# INSS 2017 Problem 10.1

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# Part 1: Why the PDG is Your Best Friend

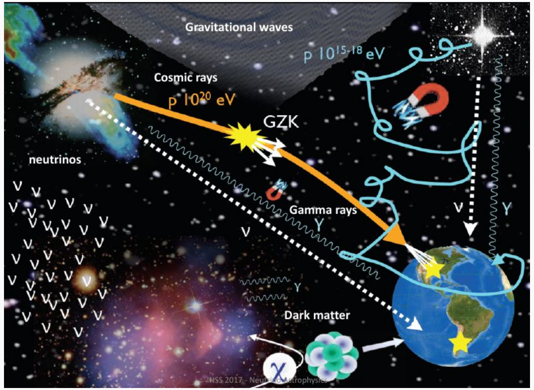
- Problem statement: what fraction of each neutrino flavor is expected from the decay of charged pions?
  - Assume neutrinos and antineutrinos are identical
- Conclusion: overwhelming majority are muon neutrinos
  - $\pi^-$ decay modes are conjugates

| $\pi^+$ DECAY MODES | | Fraction ($\Gamma_i/\Gamma$) | Confidence level | $p$ (MeV/c) |
|---|---|---|---|---|
| $\mu^+\nu_\mu$ | [b] | (99.98770±0.00004) % | | 30 |
| $\mu^+\nu_\mu\gamma$ | [c] | ( 2.00 ±0.25 ) × $10^{-4}$ | | 30 |
| $e^+\nu_e$ | [b] | ( 1.230 ±0.004 ) × $10^{-4}$ | | 70 |
| $e^+\nu_e\gamma$ | [c] | ( 7.39 ±0.05 ) × $10^{-7}$ | | 70 |

# Part 2: Back to the PDG

- Problem statement: what is produced in the decay of neutral pions? What are the implications for astrophysical neutrino detectors?
- Over 98% photons
- Implications: by identifying the brightest gamma-ray sources, we can classify the most likely neutrino sources

| $\pi^0$ DECAY MODES | Fraction ($\Gamma_i/\Gamma$) | Scale factor/ Confidence level | $p$ (MeV/$c$) |
|---|---|---|---|
| $2\gamma$ | $(98.823\pm0.034)$ % | S=1.5 | 67 |
| $e^+ e^- \gamma$ | $(1.174\pm0.035)$ % | S=1.5 | 67 |
| $\gamma$ positronium | $(1.82 \pm0.29) \times 10^{-9}$ | | 67 |
| $e^+ e^+ e^- e^-$ | $(3.34 \pm0.16) \times 10^{-5}$ | | 67 |



# Part 3: The Matrix

- Problem statement: neutrinos produced by charged pions propagate to Earth. What fraction of each flavor is expected by the time they reach Earth?
- The PMNS matrix (sort of):

$$\begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13} & c_{12}c_{23} - s_{12}s_{23}s_{13} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}s_{23}s_{13} & -c_{12}s_{23} - s_{12}c_{23}s_{13} & c_{23}c_{13} \end{pmatrix}$$

- Assumptions

$$s_{13} \to \theta_{13} \approx 0.1468 \text{ rad}$$
$$c_{13} \to 1$$
$$\delta_{CP} = 0$$

$$\theta_{12} = 0.5764$$
$$\theta_{23} = 0.7222 \text{ (normal)}, 0.8546 \text{ (inverted)}$$
$$\theta_{13} = 0.1468$$

# The Art of Approximation

- Latest greatest PMNS (according to Wikipedia)

$$\begin{bmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{bmatrix} = \begin{bmatrix} 0.82 \pm 0.01 & 0.54 \pm 0.02 & -0.15 \pm 0.03 \\ -0.35 \pm 0.06 & 0.70 \pm 0.06 & 0.62 \pm 0.06 \\ 0.44 \pm 0.06 & -0.45 \pm 0.06 & 0.77 \pm 0.06 \end{bmatrix}$$

- Using our approximations and PDG values for mixing angles,

$$\begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} = \begin{pmatrix} 0.8384 & 0.5450 & 0.1468 \\ -0.4902 & 0.5762 & 0.6610 \\ 0.2679 & -0.6143 & 0.7503 \end{pmatrix}$$

Normal

$$\begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} = \begin{pmatrix} 0.8384 & 0.5450 & 0.1468 \\ -0.4506 & 0.4901 & 0.7543 \\ 0.3302 & -0.6850 & 0.6565 \end{pmatrix}$$

Inverted

- Discrepancy due to $\delta_{CP}$ term

# How I Learned to Stop Worrying and Approximate the sin$^2$ Term

- Oscillation probability is given by

$$P_{\alpha\to\beta} = \delta_{\alpha\beta} - 4\sum_{j>i} U_{\alpha i}U_{\beta i}U_{\alpha j}U_{\beta j} sin^2\left(\frac{\Delta m_{ij}^2 L}{4E_\nu}\right)$$

- For astrophysical neutrinos, *L* is large and *E* varies over several order of magnitude
  - Another approximation: take the average

$$\sin^2\left(\frac{\Delta m_{ij}^2 L}{4E_\nu}\right) \to \frac{1}{2}$$

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(*Problem 10.1*)

(*Thetas*)
theta12 = ArcSin[Sqrt[0.297]];
theta23 = ArcSin[Sqrt[0.437]];

(*Approximation to simplify the matrix*)
s13 = Sqrt[0.0214];
c13 = 1;

(*Neutrino mixing matrix*)
uMatix = {{Cos[theta12] * c13, Sin[theta12] * c13, s13},
   {- (Sin[theta12] * Cos[theta23]) - (Cos[theta12] * Sin[theta23] * s13), Cos[theta12] * Cos[theta23] - (Sin[theta12] * Sin[theta23] * s13),
    Sin[theta23] * c13},
   {Sin[theta12] * Sin[theta23] - (Cos[theta12] * Cos[theta23] * s13), - (Cos[theta12] * Sin[theta23]) - (Sin[theta12] * Cos[theta23] * s13),
    Cos[theta23] * c13}};

(*The probability of oscillating from one flavor α to another flavor β*)
P[alpha_, beta_] := KroneckerDelta[alpha - beta] -
   (4 * Sum[
      uMatix[[alpha, i]] * uMatix[[beta, i]] * uMatix[[alpha, j]] * uMatix[[beta, j]] * 0.5
       Boole[j > i], {j, 1, 3}, {i, 1, 3}]);
(*#3*)
Print["Fraction for μ to e is ", P[2, 1], ", μ to μ is ", P[2, 2], ", and μ to τ is ", P[2, 3]];

Fraction for μ to e is 0.27682, μ to μ is 0.340218, and μ to τ is 0.388486


(*#4*)
Print["Fraction for e to e is ", P[1, 1], ", e to μ is ", P[1, 2], ", and e to τ is ", P[1, 3]];
Print["Fraction for τ to e is ", P[3, 1], ", τ to μ is ", P[3, 2], ", and τ to τ is ", P[3, 3]];

Fraction for e to e is 0.539618, e to μ is 0.27682, and e to τ is 0.174626

Fraction for τ to e is 0.174626, τ to μ is 0.388486, and τ to τ is 0.440109
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# Results

| Probability | Normal | Inverted |
|---|---|---|
| $P_{\mu\to e}$ | 0.276820 | 0.226736 |
| $P_{\mu\to\mu}$ | 0.340218 | 0.397762 |
| $P_{\mu\to\tau}$ | 0.388486 | 0.379905 |

# Part 4: “Extreme Cases” Results

- Problem statement: assume astrophysical sources other than charged pions. What are the expected resulting flavor fractions?

| Probability | Normal | Inverted |
|---|---|---|
| $P_{\mu\to e}$ | 0.276820 | 0.226736 |
| $P_{\mu\to\mu}$ | 0.340218 | 0.397762 |
| $P_{\mu\to\tau}$ | 0.388486 | 0.379905 |
| $P_{e\to e}$ | 0.539618 | 0.538818 |
| $P_{e\to\mu}$ | 0.276820 | 0.226736 |
| $P_{e\to\tau}$ | 0.174626 | 0.225343 |
| $P_{\tau\to e}$ | 0.174626 | 0.225343 |
| $P_{\tau\to\mu}$ | 0.388486 | 0.379905 |
| $P_{\tau\to\tau}$ | 0.440109 | 0.399255 |

# Thanks!

References

[1] Dawn Williams
[2] PDG
[3] Our own genius

# Emergency Slide in Case of Naysayer

- Average over an interval is given by

$$\frac{1}{b-a}\int_a^b f(x)dx$$

- Apply this to $\sin^2$:

$$\frac{1}{\pi-0}\int_0^\pi sin^2 x dx = \frac{1}{\pi}(\frac{\pi}{2}) = \frac{1}{2}$$