Chirality and Neutrinos

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The Problem:
Explain the difference between what we call neutrinos and antineutrinos in the Dirac and Majorana cases, clarifying the role of chirality in weak interactions and the difference between chirality and helicity.

Our Solution:

1. Chirality vs. Helicity
2. Chirality's role in the weak interaction
3. Dirac *vs* Majorana Neutrinos--English
4. Dirac *vs* Majorana Neutrinos--Mathematics
   a. Spinor Formalism
   b. Dirac Formalism
   c. Majorana Formalism

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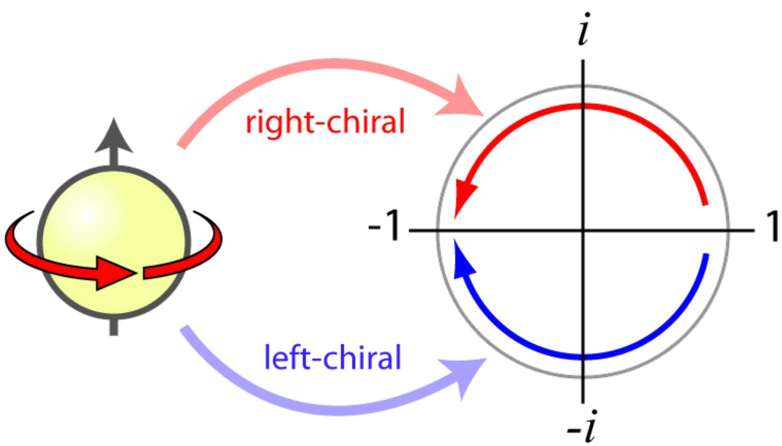
Chirality vs. Helicity

Chirality

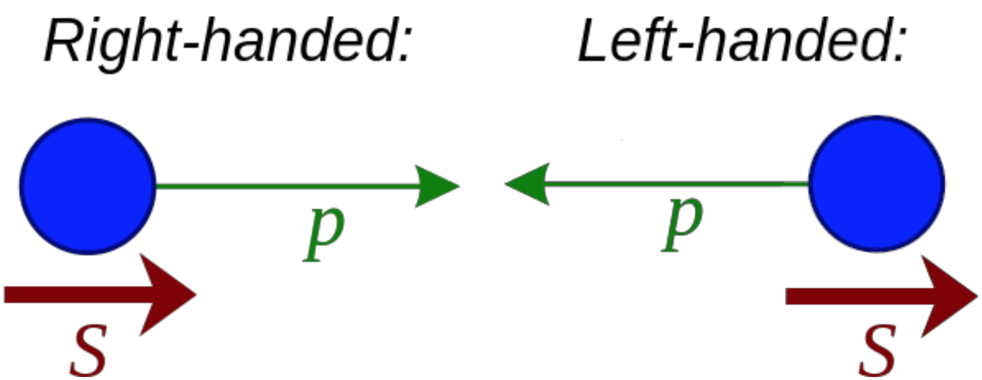
- Inherent quantum mechanical property
- Lorentz invariant
- Commutes with Hamiltonian for massless particles

Helicity

- Projection of particle's spin onto momentum
- Lorentz invariant for massless particle
- Commutes with Hamiltonian → conserved



F. Tanedo, "Helicity, Chirality, Mass, and the Higgs", Quantum Diaries



https://en.wikipedia.org/wiki/Chirality_(physics) (modified)

For a massless particle, chirality and helicity coincide.

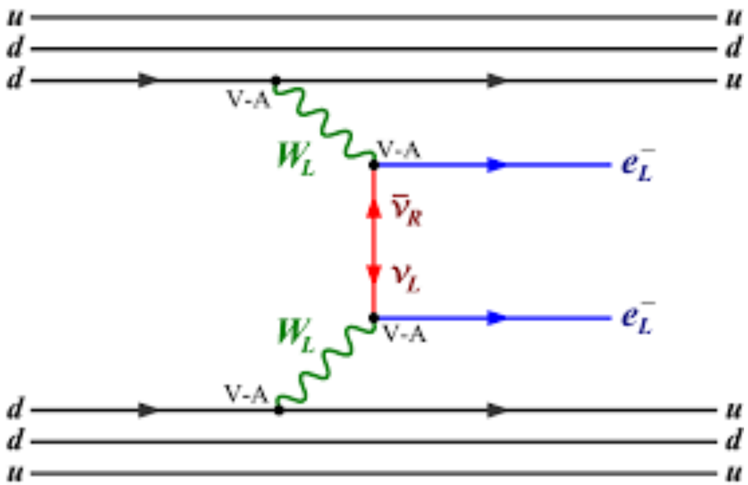
Weak Interaction and Chirality

- Charged current: $j^{CC}_{\text{weak}} = \frac{g_w}{\sqrt{2}} \bar{\psi} \gamma^\mu \frac{1}{2}(1 - \gamma^5)\psi$
    - Left-handed chiral projection operator: $P_L = \frac{1}{2}(1 - \gamma^5)$
- Charged weak interaction only couples left-handed chiral particles (or right-handed chiral antiparticles)
- Neutrinos only made in weak interactions → all created as left-handed chiral particles
- Probability of generating right-handed helicity neutrino: $\mathcal{P} \propto \left(\frac{m_\nu}{E_\nu}\right)^2$
    - Almost impossible
- Since neutrino mass very small, helicity ~ chirality

Dirac vs. Majorana Neutrinos--English
(with a picture)

In short, neutrinos and antineutrinos of the same flavor (e, μ, τ) are different particles if neutrinos are Dirac particles; if neutrinos are Majorana particles, then for a given flavor, the neutrino and antineutrino are the same particle.

As a result, neutrinoless double-β (0$\boldsymbol{\nu}$ββ) decay:

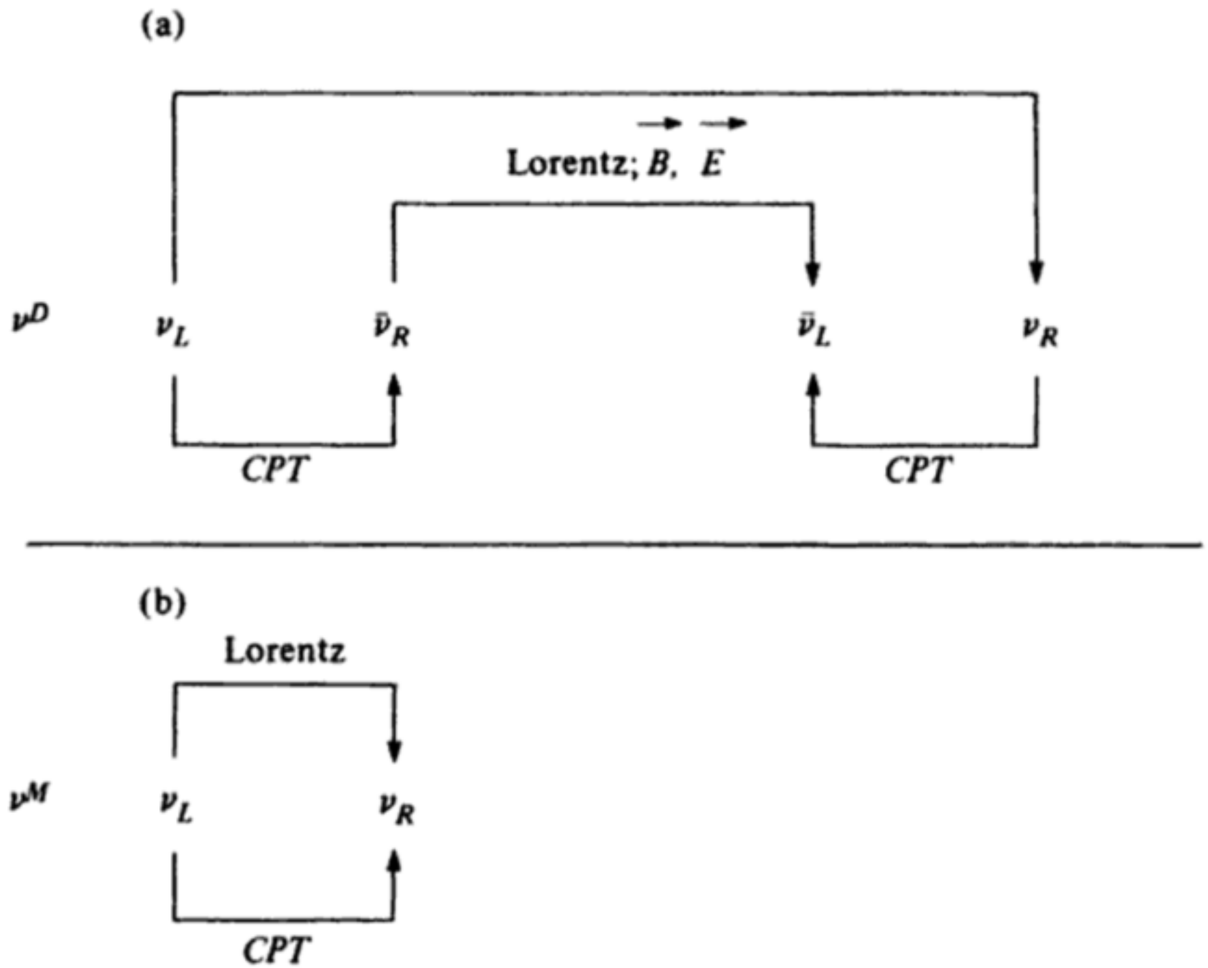


Arnold, R. *et al.*, Eur.Phys.J. C70 (2010) 927-943

is possible for Majorana neutrinos, but not for Dirac neutrinos. Thus, observation of 0$\boldsymbol{\nu}$ββ decay would demonstrate that neutrinos are Majorana particles.

Helicity States for Dirac and Majorana Neutrinos

Figure 1.1. (a) The four distinct states of a Dirac neutrino ν^D. (b) The two distinct states of a Majorana neutrino ν^M.



F. Boehm and P. Vogel. *Physics of Massive Neutrinos*. 2nd ed., Cambridge University Press, 1992

Spinor Formalism

Klein Gordon equation: $\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\psi - \nabla^2\psi + \frac{m^2c^2}{\hbar^2}\psi = 0.$

We use the Pauli matrices: $\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

The Dirac matrices are defined as: $\gamma^0 = \begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & -\mathbf{1} \end{pmatrix}, \gamma^i = \begin{pmatrix} \mathbf{0} & \sigma^i \\ -\sigma^i & \mathbf{0} \end{pmatrix}$

$$\partial\!\!\!/ = \gamma^\mu \partial_\mu = \sum_{i=0}^{3} \gamma^i \partial_i$$

$\bar{\psi} = \psi^\dagger \gamma^0$ is the Dirac adjoint adjoint of the spinor ψ

$\gamma^5 = \begin{pmatrix} \mathbf{0} & \mathbf{1} \\ \mathbf{1} & \mathbf{0} \end{pmatrix}$ is the Chirality observable in the Dirac formalism

Dirac Formalism - Part 1

$$\mathcal{L} = \mathcal{L}_{\text{free}} + \mathcal{L}_{\text{int}}$$

$$\mathcal{L}_{\text{free}} = \bar{L} i \partial\!\!\!/ L + \bar{\psi}_{eR} i \partial\!\!\!/ \psi_{eR} + \bar{\psi}_{\nu_e R} i \partial\!\!\!/ \psi_{\nu_e R} + \bar{R}' i \partial\!\!\!/ R' + \bar{\psi}_{e+L} i \partial\!\!\!/ \psi_{e+L} + \bar{\psi}_{\bar{\nu}_e L} i \partial\!\!\!/ \psi_{\bar{\nu}_e L}$$

$$\mathcal{L}_{\text{int}} = -\frac{g_w}{\sqrt{2}}\left(\bar{L}\gamma^\mu \frac{\sigma_+}{2} L W_\mu^+ + \bar{L}\gamma^\mu \frac{\sigma_-}{2} L W_\mu^-\right) - g_w \bar{L}\gamma^\mu \frac{\sigma_3}{2} L W_\mu^3$$
$$- \frac{g_w}{\sqrt{2}}\left(\bar{R}'\gamma^\mu \frac{\sigma_+}{2} R' W_\mu^- + \bar{R}'\gamma^\mu \frac{\sigma_-}{2} R' W_\mu^+\right) - g_w \bar{R}\gamma^\mu \frac{\sigma_3}{2} R' W_\mu^3$$

$$L = \begin{pmatrix} \psi_{\nu_e L} \\ \psi_{eL} \end{pmatrix} \qquad R' = \begin{pmatrix} \psi_{\bar{\nu}_e R} \\ \psi_{e+R} \end{pmatrix}$$

By breaking the electroweak Lagrangian into a free Lagrangian and an electroweak interaction Lagrangian we can see:

- The right-handed fermions and left-handed anti-fermions only propagate and do not interact via the weak interaction
- The neutrinos and anti-neutrinos are electrically neutral and not coloured, and thus do not interact via electromagnetism nor strong interaction
- Thus the right-handed neutrinos and left-handed anti-neutrinos exclusively propagate

Dirac Formalism - Part 2

- Spinors are solutions to (1) and the anti-spinors to (2). Hence, they are a free-basis for the Klein-Gordon equation.

$$(i\partial\!\!\!/ - m)\psi = 0 \qquad (1)$$

$$(i\partial\!\!\!/ + m)\psi = 0 \qquad (2)$$

- $\psi \to i\gamma^2\psi^*$ takes a spinor ↔ an anti-spinor

- The interaction Lagrangian is invariant under electroweak gauge symmetry group SU(2)

- Furthermore, we see that only left-handed particles and right-handed anti-particles are involved in the electroweak interaction.

Majorana Formalism - Part 1

Dirac and Majorana spinor fields are defined with Weyl formalism:

$$\Psi_e = \begin{pmatrix} \chi_{ea} \\ \xi_e^{\dagger a} \end{pmatrix} \qquad \Psi_{\nu_e} = \begin{pmatrix} \psi_{\nu_e a} \\ \psi_{\nu_e}^{\dagger a} \end{pmatrix}$$

$$\mathcal{L} = \mathcal{L}_{\text{free}} + \mathcal{L}_{\text{int}}$$

$$\mathcal{L}_{\text{free}} = L^\dagger i\sigma^\mu \partial_\mu L + \xi_e^\dagger i\sigma^\mu \partial_\mu \xi_e + R'^\dagger i\sigma^\mu \partial_\mu R' + \chi_{e+}^\dagger i\sigma^\mu \partial_\mu \chi_{e^+}$$

$$\mathcal{L}_{\text{int}} = -\frac{g_w}{\sqrt{2}}\left(L^\dagger \sigma^\mu \frac{\sigma_+}{2} L W_\mu^+ + L^\dagger \sigma^\mu \frac{\sigma_-}{2} L W_\mu^-\right) - g_w L^\dagger \sigma^\mu \frac{\sigma_3}{2} L W_\mu^3$$
$$- \frac{g_w}{\sqrt{2}}\left(R'^\dagger \sigma^\mu \frac{\sigma_+}{2} R' W_\mu^- + R'^\dagger \sigma^\mu \frac{\sigma_-}{2} R' W_\mu^+\right) - g_w R'^\dagger \sigma^\mu \frac{\sigma_3}{2} R' W_\mu^3$$

$$L = \begin{pmatrix} \psi_{\nu_e} \\ \chi_e \end{pmatrix} \qquad R' = \begin{pmatrix} \psi_{\nu_e} \\ \xi_{e+} \end{pmatrix}$$

By breaking down the electroweak Lagrangian into a free Lagrangian and an electroweak interaction Lagrangian we can see:

- The right-handed Dirac-fermions and left-handed Dirac-anti-fermions only propagate and do not interact via weak interaction
- The neutrino field is neutral and not coloured, thus do not interact via electromagnetism nor strong interaction

Majorana Formalism - Part 2

- There is no sterile neutrino in this formalism.
- All the fields, independently of their handedness are coupled to the electroweak boson fields.
- The electron and positron are coupled to the same field by the electroweak symmetry. That means neutrinos and anti-neutrinos are represented by the same field.
- A charge conjugation leaves the neutrino field unchanged, which proves that neutrinos and antineutrinos are the same particle: $\nu_{eL} = \bar{\nu}_{eR} \qquad \nu_{eR} = \bar{\nu}_{eL}$
- The interaction lagrangian is invariant by the electroweak gauge symmetry group
- Furthermore, we see that only left-handed Dirac-particles, the right-handed Dirac-antiparticles and the Majorana neutral particle are involved in the electroweak interaction.

Conclusions

- Chirality is a fundamental property of a particle whereas helicity is frame-dependent

- Dirac neutrinos and anti-neutrinos are distinct particles

- Majorana neutrinos and anti-neutrinos are not distinct particles