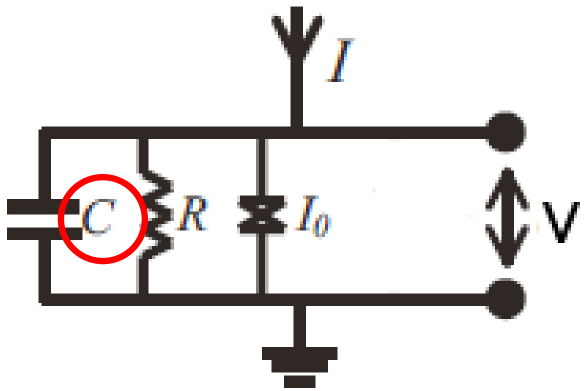


- Smaller junction area reduces C (good) but Nb trilayer junctions can only be made so tiny before reliability suffers.
  We choose a junction area of 6.25 μm^2
- We want $\Gamma \equiv \frac{2\pi k_B T}{I_0 \Phi_0}$ not be larger than 0.1 or so, and ADMX requires operation at T as high as 4.2K
  @ T = 4.2K, $I_0 > 1.7\ \mu A$
- Considering fabrication practicalities, we chose a conservative **I_0 = 2.5 μA**, with very good reliability and repeatability (too conservative?)

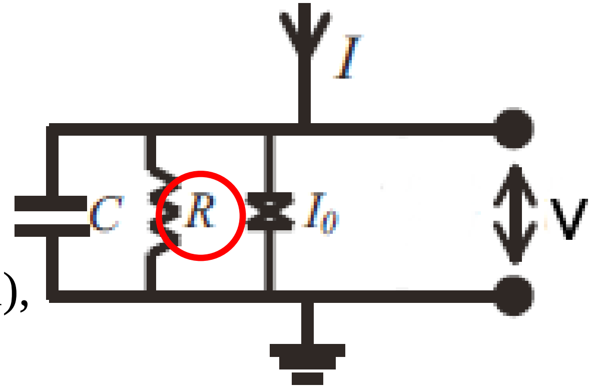
Choosing Junction Parameters: C

Once the area and critical current are chosen, C is not adjustable.
For our design parameters, C = 300fF



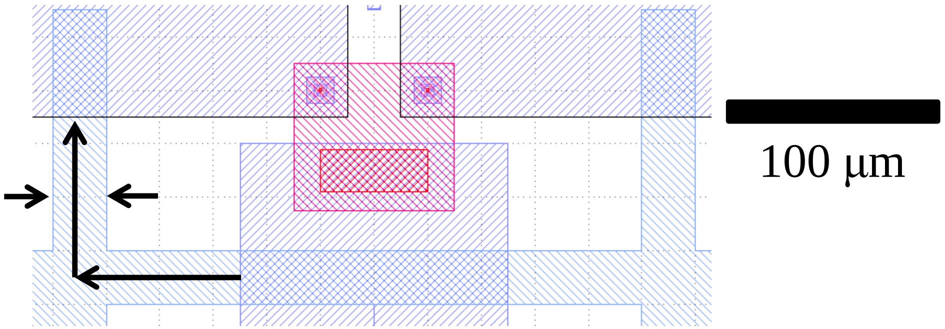
Choosing Junction Parameters: R

Once the area and critical current are chosen, C is not adjustable.
For our design parameters, C = 300fF



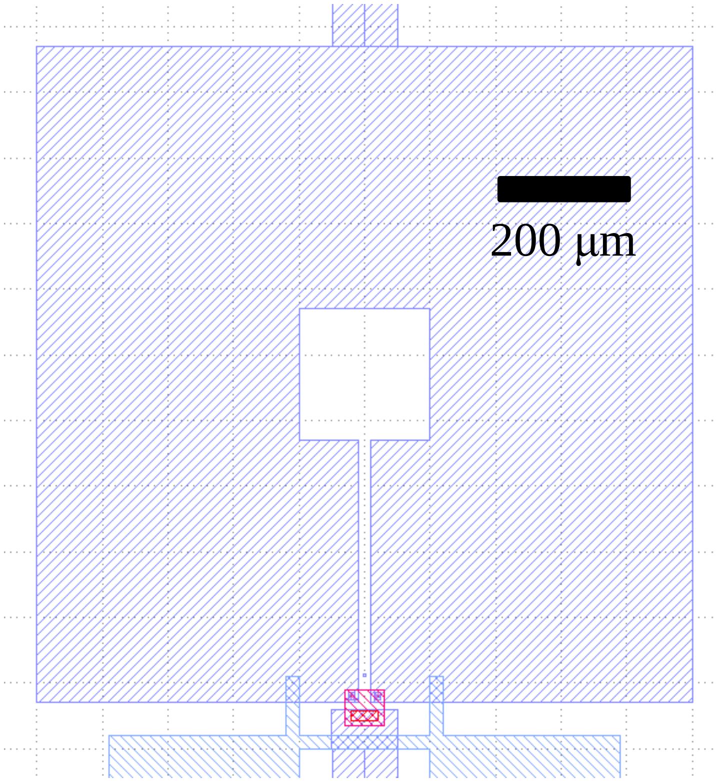
R can be made small to ensure non-hysteretic operation (critical), but large R will increase dV/dΦ (nice)

R is set by the geometry of the shunts



We chose a conservative R = 10Ω, for $\beta_c = \frac{2\pi}{\Phi_0} I_0 R^2 C = 0.24$
(too conservative?)

SQUID Inductance



$d = 200\mu m$
$l = 390\mu m$

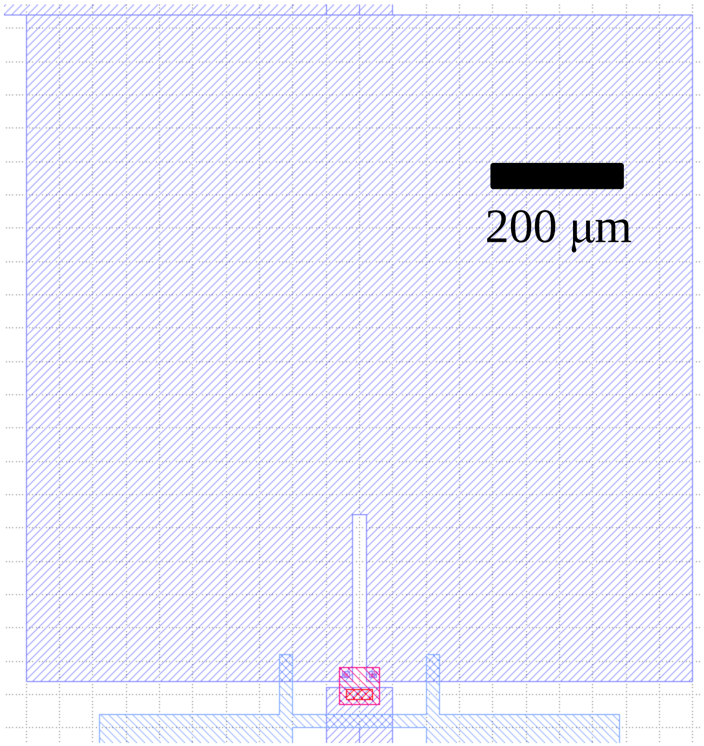
A traditional SQUID design has a square hole, narrow slit, and junctions at the outer edge.
Semi-empirical formula for this configuration is:

$$L = 1.25\mu_0 d + \frac{0.3\text{pH}}{\mu m} l$$

where *d* is the hole diameter and
l is the slit length

In one practical design (pictured)
L = 431 pH
I_0= 2.5 µA
β_L = 1.04

SQUID Inductance



$d = 5\mu m$
$l = 240\mu m$

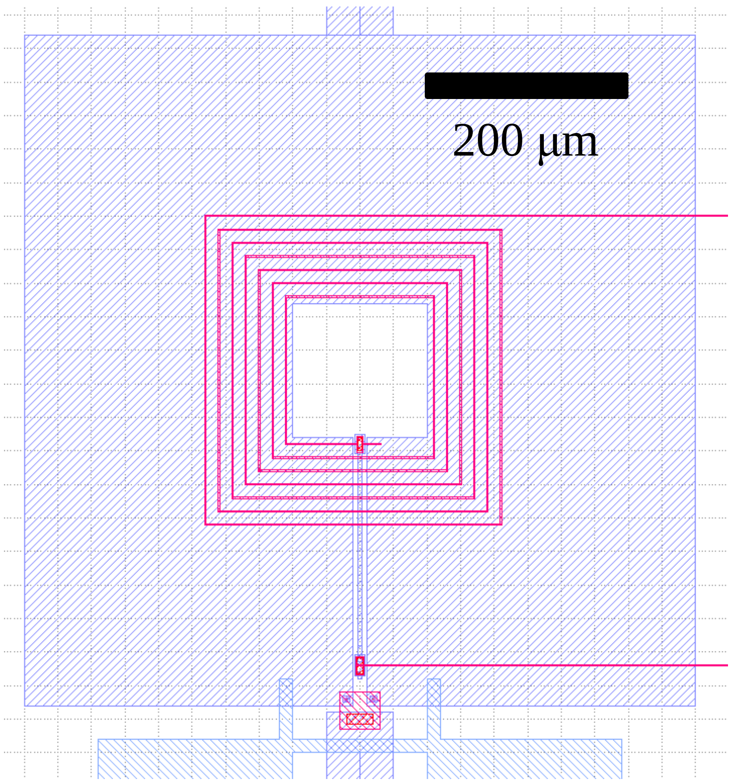
A traditional SQUID design has a square hole, narrow slit, and junctions at the outer edge.
Semi-empirical formula for this configuration is:

$$L = 1.25\mu_0 d + \frac{0.3\text{pH}}{\mu m} l$$

where *d* is the hole diameter and l is the slit length

In one practical design (pictured)
L = 80 pH
$I_0 = 2.5\ \mu A$
$\beta_L = 0.2$

MSA Input Coil



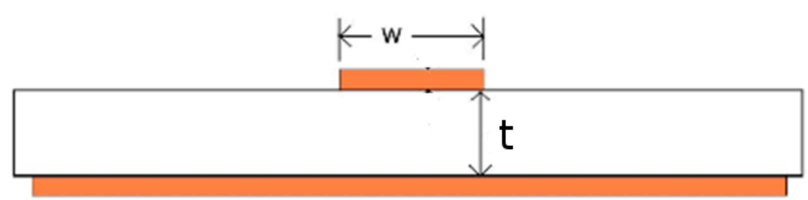
$W = 2\mu m$
$t = 350nm$

To couple the microwave signal into the SQUID:

- Cover the washer with an insulating layer (350nm of SiO_2)
- Add a spiral path of conductor around the central hole

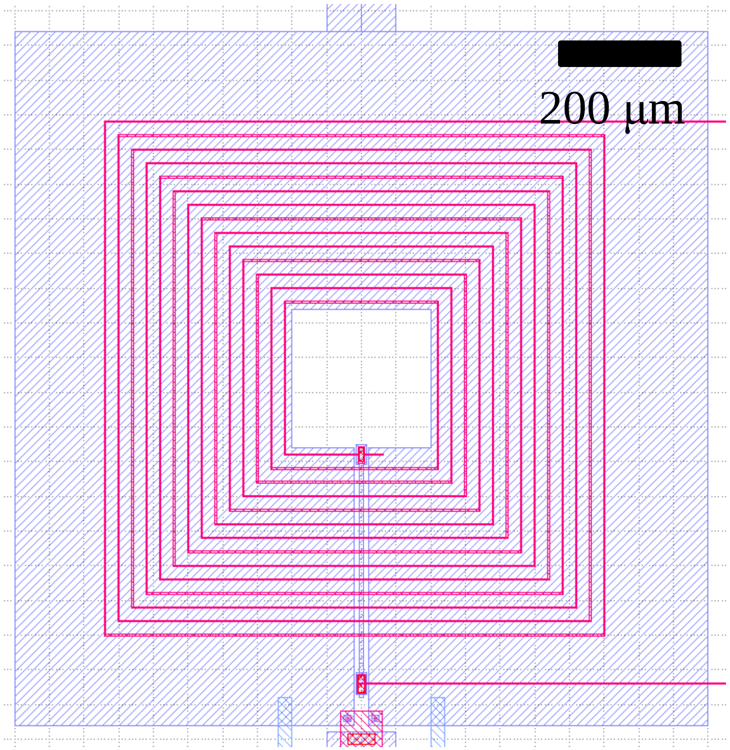
This creates a microstrip transmission line between the input coil and SQUID washer

Cross section:



Microstrip

MSA Input Coil



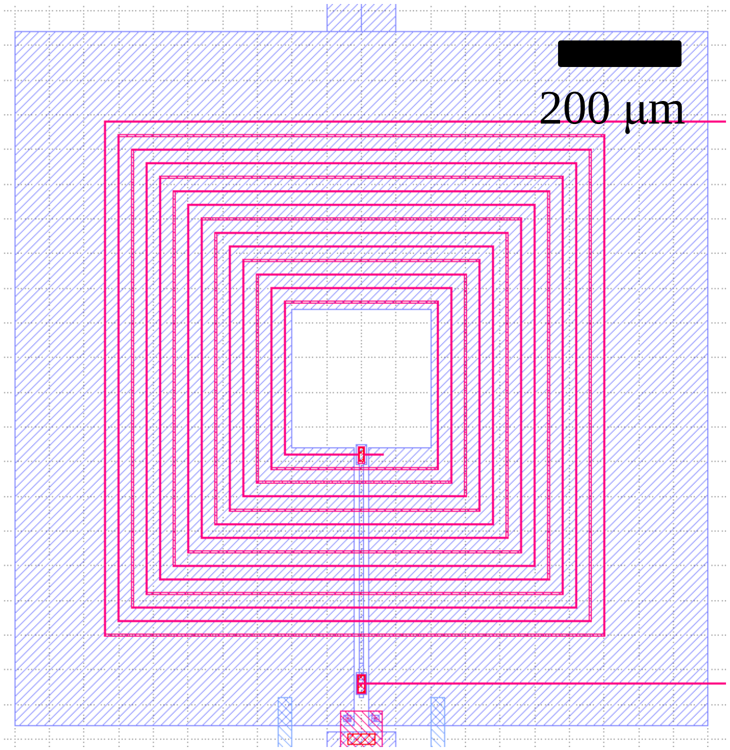
With the ends open, the microstrip is a ½-wave resonator, with the frequency set by L_l, C_l, and l

- Capacitance is well-approximated by the parallel-plate formula.
- Inductance is composed of microstrip, kinetic, and SQUID inductances, but due to strong flux-coupling between the coil and SQUID loop, the **SQUID inductance term is dominant by far.**

$$C_l = \frac{A_{coil} \cdot \epsilon_{SiO_2}}{t \cdot l}$$

$$L_l = \frac{\alpha \cdot L_{SQUID} \cdot N^2}{l}$$

MSA Input Coil



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$$C_\mathrm{l} = \frac{A_{coil} \cdot \epsilon_{SiO_2}}{t \cdot \mathrm{l}}$$

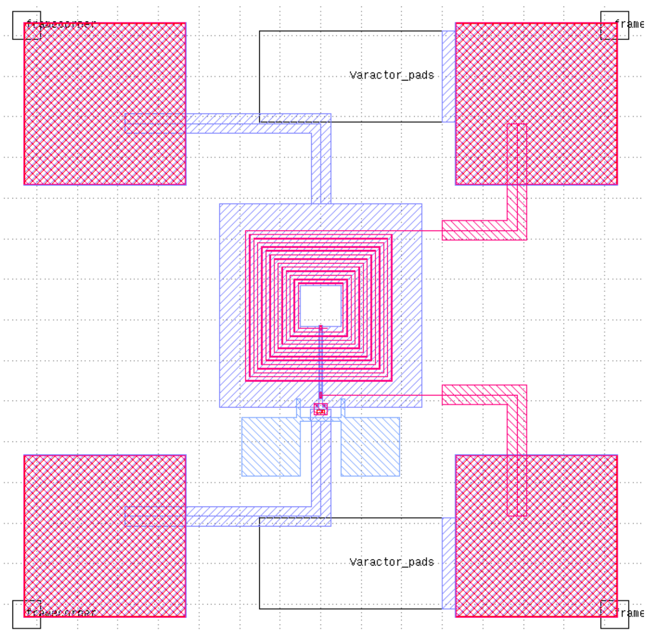
$A_{coil} = 18{,}500\ \mu\mathrm{m}^2$
$\epsilon_{SiO_2} = 3.5\ \epsilon_0$
$H = 350$ nm
$\alpha = 1$
$N = 14$
$L_{SQUID} = 431$ pH
$\mathrm{l} = 8736\ \mu$m

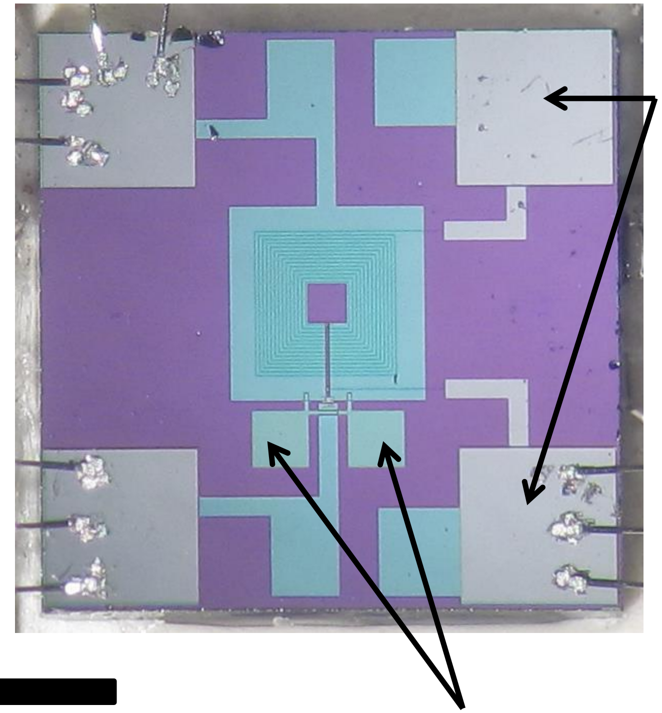
$$v = \frac{1}{\sqrt{L_\mathrm{l} \cdot C_\mathrm{l}}} \approx 0.13c$$

$$L_\mathrm{l} = \frac{\alpha \cdot L_{SQUID} \cdot N^2}{\mathrm{l}}$$

$$f_0 = \frac{v}{2l} = 798\ \mathrm{MHz} \qquad Z_0 = \sqrt{\frac{L_\mathrm{l}}{C_\mathrm{l}}} = 135\Omega$$

Connect to the Real World





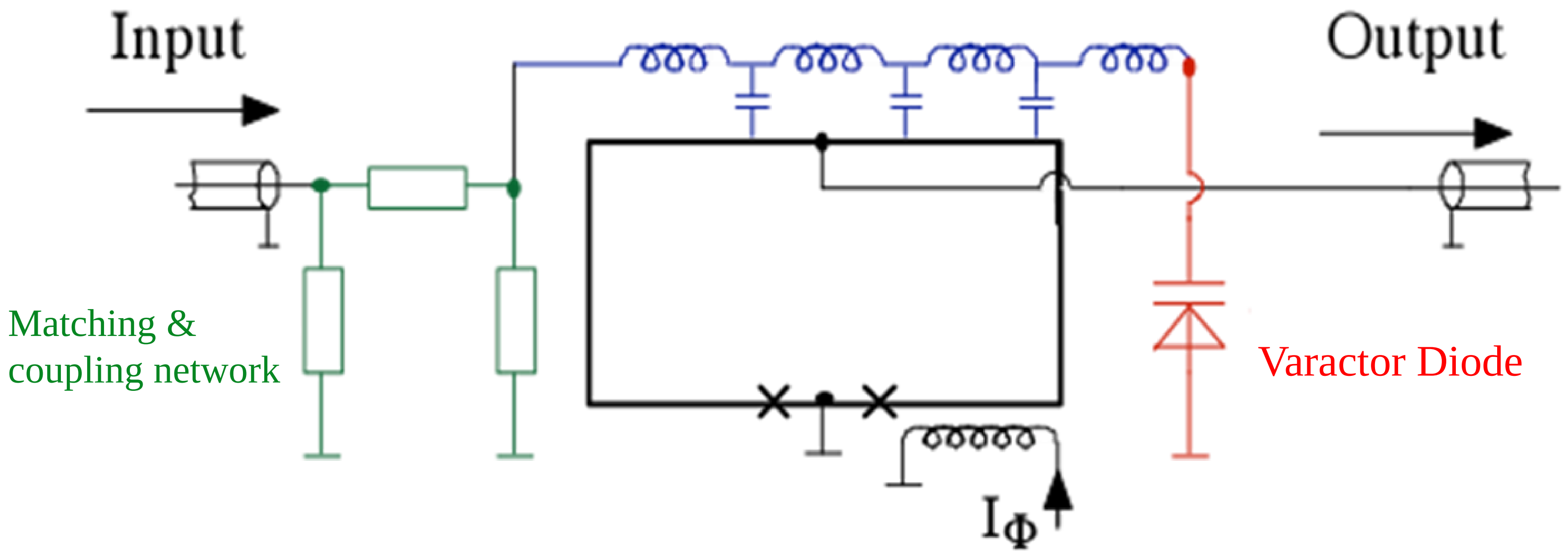
Bonding pads

1 mm

Resistor cooling fins

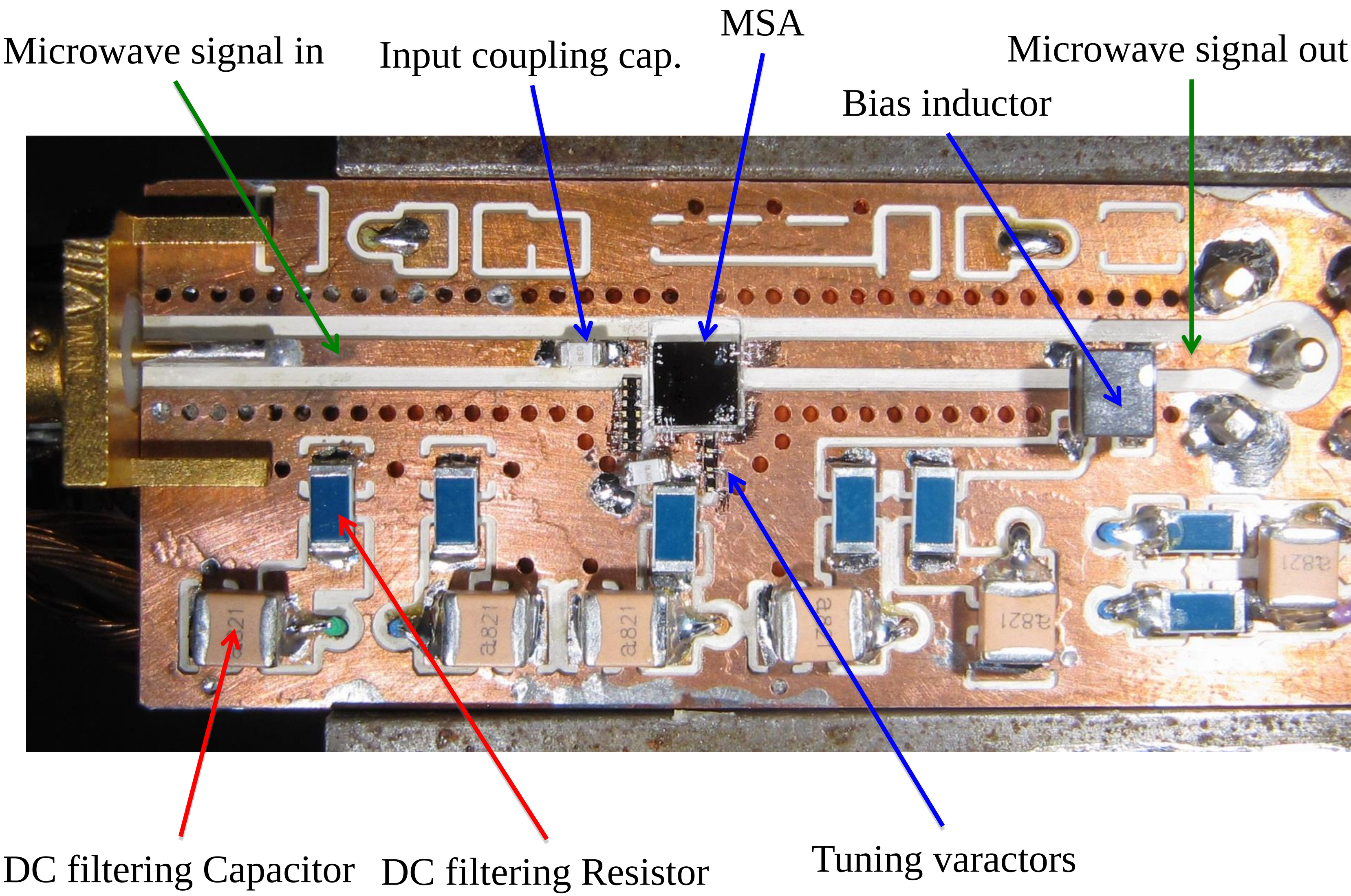
Blue: Metal covered with SiO_2
Purple: Si substrate covered with SiO_2
Silver: Bare metal

MSA RF Schematic

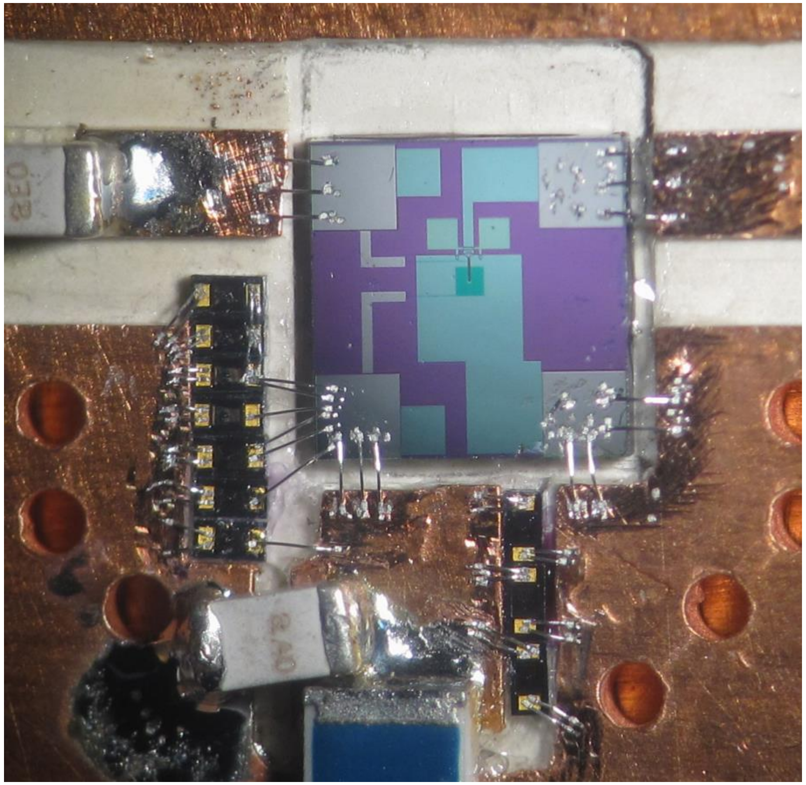


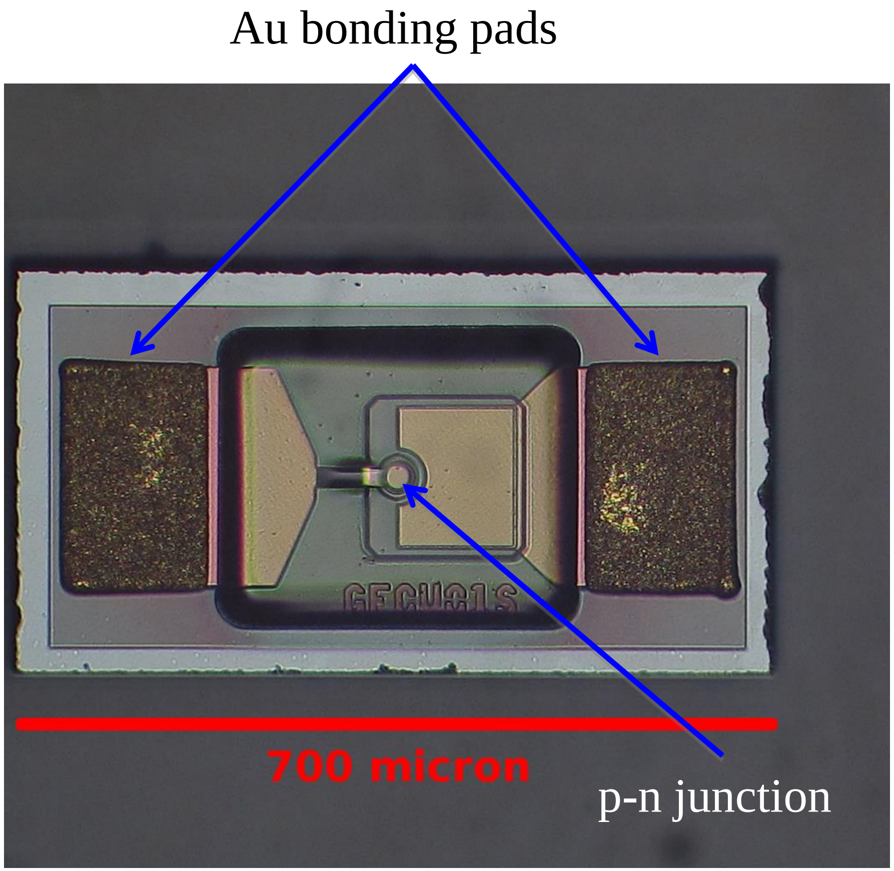
- Varying the capacitance modifies the phase change on reflection, effectively changing the length of the microstrip
- As the phase changes from a node to anti-node, the standing wave changes from $\lambda/2$ to $\lambda/4$, and the resonant frequency varies by a factor of 2
- Varactors must be GaAs (Si freezes out), high Q, very low inductance

MSA in a Working Circuit

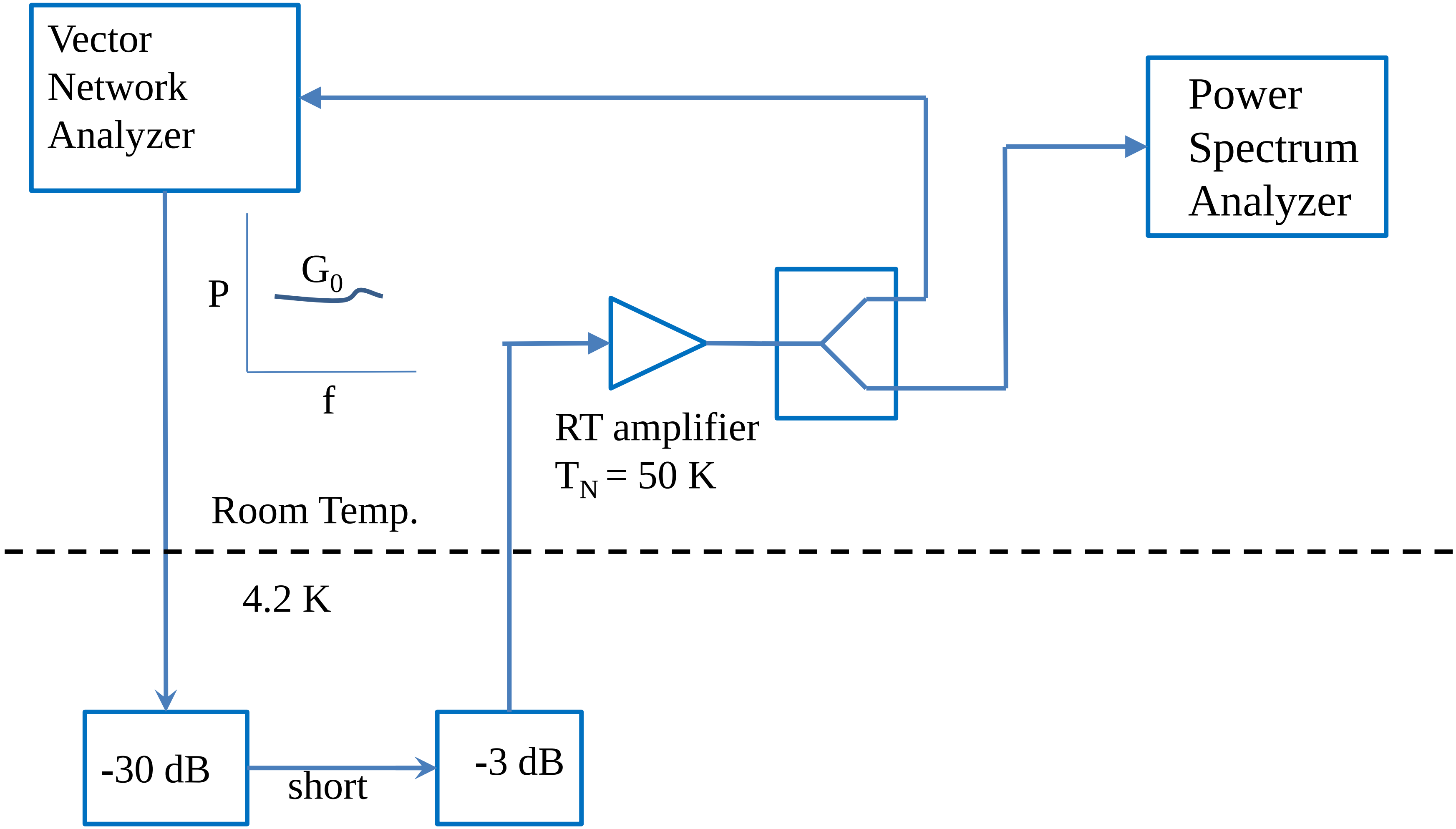


MSA in a Working Circuit

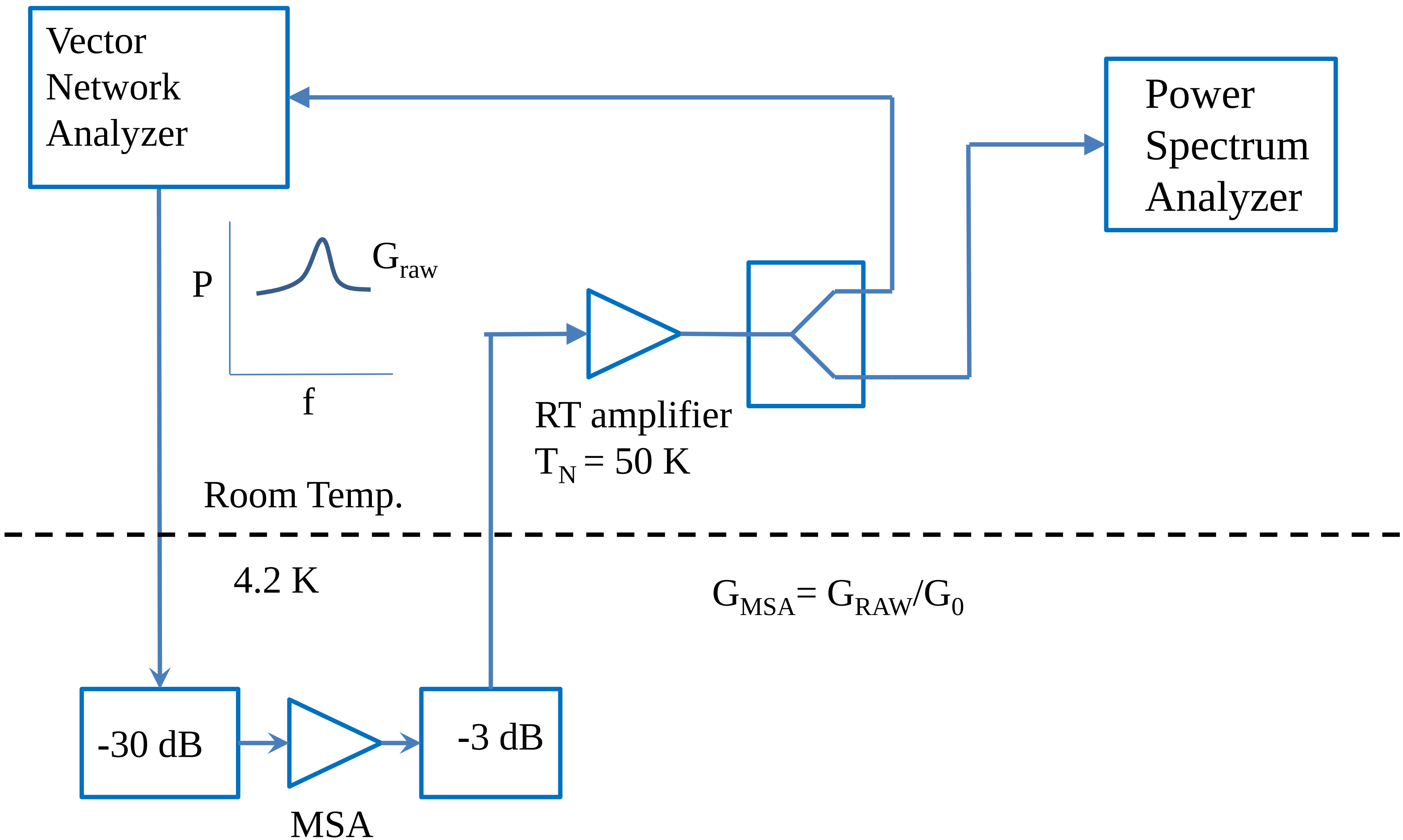




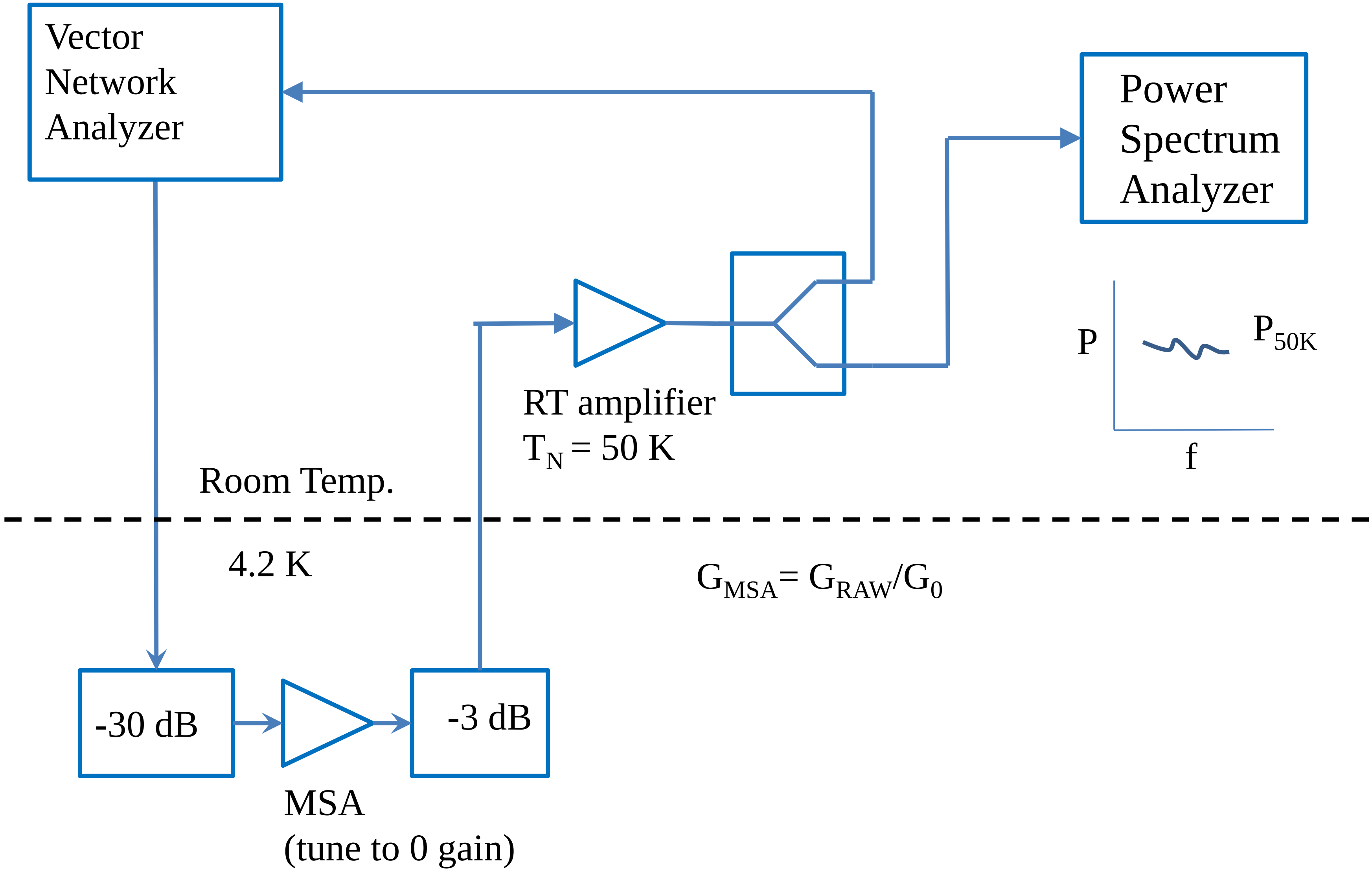
Measuring MSA Gain and T_N



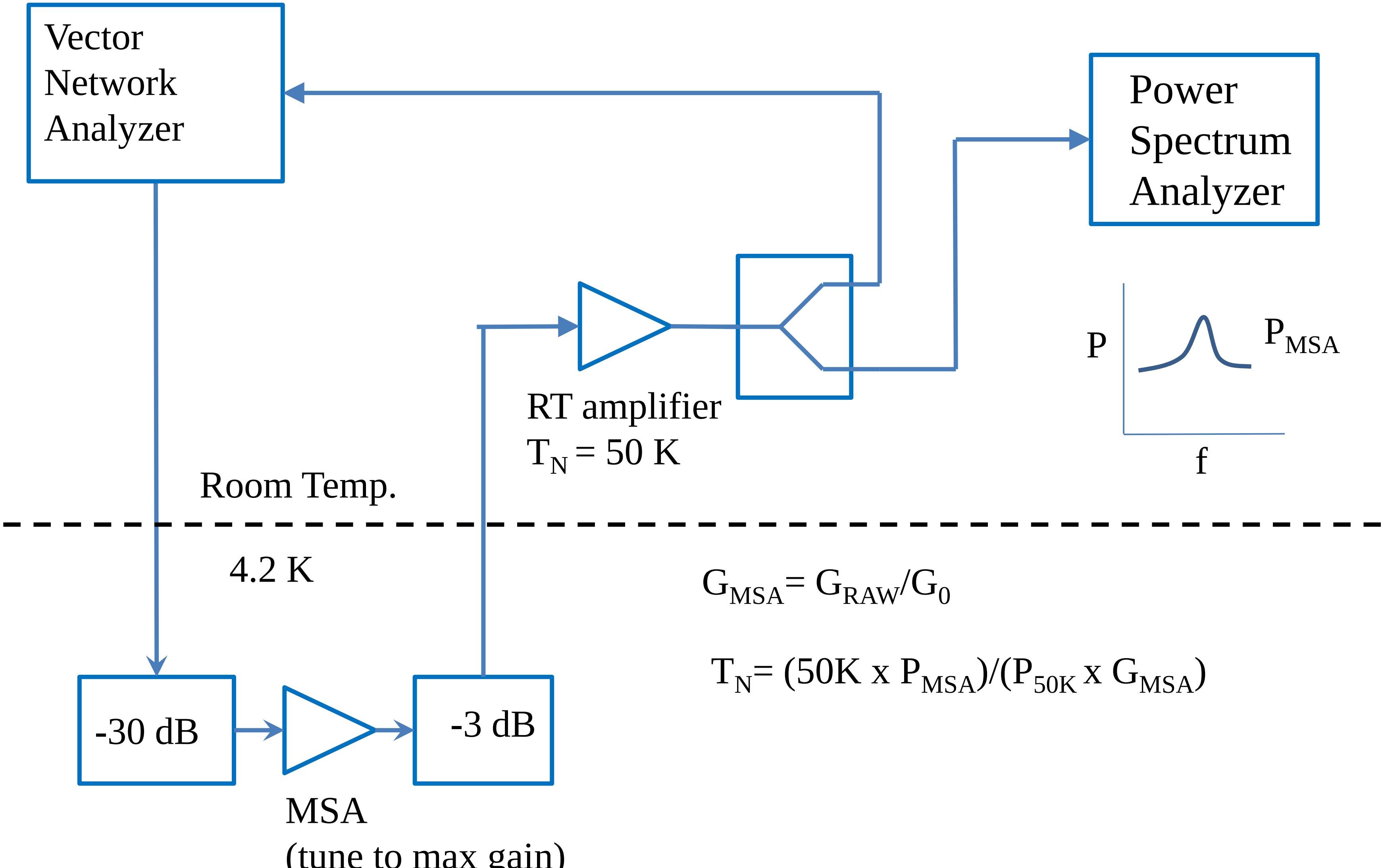
Measuring MSA Gain and T_N



Measuring MSA Gain and T_N



Measuring MSA Gain and T_N

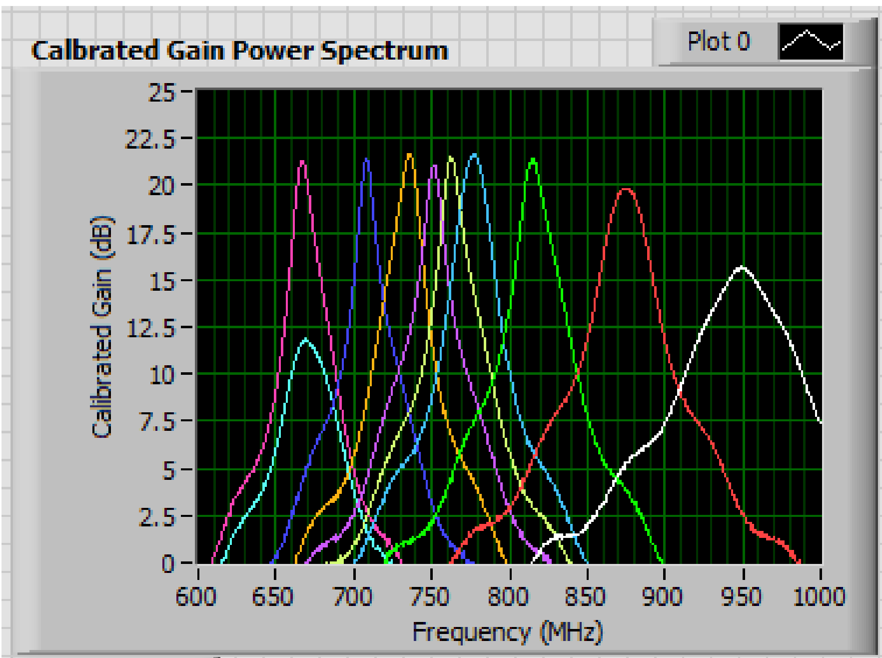


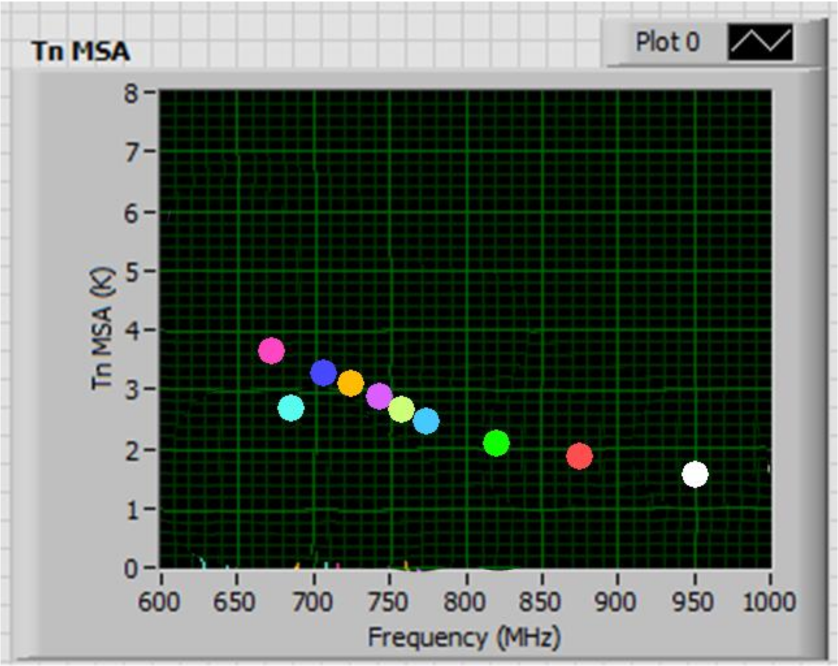
$$G_{MSA} = G_{RAW}/G_0$$

$$T_N = (50K \times P_{MSA})/(P_{50K} \times G_{MSA})$$

MSA Gain, Tunability, and Tn

Yes, it works!





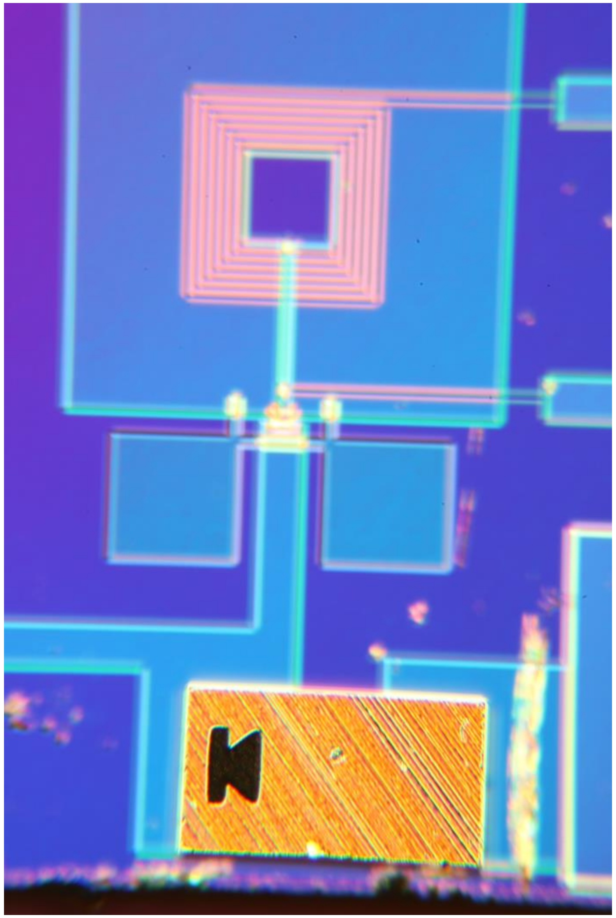
Gain ≈ 20dB
Tn < T (4.2K)

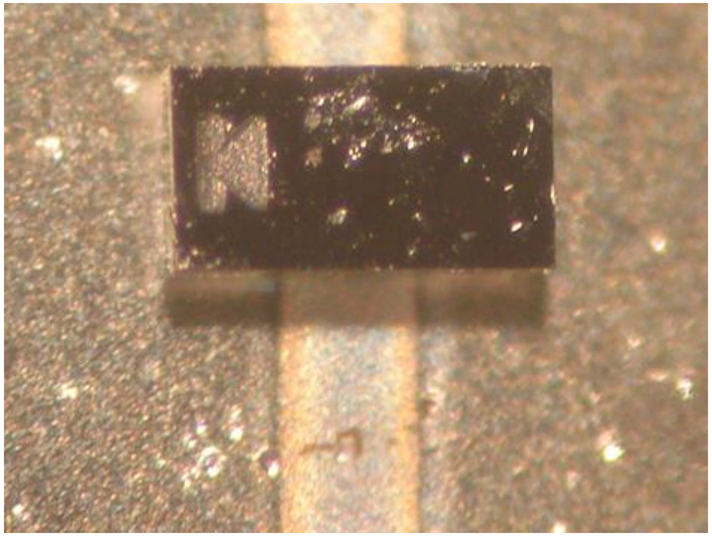
Outline

- 5 Minute Overview
- Dark Matter: The Majority Universe
- The Axion Dark Matter Candidate
- SQUIDs as microwave amplifiers
- MSA design and optimization
- Planned work

Low Inductance Varactor Mounting

Eliminate long bonds with direct varactor mounting





- Evaporate 2 μm of In on varactor pads and chip
- Press In films together to form cold weld
- Bonds are stable to thermal cycling (300 K to 4 K)
- Varactor characteristics are unchanged at 4.2 K
- Very low inductance achieved

Next- Generation MSA design

- Reduced junction I_0 and C, greater flux sensitivity
- Increased shunt resistance afforded by I_0 and C reduction and existing overhead in current conservative design for greater $dV/d\Phi$, greater gain

$$\beta_c = \frac{2\pi}{\Phi_0} I_0 R^2 C = 0.24$$

- Narrower input coil linewidth for reduced $C_{l,}$ allowing more turns, greater coupling, greater gain for the same frequency

$$\nu = \frac{1}{\sqrt{L_l \cdot C_l}}$$

- More turns on the input coil for greater gain, lower SQUID inductance for higher frequencies needed by ADMX

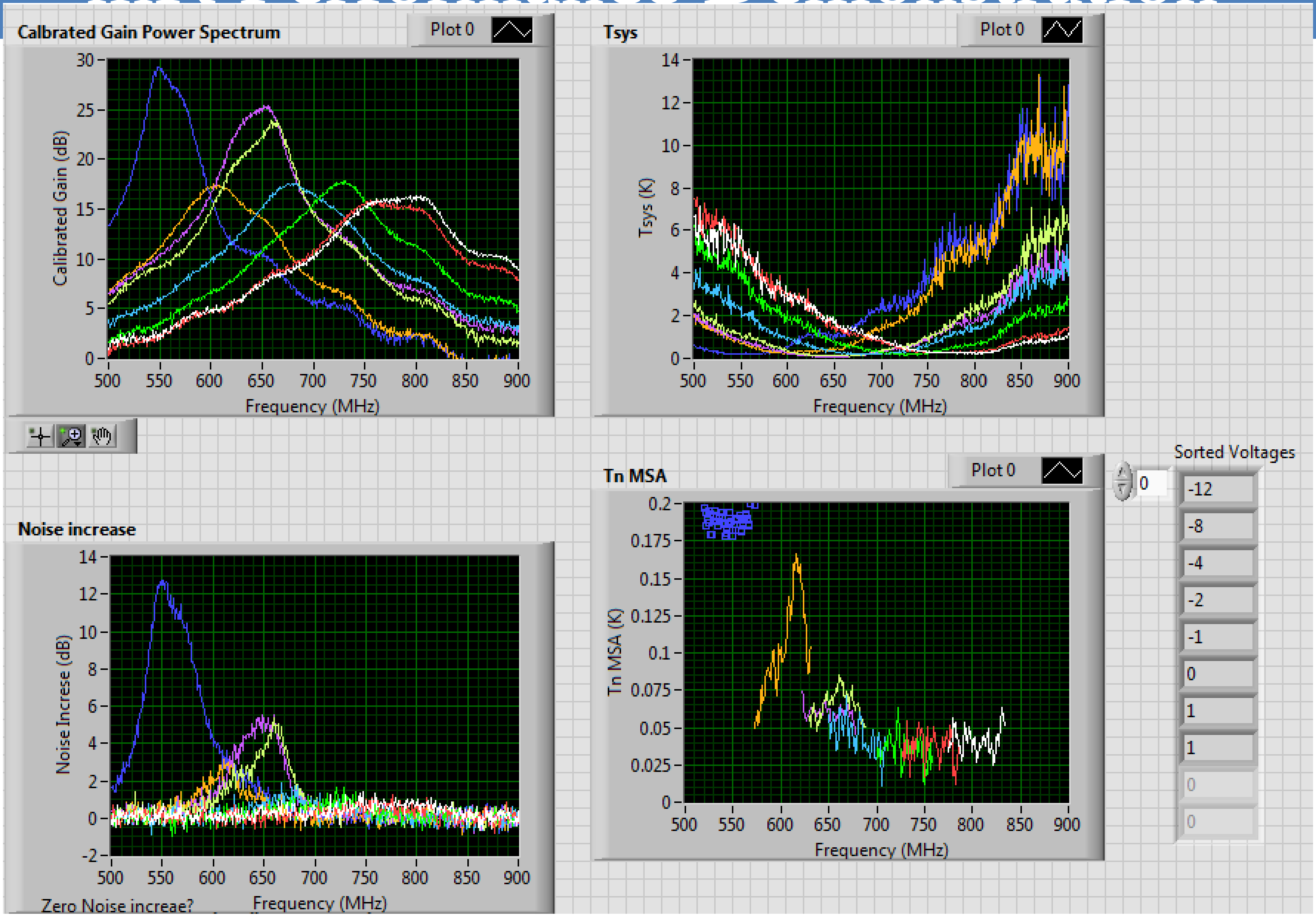
$$Z_0 = \sqrt{\frac{L_l}{C_l}}$$

- Increased $Z_{0,}$ for greater tunability for a given capacitance (fewer varactors)

mK Performance Demonstration

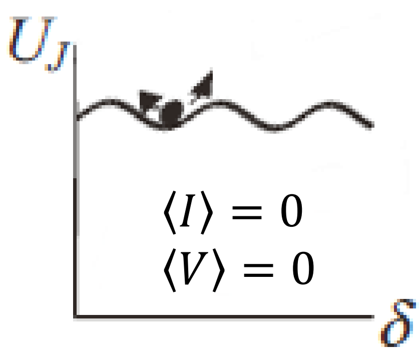
- 4K testing allows for fast turnaround and design iteration, and ADMX has been running at pumped He_4 temperatures
- ADMX is currently upgrading for mK temperatures.
- Only a few mK tests of the MSA's have been done so far.
- While those results were encouraging, comprehensive proof of performance is still needed.

mK Performance Demonstration



How high in frequency is "DC"?

The Josephson junctions have their own inductance and capacitance, which defines the junction plasma frequency ω_p.
The DC SQUID model is valid only for flux signals well below ω_p.



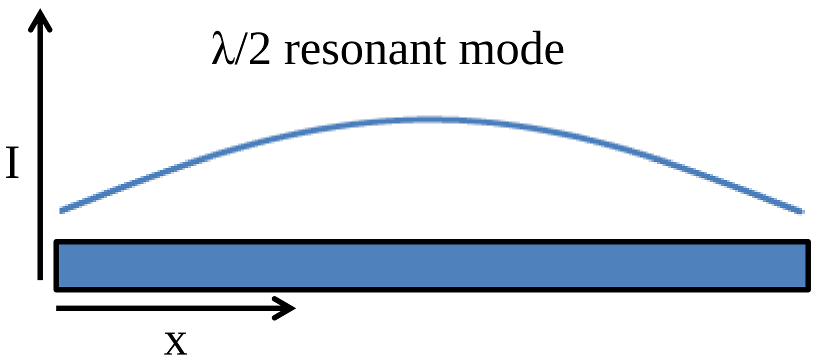
Plasma frequency $\omega_p = \sqrt{\frac{1}{L_j C_j}} = \sqrt{\frac{2\pi I_0}{\Phi_0 C}}$

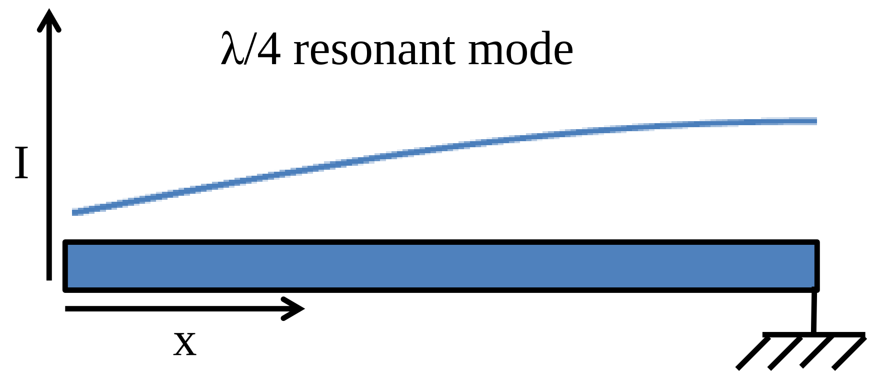
For typical values I_0 = 2.5 uA and C=300 fF
$f_p \approx$ 1THz

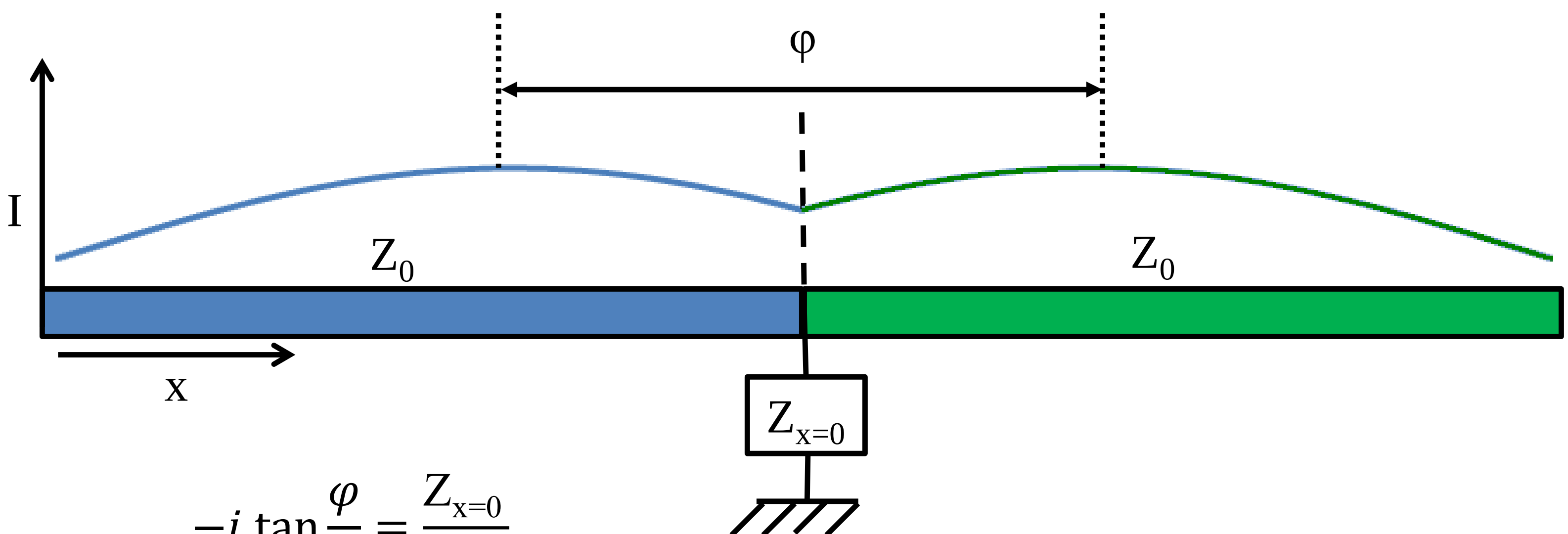
The "DC" SQUID is not limited by the junction plasma frequency.

But what about when operating in the Voltage state?

Coupling to the Microstrip

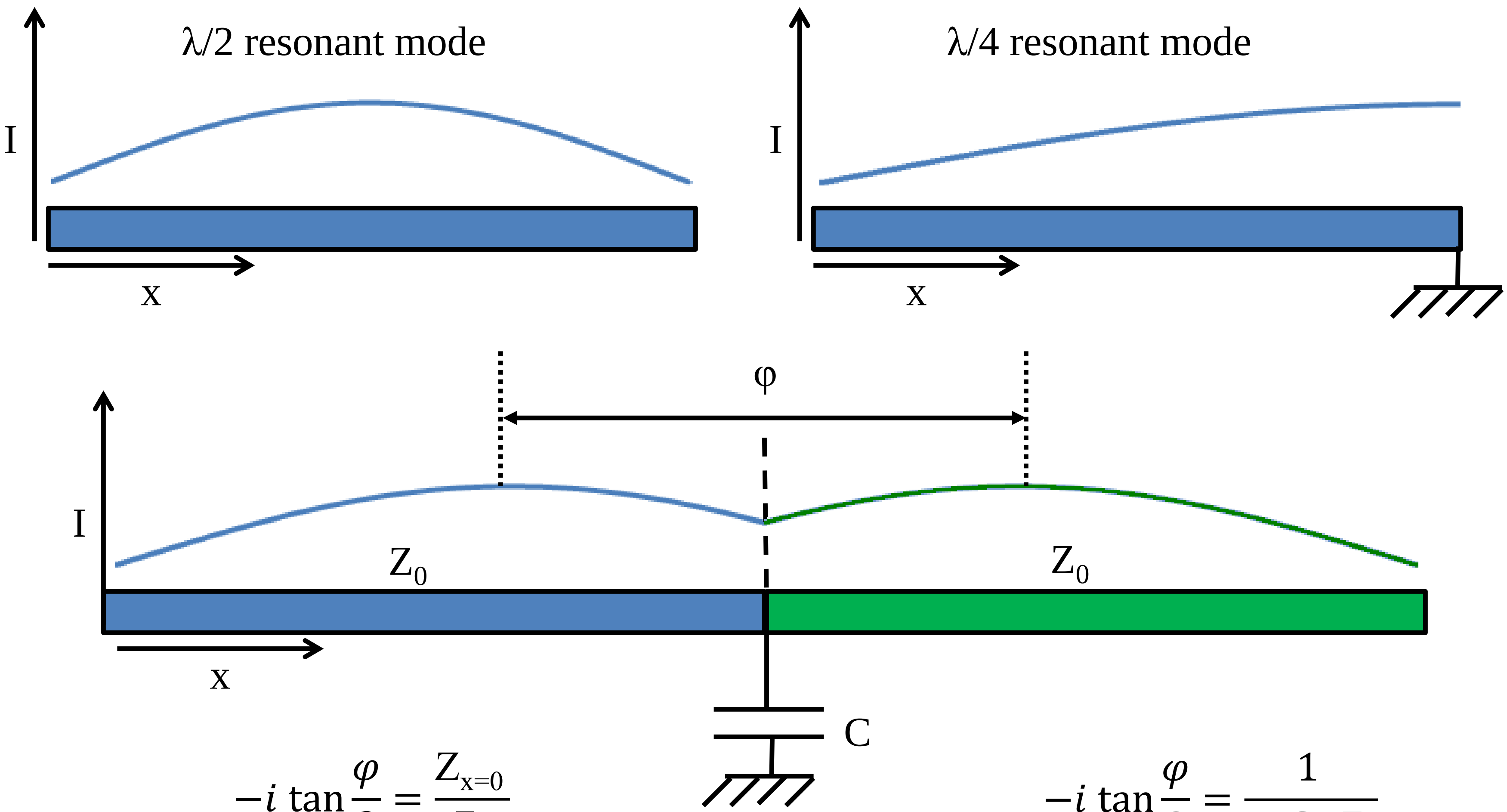






$$-i\tan\frac{\varphi}{2}=\frac{Z_{x=0}}{Z_0}$$

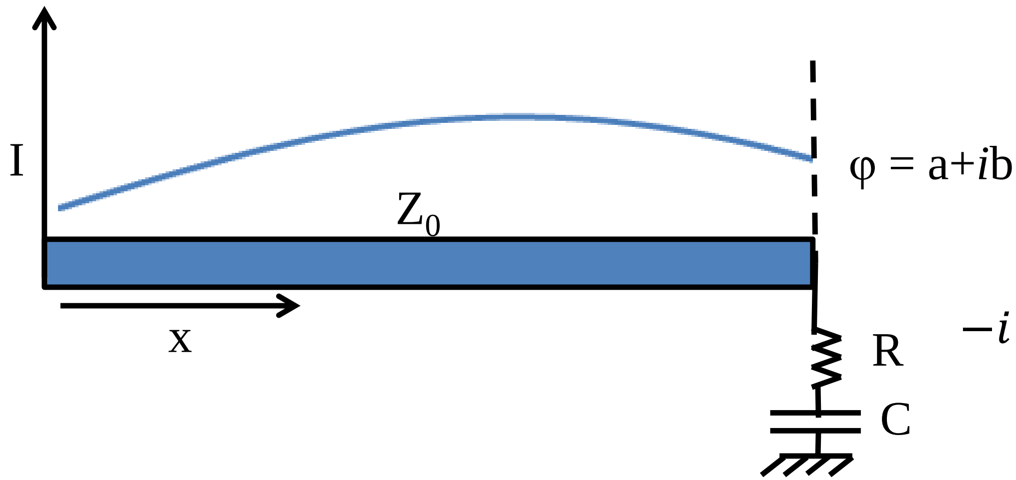
Coupling to the Microstrip



$$-i\tan\frac{\varphi}{2}=\frac{Z_{x=0}}{Z_0}$$

$$-i\tan\frac{\varphi}{2}=\frac{1}{i\omega C\cdot Z_0}$$

Coupling to the Microstrip: φ



$$-i\tan\frac{\varphi}{2} = \frac{Z_{x=0}}{Z_0}$$

$$\tan\frac{a+ib}{2} = \frac{\sin a}{\cos a + \cosh b} + i\frac{\sinh b}{\cos a + \cosh b}$$

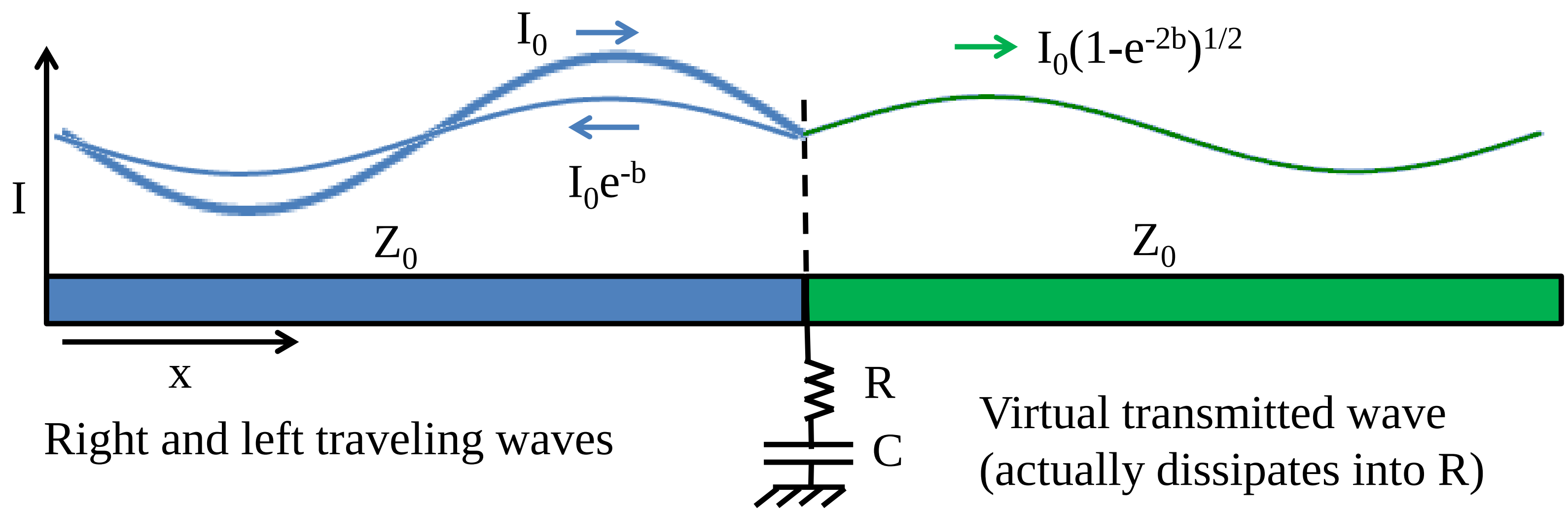
$$\frac{Z_{x=0}}{Z_0} = \frac{1}{i\omega C Z_0} + \frac{R}{Z_0}$$

$$\frac{1}{\omega C Z_0} = \frac{\sin a}{\cos a + \cosh b} \qquad \frac{R}{Z_0} = \frac{\sinh b}{\cos a + \cosh b}$$

Solve for a and b:
a gives the reflected phase, and thus the resonant frequency
b gives the loss rate, and thus the Q

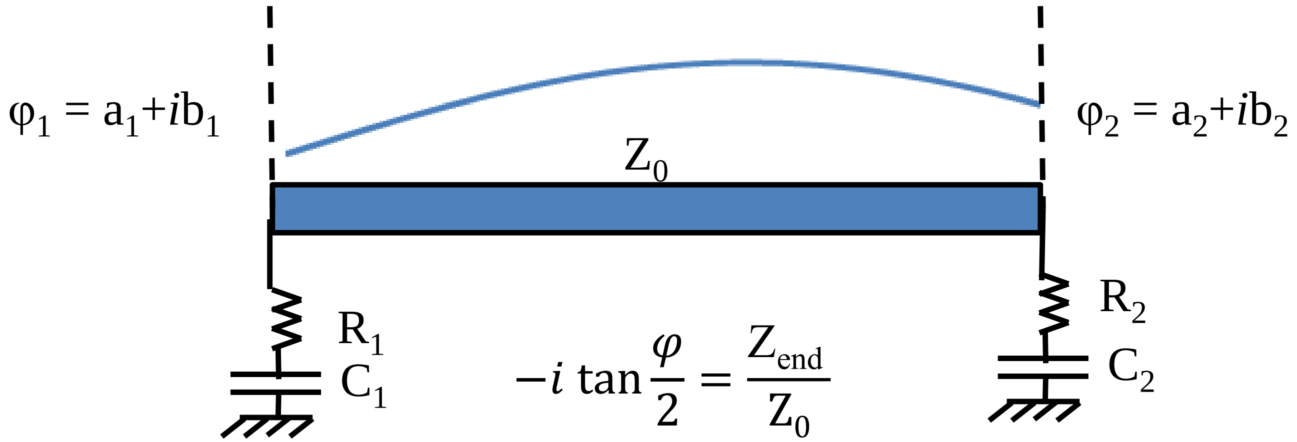
Coupling to the Microstrip: Q

$$Q = 2\pi \frac{total\ energy\ stored}{energy\ lost\ per\ cycle}$$



$$Q = 2\pi \frac{{I_0}^2\left(1 + e^{-2b}\right)}{{I_0}^2\left(1 - e^{-2b}\right)} = 2\pi \coth b$$

Accounting for Both Ends



$$-i\tan\frac{\varphi}{2} = \frac{Z_{end}}{Z_0}$$

$$\frac{f}{f_0} = \frac{a_1 + a_2}{2\pi}$$

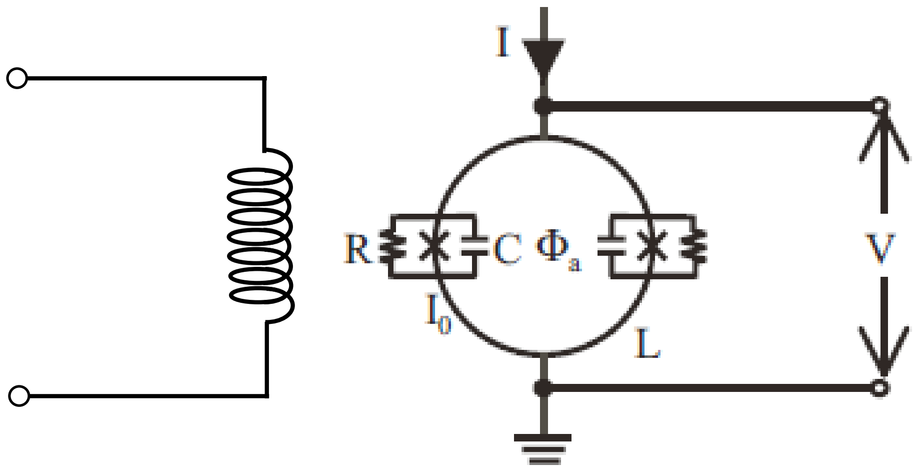
$$Q_{coupling} = \frac{2\pi}{\tanh b_1 + \tanh b_2}$$

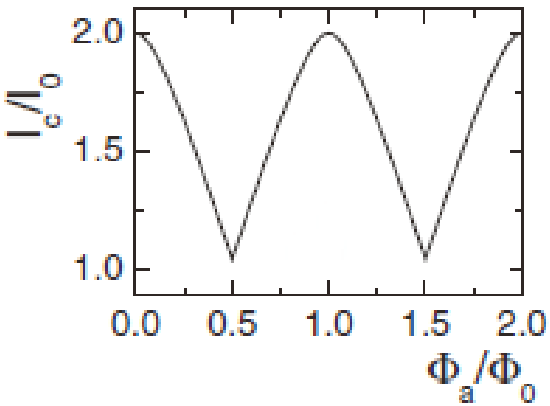
	Input	End
R	50Ω	<< 50Ω
C	fixed ~1pF (160Ω @ 1GHz)	1.3 to 0.1 pF per varactor

- f/f_0 can be < ½ with a large input capacitor
- Optimal power coupling when $Q_{coupling} = Q_{int}$

The DC SQUID

Two Josephson junctions on a superconducting ring





Critical Current I_c is modulated by magnetic flux

A flux through the SQUID loop (Φ_a) induces a circulating current to satisfy the flux quanitzation condition, adding to the current through one junction, subtracting from the other, and inducing a difference in the phases across the junctions.

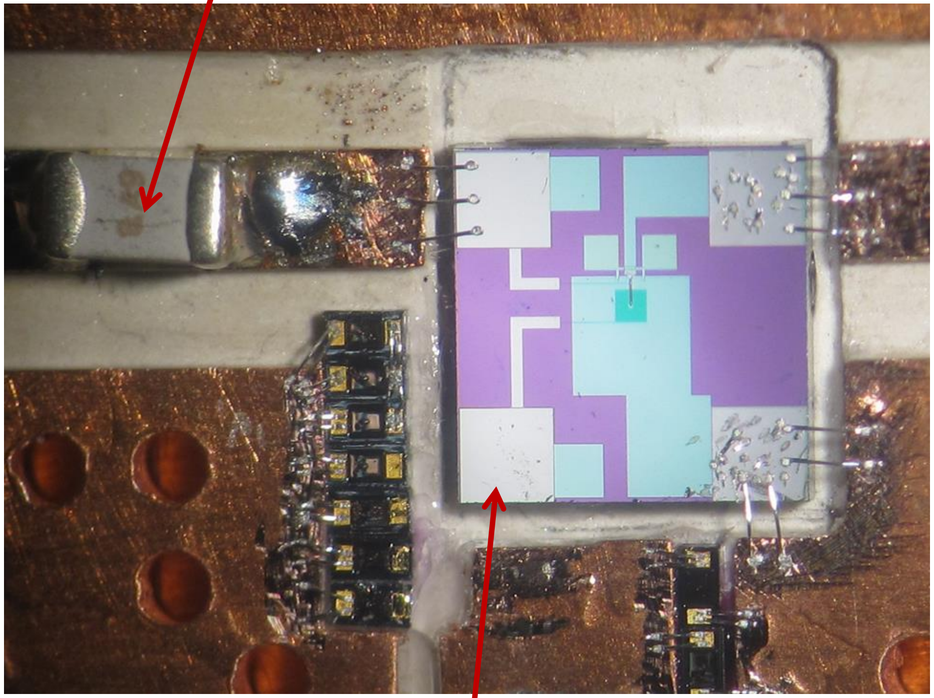
Interference of the superconducting wave functions in the two SQUID arms sets the maximum current Ic that can flow at V = 0

With some simplifying assumptions (like symmetric junctions) the DC SQUID can be treated as a single, flux-modulated Josephson junction

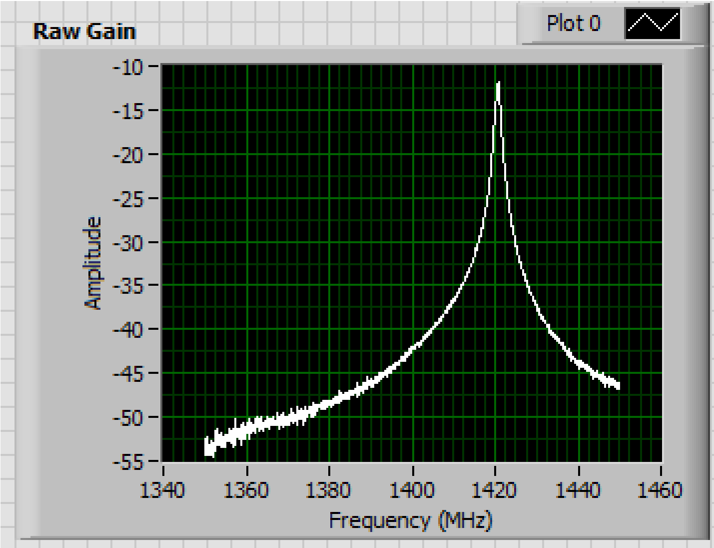
Optimization Walkthrough

Step 1: couple weakly to the input , leave end of coil open to measure f_0 and Q

0.1pF input cap



Coil end open

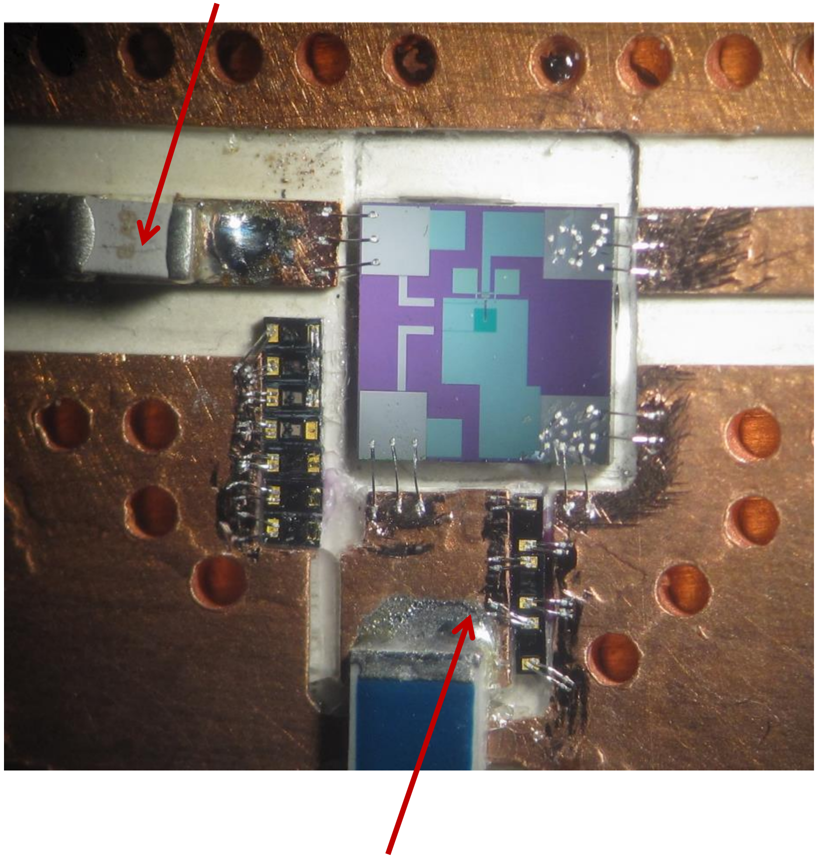


f_0 = 1420 MHz
Q = 570

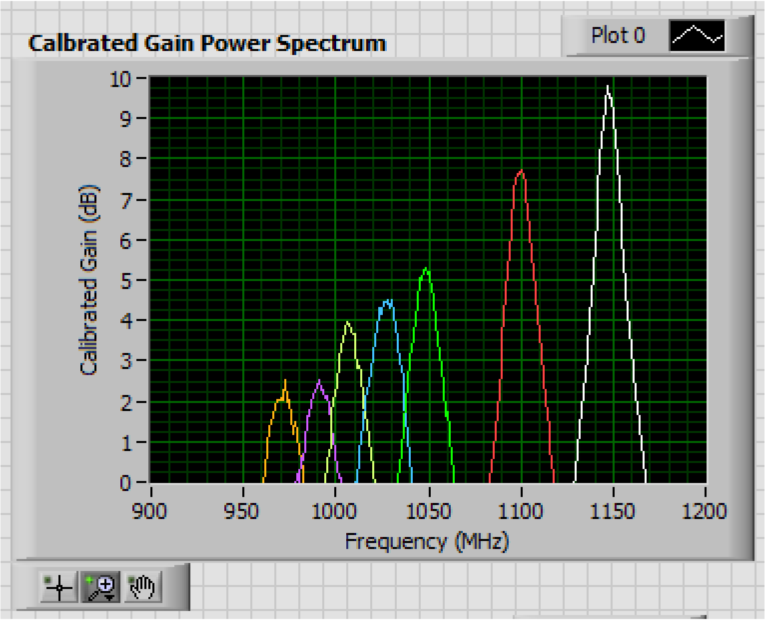
Optimization Walkthrough

Step 2: attach varactors, note frequency shift to estimate Z_0 and new Q_2

0.1pF input cap



Coil end connected to 3 varactors



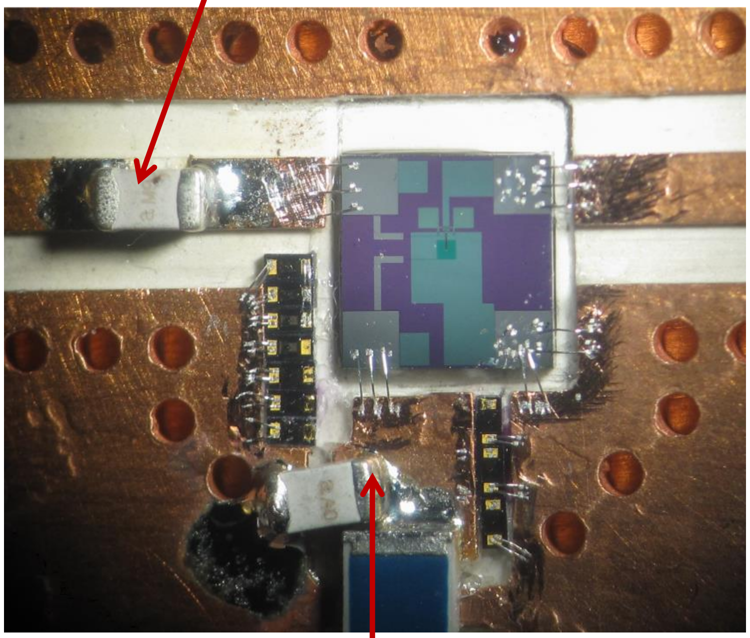
$Z_0 \approx 95\ \Omega$

Q = 115 (much lower!)

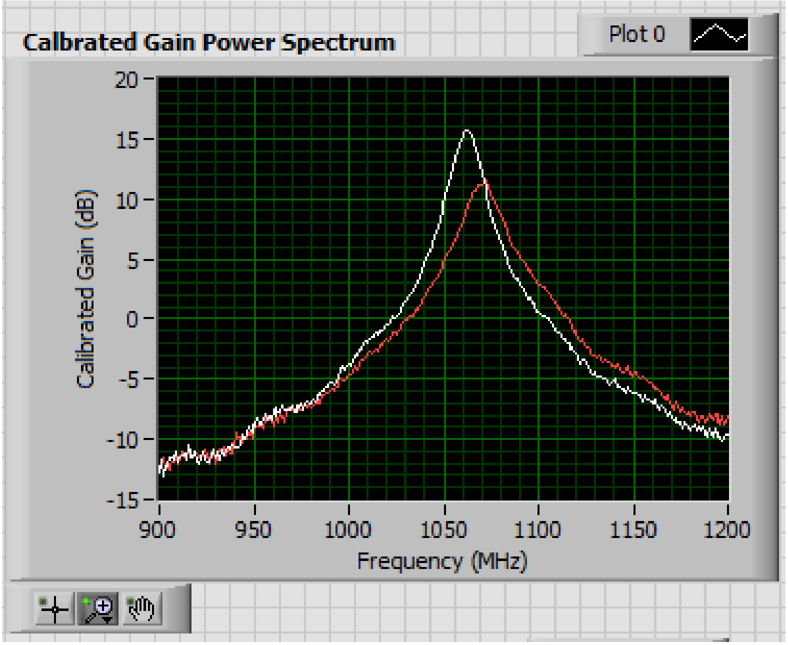
Optimization Walkthrough

Step 3: Choose input coupling capacitor for optimal coupling

0.3 pF input cap



Coil end connected to fixed cap.

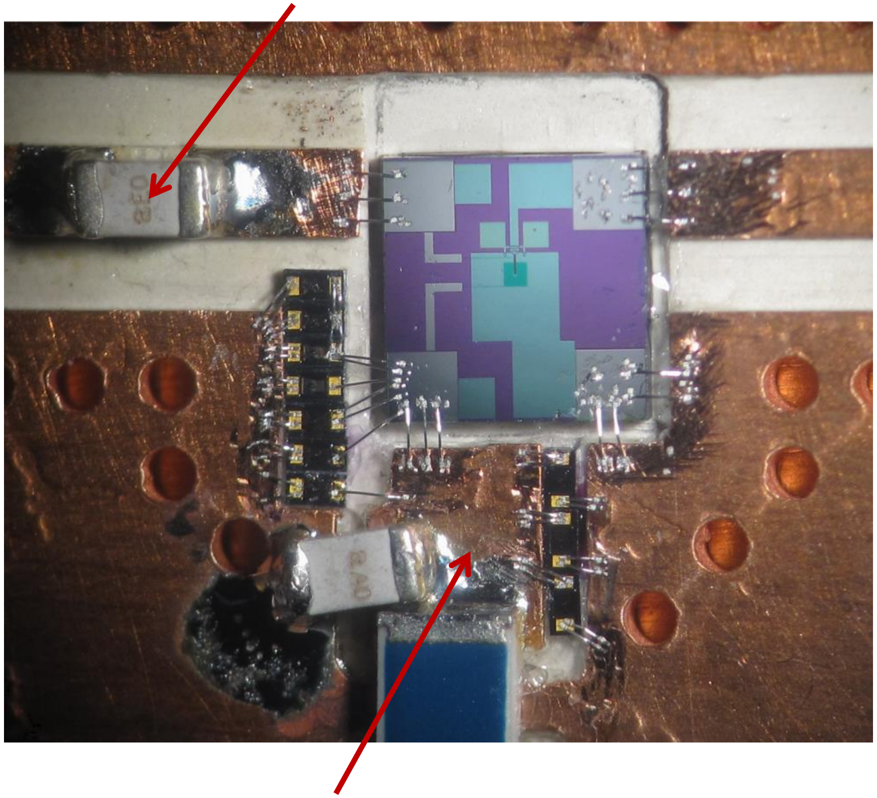


Q = 60
Gain about 6dB greater

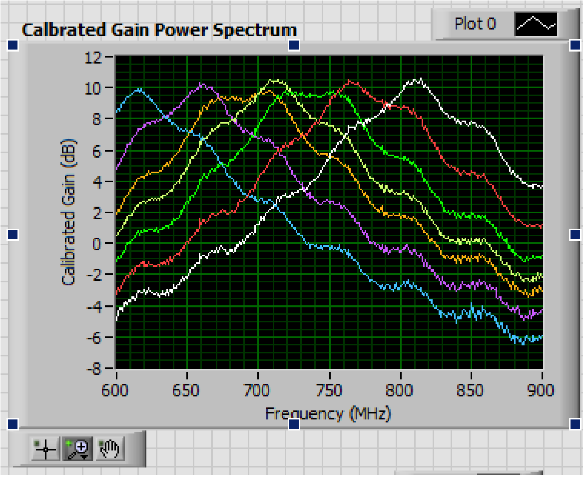
Optimization Walkthrough

Step 4: Add varactors and alter input cap to achieve desired frequency range

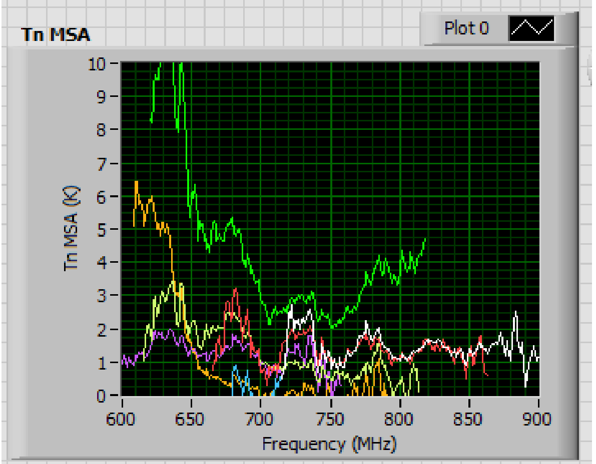
1.5 pF input cap



Coil end connected to fixed 1pF cap.
and 10 varactors



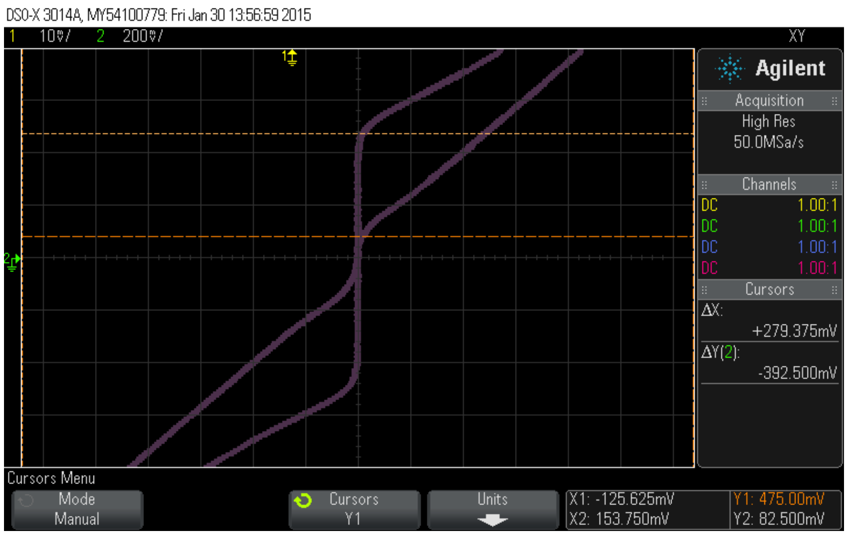
Q ≈ 9, gain reduced to 10dB

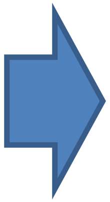


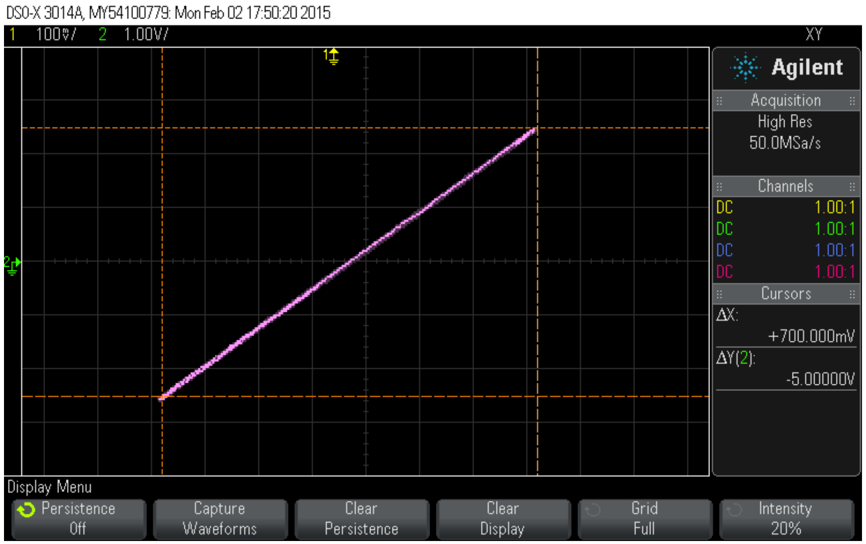
Tn ≈ T/2

Optimization Walkthrough

Step 5: Blow out the MSA and contemplate how to do this better







Thank goodness we have replacements!