VALOR/DUNE
Technique and Motivation for Near Detector Analysis

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Outline

- Our role in ND evaluation studies.
- The VALOR analysis strategy.
- Motivation for this approach.
- A few example plots from the FGT.
- Outlook and summary.

ND performance evaluation considerations

- ND **performance evaluation is debated extensively** in DUNE.
- Problem: Stringent requirements + limited time/manpower to evaluate
- Several colleagues favour **simple metrics** based on a narrow choice of channels and/or performance on the reduction of single systematics.
  - E.g. what is the efficiency and angular resolution for νe^- elastic scattering, and how well you constrain the absolute flux.
- But ND design is a complex, multi-dimensional optimization problem.
- Every metric will bias the design choices towards a different direction.
  - Obviously, νe^- is a key channel to be studied!
  - But, isn't it clear that if this is the only metric it will place perhaps undue weight on detector mass?
- How to make a balanced choice between different ND concepts?

ND performance evaluation considerations

- We took a very **broad and inclusive approach**.
- At the systematic error regime of DUNE, **any of a large number of systematics** can limit the sensitivity substantially.
  For each proposed ND concept:
  - Demonstrate adequate error reduction across the board.
  - $\rightarrow$ Employ a multi-channel analysis (VALOR)
- Not sufficient to just optimise a resolution or efficiency
  - Use oscillation physics driven metrics.
- Use **realistic reconstruction**.
  - *'Cheating but not lying'* (Steve Brice)
  - Try to reflect evaluate the physics capability after years of development.

A joint multi-channel analysis for ND design evaluation

- Different samples "*speak*" to different physics.
- A simultaneous fit of several exclusive event samples **maximizes physics sensitivity** by
  - breaking flux, cross-section and efficiency degeneracies, and
  - providing in-situ constraint on systematic uncertainties
- The method is statistically robust
  - Provides **correlations between physics parameters**.
  - Uses each event once
    (not always the case with more piece-wise approaches)
- It exploits the **complementarity and redundancy of information** that is brought about by this new generation of highly-capable NDs.

Which ND event samples are we looking at?

The VALOR analysis used for design optimization studies in DUNE considers **46 ND samples**. 23 samples for the neutrino-enhanced (FHC) beam configuration:

- ν_μ **CC**
  1. 1-track 0π (μ^- only)
  2. 2-track 0π (μ^- + nucleon)
  3. N-track 0π (μ^- + (>1) nucleons)
  4. 3-track Δ-enhanced ($\mu^- + \pi^+ +$ p, $W_{reco} \approx 1.2$ GeV)
  5. $1\pi^\pm$ ($\mu^- + 1\pi^\pm + X$)
  6. $1\pi^0$ ($\mu^- + 1\pi^0 + X$)
  7. $1\pi^\pm + 1\pi^0$ ($\mu^- + 1\pi^\pm + 1\pi^0 + X$)
  8. Other
- **Wrong-sign** ν_μ **CC**
  9. 0π ($\mu^+ + X$)
  10. $1\pi^\pm$ ($\mu^+ + \pi^\pm + X$)
  11. $1\pi^0$ ($\mu^+ + \pi^0 + X$)
  12. Other
- ν_e **CC**
  13. 0π ($e^- + X$)
  14. $1\pi^\pm$ ($e^- + \pi^\pm + X$)
  15. $1\pi^0$ ($e^- + \pi^0 + X$)
  16. Other
- **Wrong-sign** ν_e **CC**
  17. Inclusive
- **NC**
  18. 0π (nucleon(s))
  19. $1\pi^\pm$ ($\pi^\pm + X$)
  20. $1\pi^0$ ($\pi^0 + X$)
  21. Other
- ν**-e**
  22. $\nu_e + e^-$ elastic
  23. Inverse μ decay $\nu_\mu + e^- \rightarrow \mu^- + \nu_e$ and $\bar{\nu}_e + e^- \rightarrow \mu^- + \bar{\nu}_\mu$ (annih.)

and a similar set of 23 samples for the antineutrino enhanced (RHC) beam configuration.

How do we use all these samples?

We **perform a likelihood fit of $\approx$250 physics systematics**.
They are systematics controlling our estimates of neutrino fluxes, neutrino cross-sections, and hadron re-interaction probabilities.

$\approx$300 detector systematics are taken into account and are allowed to degrade our physics sensitivity.

We fit event rate histograms. The event rate is binned in true interaction type, as well as:

- $\{ E_{\nu;reco}, y_{reco} \}$ 2-D space for **CC-like** events, and
- $\{ E_{vis} \}$ 1-D space for **NC-like** events

where

- $E_{\nu;reco}$: reconstructed neutrino energy
- y_{reco} $(= \frac{E_{had;reco}}{E_{\nu;reco}})$: reconstructed inelasticity
- $E_{had;reco}$: reconstructed hadronic energy
- E_{vis}: visible energy

Physics systematics in the VALOR fit

Neutrino flux systematics: 208 normalization factors for "bins" in the 4-D space of (detector hall, beam configuration, neutrino species, energy range).

- 104 **ND hall** parameters

- 52 **FHC** parameters
    - 19 ν_μ parameters: Energy bins defined by (0, 0.5, 1., 1.5, 2., 2.5, 3., 3.5, 4., 4.5, 5., 5.5, 6., 7., 8., 12., 16., 20., 40., 100.) GeV.
    - 19 $\bar{\nu}_\mu$ parameters: as above
    - 7 ν_e parameters: Energy bins defined by (0., 2., 4., 6., 8., 10., 20., 100.) GeV.
    - 7 $\bar{\nu}_e$ parameters: as above
- 52 **RHC** parameters
    - Same decomposition as for ND/FHC

- 104 **FD hall** parameters

- Same decomposition as for ND

Physics systematics in the VALOR fit

Neutrino cross-section systematics:

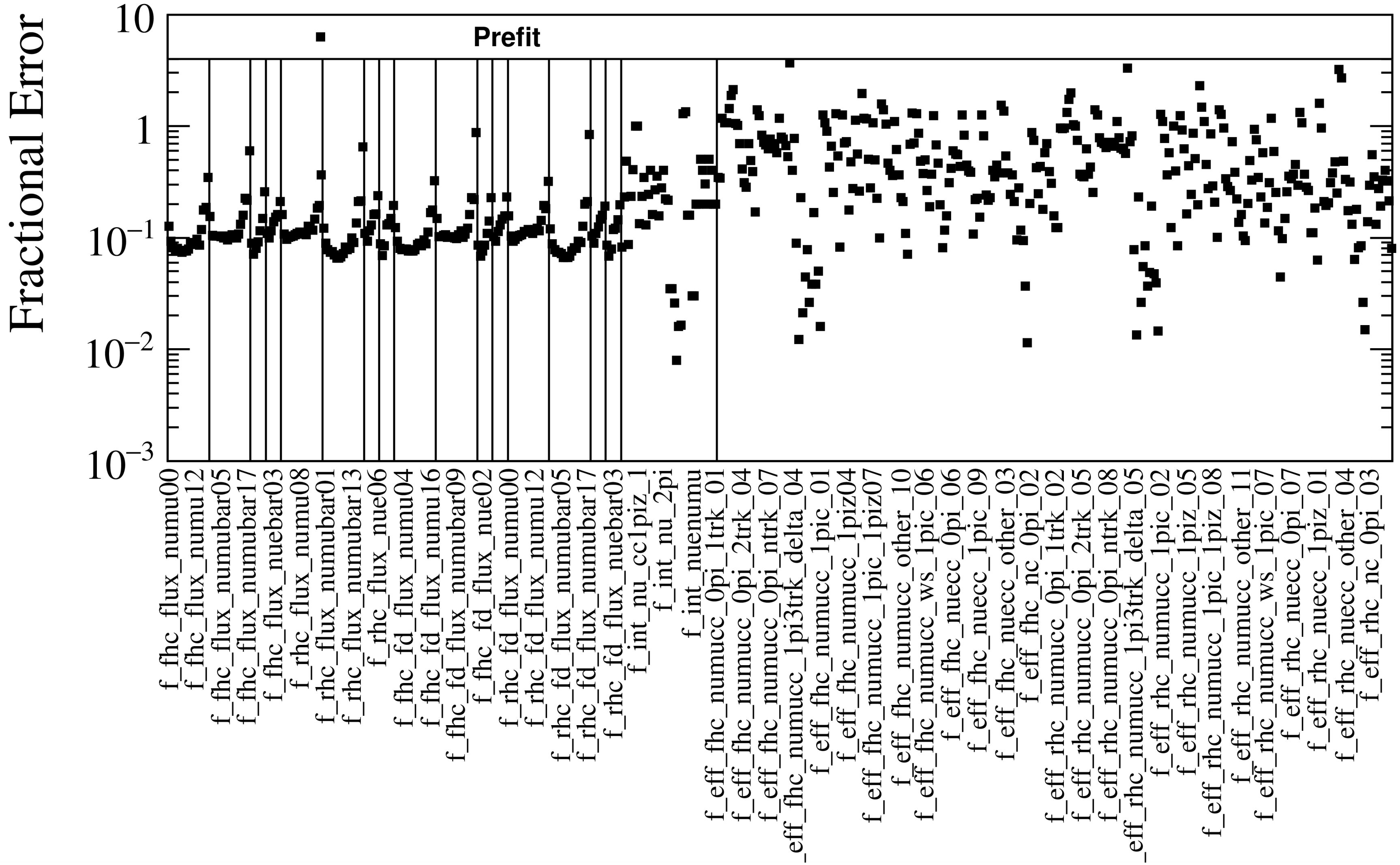
- 6 Q^2-dependent systematics for ν and $\bar{\nu}$ CC QE,
- 2 systematics for ν and $\bar{\nu}$ CC MEC,
- 6 Q^2-dependent systematics for ν and $\bar{\nu}$ CC $1\pi^{\pm}$,
- 6 Q^2-dependent systematics for ν and $\bar{\nu}$ CC $1\pi^{0}$,
- 2 systematics for ν and $\bar{\nu}$ CC 2π
- 6 energy-dependent systematics for ν and $\bar{\nu}$ CC DIS ($> 2\pi$)
- 2 systematics for ν and $\bar{\nu}$ CC coherent production of pions,
- 2 overall systematics for ν and $\bar{\nu}$ NC, and
- 1 ν_e/ν_μ cross-section ratio systematic.

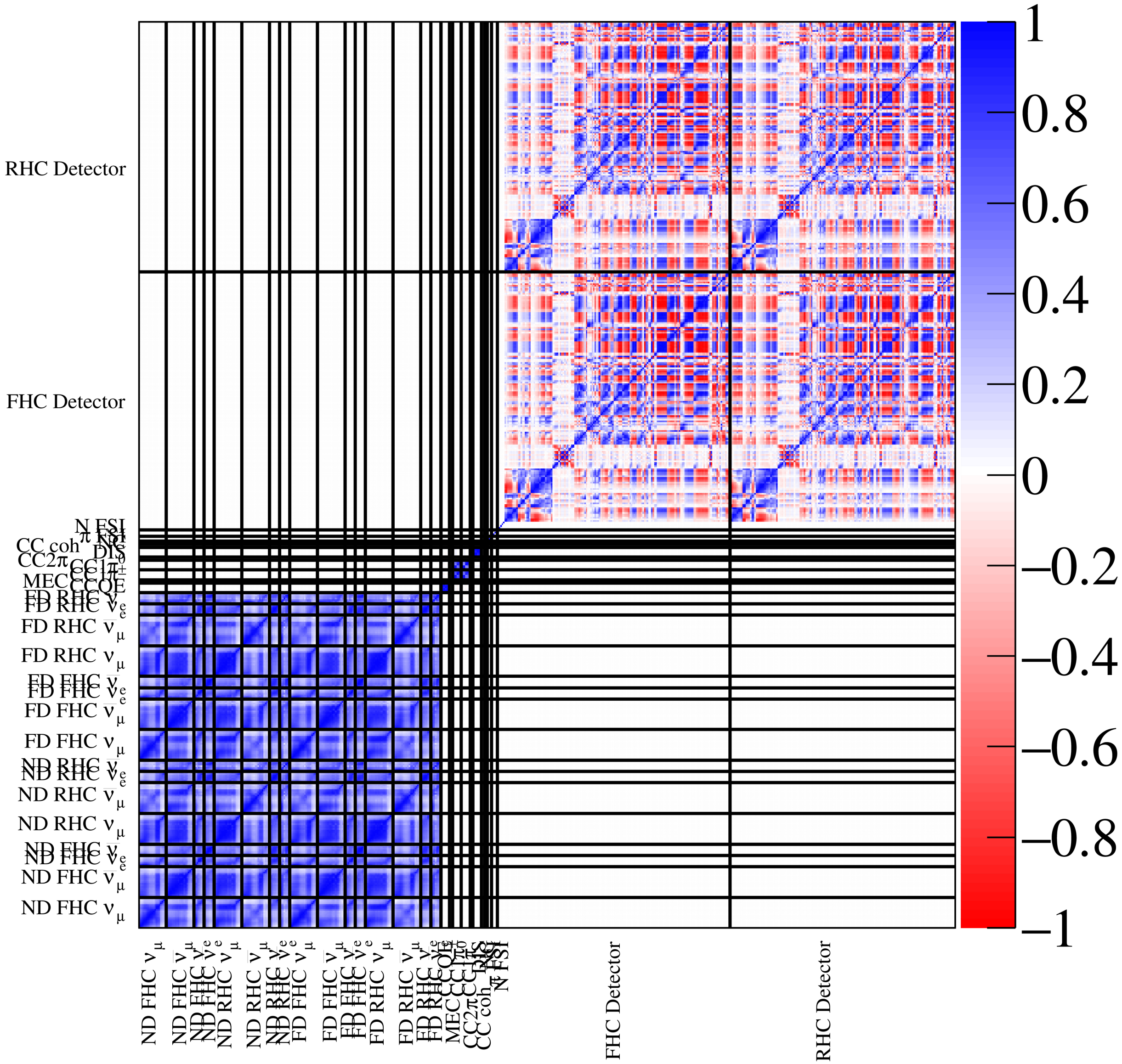
Hadronic re-interaction (FSI) systematics:

- 2 systematics on the overall re-interaction rate for pions and nucleons, and
- 8 systematics on the relative strength of different rescattering mechanisms (chg. exch., inelastic, absorption, pion production) for pions and nucleons.

Prior uncertainties

1σ **fractional error** for all ≈250 **physics** and ≈300 **detector** systematics.





A block diagonal matrix.

Three blocks:

- Flux (208×208)
- Interaction (43×43)
- Detector (310×310)

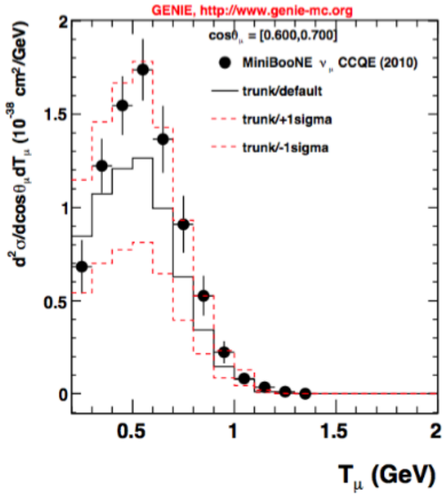
Prior uncertainties: Physics

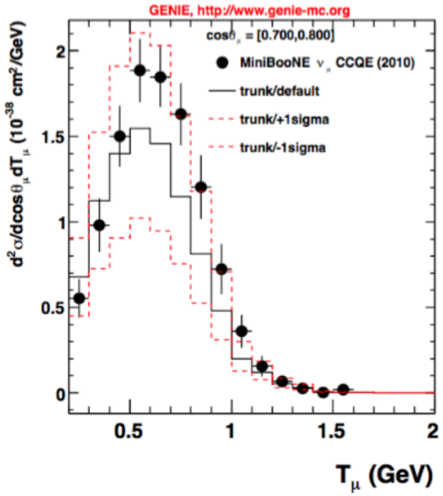
- **Flux uncertainties** come in the form of a 208×208 covar. matrix.
- We take a sum of two matrices which separately describe:
  - **Hadron production** uncertainties
    Error estimation derives from MINERvA work (L.Aliaga)
    http://lss.fnal.gov/archive/thesis/2000/fermilab-thesis-2016-03.pdf
    Caveat:
    Hadron production uncertainties are evaluated for the flux at the centre of the detector. This may result to too strong correlation between the near and far flux.
  - **Beam alignment** uncertainties.
    Evaluated using several MC runs with varied conditions.

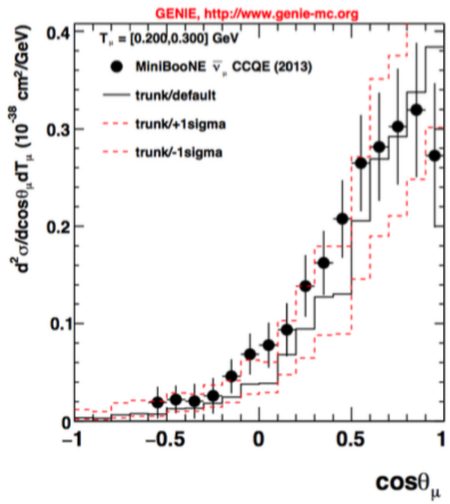
- Conservative prior **neutrino interaction** systematics assignments were supported by a series of data / GENIE MC comparisons.
  - These estimates are now further informed from the new GENIE global fit to neutrino scattering data.

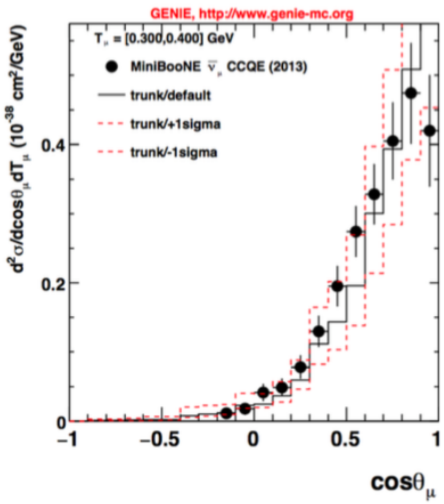
Prior uncertainties: Physics

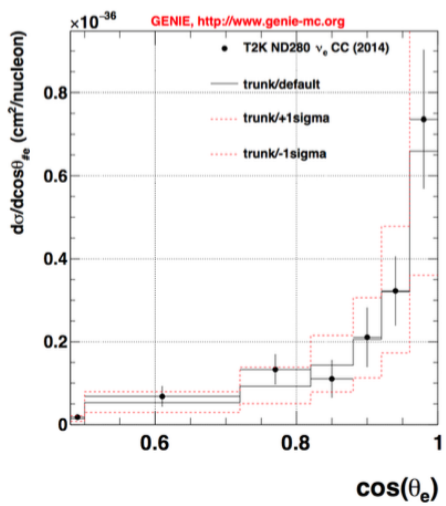
Conservative prior neutrino interaction systematics assignments were supported by a series of data / GENIE MC comparisons. More studies are in progress.

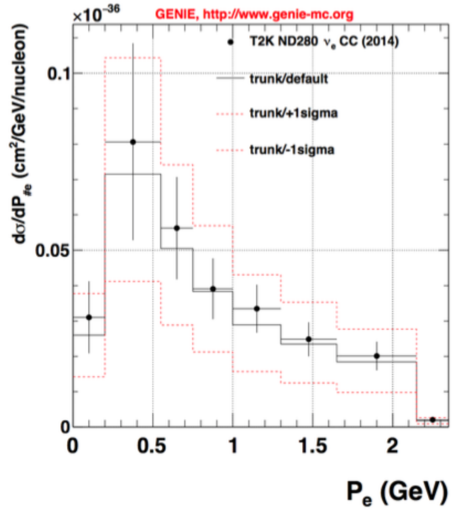


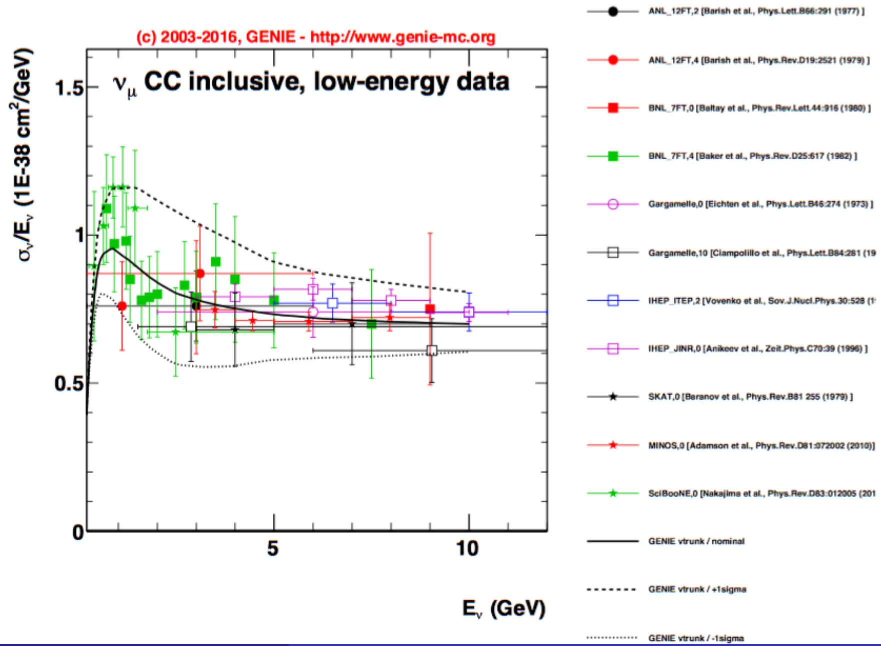












Prior uncertainties: Detector

$\approx$300 systematics encapsulating detector effects in various bin groups of the fitted distributions.

Capturing the uncertainty on event migration

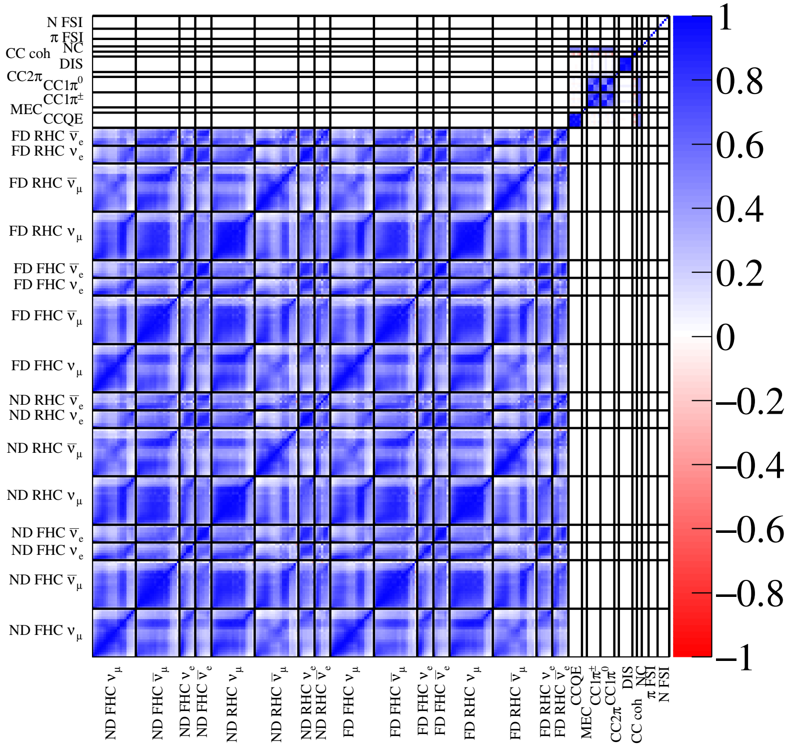
- between different samples, and
- between different kinematical bins.

Uncertainty in each fit bin was evaluated by variations of

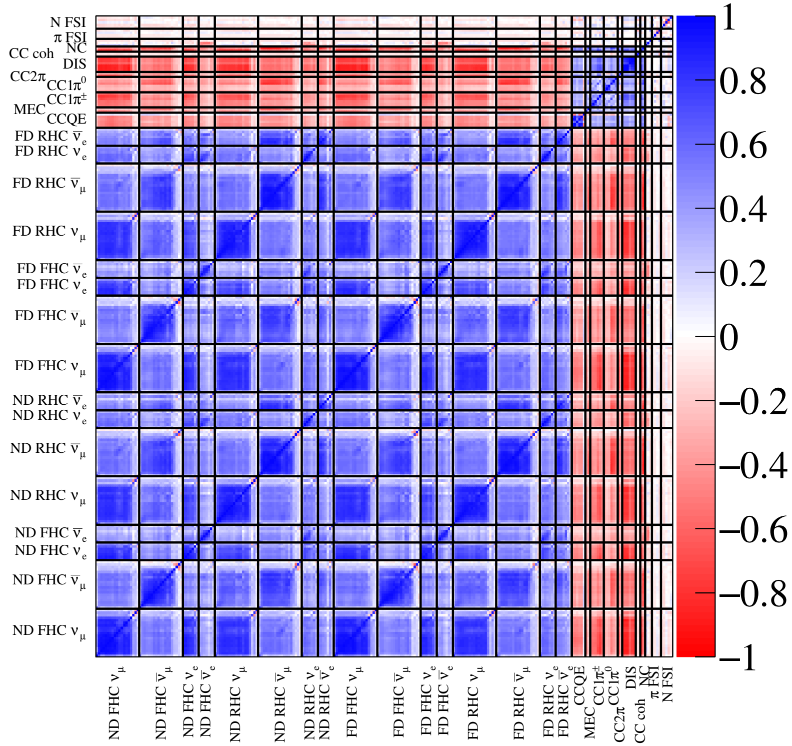
- Electron, muon and hadronic **energy scale**
- Electron, muon, proton, charged pion and neutral pion **efficiency**

Prior vs FGT post-fit uncertainties

Joint multi-channel fit **breaks systematic parameter correlations**.
As expected (experimental constraint is an event rate), flux and cross-section parameters become anti-correlated.

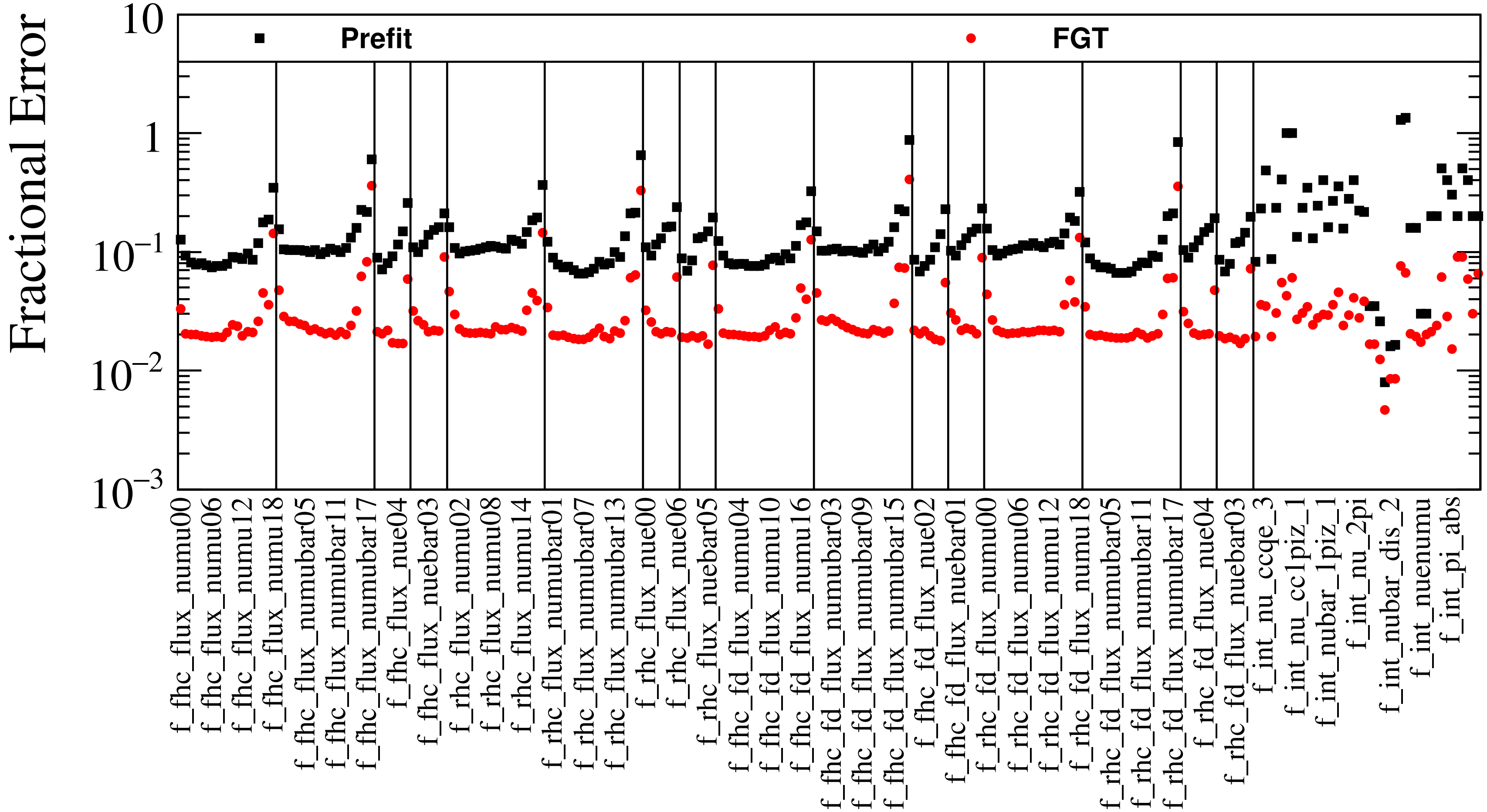


Pre-fit correlations



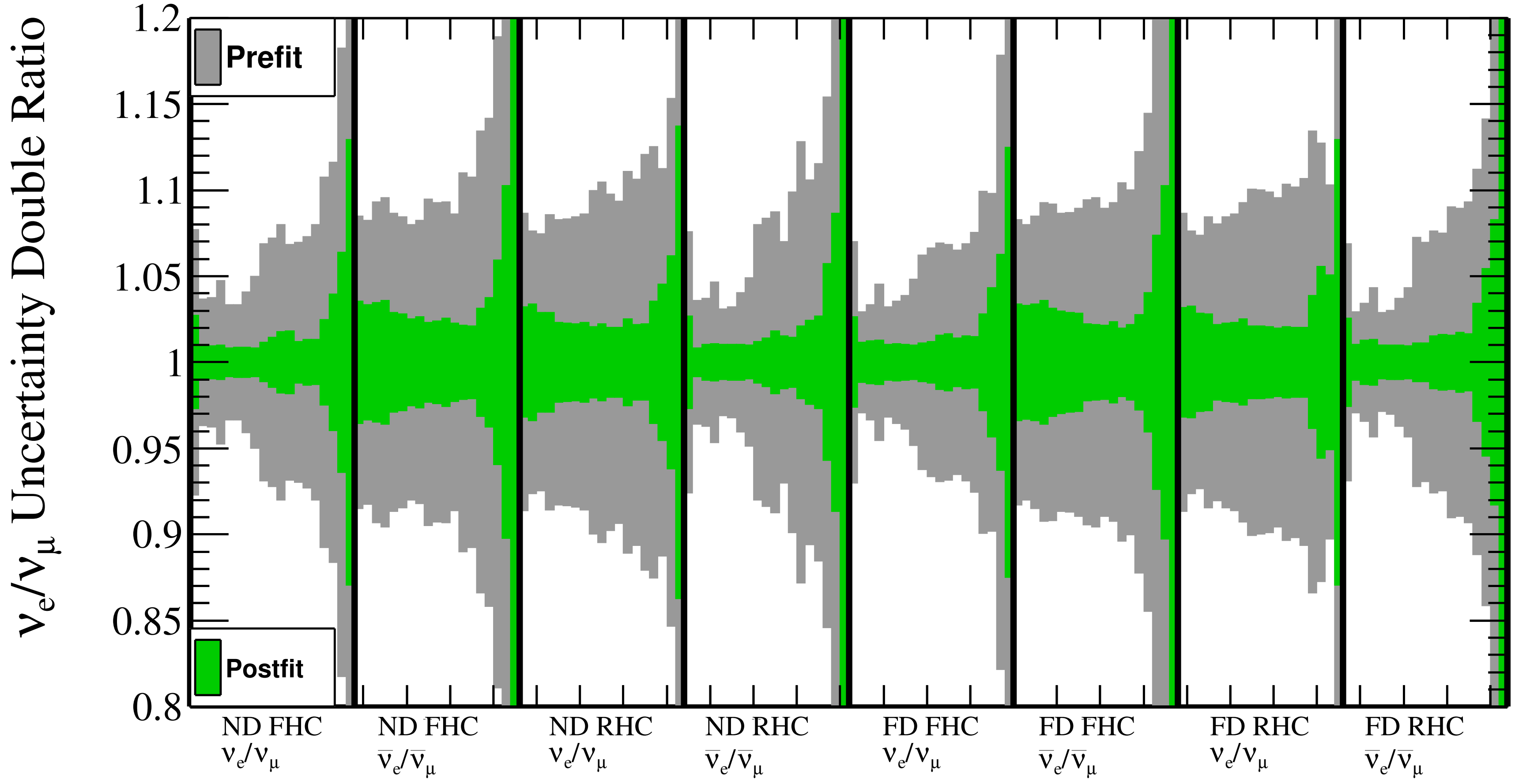
Post-fit correlations

Systematic error reduction with FGT fit



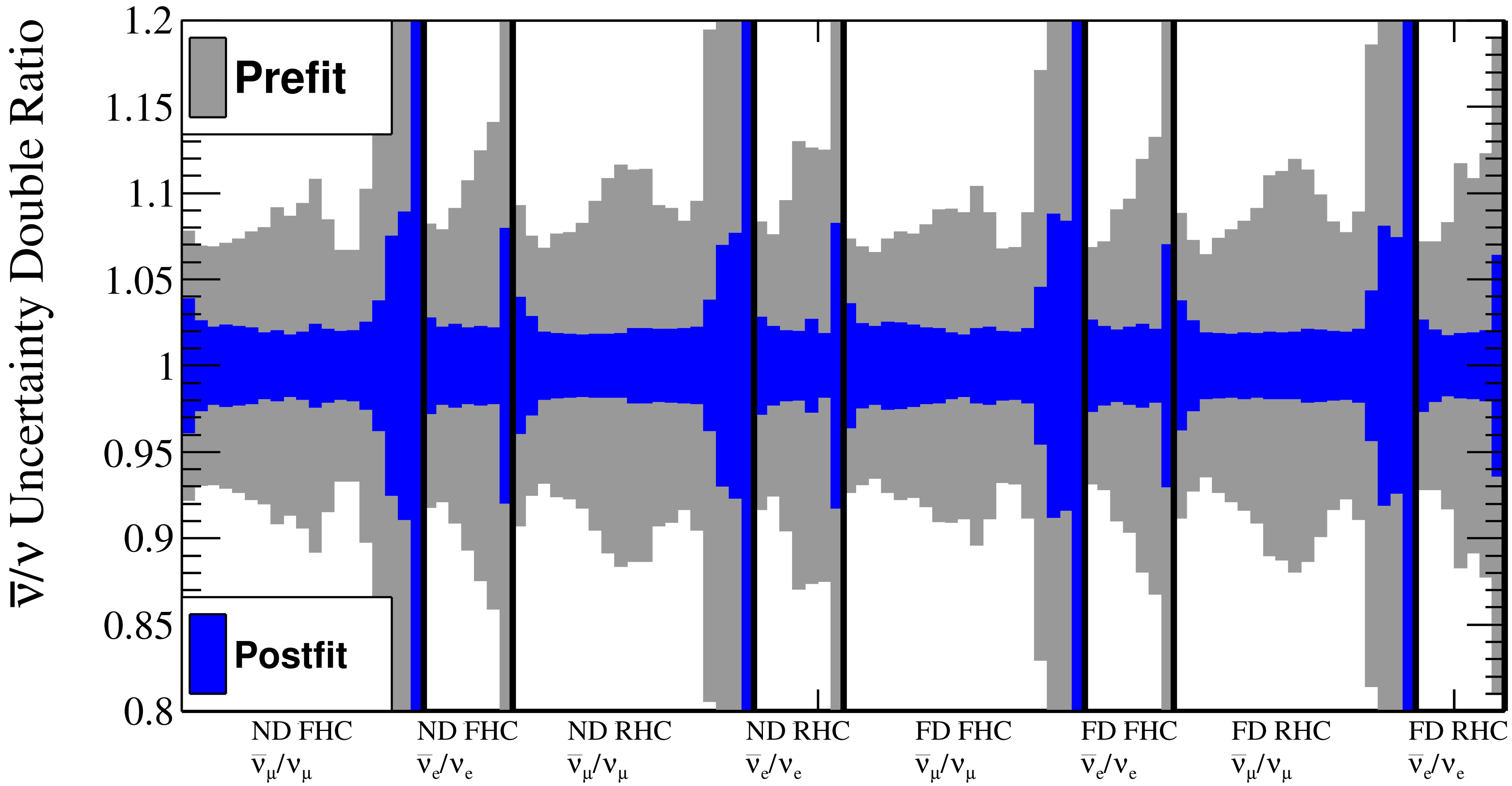
PRELIMINARY

Relative flux constraints: ν_e/ν_μ



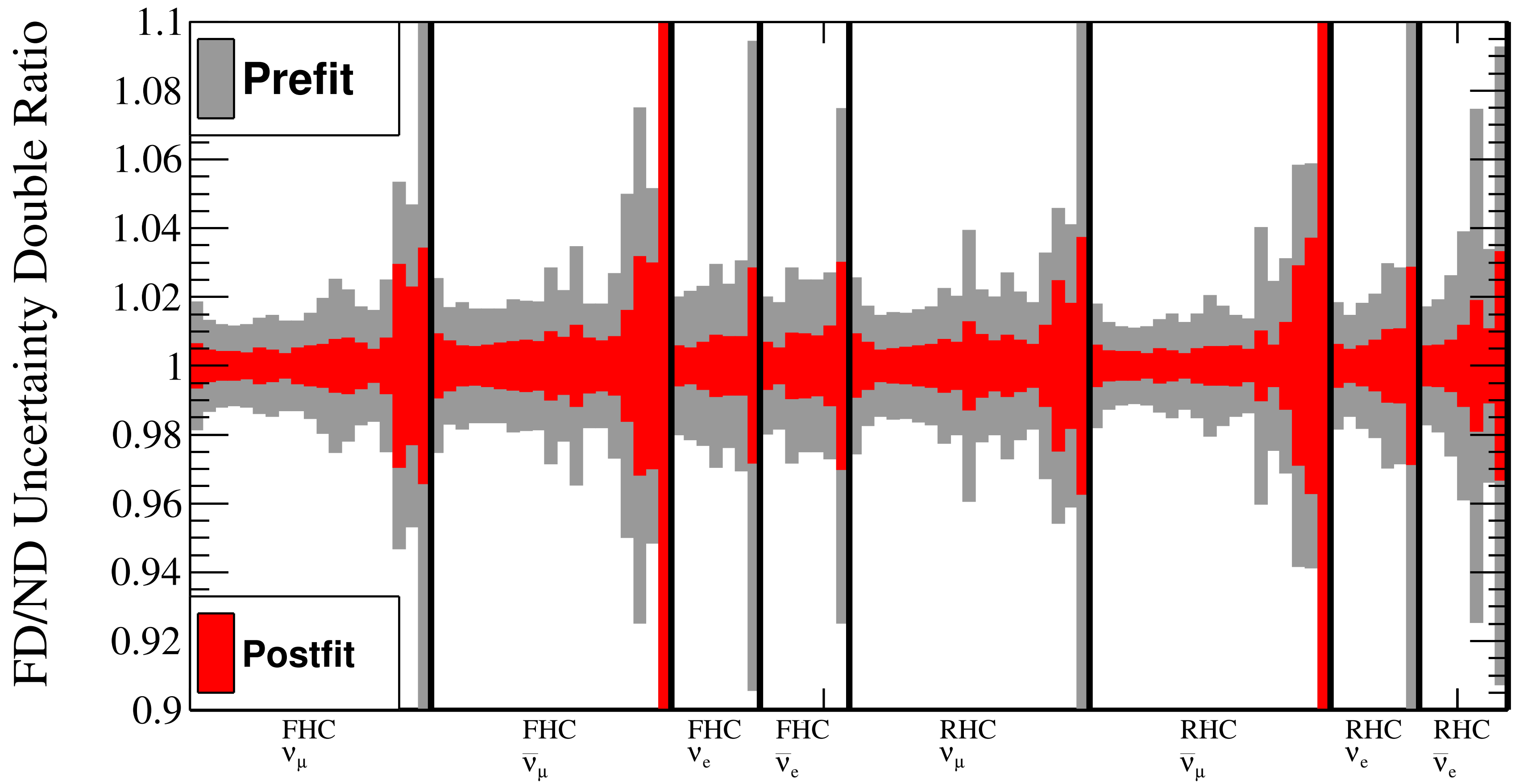
Relative flux constraints: $\bar{\nu}/\nu$

Spread of $\frac{(\bar{\nu}_{tweaked}/\nu_{tweaked})}{(\bar{\nu}_{nominal}/\nu_{nominal})}$ for different halls, beam configurations and ν species.



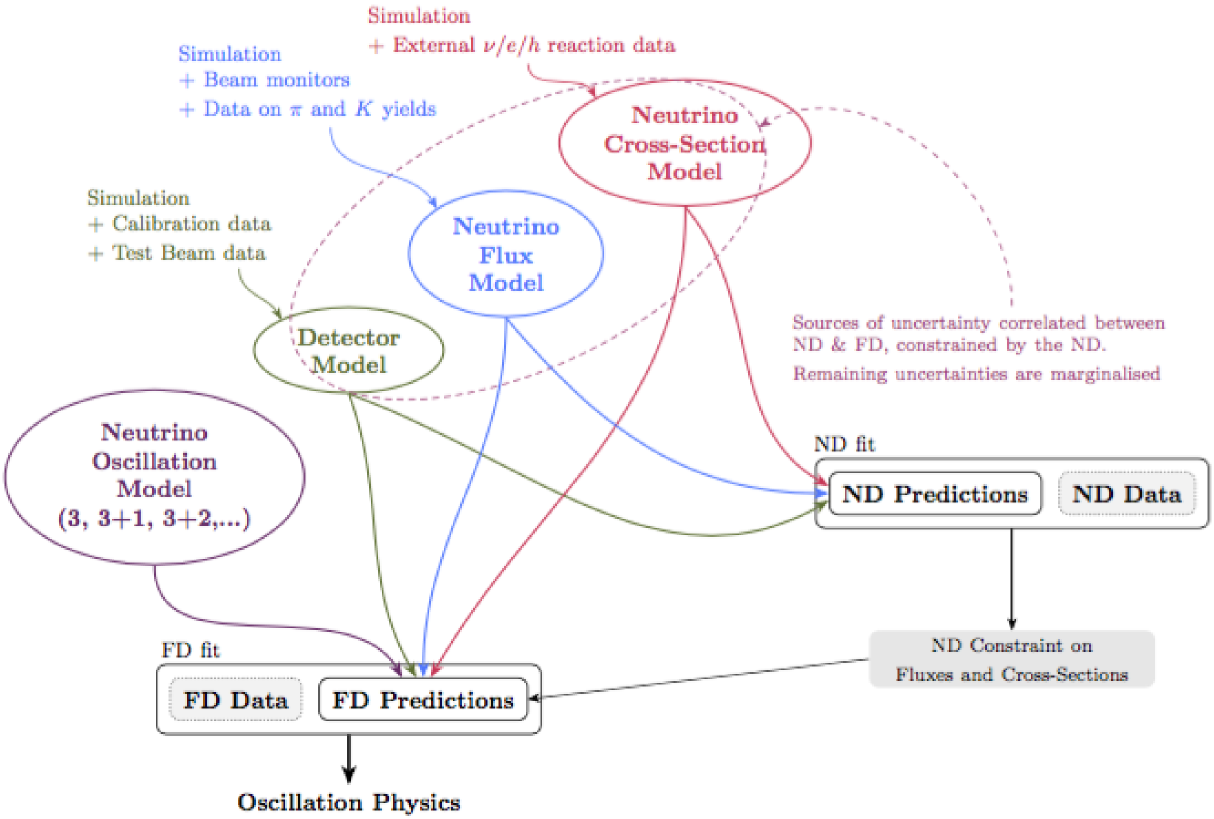
Relative flux constraints: Far/Near

Spread of $\frac{(FD_{tweaked}/ND_{tweaked})}{(FD_{nominal}/ND_{nominal})}$ for different configurations and neutrino species.



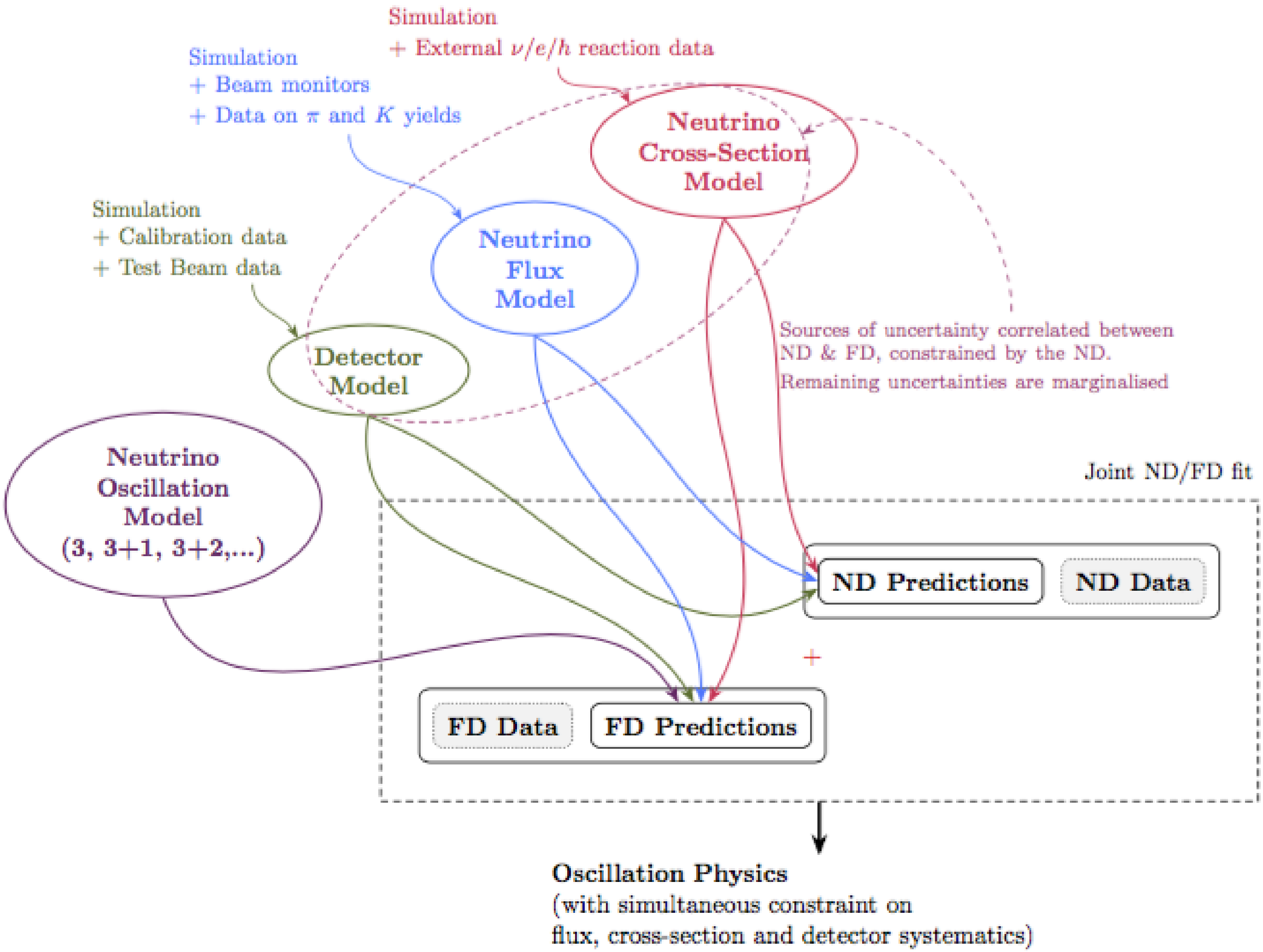
Oscillation analysis strategy implemented in VALOR

A two-step procedure used in (eg) T2K and available in DUNE: ND constraint followed by FD oscillation fit. VALOR capable of making an ND matrix propagatable to VALOR/LOAF FD-only fits.

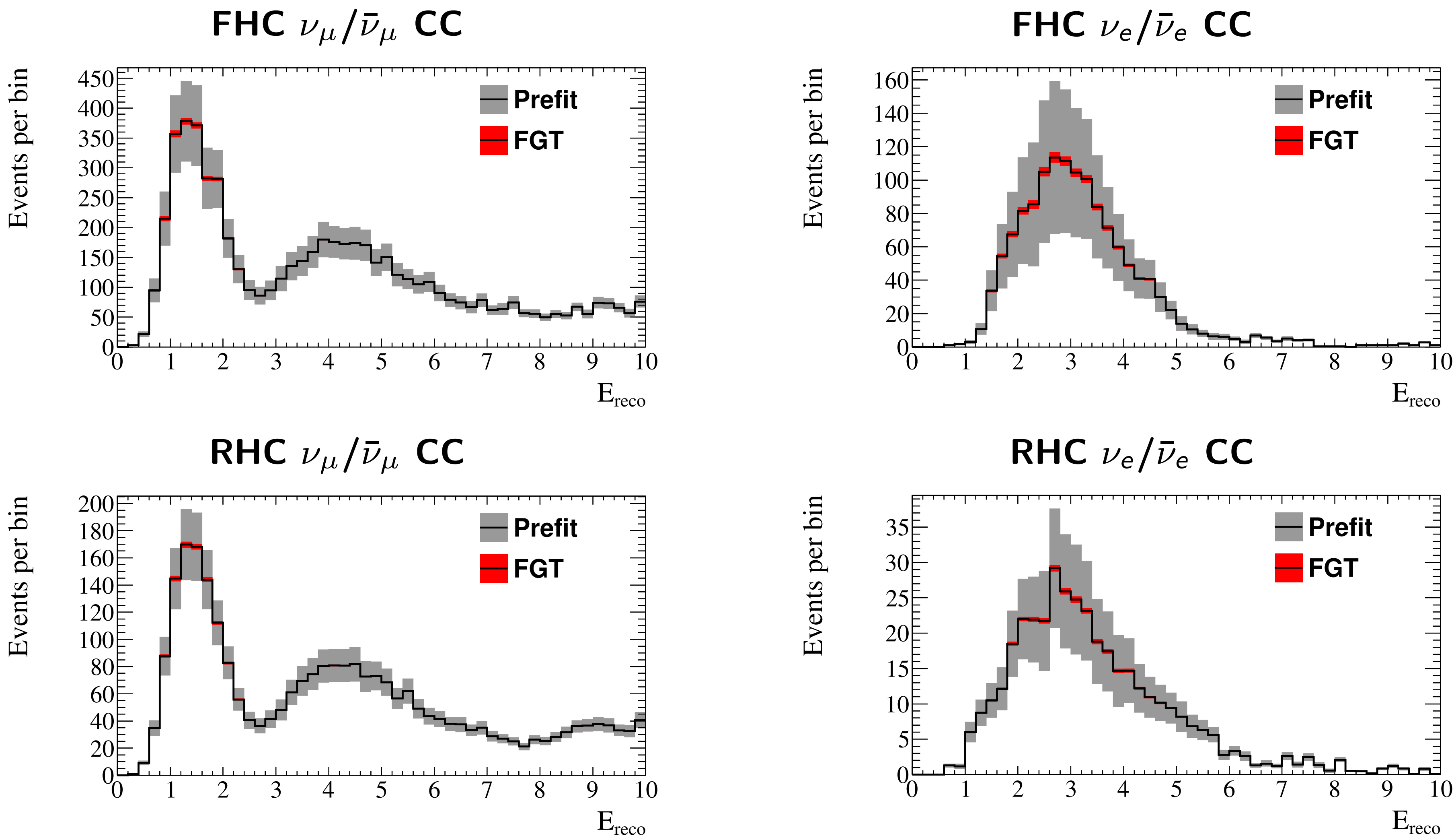


Oscillation analysis strategy implemented in VALOR

VALOR analysis for DUNE: A joint oscillation and systematics constraint fit was implemented.



Impact on FD event rate predictions



Impact on FD event rate predictions (1-6 GeV

	FHC		RHC	
	μ-like	e-like	μ-like	e-like
Flux + interaction w/o ND	16.8%	36.3%	15.0%	28.3%
Flux + interaction w/ FGT	1.0%	2.4%	0.9%	1.5%
Flux w/ FGT	1.9%	1.9%	1.8%	1.7%
Interaction w/ FGT	2.0%	3.9%	1.6%	2.7%

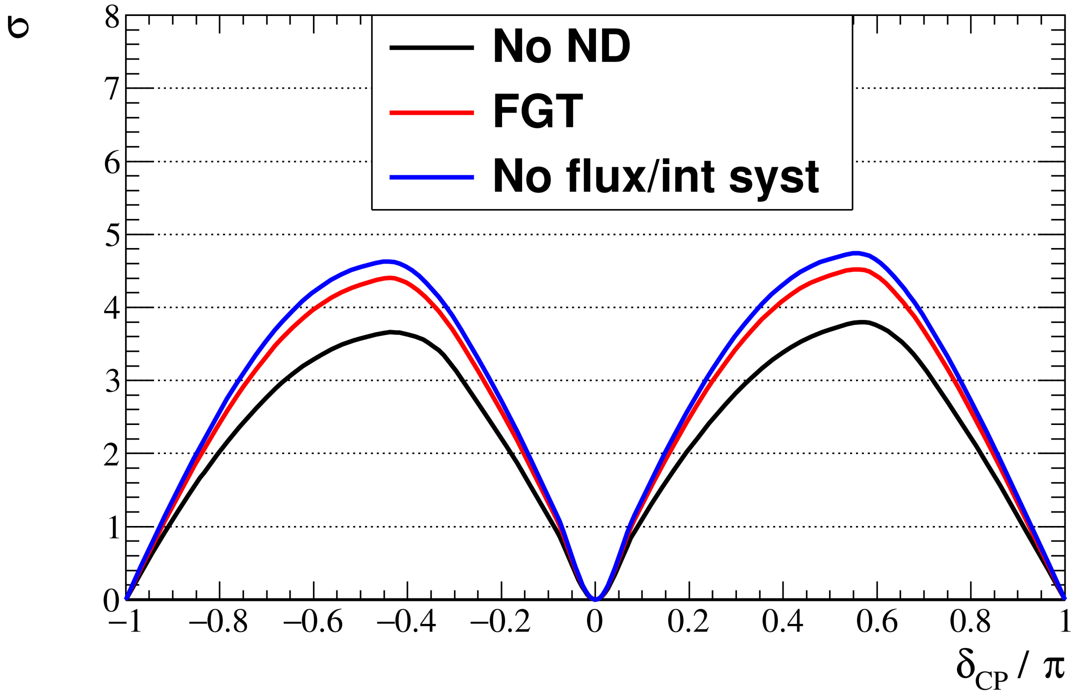
Note current huge effect of anticorrelations - making fitting a limited subset of systematics misleading.
Interesting to compare with CDR requirements - 2% on electrons, 5% on muons.

PRELIMINARY

Impact on the DUNE CP sensitivity

DUNE CP discovery sensitivity (for NuFit2016 best-fit parameters)
Exposure: $\approx$ 10-yr FHC + 10-yr RHC running (1.47×10^{21} POT/yr) with 40-kt fiducial FD)

Black: Full prior flux and interaction error. Red: With FGT constraint.



Note: Using **real FD reconstruction** (in its current state), hence reduced sensitivity (high NC bkg to e-like samples. Inputs not consistent w/ the LOAF plot showed in Kendall's talk - not expected to match.

PRELIMINARY

Outlook and summary

- Developed framework for:
  - ND **performance evaluation**, and
  - **oscillation physics-driven design optimization**.
- Final runs towards evaluating the performance of **HPGArTPC**, **LArTPC** and **FGT** DUNE ND concepts.
- **DUNE ND task force report** due shortly after we receive the final detector productions and uncertainties.
  - Lots of potential future studies on value/effect of improved detector performance.

Backup slides

The VALOR group

VALOR is a well-established neutrino fitting group.

(2010 - present); https://valor.pp.rl.ac.uk

Costas Andreopoulos[1,2], Chris Barry[1], Francis Bench[1], Andy Chappell[3], Thomas Dealtry[4], Steve Dennis[1], Lorena Escudero[5], Rhiannon Jones[1], Nick Grant[3], Marco Roda[1], Davide Sgalaberna[6], Raj Shah[2,7]

[*Faculty*, *Postdocs (former PhD students with VALOR T2K PhD theses)*, *Postdocs*, *Current PhD students*]

[1] University of Liverpool, [2] STFC Rutherford Appleton Laboratory, [3] University of Warwick, [4] Lancaster University, [5] University of Cambridge, [6] University of Geneva, [7] University of Oxford

VALOR fit
Physics parameterization

VALOR fit: Construction of likelihood

A joint VALOR fit considers simultaneously:

- A flexibly-defined **set of detectors d**. *E.g.* $d \in \{SBND, \mu BooNE, ICARUS\}$.
- A flexibly-defined **set of beam configurations b** (for each d). *E.g.* $b \in \{FHC, RHC, ...\}$
- A flexibly-defined **set of event selections s** (for each d and b). *E.g. see page 11.*

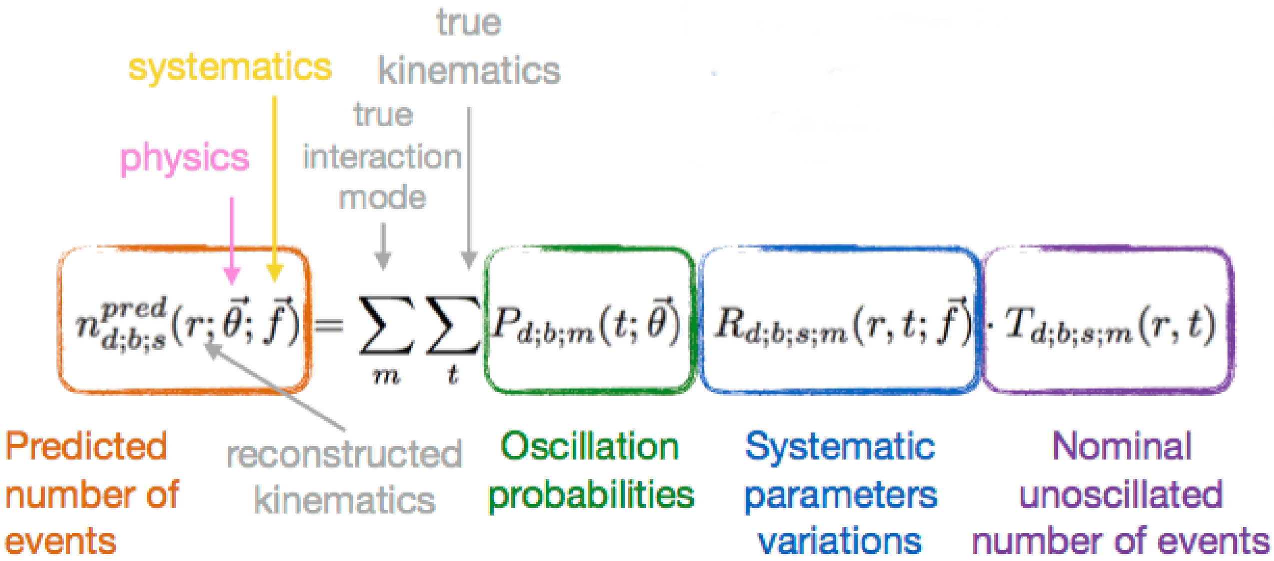
For each (d,b,s):

- Experimental information is recorded in a number of **multi-dim. reco. kinematical bins r**
  E.g. $r \equiv \{\ E_{\nu;reco}\ \}, \{E_{\nu;reco},\ y_{reco}\ \}, \{\ p_{\ell;reco},\ \theta_{\ell;reco}\ \}, \{\ E_{vis;reco}\ \}, \ldots$

Our predictions for

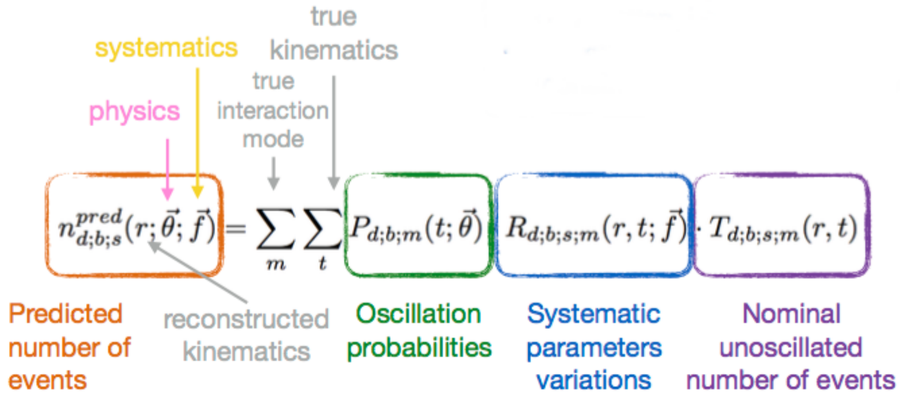
- a set of **interesting physics params** $\vec{\theta}$ (*e.g.* $\{\theta_{23},\ \delta_{CP},\ \Delta m^2_{31}\}$ *or* $\{\theta_{\mu e},\ \theta_{\mu\mu},\ \Delta m^2_{41}\}$), and
- a set of O(10^2)-O(10^3) **systematic (nuisance) params** $\vec{f}$

are constructed as follows:



VALOR fit: Construction of likelihood

Predictions are built using **MC templates** $T_{d;b;s;m}(r,t)$ constructed by applying event selection code to the output of a full event simulation and reconstruction chain.



For each (d,b,s), MC templates are constructed for a set of **true reaction modes m**.

- Currently, templates are constructed for the 52 true reaction modes shown on the right.

The templates store the mapping between reconstructed and truth information (as derived from full simulation and reconstruction).

- E.g. $\{ E_{\nu;true},\ Q^2_{true},\ W_{true} \} \leftrightarrow \{ p_{\ell;reco},\ \theta_{\ell;reco} \}$

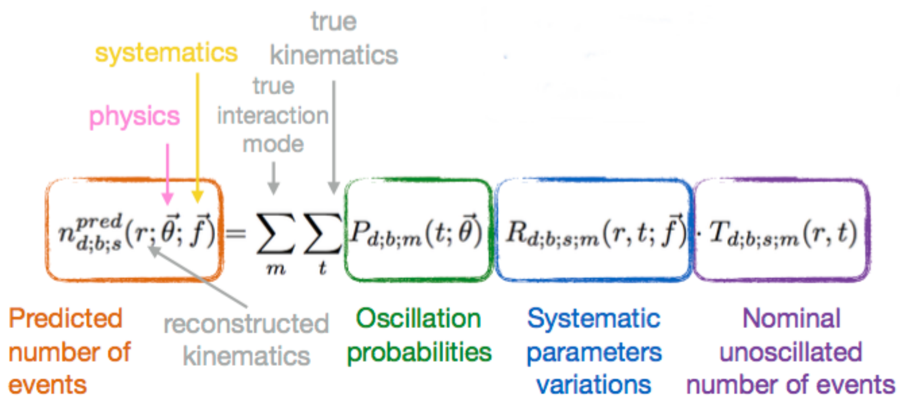
The choice of true kinematical space { t } and true reaction modes m is **highly configurable** for each (d,b,s) independently.

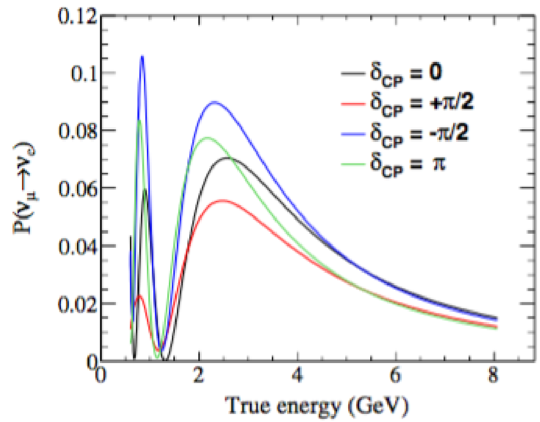
- Main consideration: **Sufficient granularity to apply desired physics and systematic effects** (function of truth quantities).

- ν_μ CC QE
- ν_μ CC MEC
- ν_μ CC $1\pi^\pm$
- ν_μ CC $1\pi^0$
- ν_μ CC $2\pi^\pm$
- ν_μ CC $2\pi^0$
- ν_μ CC $1\pi^\pm + 1\pi^0$
- ν_μ CC coherent
- ν_μ CC other
- ν_μ NC $1\pi^\pm$
- ν_μ NC $1\pi^0$
- ν_μ NC coherent
- ν_μ NC other
- **similarly for** $\bar{\nu}_\mu$
- **similarly for** ν_e
- **similarly for** $\bar{\nu}_e$

VALOR fit: Construction of likelihood

Finally, the effect of **neutrino oscillations** is included in $P_{d;b;m}(t;\vec{\theta})$.



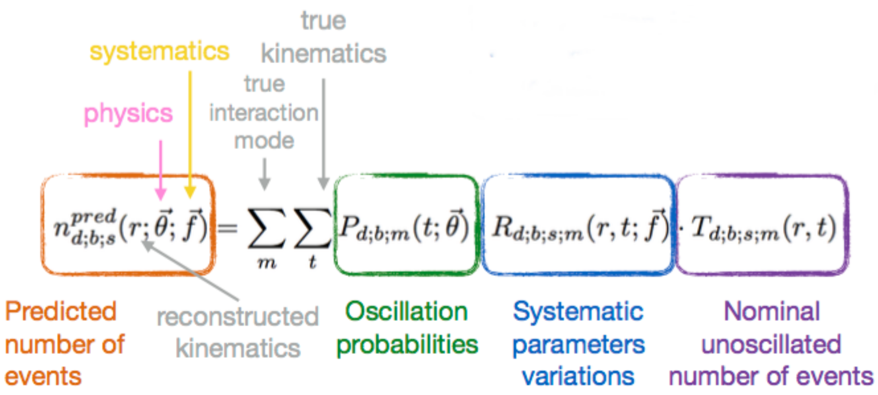


- Using bespoke library for calculation of osc. probabilities.
- **Very fast!**
- **Extensively validated** against GloBES and Prob3++.
- Supports 3-flavour calculations (incl. standard matter / NSI effects) and, also, calculations in 3+1, 3+2, 1+3+1 schemes.
- **Flexibility** provided by bespoke library is immensely useful (tuning performance, moving between different parameter conventions, trying out different oscillation frameworks).

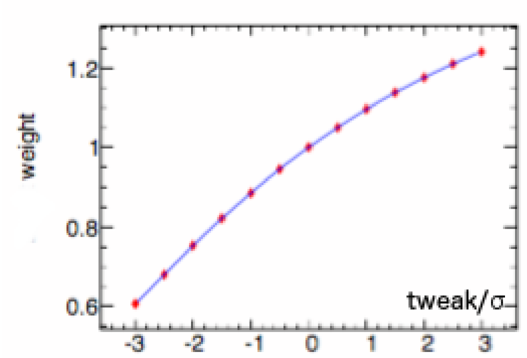
- $\sin^2(\theta_{12}) = 0.3$
- $\sin^2(\theta_{13}) = 0.025$
- $\sin^2(\theta_{23}) = 0.5$
- $\Delta m^2_{21} = 7.5\times10^{-5}$ eV^2/c^4
- $\Delta m^2_{32} = 2.5\times10^{-3}$ eV^2/c^4
- Normal ordering
- Earth matter density = 2.7 g/cm^3
- Baseline = 1300 km

VALOR fit: Construction of likelihood

Systematic variations are applied using the **response functions** $\boldsymbol{R_{d;b;s;m}(r, t; \vec{f})}$.



Example of a non-linear response function.



Typically, but not always, the response $R_{d;b;s;m}(r, t; \vec{f})$ factorises and it can be written as

$$R_{d;b;s;m}(r, t; \vec{f}) = \prod_{i=0}^{N-1} R^i_{d;b;s;m}(r, t; f_i)$$

For several systematics the response is linear and, therefore,

$$R^i_{d;b;s;m}(r, t; f_i) \propto f_i$$

For non linear systematics, the response function $R^i_{d;b;s;m}(r, t; f_i)$ is pre-computed (for every detector, beam, sample, mode, true kinematical bin and reconstructed kinematical bin) using event reweighting libraries in the [-5σ, +5σ] range of the parameter f_i and it is represented internally using an Akima spline.

VALOR fit: Construction of likelihood

Once we have estimates of $n^{pred}_{d;b;s}(r;\vec{\theta};\vec{f})$, VALOR computes a **likelihood ratio**:

$$ln\ \lambda_{d;b;s}(\vec{\theta};\vec{f}) = -\sum_{r}\left\{\left(n^{pred}_{d;b;s}(r;\vec{\theta};\vec{f}) - n^{obs}_{d;b;s}(r)\right) + n^{obs}_{d;b;s}(r)\cdot ln\frac{n^{obs}_{d;b;s}(r)}{n^{pred}_{d;b;s}(r;\vec{\theta};\vec{f})}\right\}$$

$$\lambda_{SBN}(\vec{\theta};\vec{f}) = \prod_{d}\prod_{b}\prod_{s}\lambda_{d;b;s}(\vec{\theta};\vec{f})$$

Most parameters in the fit come with prior constraints from external data. Where needed, the following Gaussian penalty term is computed:

$$ln\ \lambda_{prior}(\vec{\theta};\vec{f}) = -\frac{1}{2}\left\{(\vec{\theta}-\vec{\theta}_0)^T C_\theta^{-1}(\vec{\theta}-\vec{\theta}_0) + (\vec{f}-\vec{f}_0)^T C_f^{-1}(\vec{f}-\vec{f}_0)\right\}$$

and combined likelihood ratio is given by:

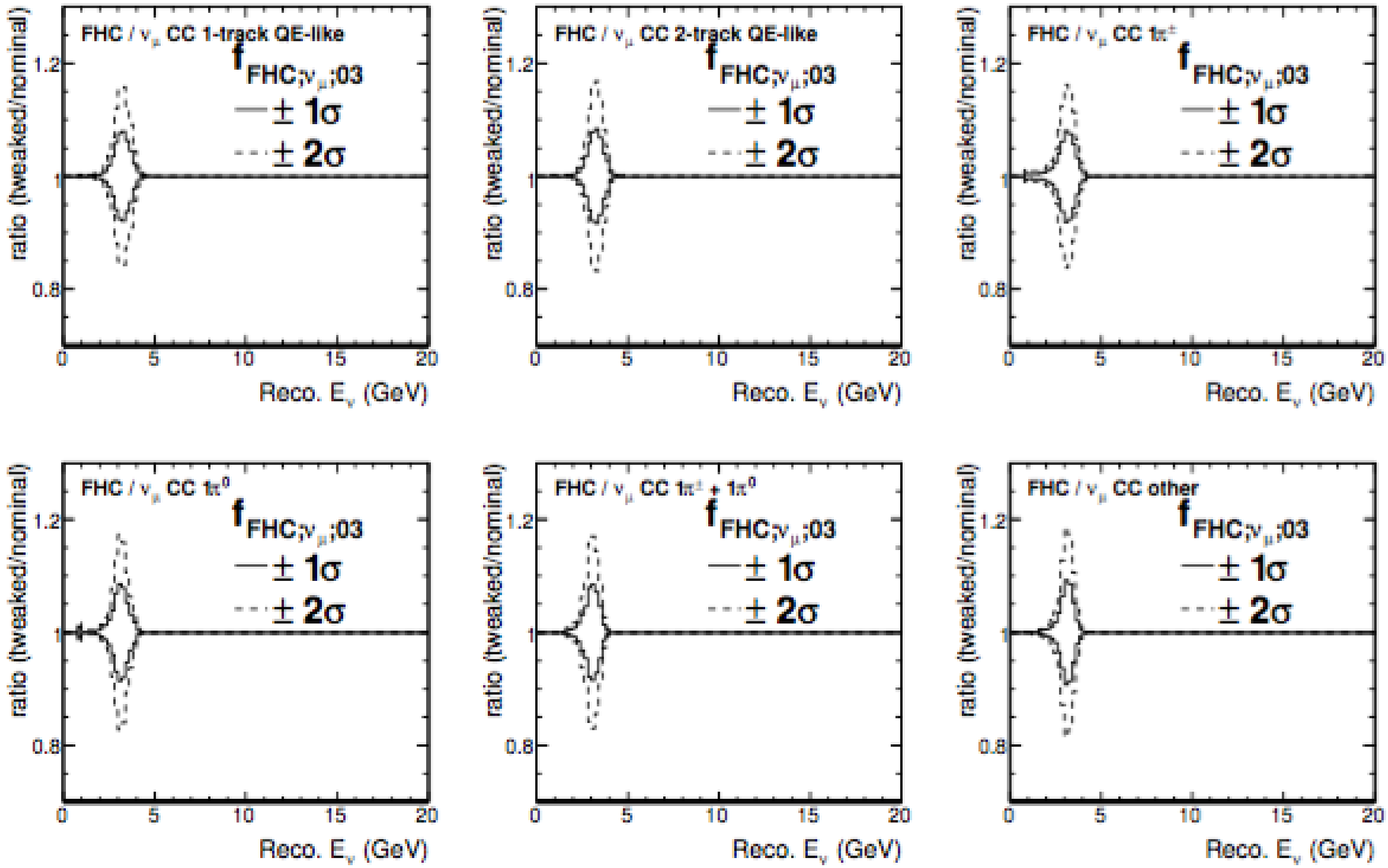
$$\lambda(\vec{\theta};\vec{f}) = \lambda_{SBN}(\vec{\theta};\vec{f})\cdot\lambda_{prior}(\vec{\theta};\vec{f})$$

In the large-sample limit, the quantity $-2\lambda(\vec{\theta};\vec{f})$ has a χ^2 distribution and it can therefore be used as a goodness-of-fit test.

Systematics in the VALOR fit - Example variation

Pre-fit effect of a flux systematic [ν_μ FHC at 3.0-3.5 GeV] on selected VALOR/DUNE samples.

The ratios of tweaked/nominal spectra for $\pm 1\sigma$ and $\pm 2\sigma$ variations are shown.



Physics systematics in the VALOR fit I

Idx	Name	Physics quantity
0-18	$f_{ND;FHC;\nu_\mu;00}$ - $f_{ND;FHC;\nu_\mu;18}$	FHC ν_μ flux at the ND hall in the 18 true energy bins defined by the following bin edges: (0.0, 0.5, 1.0, 1.5, 2.0, 2.5, 3.0, 3.5, 4.0, 4.5, 5.0, 5.5, 6.0, 7.0, 8.0, 12.0, 16.0, 20.0, 40.0, 100.0) GeV.
19-37	$f_{ND;FHC;\bar{\nu}_\mu;00}$ - $f_{ND;FHC;\bar{\nu}_\mu;18}$	FHC $\bar{\nu}_\mu$ flux at the ND hall in same 18 true energy bins listed above.
38-44	$f_{ND;FHC;\nu_e;00}$ - $f_{ND;FHC;\nu_e;06}$	FHC ν_e flux at the ND hall in the 7 true energy bins defined by the following bin edges: (0.0, 2.0, 4.0, 6.0, 8.0, 10.0, 20.0, 100.0) GeV.
45-51	$f_{ND;FHC;\bar{\nu}_e;00}$ - $f_{ND;FHC;\bar{\nu}_e;06}$	FHC $\bar{\nu}_e$ flux at the ND hall in same 7 true energy bins listed above.
52-70	$f_{ND;RHC;\nu_\mu;00}$ - $f_{ND;RHC;\nu_\mu;18}$	RHC ν_μ flux at the ND hall in the 18 true energy bins defined by the following bin edges: (0.0, 0.5, 1.0, 1.5, 2.0, 2.5, 3.0, 3.5, 4.0, 4.5, 5.0, 5.5, 6.0, 7.0, 8.0, 12.0, 16.0, 20.0, 40.0, 100.0) GeV.
71-89	$f_{ND;RHC;\bar{\nu}_\mu;00}$ - $f_{ND;RHC;\bar{\nu}_\mu;18}$	RHC $\bar{\nu}_\mu$ flux at the ND hall in same 18 true energy bins listed above.
90-96	$f_{ND;RHC;\nu_e;00}$ - $f_{ND;RHC;\nu_e;06}$	RHC ν_e flux at the ND hall in the 7 true energy bins defined by the following bin edges: (0.0, 2.0, 4.0, 6.0, 8.0, 10.0, 20.0, 100.0) GeV.
97-103	$f_{ND;RHC;\bar{\nu}_e;00}$ - $f_{ND;RHC;\bar{\nu}_e;06}$	RHC $\bar{\nu}_e$ flux at the ND hall in same 7 true energy bins listed above.
104-122	$f_{FD;FHC;\nu_\mu;00}$ - $f_{FD;FHC;\nu_\mu;18}$	FHC ν_μ flux at the FD hall in the 18 true energy bins defined by the following bin edges: (0.0, 0.5, 1.0, 1.5, 2.0, 2.5, 3.0, 3.5, 4.0, 4.5, 5.0, 5.5, 6.0, 7.0, 8.0, 12.0, 16.0, 20.0, 40.0, 100.0) GeV.
123-141	$f_{FD;FHC;\bar{\nu}_\mu;00}$ - $f_{FD;FHC;\bar{\nu}_\mu;18}$	FHC $\bar{\nu}_\mu$ flux at the FD hall in same 18 true energy bins listed above.
142-148	$f_{FD;FHC;\nu_e;00}$ - $f_{FD;FHC;\nu_e;06}$	FHC ν_e flux at the FD hall in the 7 true energy bins defined by the following bin edges: (0.0, 2.0, 4.0, 6.0, 8.0, 10.0, 20.0, 100.0) GeV.

Physics systematics in the VALOR fit II

149-155	$f_{FD;FHC;\bar{\nu}_e;00}$ $f_{FD;FHC;\bar{\nu}_e;06}$	-	FHC $\bar{\nu}_e$ flux at the FD hall in same 7 true energy bins listed above.
156-174	$f_{FD;RHC;\nu_\mu;00}$ $f_{FD;RHC;\nu_\mu;18}$	-	RHC ν_μ flux at the FD hall in the 18 true energy bins defined by the following bin edges: (0.0, 0.5, 1.0, 1.5, 2.0, 2.5, 3.0, 3.5, 4.0, 4.5, 5.0, 5.5, 6.0, 7.0, 8.0, 12.0, 16.0, 20.0, 40.0, 100.0) GeV.
175-193	$f_{FD;RHC;\bar{\nu}_\mu;00}$ $f_{FD;RHC;\bar{\nu}_\mu;18}$	-	RHC $\bar{\nu}_\mu$ flux at the FD hall in same 18 true energy bins listed above.
194-200	$f_{FD;RHC;\nu_e;00}$ $f_{FD;RHC;\nu_e;06}$	-	RHC ν_e flux at the FD hall in the 7 true energy bins defined by the following bin edges: (0.0, 2.0, 4.0, 6.0, 8.0, 10.0, 20.0, 100.0) GeV.
201-207	$f_{FD;RHC;\bar{\nu}_e;00}$ $f_{FD;RHC;\bar{\nu}_e;06}$	-	RHC $\bar{\nu}_e$ flux at the FD hall in same 7 true energy bins listed above.
208 - 210	$f_{\nu CCQE;1}$ $f_{\nu CCQE;3}$	-	ν_μ CC QE cross-section for the 3 true Q^2 bins defined by the following bin edges: (0, 0.2, 0.55, ∞) GeV^2.
211 - 213	$f_{\bar{\nu} CCQE;1}$ $f_{\bar{\nu} CCQE;3}$	-	$\bar{\nu}_\mu$ CC QE cross-section for the same 3 true Q^2 bins defined above.
214	$f_{\nu CCMEC}$		ν_μ CC MEC cross-section
215	$f_{\bar{\nu} CCMEC}$		$\bar{\nu}_\mu$ CC MEC cross-section
216 - 218	$f_{\nu CC1\pi^0;1}$ $f_{\nu CC1\pi^0;3}$	-	ν CC1π^0 cross-section for the 3 true Q^2 bins defined by the following bin edges: (0, 0.35, 0.9, ∞) GeV^2.
219 - 221	$f_{\nu CC1\pi^\pm;1}$ $f_{\nu CC1\pi^\pm;3}$	-	ν CC1$\pi^\pm$ cross-section for the 3 true Q^2 bins defined by the following bin edges: (0, 0.3, 0.8, ∞) GeV^2.
222 - 224	$f_{\bar{\nu} CC1\pi^0;1}$ $f_{\bar{\nu} CC1\pi^0;3}$	-	$\bar{\nu}$ CC1π^0 cross-section for the same 3 true Q^2 bins used for ν CC1π^0.

Physics systematics in the VALOR fit III

225 - 227	$f_{\bar{\nu} CC1\pi^{\pm};1}$ - $f_{\bar{\nu} CC1\pi^{\pm};3}$	$\bar{\nu}$ CC1$\pi^{\pm}$ cross-section for the same 3 true Q^2 bins used for ν CC1$\pi^{\pm}$.
228	$f_{\nu CC2\pi}$	ν CC2π cross-section.
229	$f_{\bar{\nu} CC2\pi}$	$\bar{\nu}$ CC2π cross-section.
230 - 232	$f_{\nu CCDIS;1}$ - $f_{\nu CCDIS;3}$	CCDIS ($>$ 2π) cross-section for the 3 true neutrino energy bins defined by the following bin edges: (0, 7.5, 15.0, ∞) GeV.
233 - 235	$f_{\bar{\nu} CCDIS;1}$ - $f_{\bar{\nu} CCDIS;3}$	$\bar{\nu}$ CCDIS ($>$ 2π) cross-section for the 3 true neutrino energy bins defined above.
236	$f_{\nu CCCoh}$	ν CC coherent π production cross-section.
237	$f_{\bar{\nu} CCCoh}$	$\bar{\nu}$ CC coherent π production cross-section.
238	$f_{\nu NC}$	ν NC inclusive cross-section.
239	$f_{\bar{\nu} NC}$	$\bar{\nu}$ NC inclusive cross-section.
240	f_{ν_e/ν_μ}	ν_e/ν_μ cross-section ratio.
241	$f_{FSI;\pi;MFP}$	π mean free path in nucleus.
242	$f_{FSI;N;MFP}$	nucleon mean free path in nucleus.
243	$f_{FSI;\pi;CEx}$	π-nucleus charge exchange cross-section fraction.
244	$f_{FSI;\pi;Inel}$	π-nucleus inelastic cross-section fraction.
245	$f_{FSI;\pi;Abs}$	π-nucleus absorption cross-section fraction.
246	$f_{FSI;\pi;\pi Prod}$	π-nucleus π production cross-section fraction.
247	$f_{FSI;N;CEx}$	nucleon-nucleus charge exchange cross-section fraction.
248	$f_{FSI;N;Inel}$	nucleon-nucleus inelastic cross-section fraction.
249	$f_{FSI;N;Abs}$	nucleon-nucleus absorption cross-section fraction.
250	$f_{FSI;N;\pi Prod}$	nucleon-nucleus π production cross-section fraction.

VALOR fit
Statistical treatment

All physics is included in the definition of $\lambda(\vec{\theta}; \vec{f})$ (see previous page).

What follows describes (briefly) the procedures used for nuisance parameter elimination, point and interval estimation, and hypothesis testing.

VALOR draws in a pragmatic way on both Bayesian and Frequentist methods. The methodology follows best HEP traditions and it was exercised repeatedly by the group in precision neutrino measurements (T2K).

E.g. see several talks and posters by group members during PHYSTAT-ν at IPMU and FNAL.

VALOR fit: Parameter elimination

The likelihood ratio $\lambda(\vec{\theta}; \vec{f})$ built for the **VALOR** **multi-detector**, **multi-channel**, **joint oscillation and systematics constraint fit** a function of O(10^2 - O(10^3 interesting physics and nuisance parameters!

Both **marginalization** and **profiling** are used for parameter elimination.

- Most parameters $\vec{f'}$ (any subset of $(\vec{\theta}; \vec{f})$) would have a **well-established prior** $\pi(\vec{f'})$ (from hadron-production measurements, external neutrino cross-section measurements, electron scattering data, calibration data etc.).
  - Eliminated by marginalization. The **marginal likelihood** $\lambda_{marg}(\vec{\theta'})$ is:

$$\lambda_{marg}(\vec{\theta'}) = \int \lambda(\vec{\theta'}; \vec{f'})\pi(\vec{f'})d\vec{f'}$$

- For other parameters ($\theta_{\mu e}$, $\theta_{\mu\mu}$, Δm^2_{41}) use of a prior may be undesirable and an uninformative prior may be problematic: Flat priors in $\theta_{\mu e}$, $sin\theta_{\mu e}$, $sin^2\theta_{\mu e}$, $sin^2 2\theta_{\mu e}$, would yield different results!
  - Eliminated by profiling (free-floating parameters included in the fit).

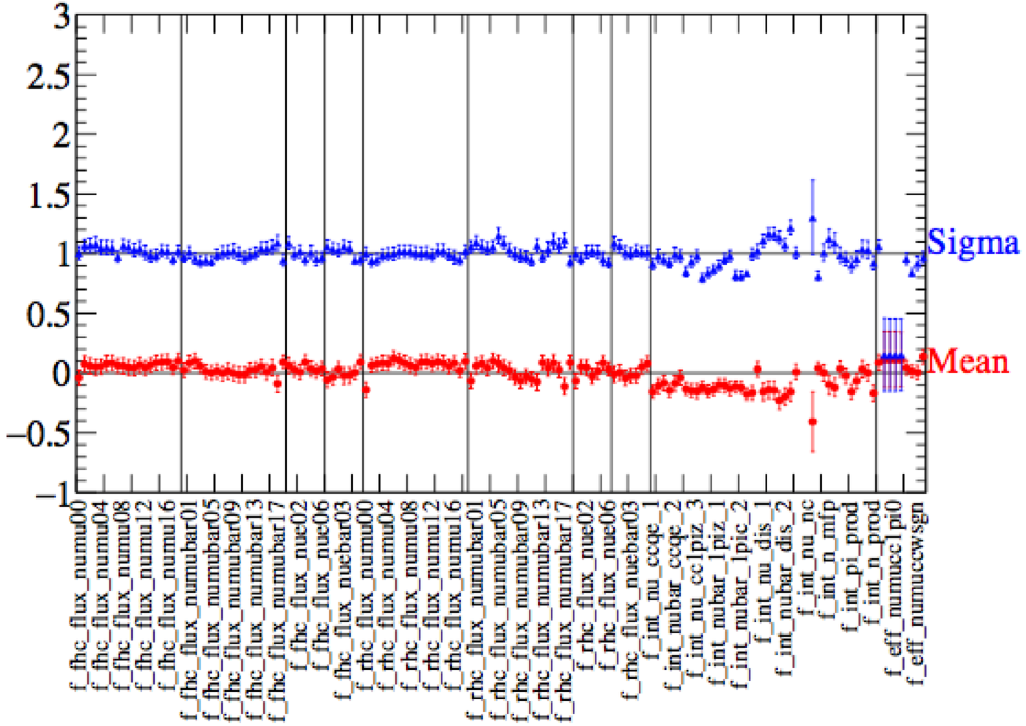
VALOR fit: Parameter estimation

To extremize the test-statistic VALOR uses the **MINUIT/MIGRAD** algorithm.

Several other methods available within VALOR via a **VALOR/GSL** interface:
Simulated annealing, Levemberg-Marquardt, Fletcher-Reeves conjugate gradient, Polak-Ribiere conjugate gradient and Vector Broyden-Fletcher-Goldfarb-Shanno.

Marginalization of systematic parameters reduces the dimensionality of the likelihood ratio dramatically. Nevertheless, would like to make the point here that much more complex fits work beautifully within VALOR:

Pulls from a O(150) parameter fit.



$$pull = \frac{f_{bf} - f_0}{\sqrt{\sigma^2_{prior} - \sigma^2_{post-fit}}}$$

- f_{bf}: best-fit value of systematic parameter f
- f_0: nominal value
- σ_{prior}: prior error on f
- $\sigma_{post-fit}$: fit (MIGRAD) error on f

VALOR fit: Interval estimation

After the fit is completed, the full χ^2 $(= -2\lambda(\vec{\theta'}))$ distribution is shifted with respect to $\chi^2(\vec{\theta'}_{bf})$:

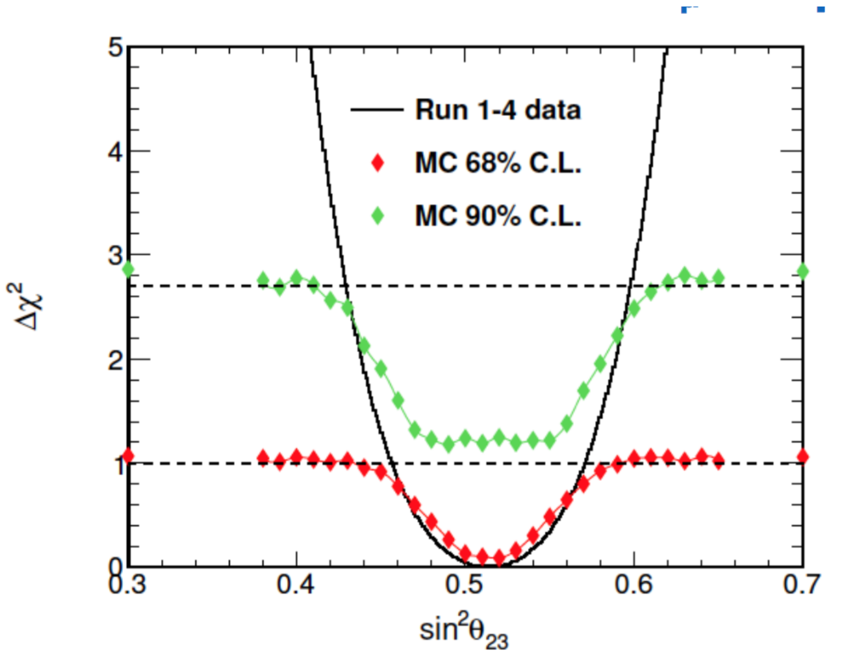
$$\Delta\chi^2(\vec{\theta'}) = \chi^2(\vec{\theta'}) - \chi^2(\vec{\theta'}_{bf})$$

Confidence intervals at X% C.L. are set on $\Delta\chi^2(\vec{\theta'})$.

$$\Delta\chi^2(\vec{\theta'}) < \Delta\chi^2_{crit;X}$$

where $\Delta\chi^2_{crit;X}$ the corresponding critical value.

In the Gaussian approximation constant values of $\Delta\chi^2_{crit}$ can be used. Usually this approximation is not reliable and the Feldman - Cousins / Cousins - Highland method is used instead.



Example from T2K Run 1-4 disappearance analysis. Comparison of $\Delta\chi^2_{crit;X}$ values from the FC method with the ones obtained under the Gaussian approximation.

The VALOR group has developed several tools to probe the severity of coverage problems.

If needed, it has the CPU muscle and efficient methods to compute corrections.

Illustration: Reduction of systematic uncertainties

Before closing, I would like to show you a beautiful example from the VALOR/DUNE analysis. It illustrates the power of a multi-channel analysis and ability to reduce systematic uncertainties.