Neutrino energy reconstruction in a quasi-elastic interaction

Problem 7.2

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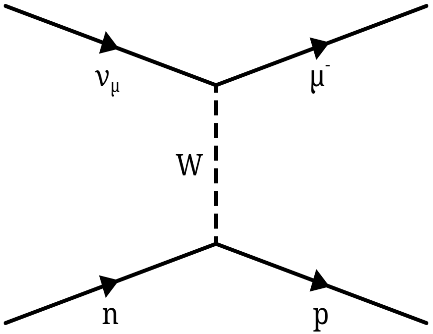
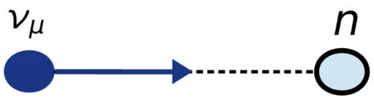
7.2.1: Derive this formula to evaluate neutrino energy in quasi-elastic (QE) neutrino scattering

$$E_\nu = \frac{{m_p}^2 - (m_n - E_b)^2 - {m_\mu}^2 + 2\,(m_n - E_b)E_\mu}{2\left(m_n - E_b - E_\mu + |\vec{p}_\mu|\cos\theta\right)}$$

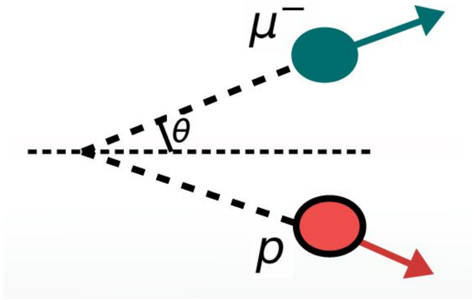
Quasi-elastic neutrino-nucleon interaction

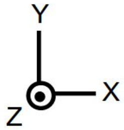


Before



After





Assuming neutron at rest and bound to the nucleus due to a potential, we can write the four-momenta of the particles in the **lab frame**

Before

$$p_\nu = (E_\nu, |\vec{p}_\nu|, 0, 0)$$

$$p_n = (E_n - E_b, 0, 0, 0)$$

After

$$p_\mu = \left(E_\mu, |\vec{p}_\mu|\cos\theta, |\vec{p}_\mu|\sin\theta, 0\right)$$

$$p_p = (E_p, |\vec{p}_p|_x, -|\vec{p}_p|_y, 0)$$

Playing with the four-momenta

$\nu_\mu + n \rightarrow \mu^- + p$

But we already know beforehand that

$$E_\nu{}^2 = |\vec{p}_\nu|^2 \qquad E_n{}^2 = m_n{}^2 + |\vec{p}_n|^2 = m_n{}^2$$

Four-momentum conservation requires

$$p_\nu + p_n = p_\mu + p_p$$

$$(p_\nu - p_\mu)^2 = (p_p - p_n)^2$$

$$p_\nu{}^2 + p_\mu{}^2 - 2\,p_\nu \cdot p_\mu = p_p{}^2 + p_n{}^2 - 2\,p_p \cdot p_n$$

Using relativistic kinematics

$$\nu_\mu + n \rightarrow \mu^- + p$$

The squared four-momenta can be easily found

- For "massless" neutrino: $${p_\nu}^2 = {E_\nu}^2 - |\vec{p}_\nu|^2 = 0$$
- For massive free particles: (Lorentz invariant) $${p_\mu}^2 = {m_\mu}^2 \quad , \quad {p_p}^2 = {m_p}^2$$
- For bound neutron: $${p_n}^2 = (m_n - E_b)^2$$

Crossed terms are more interesting, but still solvable if working in our previously defined lab frame

$$p_\nu \cdot p_\mu = E_\nu \left(E_\mu - |\vec{p}_\mu| \cos\theta\right)$$

$$p_p \cdot p_n = E_p \left(m_n - E_b\right)$$

Getting the neutrino energy

$$\nu_\mu + n \rightarrow \mu^- + p$$

Based on conservation of energy $E_p = E_\nu - E_\mu + (m_n - E_b)$

The second crossed term can be rewritten in terms of the other variables

$$p_p \cdot p_n = (m_n - E_b)\left[E_\nu - E_\mu + (m_n - E_b)\right]$$

And rearranging everything, we get the neutrino energy that can be reconstructed from the other variables:

$$E_\nu = \frac{{m_p}^2 - (m_n - E_b)^2 - {m_\mu}^2 + 2\,(m_n - E_b)E_\mu}{2\left(m_n - E_b - E_\mu + |\vec{p}_\mu|\cos\theta\right)}$$

Dependence of neutrino energy respect to binding energy is linear; so then **with higher binding energy, the neutrino reco energy will be larger**.

What if neutrinos are not massless?

Problem 1.1

$$\pi^- \rightarrow \mu^- + \nu_\mu$$

We can use another problem to illustrate what happens when we consider the neutrinos to be massive, for instance pion decay:

Before

After

Starting in the **center of mass frame**, where pion is at rest and all three-momenta cancel out with each other, we get

Before

$$p_\pi = \left(m_\pi, \vec{0}\right)$$

After

$$p_\mu = (E_\mu, \vec{p}_\mu)$$
$$p_\nu = (E_\nu, \vec{p}_\nu)$$

What if neutrinos are not massless?

Problem 1.1

$\pi^- \to \mu^- + \nu_\mu$

Using four-momentum conservation once again, we get

$$p_\pi = p_\mu + p_\nu$$

$$(p_\pi - p_\mu)^2 = {p_\nu}^2$$

This time the four-momentum squared of the neutrino is not zero, but a Lorentz invariant related to its mass

$${p_\pi}^2 = {m_\pi}^2 \quad , \quad {p_\mu}^2 = {m_\mu}^2 \quad , \quad {p_\nu}^2 = {m_\nu}^2$$

After some algebra, we can find the muon energy

$$E_\mu = \frac{{m_\pi}^2 + {m_\mu}^2 - {m_\nu}^2}{2\, m_\pi}$$

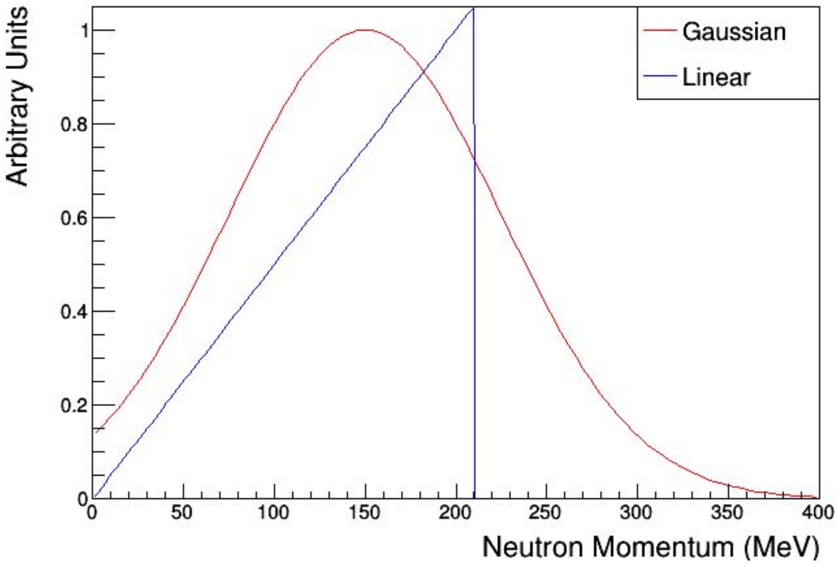
Compared to the typical result assuming a massless neutrino

$$E_\mu = \frac{{m_\pi}^2 + {m_\mu}^2}{2\, m_\pi}$$

7.2.2: Consider the case of monochromatic neutrino flux of 1GeV. How does the distribution of reconstructed neutrino change if initial state neutron has some given momentum distribution ?

Neutron Momentum

- ~~Monochromatic flux of 1 GeV neutrinos~~ ⇨ NOvA "beam"
- Reconstructed neutrino energy if the neutron has a given momentum?



- Linear ~ Relativistic Fermi Gas (RFG) model
- Gaussian ~ Local Fermi Gas (LFG) model or Spectral Function

Neutron Momentum

What to do?

$$E_\nu = \frac{m_p^2 - (m_n - E_b)^2 - m_\mu^2 + 2(m_n - E_b)E_\mu}{2(m_n - E_b - E_\mu + p_\mu cos\theta)}$$

- Lorentz boost to lab?
  - Basic eqn is in neutron rest frame
  - Angles change, neutrino beam is no longer horizontal
  - Couldn’t figure out neutron directional momentum

- Rederive the energy formula

Math 1, Uprising

$$\mathbf{p}_\nu = (E_\nu,\, p_\nu, 0, 0) \quad \mathbf{p}_n = (E_n,\, p_{nx},\, p_{ny},\, p_{nz}) \quad \mathbf{p}_p = (E_p,\, p_{px},\, p_{py},\, p_{pz})$$

$$\mathbf{p}_\mu = (E_\mu,\, p_\mu\, cos\theta\, sin\phi,\, p_\mu\, sin\theta\, sin\phi,\, p_\mu\, cos\phi)$$

Conservation of momentum

$$p_{px} = p_{nx} - p_\mu\, cos\theta\, sin\phi$$

$$p_{py} = p_{ny} - p_\mu\, sin\theta\, sin\phi$$

$$p_{pz} = E_\nu + p_{nz} - p_\mu\, cos\phi$$

Conservation of energy

$$E_p = E_\nu + E_n - E_\mu$$

Proton energy

$$E_p^2 = m_p^2 + p_p^2$$

Math 3, The Search For Math 2

Take muon direction to define xy plane $\Rightarrow$ $\phi = 0$

ALGEBRA

$$E_\nu = \frac{m_p^2 + p_n^2 - E_n^2 - m_\mu^2 + 2\,E_n\,E_\mu - 2\,p_\mu(p_{nx}\,cos\theta + p_{ny}\,sin\theta)}{2(E_n - E_\mu + p_\mu\,cos\theta - p_{nx})}$$

Neutron Energy $\quad E_n = E_{nf} - E_b$

Free Neutron Energy $\quad E_{nf}^2 = m_n^2 + p_n^2$

Cool, Now What?

$$E_\nu = \frac{m_p^2 - (m_n - E_b)^2 - m_\mu^2 + 2(m_n - E_b)E_\mu}{2(m_n - E_b - E_\mu + p_\mu\, cos\theta)}$$

Reco eqn

$$E_\nu = \frac{m_p^2 + p_n^2 - E_n^2 - m_\mu^2 + 2\, E_n\, E_\mu - 2\, p_\mu (p_{nx}\, cos\theta + p_{ny}\, sin\theta)}{2(E_n - E_\mu + p_\mu\, cos\theta - p_{nx})}$$

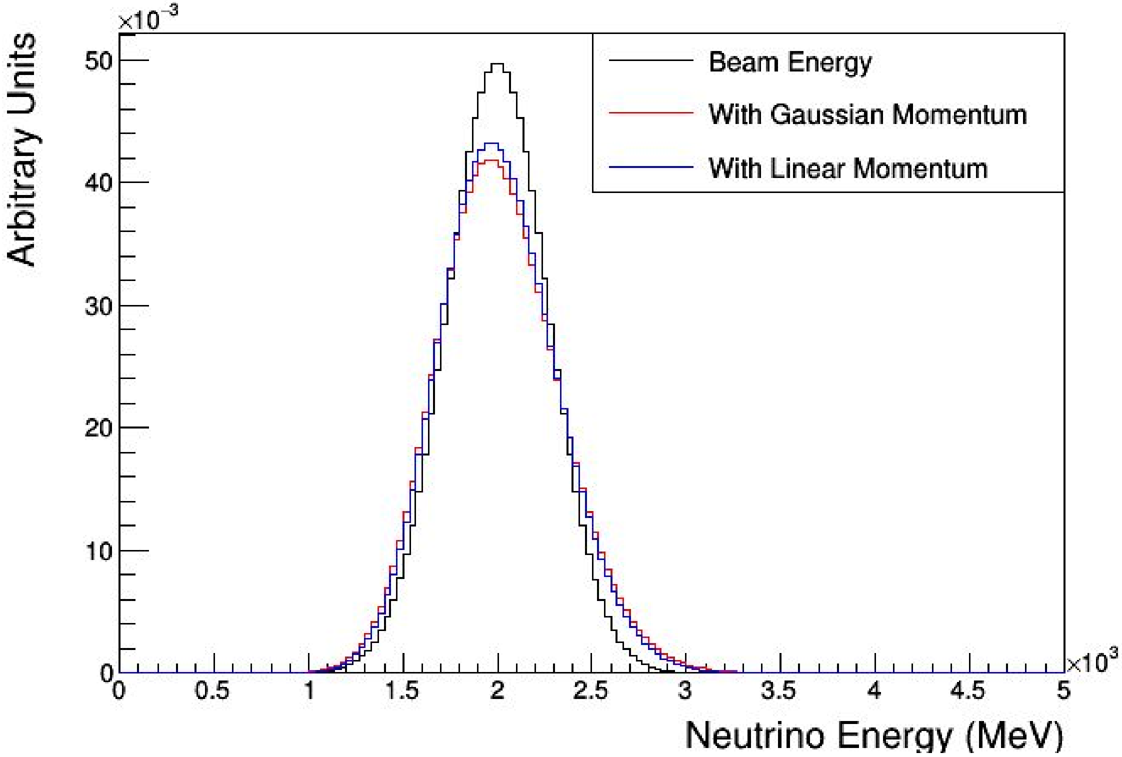
Real eqn

$$E_\mu \approx p_\mu \quad \Longrightarrow \quad p_\mu = \frac{m_p^2 + p_n^2 - m_\mu^2 - E_n^2 + 2\, E_\nu\, p_{nx} - 2\, E_\nu\, E_n}{2(E_\nu\, cos\theta - E_\nu - 2\, E_n + p_{nx}\, cos\theta + p_{ny}\, sin\theta)}$$

Whatcha Gonna Do?

- Take outgoing muon angle as 35°
- Randomly sample from momentum distribution, isotropic to get random x and y components, randomly sample from “beam” energy distribution
- Use the real eqn to get muon momentum
- Use muon momentum in reco eqn

Results (aka A Nice Plot)



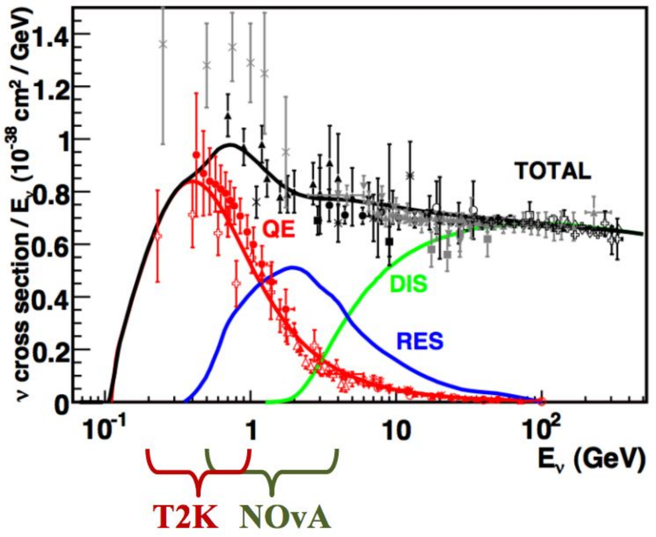
- It smears!
- The two distributions give nearly the same result

7.2.3: What do you need to measure in order to reconstruct neutrino energy ?

Reconstruct neutrino energy

Two methods

1.) **Measure outgoing particle kinematics** e.g in T2K - Region of interest is dominant with QuasiElastic (QE) events.
2.) **Measure Calorimetric energy** e.g in NOvA, MINOS - Accessible to all interaction types (QE, RES & DIS)



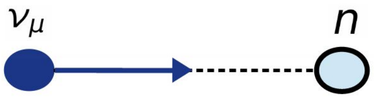
Kinematic Method

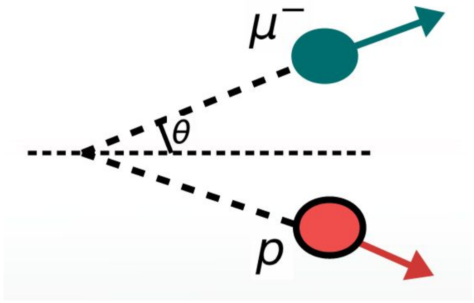
a.) E_ν is estimated by studying the kinematics of the outgoing charged lepton e.g Kinetic Energy and angle of the lepton.

b.) This technique assumes that the beam interacts with a single bound nucleon at rest and so no other nucleons are knocked out from the nucleus (**Mostly applied to QE events)** (Use 7.2.1 to calculate E_ν)

After

Before



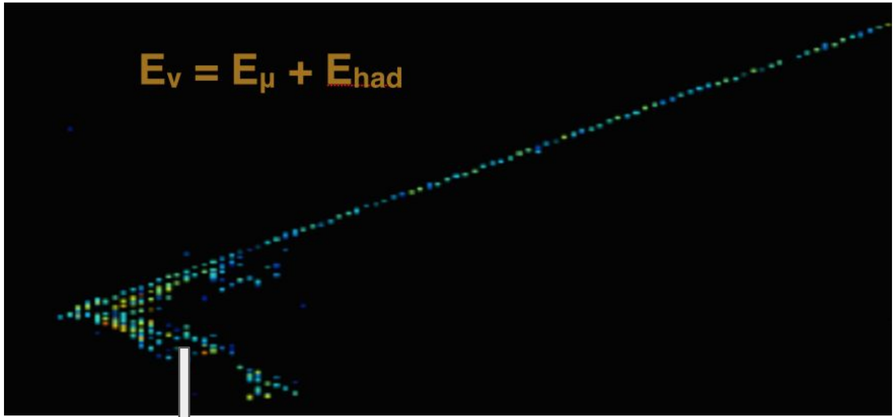


Calorimetric Method

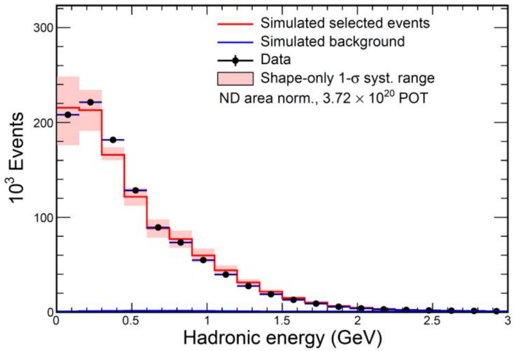
a.) Energy deposited by the final state particles is used to measure reconstructed neutrino energy: Measure the visible energy associated with each event.

b.) This method can be applied to any type of CC interaction.

c.) Challenge is to accurately reconstruct the Hadronic energy (Need to consider Nuclear effects)



Hadronic energy

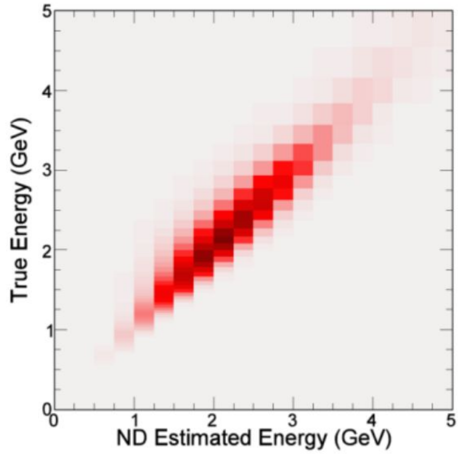


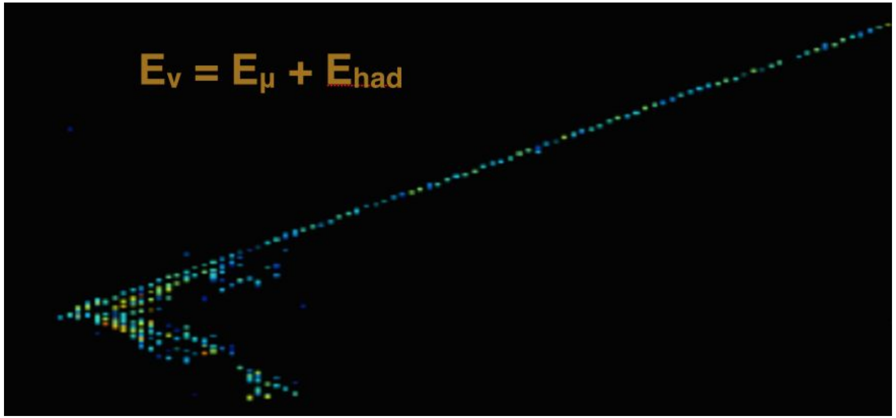
Plot from NOvA as an example

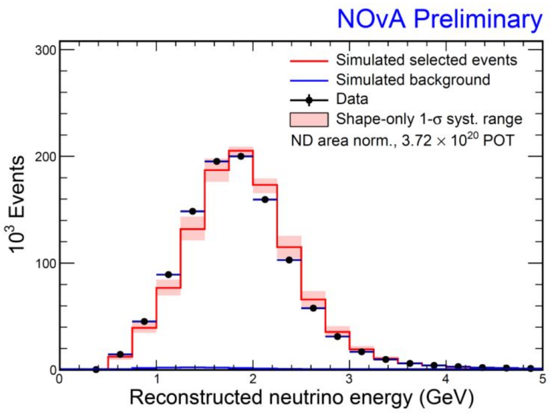
Calorimetric Method

a.) Energy deposited by the final state particles is used to measure reconstructed neutrino energy.

b.) Then, to predict energy spectrum at FD, we do ND to FD extrapolation using <u>migration matrix M_{ij}</u>.

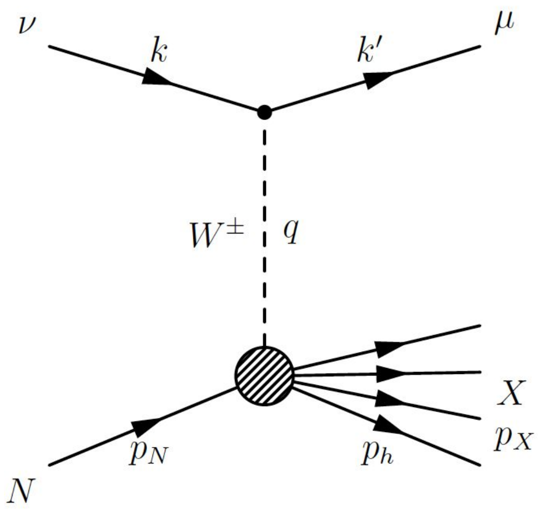


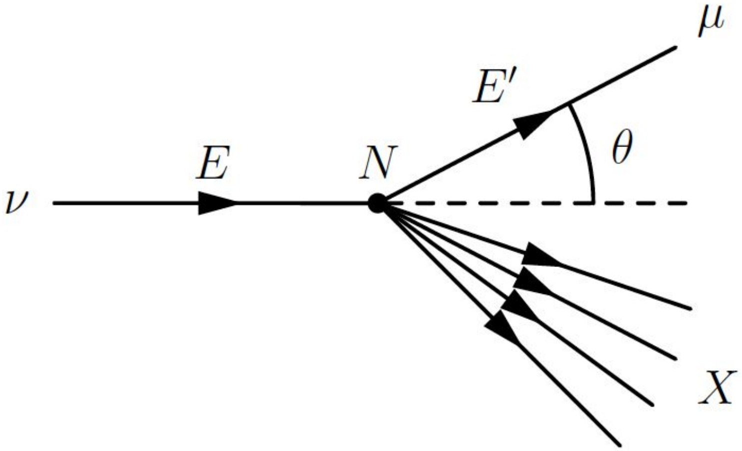




Plot from NOvA as an example

7.2.4: What happens if the event is not QE but there is another particle outgoing which escapes detection?





- Escaped particle’s linear momentum (**P**) is unknown
- Reconstructing neutrino energy accurately is not possible
- Include systematic uncertainties

References

Quasi Elastic Charm Production in Neutrino-Nucleon Scattering by Markus Bischofberger

Thank you.!