Reactor neutrino oscillations

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Introduction

Reactor neutrinos

- General survival probability
- Approximations
- Results and uncertainties

Survival probability basics

Neutrino mixing:

$$|\nu_\alpha\rangle = \sum_{i=1}^{3} U_{\alpha i} |\nu_i\rangle \quad (1)$$

Time evolution:

$$|\nu_\alpha(t)\rangle = \sum_{i=1}^{3} U_{\alpha i} e^{-iE_i t\hbar} |\nu_i\rangle \quad (2)$$

Here $\alpha = e, \mu, \tau$ represent the flavor eigenstates and $i = 1, 2, 3$ represent the mass eigenstates.

Survival probability:

$$\begin{aligned} P_{\alpha\alpha} &= | \langle \nu_\alpha | \nu_\alpha(t) \rangle |^2 \\ &= | U_{\alpha 1} U^*_{\alpha 1} e^{-iE_1 t\hbar} + U_{\alpha 2} U^*_{\alpha 2} e^{-iE_2 t\hbar} + U_{\alpha 3} U^*_{\alpha 3} e^{-iE_3 t\hbar} | \end{aligned} \quad (3)$$

Electron antineutrino survival probability

If we plug in the values of the PMNS matrix, where only the electron related entries matter, we obtain the following survival probability.

$$U = \begin{pmatrix} c_{13}c_{12} & c_{13}s_{12} & s_{13}e^{-i\delta} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}$$

Figure: PMNS matrix with filled electron neutrino mixing entries

$$\begin{aligned} P(\overline{\nu}_e \rightarrow \overline{\nu}_e) = 1 &- \sin^2 2\theta_{13} * \cos^2\theta_{12} * \sin^2\left(\frac{(E_1 - E_3)t}{2\hbar}\right) \\ &- \sin^2 2\theta_{13} * \sin^2\theta_{12} * \sin^2\left(\frac{(E_2 - E_3)t}{2\hbar}\right) \\ &- \cos^4\theta_{13} * \sin^2 2\theta_{12} * \sin^2\left(\frac{(E_1 - E_2)t}{2\hbar}\right) \end{aligned} \tag{4}$$

Electron antineutrino survival probability

In the approximation that neutrinos have small masses and are extremely relativistic:

$$(E_1 - E_2)t \approx \frac{(m_1^2 - m_2^2)c^3 L}{2E_\nu} \tag{5}$$

With that, we ultimately end up with the probability written in the desired form:

$$\begin{aligned} P(\overline{\nu}_e \rightarrow \overline{\nu}_e) = 1 &- \sin^2 2\theta_{13} * \cos^2\theta_{12} * \sin^2(\frac{\Delta m_{31}^2 L}{4E}) \\ &- \sin^2 2\theta_{13} * \sin^2\theta_{12} * \sin^2(\frac{\Delta m_{32}^2 L}{4E}) \\ &- \cos^4\theta_{13} * \sin^2 2\theta 12 * \sin^2(\frac{\Delta m_{21}^2 L}{4E}) \end{aligned} \tag{6}$$

Approximations to the survival probability

We introduce a new symbol denoted by:

$$\Delta m_{ee}^2 = \cos^2\theta_{12}\Delta m_{31}^2 + \sin^2\theta_{12}\Delta m_{32}^2 \tag{7}$$

Note also that only two mass differences are independent:

$$\Delta m_{31}^2 = \Delta m_{21}^2 + \Delta m_{32}^2$$

Then we can see that:

$$\begin{aligned}\Delta m_{31}^2 &= m_{ee}^2 + \sin^2\theta_{12}\Delta m_{12}^2 \\ \Delta m_{32}^2 &= m_{ee}^2 - \cos^2\theta_{12}\Delta m_{12}^2\end{aligned} \tag{8}$$

Further noting that

$$\Delta m_{12}^2 << m_{ee}^2 \approx \Delta m_{13}^2 \approx \Delta m_{23}^2$$

with

$$\frac{\Delta m_{12}^2}{\Delta m_{ee}^2} \approx 0.03$$

Approximations to the survival probability

Applying those approximations to the mass squared terms in the survival probability:

$$\begin{aligned}
\sin\frac{\Delta m_{13}^2 L}{E} &\approx \sin\frac{\Delta m_{ee}^2 L}{E} + \sin^2\theta_{12} * \frac{\Delta m_{12}^2 L}{E}\cos\frac{\Delta m_{ee}^2 L}{E} \\
\sin\frac{\Delta m_{23}^2 L}{E} &\approx \sin\frac{\Delta m_{ee}^2 L}{E} - \cos^2\theta_{12} * \frac{\Delta m_{12}^2 L}{E}\cos\frac{\Delta m_{ee}^2 L}{E}
\end{aligned} \tag{9}$$

If we look at what this means for the terms with Δm_{13}^2 and Δm_{13}^2

$$\begin{aligned}
&\cos^2\theta_{12}\sin^2\frac{\Delta m_{13}^2 L}{E} + \sin^2\theta_{12} * \sin^2\frac{\Delta m_{23}^2 L}{E} \\
&\approx \sin^2\frac{\Delta m_{ee}^2 L}{E} + \frac{1}{4}\left(\frac{\Delta m_{13}^2 L}{E}\right)^2\sin^2(2\theta_{12})\cos^2\frac{\Delta m_{ee}^2 L}{E}
\end{aligned} \tag{10}$$

Approximations to the survival probability

With that, we ultimately end up with the probability written in the desired form:

$$P(\overline{\nu}_e \to \overline{\nu}_e) = 1 - \cos^4\theta_{13} * \sin^2 2\theta_{12} * \sin^2(\frac{\Delta m^2_{12} L}{4E})$$
$$- \sin^2 2\theta_{13}[\sin^2(\frac{\Delta m^2_{ee} L}{4E}) + \frac{1}{4}(\frac{\Delta m^2_{12} L}{4E})^2 \sin^2 2\theta_{12} * \cos^2(\frac{\Delta m^2_{ee} L}{4E})] \quad (11)$$

If we ignore the last part with the $\frac{1}{4}$ fraction, we obtain:

$$P(\overline{\nu}_e \to \overline{\nu}_e) = 1 - \cos^4\theta_{13} * \sin^2 2\theta_{12} * \sin^2(\frac{\Delta m^2_{12} L}{4E})$$
$$- \sin^2 2\theta_{13}[\sin^2(\frac{\Delta m^2_{ee} L}{4E})] \quad (12)$$

Mental break

Figure: Ducks

Concluding survival probability

If we start off with that form of the probability:

$$P(\overline{\nu}_e \rightarrow \overline{\nu}_e) = 1 - \cos^4\theta_{13} * \sin^2 2\theta_{12} * \sin^2(\frac{\Delta m^2_{12} L}{4E}) - \sin^2 2\theta_{13}[\sin^2(\frac{\Delta m^2_{ee} L}{4E})] \tag{13}$$

and we realize that the ratio of

$$\frac{\sin^2(\frac{\Delta m^2_{12} L}{4E})}{\sin^2(\frac{\Delta m^2_{ee} L}{4E})} \approx 0.06\%$$

Then we can understand that for short baselines (say L < 2km), we can neglect the Δm^2_{12} term:

$$\boxed{P(\overline{\nu}_e \rightarrow \overline{\nu}_e) \approx 1 - \sin^2 2\theta_{13}[\sin^2(\frac{\Delta m^2_{ee} L}{4E})]} \tag{14}$$

Comparison of the survival probabilities

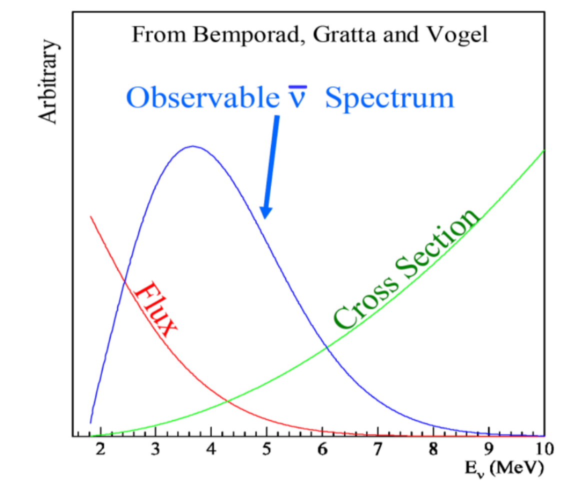

Figure: Neutrino energy spectrum

Comparison of the survival probabilities

$$P(\overline{\nu}_e \rightarrow \overline{\nu}_e) = 1 - \cos^4\theta_{13} * \sin^2 2\theta_{12} * \sin^2(\frac{\Delta m^2_{12} L}{4E}) - \sin^2 2\theta_{13}[\sin^2(\frac{\Delta m^2_{ee} L}{4E}) + \frac{1}{4}(\frac{\Delta m^2_{12} L}{4E})^2 \sin^2 2\theta_{12} * \cos^2(\frac{\Delta m^2_{ee} L}{4E})] \quad (15)$$

$$P(\overline{\nu}_e \rightarrow \overline{\nu}_e) = 1 - \cos^4\theta_{13} * \sin^2 2\theta_{12} * \sin^2(\frac{\Delta m^2_{12} L}{4E}) - \sin^2 2\theta_{13}[\sin^2(\frac{\Delta m^2_{ee} L}{4E})] \quad (16)$$

$$P(\overline{\nu}_e \rightarrow \overline{\nu}_e) \approx 1 - \sin^2 2\theta_{13}[\sin^2(\frac{\Delta m^2_{ee} L}{4E})] \quad (17)$$

Comparison of the survival probabilities

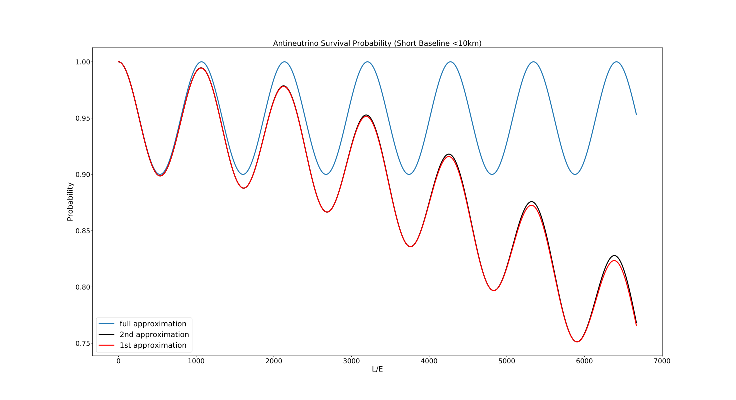

Figure: Oscillation probabilities for different levels of approximation

Possible reactor measurements

If we go back to the simplest form of the probability:

$$\boxed{P(\overline{\nu}_e \rightarrow \overline{\nu}_e) \approx 1 - \sin^2 2\theta_{13}[\sin^2(\frac{\Delta m^2_{ee} L}{4E})]} \quad (18)$$

We can separate the second term in two pieces: The oscillation amplitude and the frequency.

$$A = \sin^2 2\theta_{13} \quad (19) \qquad F = \sin^2(\frac{\Delta m^2_{ee} L}{4E}) \quad (20)$$

By measuring the oscillation probability at fixed L for certain energies, the amplitude of the signal will be related to θ_{13} and the frequency will be related to Δm^2_{ee}

Systematic uncertainties

$$P_{\nu_\alpha \to \nu_{\alpha'}} \approx \frac{N_{\nu_{\alpha'}}^{measured-at-FD}}{N_{\nu_\alpha}^{measured-at-ND}} \times \frac{\epsilon^{ND}}{\epsilon^{FD}} \times \frac{p^{FD}}{p^{ND}}$$

Figure: Naive probability

$$\frac{N_{\nu_{\alpha'}}^{FD}(E_\nu)}{N_{\nu_\alpha}^{ND}(E_\nu)} \approx P_{\nu_\alpha \to \nu_{\alpha'}}(E_\nu) \times \frac{\varphi_{\nu_{\alpha'}}^{FD}(E_\nu)}{\varphi_{\nu_\alpha}^{ND}(E_\nu)} \times \frac{\sigma_{\nu_{\alpha'}}^{FD}(E_\nu)}{\sigma_{\nu_\alpha}^{ND}(E_\nu)} \times \frac{\epsilon^{ND}}{\epsilon^{FD}} \times \frac{p^{FD}}{p^{ND}}$$

Figure: True probability