Testing Flavour Symmetries with Oscillation Experiments

Peter Ballett
IPPP, Durham

Exotic neutrinos workshop, Lancaster University
5 December 2016

Neutrinos as a window on flavour

How the neutrino sector both worsens the flavour problem and offers hope for its resolution

The flavour problem (… who ordered that?)

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + i\bar{\Psi} \not{D} \Psi + h.c. + \bar{\Psi}_i y_{ij} \Psi_j \phi + h.c. + |D_\mu \phi|^2 - V(\phi)$$

There are three central aspects to the flavour problem:

- Why 3 families of fermions?
- What dictates the pattern of masses?
- Why mixing? Or, why *this* mixing?

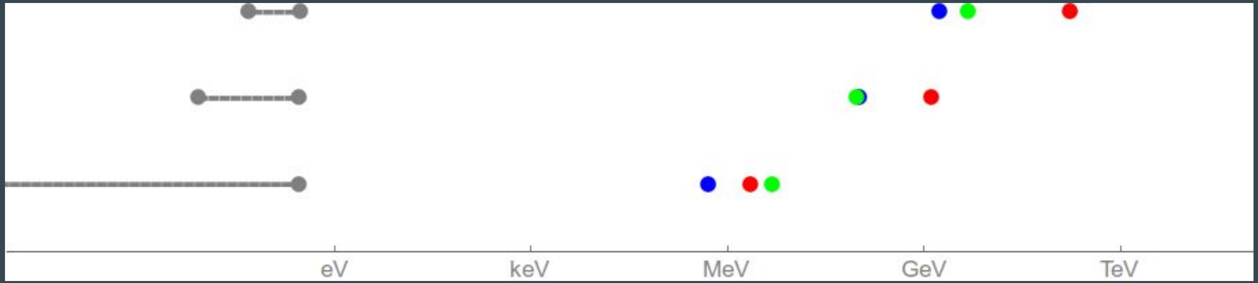
It is the origin of mass which leads to the observable differences between families.

Flavour is intrinsically linked to mass generation.

How bad is it?

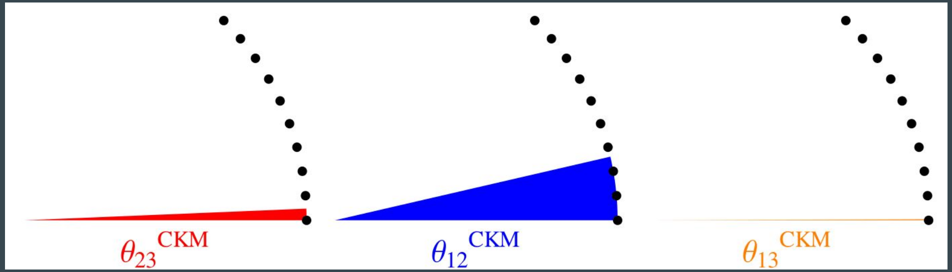
- Neutrinos make the intra-generational hierarchy much worse

$$\frac{m_\nu}{m_e} \lesssim 10^{-6}$$

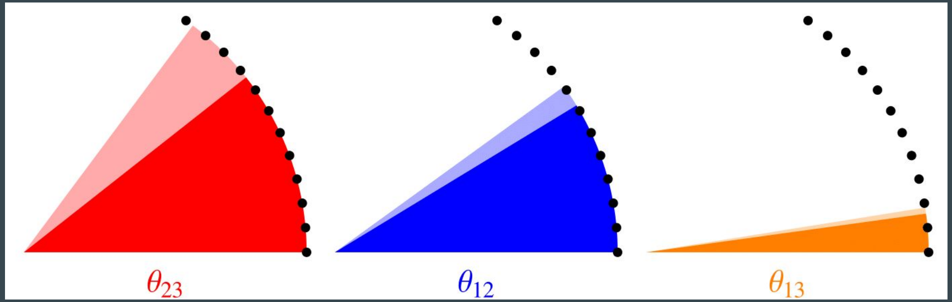


- The CKM matrix is close to the identity matrix

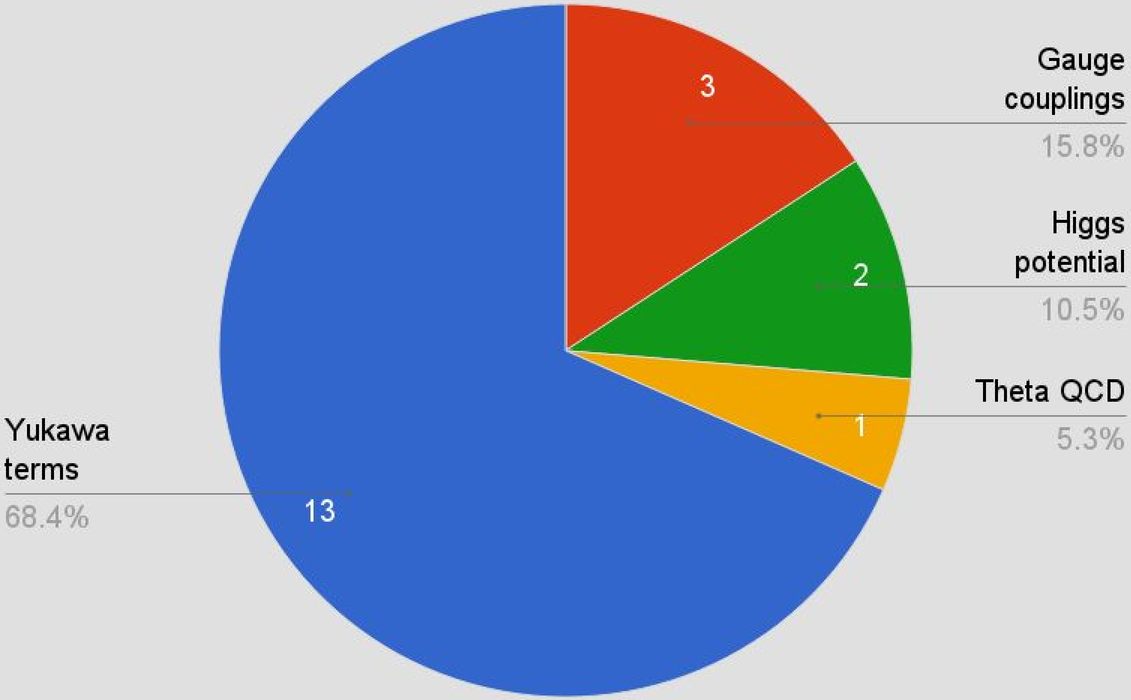
$$V_{\mathrm{CKM}} = I + \mathcal{O}(\theta_{\mathrm{C}})$$



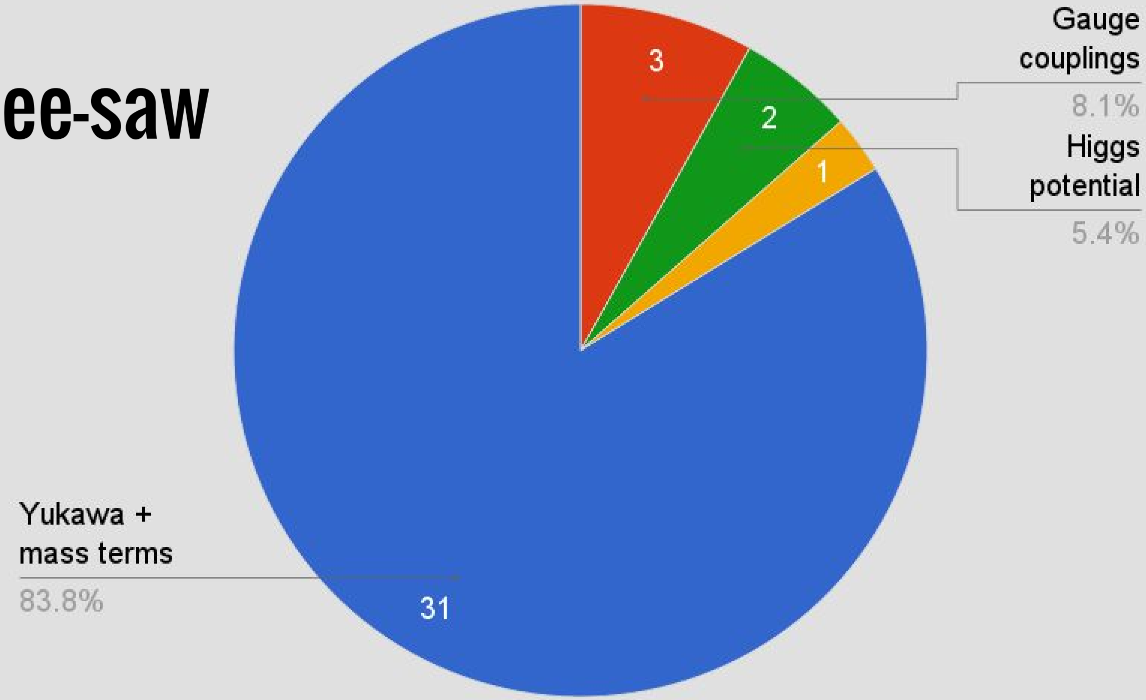
- The PMNS matrix is the opposite
  - Closer to maximal mixing, or democratic mixing, than the identity



Free parameters of the SM



Free parameters of the SM
+ Type I see-saw

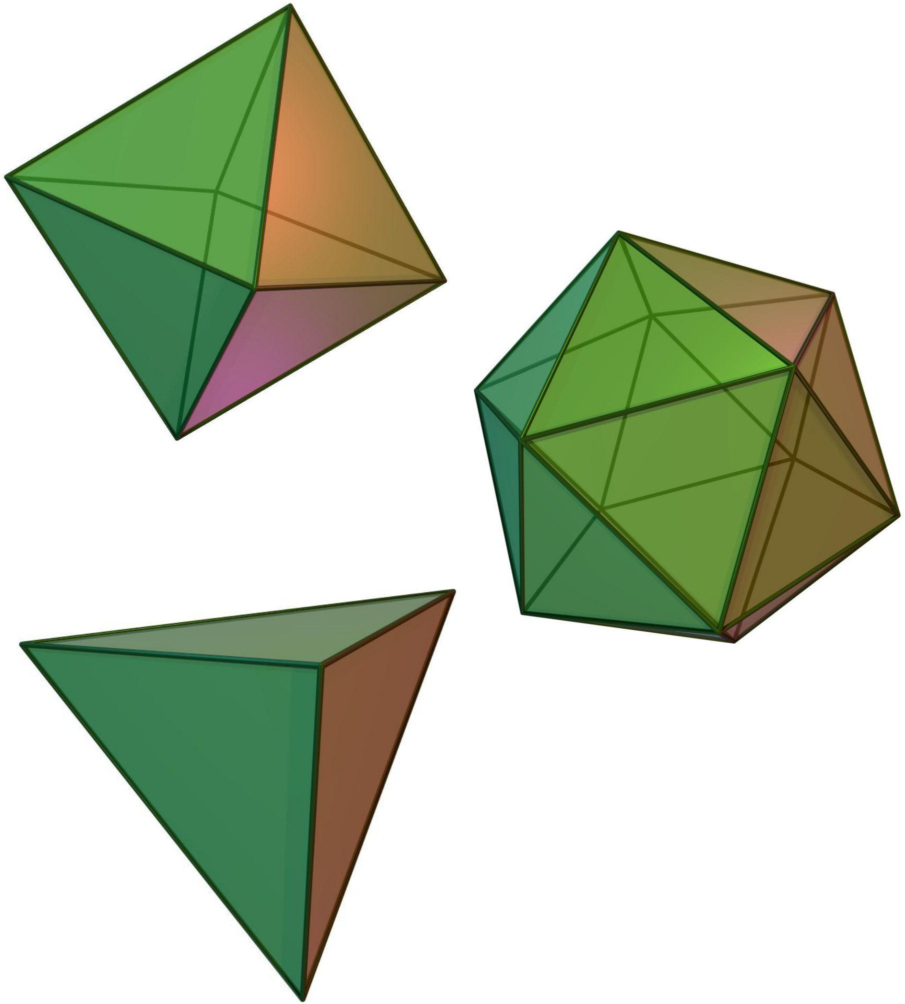


Only hope was left…

- Any neutrino mass mechanism will exacerbate the problem of flavour: more arbitrary parameters, more complicated flavour patterns, more scales
- Its exploration (theoretical and experimental) offers new opportunities to investigate and address the flavour problem
- However, for this talk, we assume that no novel low-scale dynamics will be discovered. Clearly, it would be a game changer were this to occur.
- The primary means of studying flavour will therefore be via the PMNS matrix and neutrino oscillation.

Leptonic flavour models

How we introduce structure to the flavour parameters of the SM and predict the PMNS



How to constrain Yukawas

- Continuous symmetries
  - Subgroups of $U(3)^2$ and SSB
    - Leptonic Minimal Flavour Violation [Cirigliano et al. 0507001; Davidson 0607329; Gavela et al. 0906.1461;
    - Naturalness/Extremal configurations [Alonso et al. 1306.5927]
- Discrete symmetries [For a review see e.g. King & Luhn 1301.1340]
  - Simplest means of forbidding terms in lagrangian
  - Motivated by large mixing angles of PMNS
    - Direct, semi-direct, indirect models
    - gCP and phase predictions [Feruglio et al. 1211.5560; Holthausen et al. 1211.6953; Chen et al. 1402.0507]
    - Predictions with corrections
- Bottom up approaches
  - Texture zeros [Weinberg, Wilczek & Zee, Fritzsch 1977; see also Frampton et al. 0201008]
  - "Symmetry model building" [Hernandez & Smirnov 1204.0445, 1212.2149, 1304.7738]

Residual discrete symmetries

- Mechanism behind many previous (semi-)complete models [Review: King & Luhn 1301.1340; see also de Adelhart Toorop 1112.1340]
  - Can be treated bottom-up in a (rather) model independent way [Hernandez & Smirnov 1204.0445, 1212.2149]
  - Provides a connection between # of families and flavour by unifying leptons.
  - Generally does not predict PMNS matrix completely

- Leads to testable predictions for mixing angles and phases
  - Some are predicted absolutely (e.g. $\delta = 0$ or $\theta 23 = \pi/4$)
  - Others are constrained by (mixing) sum rules

- Does not address the values of masses themselves
  - Mass hierarchies can be dictated by another mechanism (e.g. see-saw)
  - Decouples mixing from absolute mass scales

$$\mathcal{G} = [\mathrm{SU}(3) \otimes \mathrm{SU}(2) \otimes U(1)] \otimes G_{\mathrm{F}}$$

High-scale UV complete theory

Flavour breaking

EW breaking

$$\mathcal{G} \supset G_{\mathrm{low}} = [\mathrm{SU}(3) \otimes U(1)] \otimes G_l \otimes G_\nu$$

Effective symmetry of low-energy lagrangian

The parameters of the low-scale lagrangian are constrained by the residual symmetry.

Charged leptons and neutrinos see a different residual symmetry, leading to non-trivial PMNS matrices.

$$T^\dagger (m_l m_l^\dagger) T = m_l m_l^\dagger$$

$$S^T m_\nu S = m_\nu$$

$$U_{\mathrm{PMNS}} = U_l^\dagger(T) U_\nu(S)$$

Patterns for PMNS

Bimaximal

$$\theta_{12}^{\nu} = 45^{\circ}$$

$$\begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

Tribimaximal

$$\theta_{12}^{\nu} = 35.3^{\circ}$$

$$\begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

Golden Ratio A and B

$$\theta_{12}^{\nu} = 31.7^{\circ}$$

$$\begin{pmatrix} \frac{\varphi}{\sqrt{2+\varphi}} & \frac{1}{\sqrt{2+\varphi}} & 0 \\ -\frac{1}{\sqrt{4+2\varphi}} & \frac{\varphi}{\sqrt{4+2\varphi}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{4+2\varphi}} & -\frac{\varphi}{\sqrt{4+2\varphi}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

$$\theta_{12}^{\nu} = 20.9^{\circ}$$

$$\begin{pmatrix} \frac{\varphi}{\sqrt{3}} & \frac{\varphi_g}{\sqrt{3}} & 0 \\ -\frac{\varphi_g}{\sqrt{6}} & \frac{\varphi}{\sqrt{6}} & \frac{1}{\sqrt{2}} \\ \frac{\varphi_g}{\sqrt{6}} & -\frac{\varphi}{\sqrt{6}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

- Fonseca & Grimus (1405.3678) have impressively exhausted this paradigm, deriving all possible PMNS matrices.
  - 17 sporadic forms of PMNS matrix
  - 1 infinite family of matrices

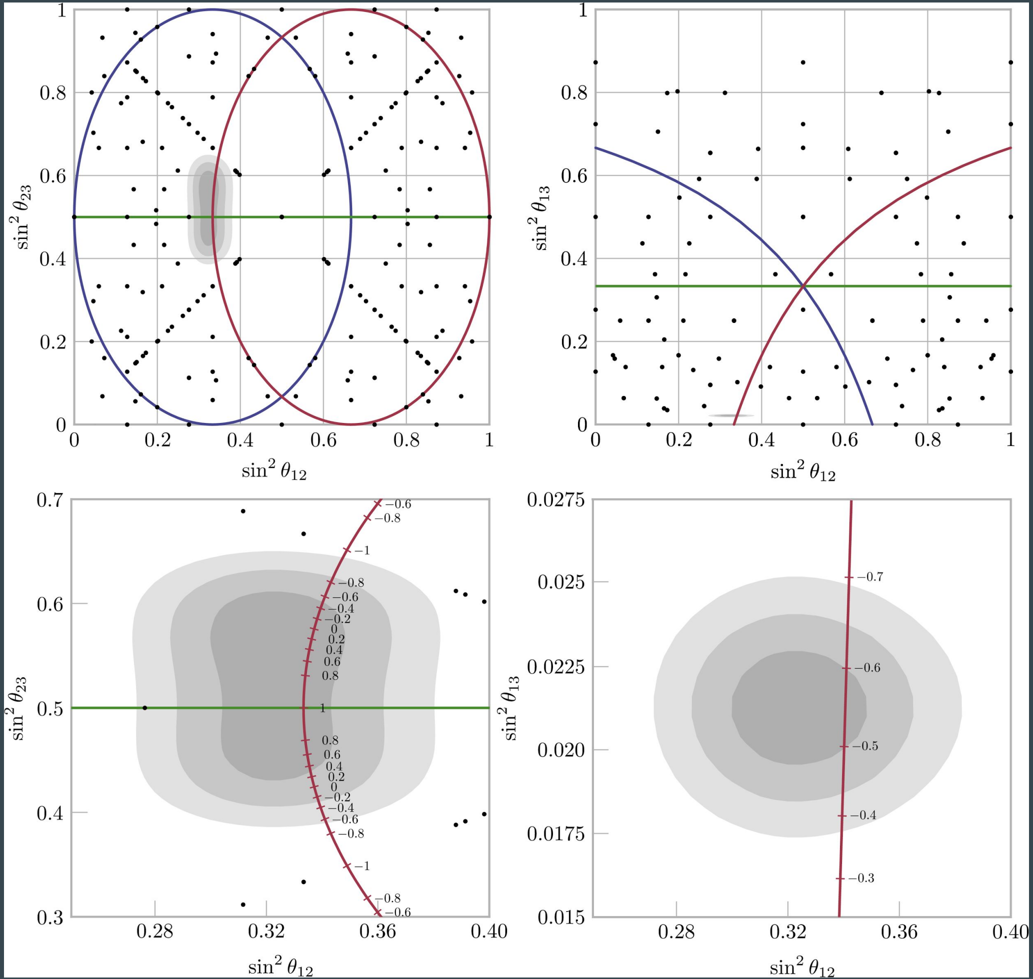
- Only the infinite family can fit the data (red curve).

$$|U|^2 = \frac{1}{3}\begin{pmatrix} 1 & 1+\mathrm{Re}\,\sigma & 1-\mathrm{Re}\,\sigma \\ 1 & 1+\mathrm{Re}\,(\omega\sigma) & 1-\mathrm{Re}\,(\omega\sigma) \\ 1 & 1+\mathrm{Re}\,(\omega^2\sigma) & 1-\mathrm{Re}\,(\omega^2\sigma) \end{pmatrix}$$

For the range:

$$-0.69 \lesssim \mathrm{Re}(\sigma^6) \lesssim -0.37$$

- Delta is always zero for this pattern!



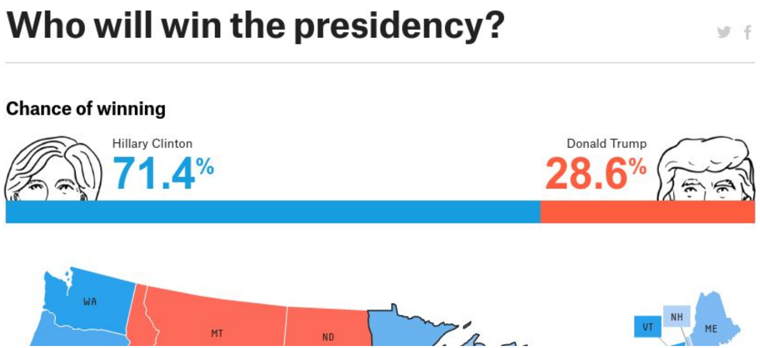
[Fonseca & Grimus 1405.3678]

Correlations from realistic models

- However, we expect these patterns to receive corrections:
  - Insufficient residual symmetry ("semi-direct models")
    - Atmospheric sum rules (ASR) [King & Luhn 1301.1340; PB et al. 1308.4314]
  - Charged-lepton corrections [Xing 0107005; Giunti & Tanimoto 0207096]
    - Solar sum rule (SSR)
    - Generalised SSRs [Petcov 1405.6006; PB et al. 1410.7573; Girardi et al. 1410.8056, 1504.00658, 1509.02502]
  - Radiative corrections
    - We expect RG effects to mix the sectors with different residual symmetries, producing deviations from the simple patterns.
    - Highly model dependent, but if we assume that no new dynamics occurs below the GUT scale, we see negligible effects [Antusch et al. 0305273, PB et al. 1410.7573, Zhang & Zhou 1604.03039, Gehrlein et al. 1608.08409]
  - VEV mis-alignment, higher-dimension operators ... many ideas!

Predictions for oscillation experiments

Precision targets for upcoming experiments



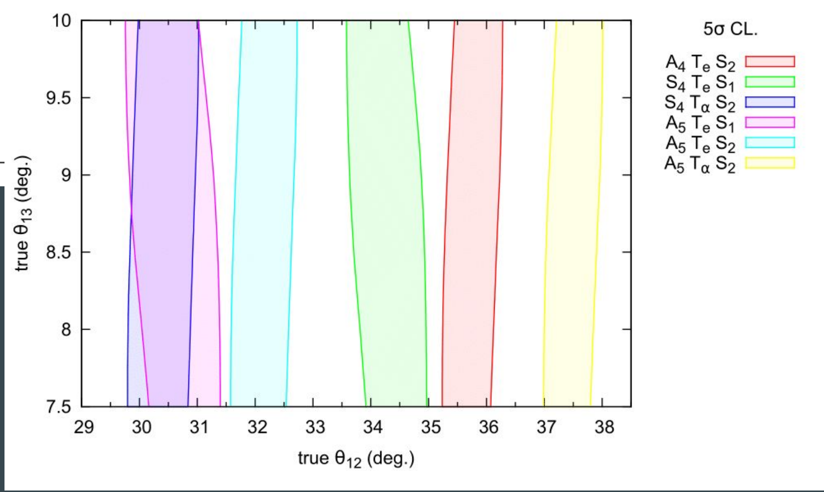
$\theta_{12} - \theta_{13}$ correlations

These arise in many models with residual flavour symmetries of the semi-direct type.

[Figures from PB et al. 1406.0308]

Model label	solar prediction	predicted θ_{12} (r in 3σ)
A_4 T_α–S_2	$s = \sqrt{\frac{2}{2-r^2}} - 1$	$[35.62,\ 35.86]$
S_4 T_e–S_1	$s = \sqrt{1 - \frac{2r^2}{2-r^2}} - 1$	$[34.05,\ 34.55]$
S_4 T_α–S_2	$s = \sqrt{\frac{3}{2(1-r^2)}} - 1$	$[30.29,\ 30.49]$
A_5 T_e–S_1	$s = \sqrt{3 + \frac{6}{(3-\varphi)(r^2-2)}} - 1$	$[30.33,\ 30.90]$
A_5 T_e–S_2	$s = \sqrt{\frac{6}{(2+\varphi)(2-r^2)}} - 1$	$[32.03,\ 32.24]$
A_5 T_α–S_2	$s = \sqrt{\frac{3\varphi}{(2\varphi-1)(2-r^2)}} - 1$	$[37.56,\ 37.62]$

$$s = \sqrt{3}\sin\theta_{12} - 1$$
$$r = \sqrt{2}\sin\theta_{13}$$



High precision measurements of θ12 can distinguish between these (medium baseline reactor experiments JUNO and RENO-50).

Delta CP

- The great unknown of the PMNS matrix is still open for predictions!
  - Atmospheric sum rule

$$\cos\delta = \frac{\sqrt{2}\sin\theta_{23} - (1 + a_0)}{\lambda\sqrt{2}\sin\theta_{13}}$$

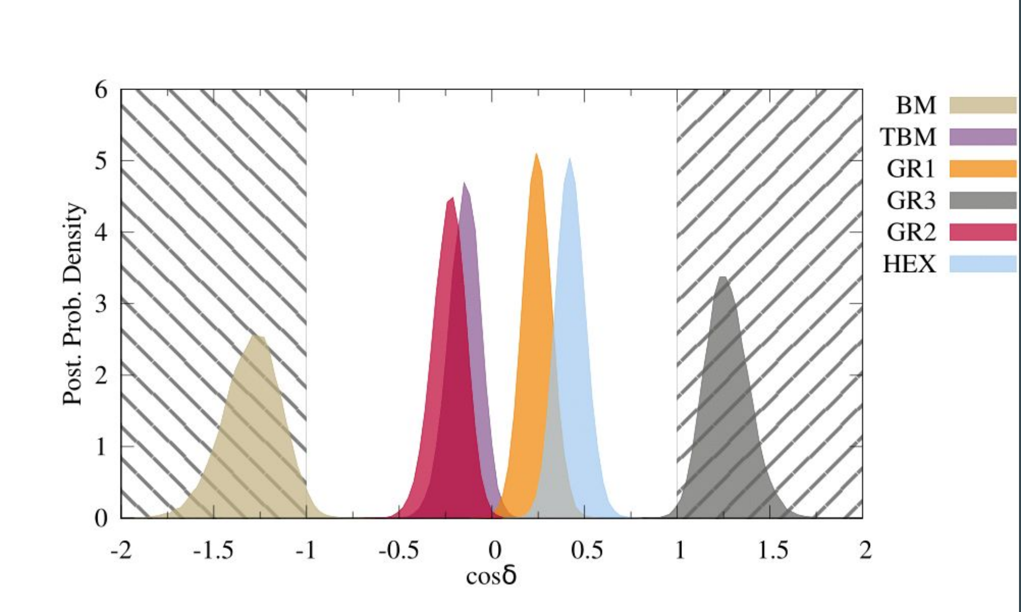
Model dependent parameters. Must be chosen from a finite set of options dictated by symmetry.

  - Solar sum rule

$$\cos\delta = \frac{t_{23}s_{12}^2 + s_{13}^2 c_{12}^2/t_{23} - \alpha^2(t_{23} + s_{13}^2/t_{23})}{\sin 2\theta_{13} s_{13}}$$

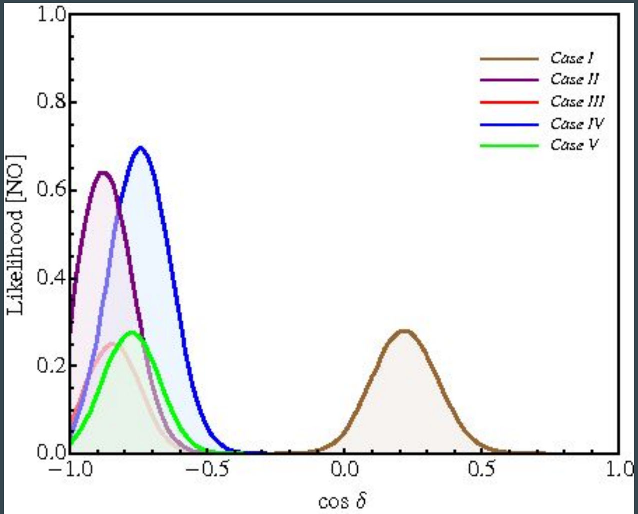
- An important aspect of these predictions is their reliance on our current knowledge of mixing parameters.
  - Improvements in e.g. θ23 precision make our predictions for delta more accurate.

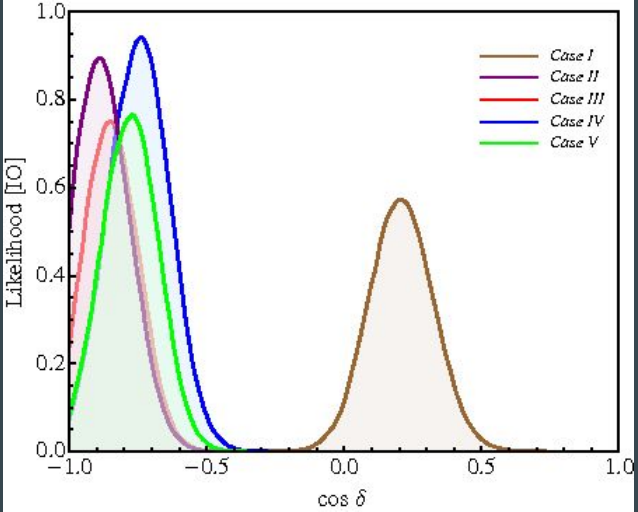
Delta CP from SSR/CLC



[From PB et al. 1410.7573]

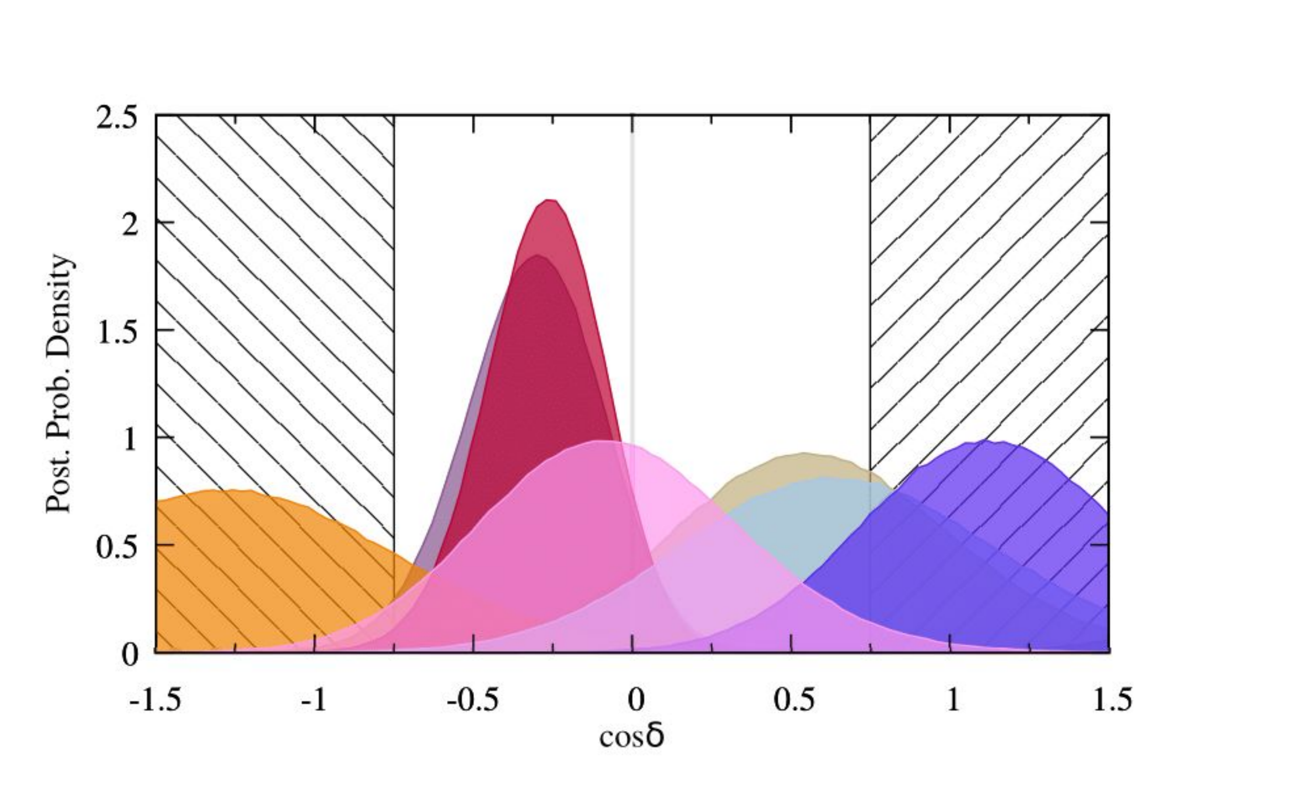
- Solar sum rule predictions for all possible leading order matrices.
- Hatched regions show where the data leads to inconsistent predictions.





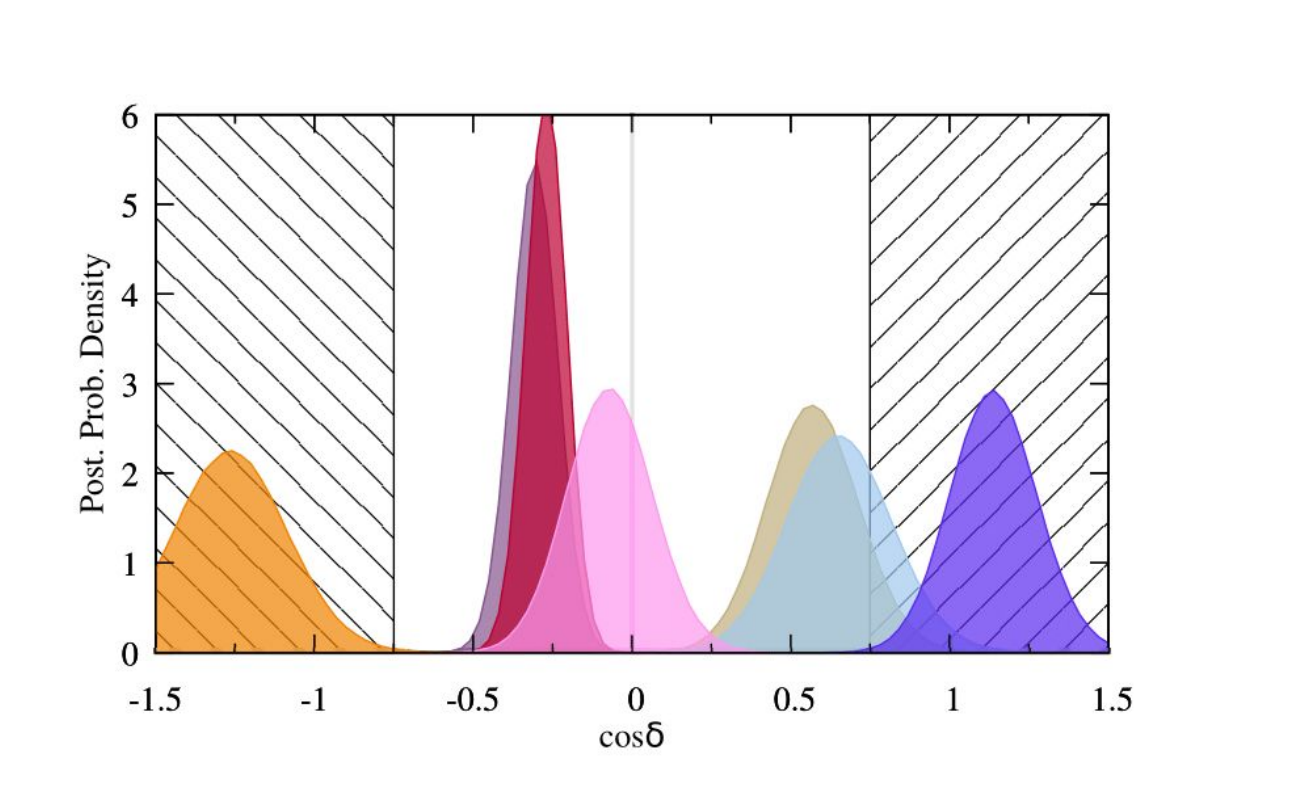
[Girardi et al 1504.00658]

Delta CP from atmospheric sum rules



[Based on relations derived in PB 1308.4314]

Delta CP from atmospheric sum rules



As our precision on θ23 improves, the correlations make sharper predictions for delta.

(Figure assumes our precision on θ23 is smaller by a factor of 3.)

[Based on relations derived in PB 1308.4314]

In summary

- The extension of the neutrino sector is fundamentally linked to our understanding of lepton flavour; its exploration will open many doors
- Discrete symmetry is a popular (albeit not necessary) way to reduce d.of.s and make predictions
- This is highly model dependent; however, there are classes of prediction which capture the essence of many models known as sum rules
- Three important questions for the future programme:
  - How are $\theta 12$ and $\theta 13$ correlated?
  - Is $\theta 23$ maximal? Or is its deviation from maximal correlated to $\theta 13$ and delta.
  - What is the precise value of delta?

Thank you

And thanks to...

Free parameters of the SM

- Dirac V