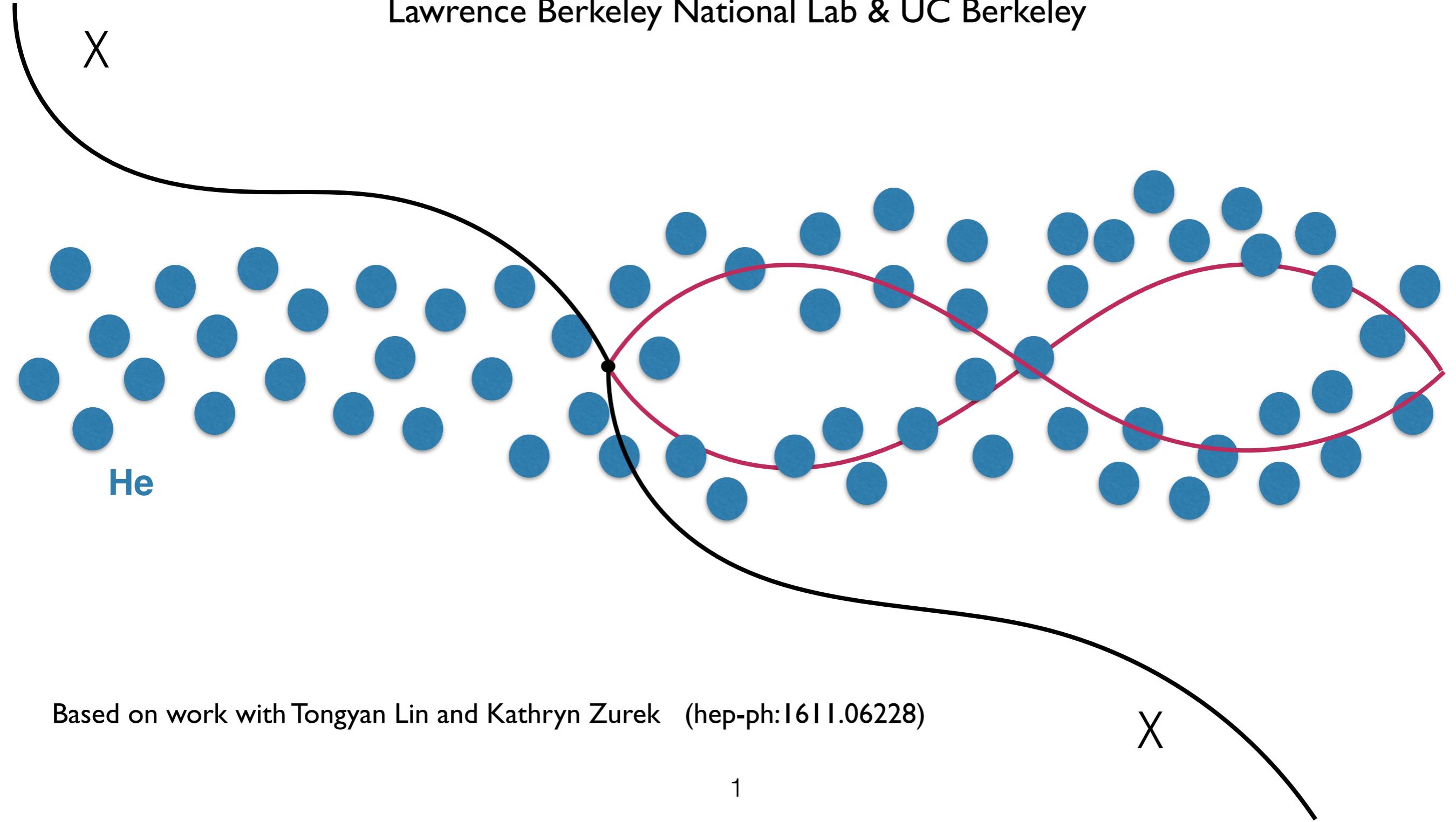


Sub-MeV Dark Matter detection with superfluid Helium

Simon Knapen

Lawrence Berkeley National Lab & UC Berkeley



Based on work with Tongyan Lin and Kathryn Zurek (hep-ph:1611.06228)

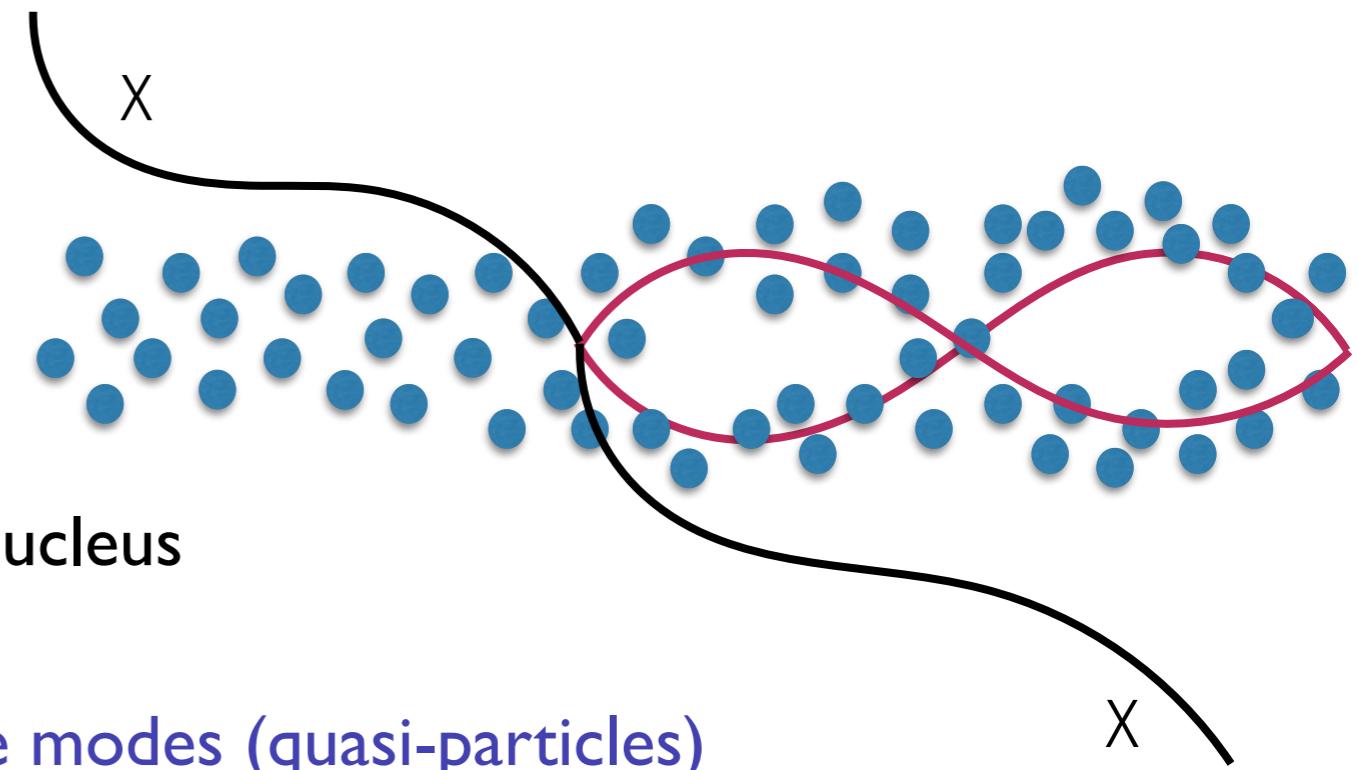
Scattering of quasi-particles

For $m_X < 1 \text{ MeV}$



$$q \sim v m_X < 1 \text{ nm}^{-1}$$

Can no longer scatter of single atom / nucleus



Scattering of collective modes (quasi-particles)

For detector technology, see talks by:

D. McKinsey

S. Hertel

M. Pyle

D. Stein / G. Seidel

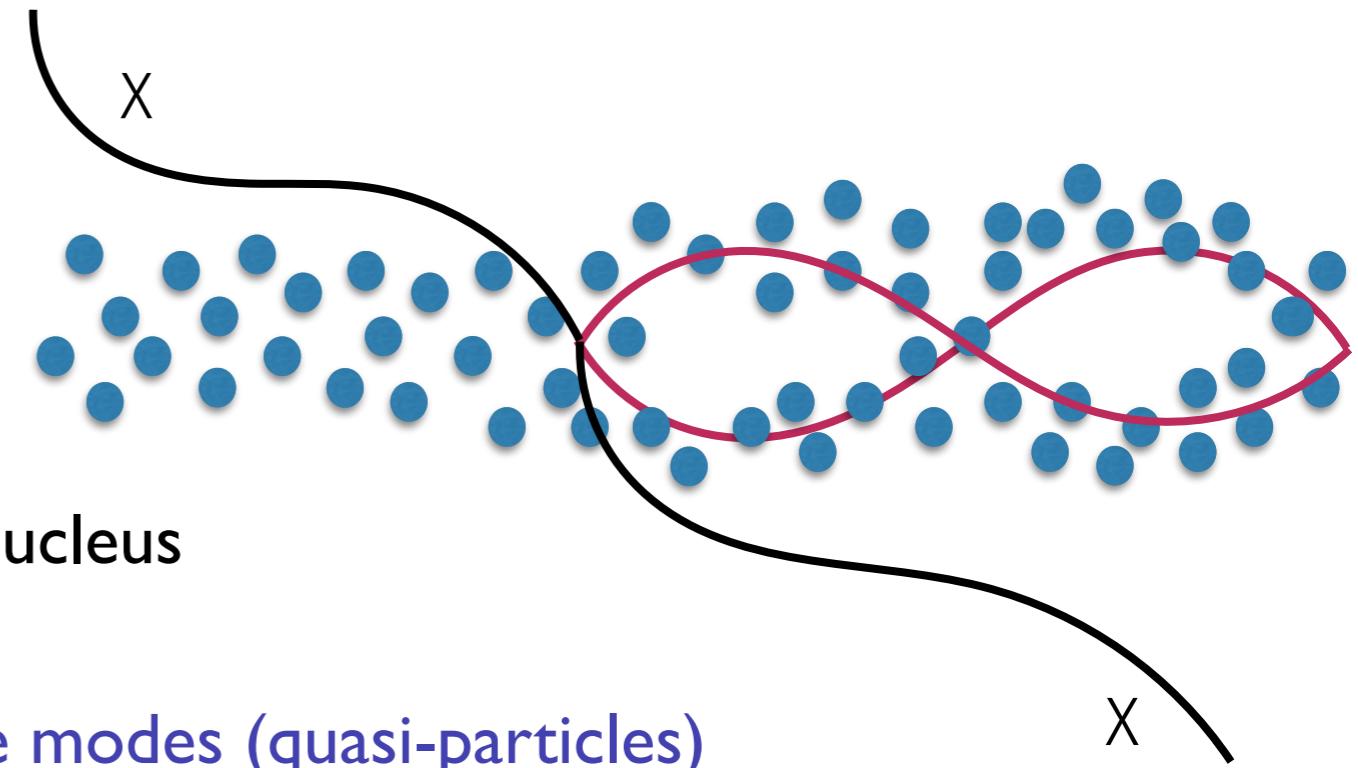
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Goal: Estimate scattering cross section with quasi-particle excitations in superfluid He

For detector technology, see talks by:

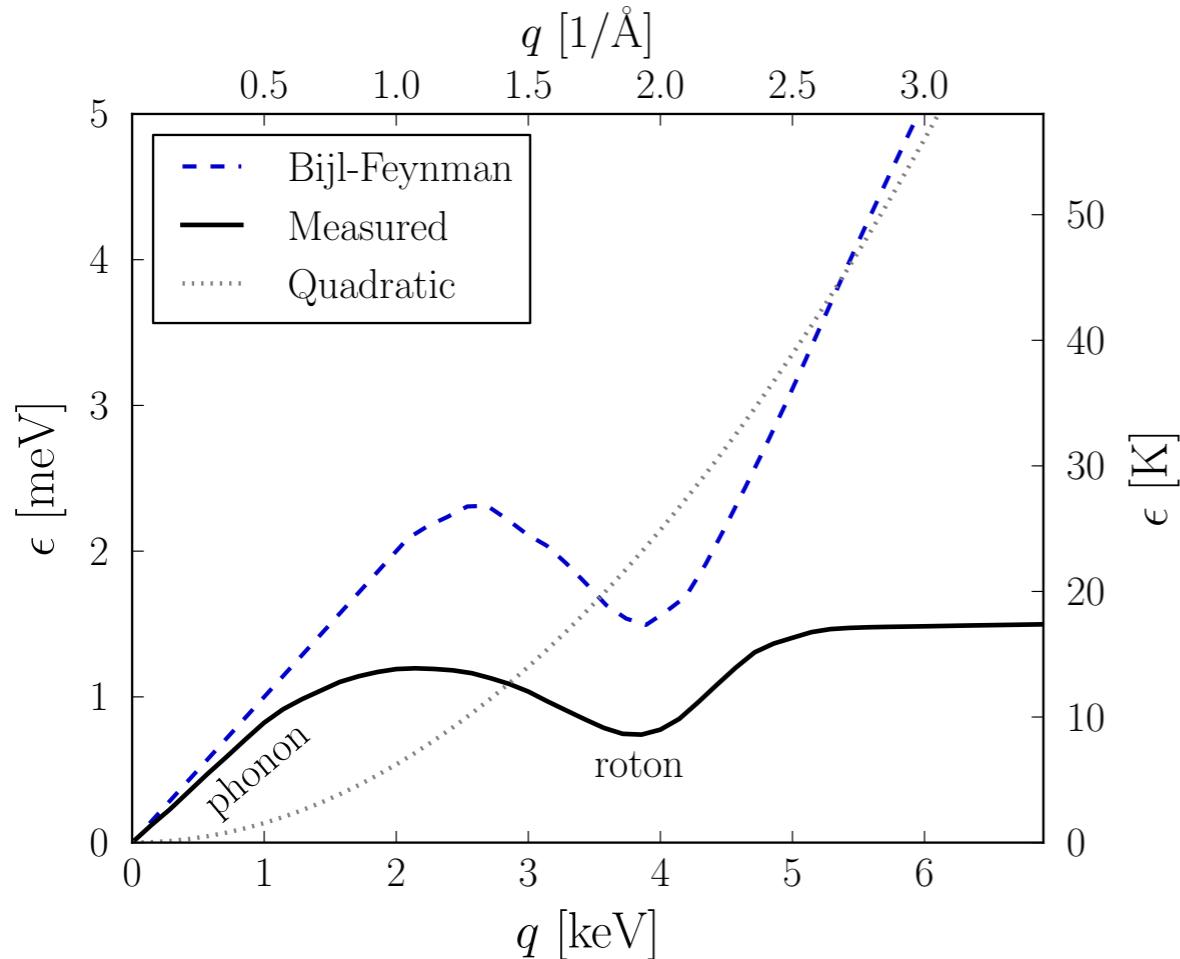
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Quasi-particles

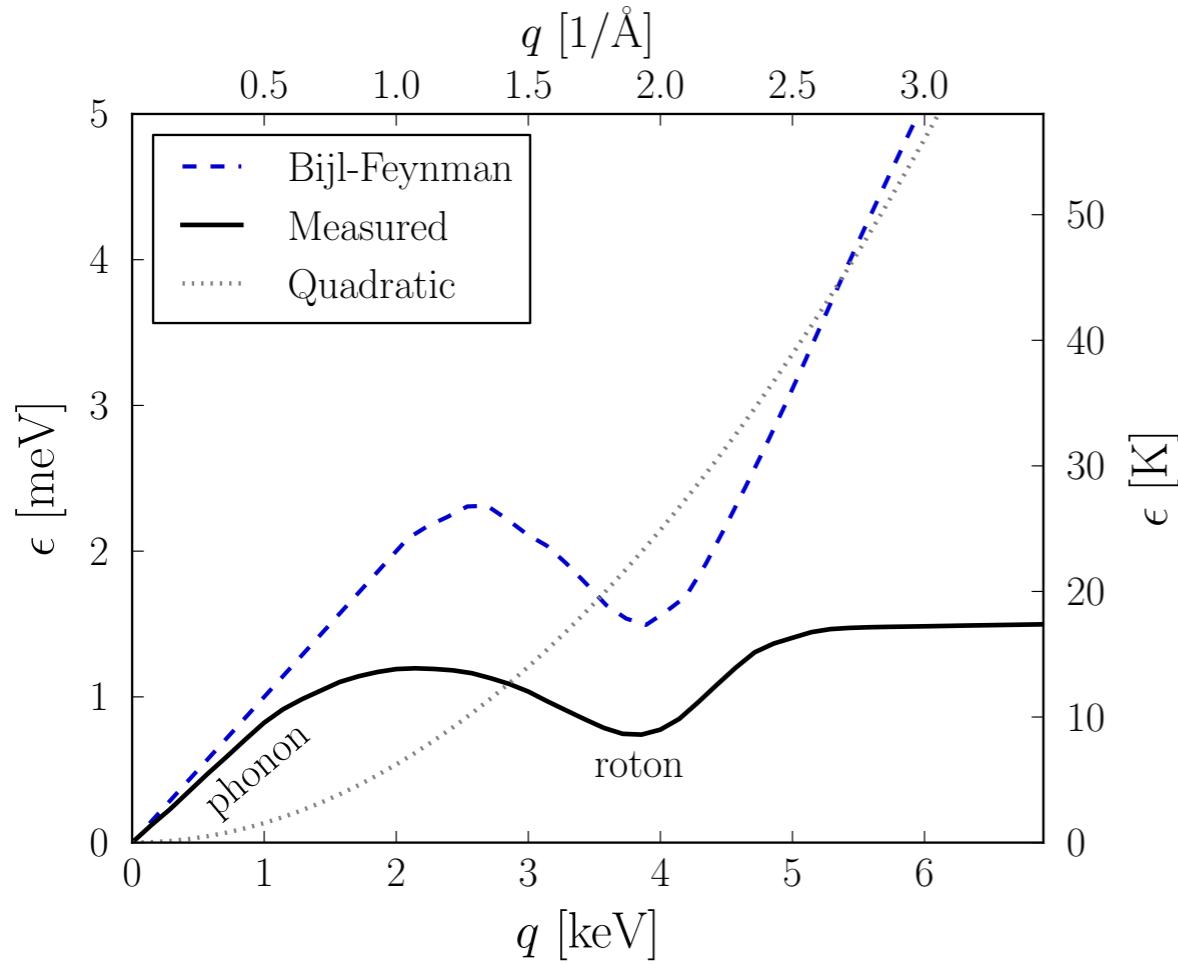


Issue: speed of DM \gg speed of sound

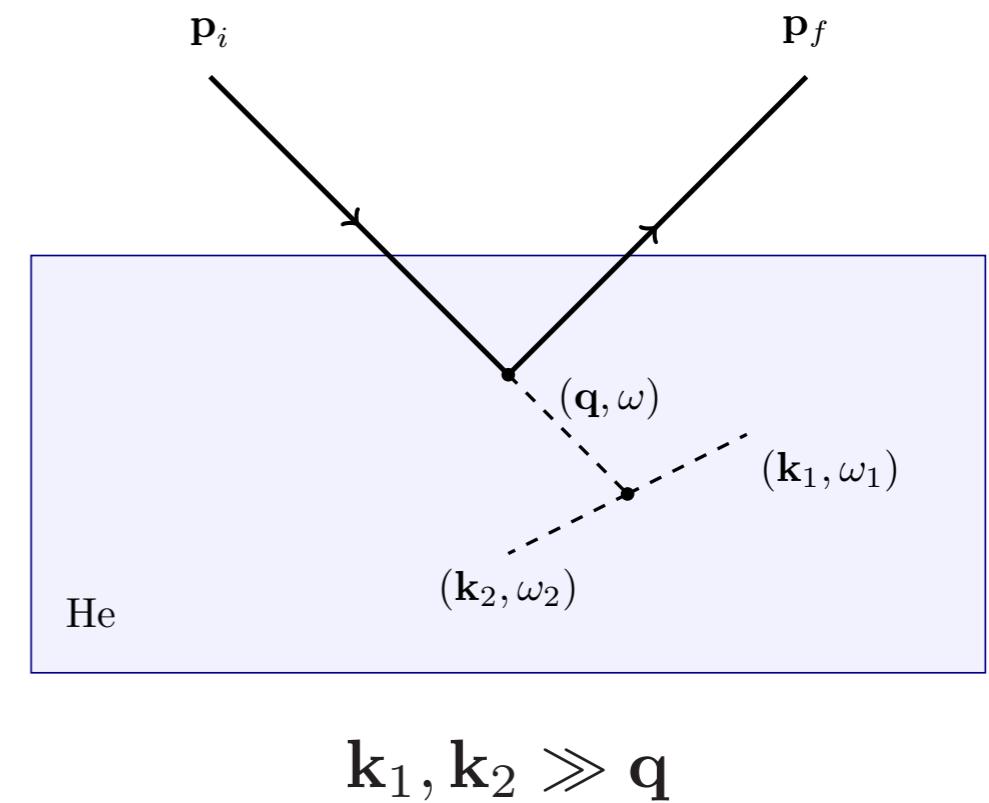


Cannot scatter against single, on shell excitation

Quasi-particles



Multi-excitation final state needed



Issue: speed of DM >> speed of sound



Cannot scatter against single, on shell excitation

(Most relevant papers written in late 60's)

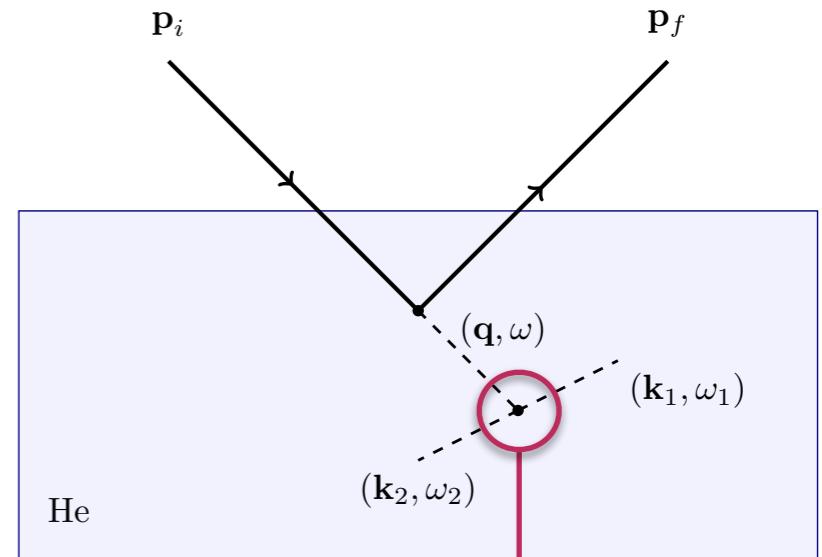
1604.08206: K. Schultz, K. Zurek
1611.06228: SK, T. Lin, K. Zurek

Response function

Cross section

$$\frac{d^2\sigma}{d\Omega d\omega} = b_n^2 \frac{p_f}{p_i} S(\mathbf{q}, \omega)$$

nucleus-DM interaction length



Dynamic response function

$$S(\mathbf{q}, \omega) \equiv \frac{1}{n_0} \sum_{\beta} |\langle \Psi_{\beta} | n_{\mathbf{q}} | \Psi_0 \rangle|^2 \delta(\omega - \omega_{\beta}),$$

density perturbation

mean density excited states ground state

Compute the 3-excitation matrix element

(Fermi's Golden Rule)

R. Feynman, 1954
 H.W.Jackson, E. Feenberg, 1962
 E. Feenberg, 1969
 M. J. Stephen, 1969

Basis of states

Basis of states

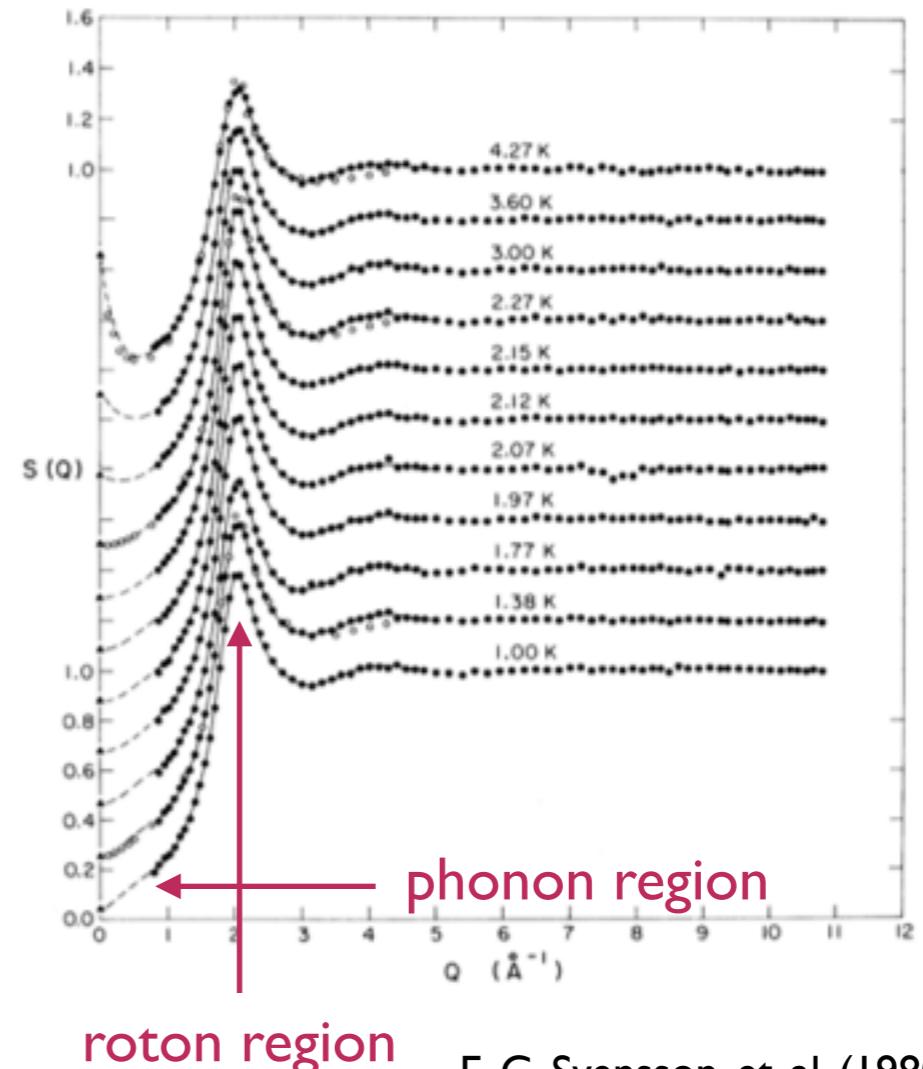
$$|\mathbf{q}\rangle^0 \equiv \frac{1}{\sqrt{n_0 S(\mathbf{q})}} n_{\mathbf{q}} |\Psi_0\rangle$$
$$|\mathbf{q}_1, \mathbf{q}_2\rangle^0 \equiv \frac{1}{\sqrt{n_0 S(\mathbf{q}_1)}} \frac{1}{\sqrt{n_0 S(\mathbf{q}_2)}} n_{\mathbf{q}_1} n_{\mathbf{q}_2} |\Psi_0\rangle$$

with

$$n_{\mathbf{q}} \equiv \frac{1}{\sqrt{V}} \sum_{i=1}^N \exp(i\mathbf{q} \cdot \mathbf{r}_i)$$

Static structure function

$$S(\mathbf{q}) \equiv \frac{1}{n_0} \langle \Psi_0 | n_{-\mathbf{q}} n_{\mathbf{q}} | \Psi_0 \rangle$$



E. C. Svensson, et. al. (1980)

Basis of states

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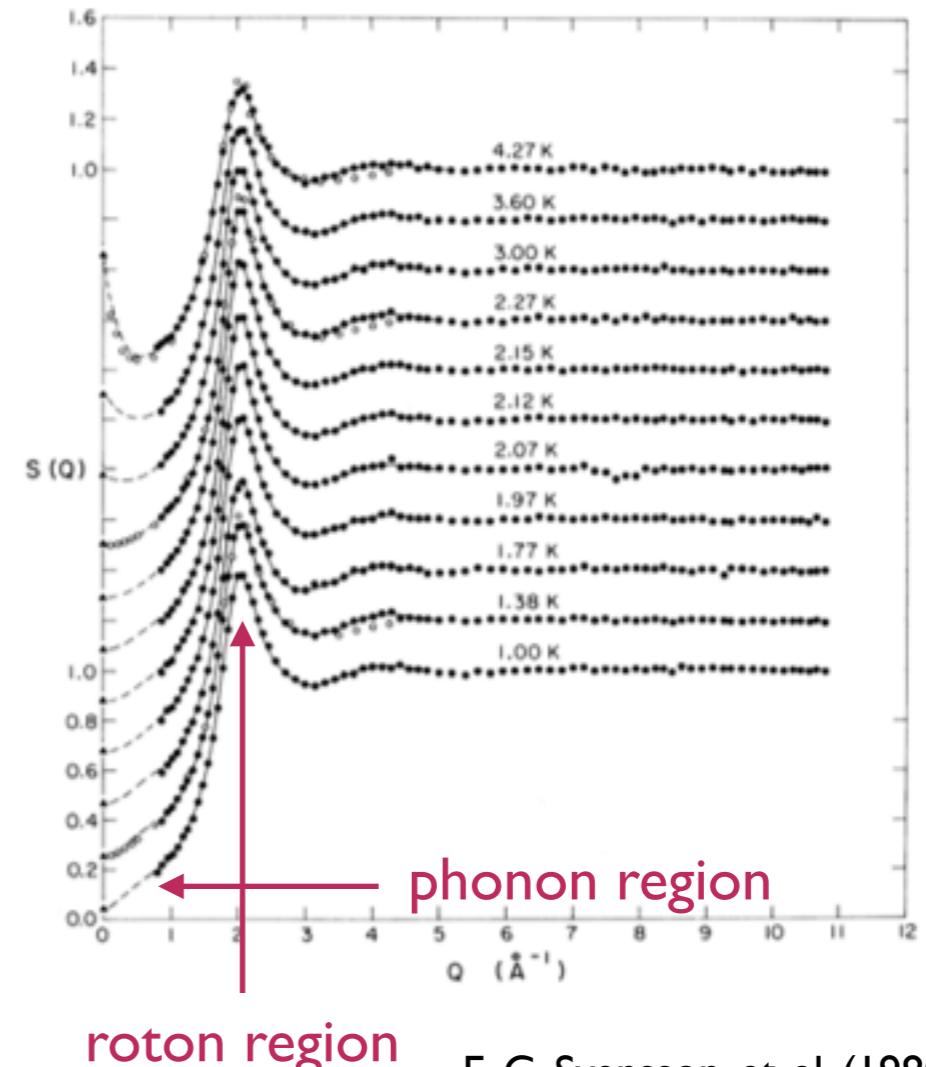
$$S(\mathbf{q}) \equiv \frac{1}{n_0} \langle \Psi_0 | n_{-\mathbf{q}} n_{\mathbf{q}} | \Psi_0 \rangle$$



In an interacting medium,
the vacuum is *active*



$|\mathbf{q}\rangle^0$ and $|\mathbf{q}_1, \mathbf{q}_2\rangle^0$ are not
orthogonal



E. C. Svensson, et. al. (1980)

Basis of states (II)

Gram-Schmidt orthogonalization

Jackson and Feenberg, 1962

$$|\mathbf{q}\rangle \equiv |\mathbf{q}\rangle^0$$

$$|\mathbf{q}_1, \mathbf{q}_2\rangle \equiv |\mathbf{q}_1, \mathbf{q}_2\rangle^0 - \sum_{\mathbf{q}'} \langle \mathbf{q}' | \mathbf{q}_1, \mathbf{q}_2 \rangle^0 |\mathbf{q}'\rangle$$



unknown overlap term
(deal with it later)

Long-lived on time-scale of the Hamiltonian → Factorization possible

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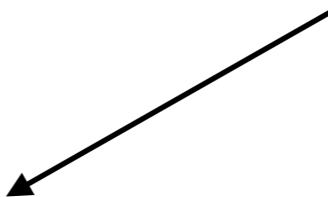
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3 excitation matrix element

$$\langle \mathbf{q} - \mathbf{k}, \mathbf{k} | H - E_0 | \mathbf{q} \rangle = \langle \mathbf{q} - \mathbf{k}, \mathbf{k} | H - E_0 | \mathbf{q} \rangle^0 - \epsilon_0(\mathbf{q}) \langle \mathbf{q} - \mathbf{k}, \mathbf{k} | \mathbf{q} \rangle$$

Compute from first principles



additional assumptions /
experimental input needed

Hamiltonian

Two equivalent pictures

Quantum hydrodynamics

$$H = \int d^3\mathbf{r} \left(\frac{1}{2} m_{\text{He}} \mathbf{v} \cdot n\mathbf{v} + \mathcal{V}(n) \right)$$

(+ continuity equation)

- ✓ Effective field theory intuition
- ✓ Explicit second quantization

Microscopic formulation

$$H = \sum_i \left(-\frac{\nabla_i^2}{2m_{\text{He}}} \right) + \mathcal{V}(\{\mathbf{r}_i\})$$

- ✓ Answer independent of potential
- ✓ Easier calculation

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Full result

$$\langle \mathbf{q} - \mathbf{k}, \mathbf{k} | H - E_0 | \mathbf{q} \rangle = \frac{\mathbf{q} \cdot (\mathbf{q} - \mathbf{k}) S(\mathbf{k}) + \mathbf{q} \cdot \mathbf{k} S(\mathbf{q} - \mathbf{k}) - q^2 S(\mathbf{k}) S(\mathbf{q} - \mathbf{k})}{2m_{\text{He}} \sqrt{N} \sqrt{S(\mathbf{q} - \mathbf{k}) S(\mathbf{k}) S(\mathbf{q})}}$$

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Expanding the structure function in small \mathbf{q}

$$S(\mathbf{q}, \omega) \approx \frac{1}{16\pi^2} \frac{\mathbf{q}^4}{n_0 m_{\text{He}}^2 \omega^2} \sum_i \tilde{\mathbf{k}}_i^2 (1 - S(\tilde{\mathbf{k}}_i))^2 \quad \epsilon_0(\tilde{k}_i) = \omega/2$$

Power law reproduced in state-of-the-art calculations

State-of-the-art calculation

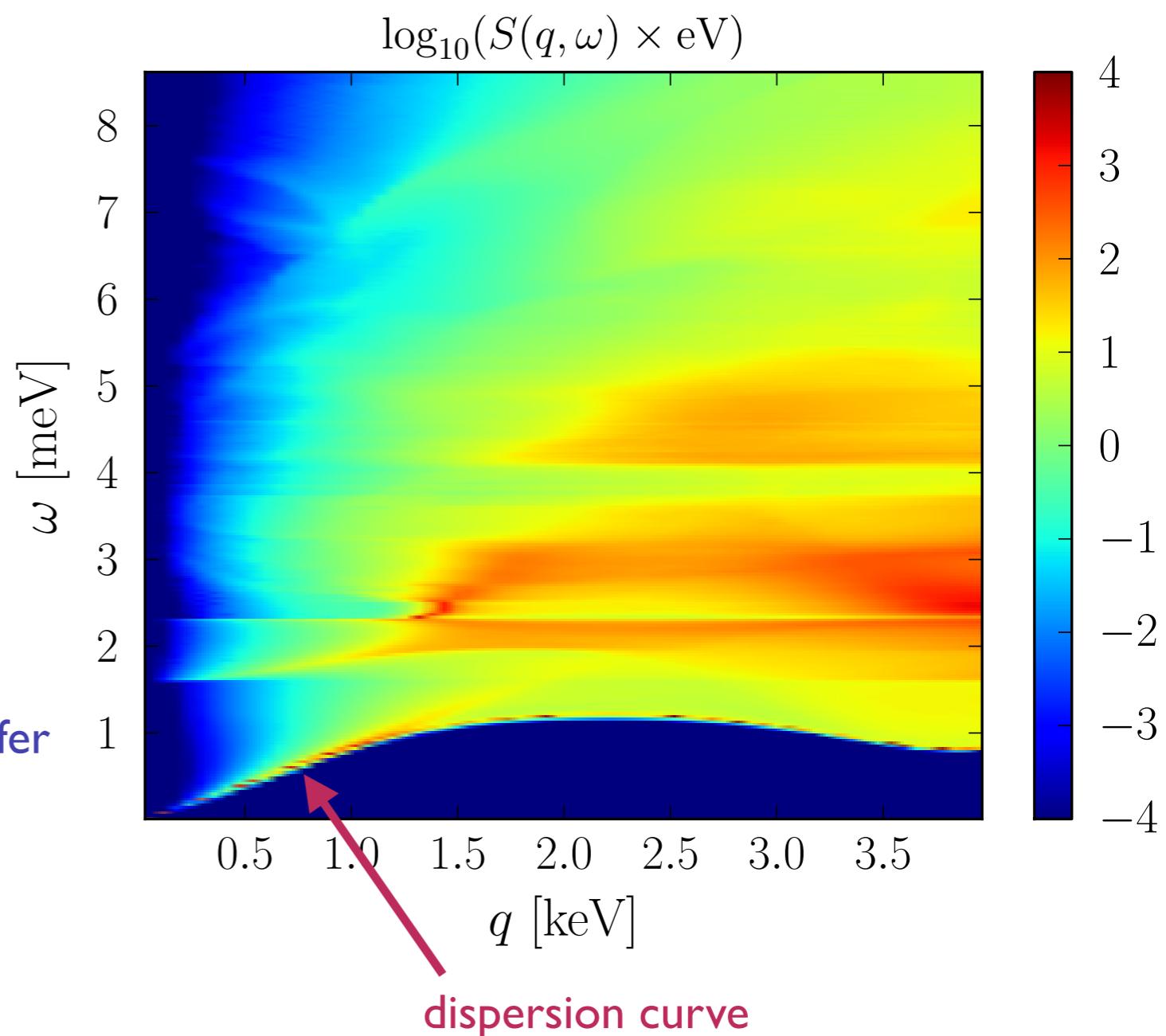
Combination of standard perturbation theory
& dynamic multiparticle fluctuations theory

- More sophisticated ansatz for the potential
- Resummed self-energies

No resolution for very low momentum transfer

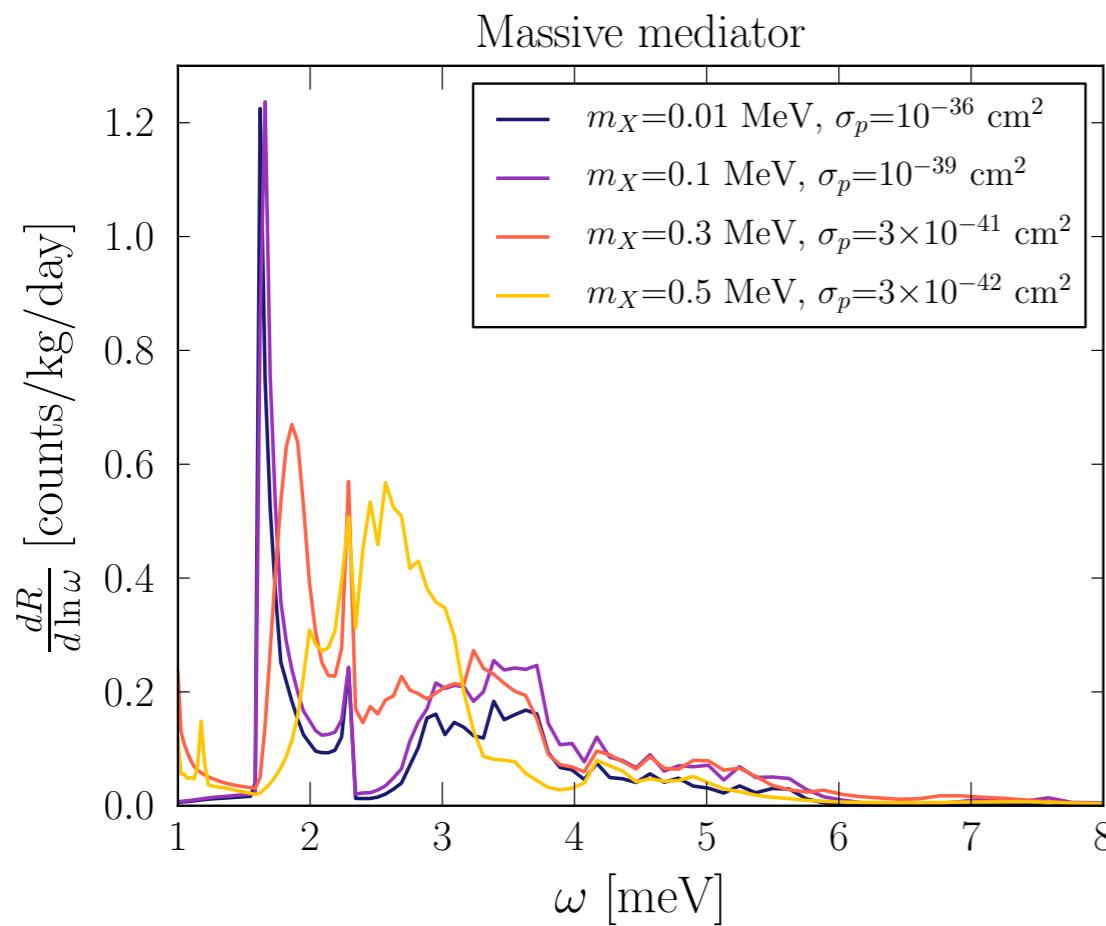


Use our analytic expressions to extrapolate

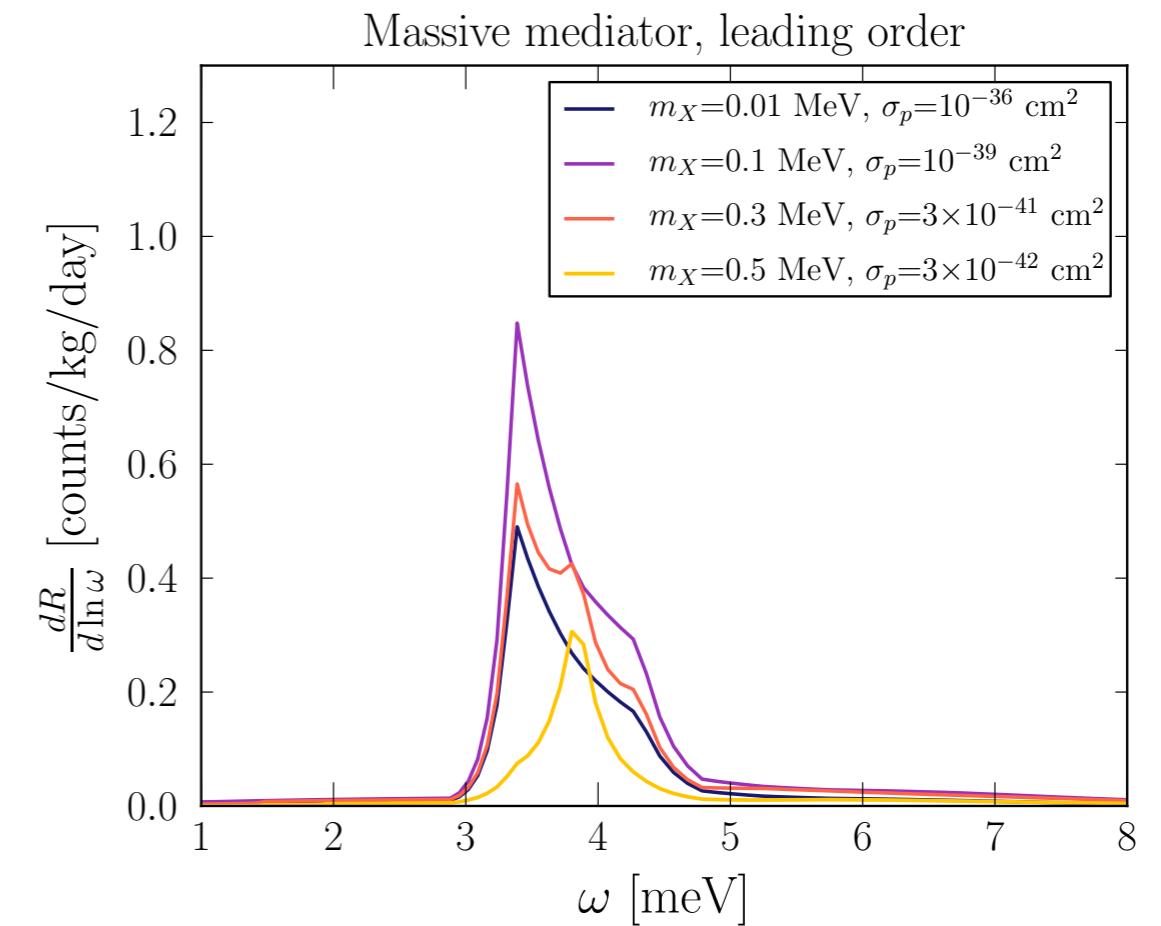


Spectrum

State-of-the-art calculation

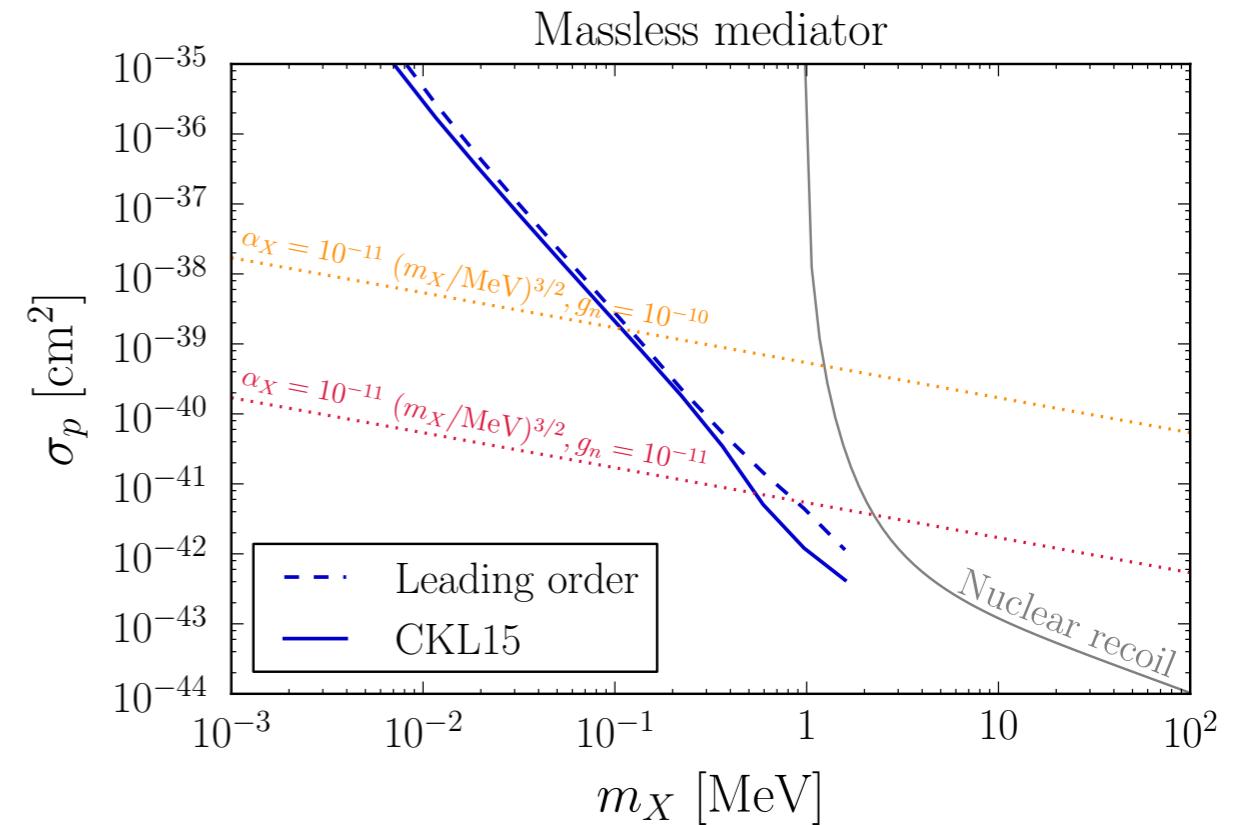
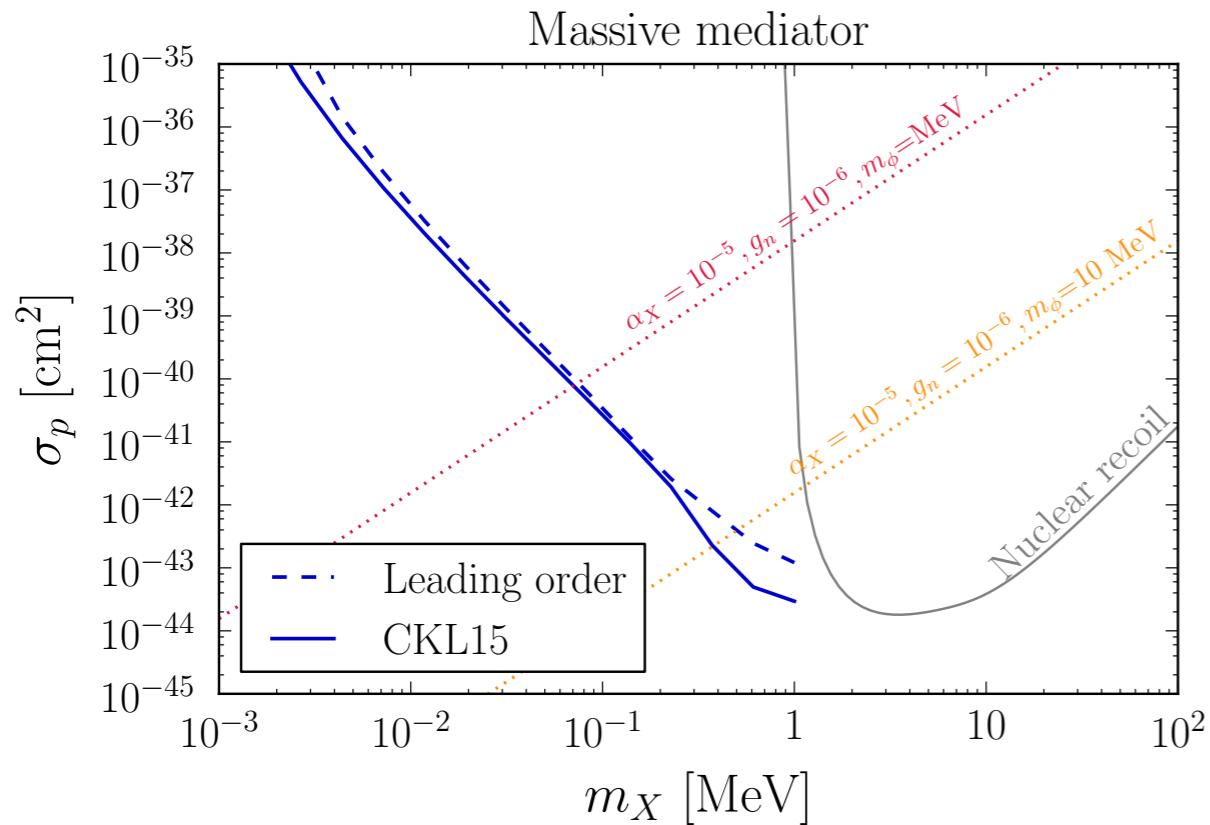


Our computation



Only qualitative agreement on the differential rate, but integrated rate roughly right

Results



1 event /kg/year, no background

Assumed sensitivity down to \sim meV

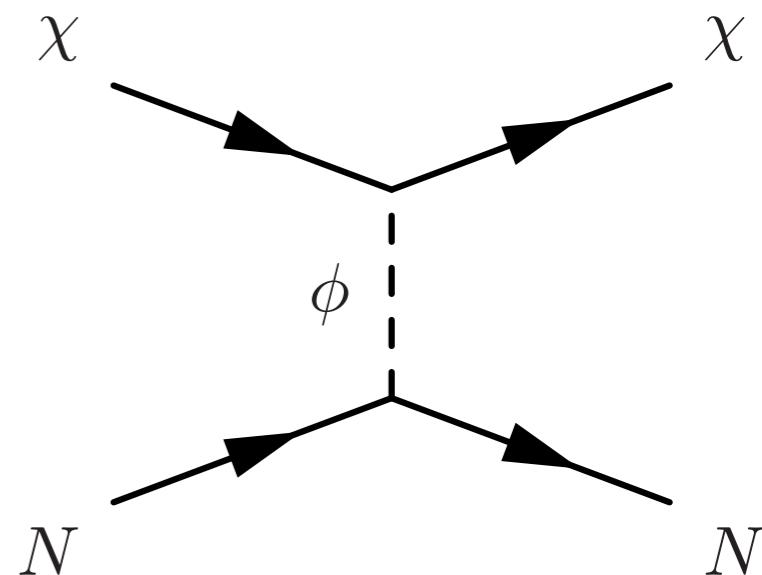
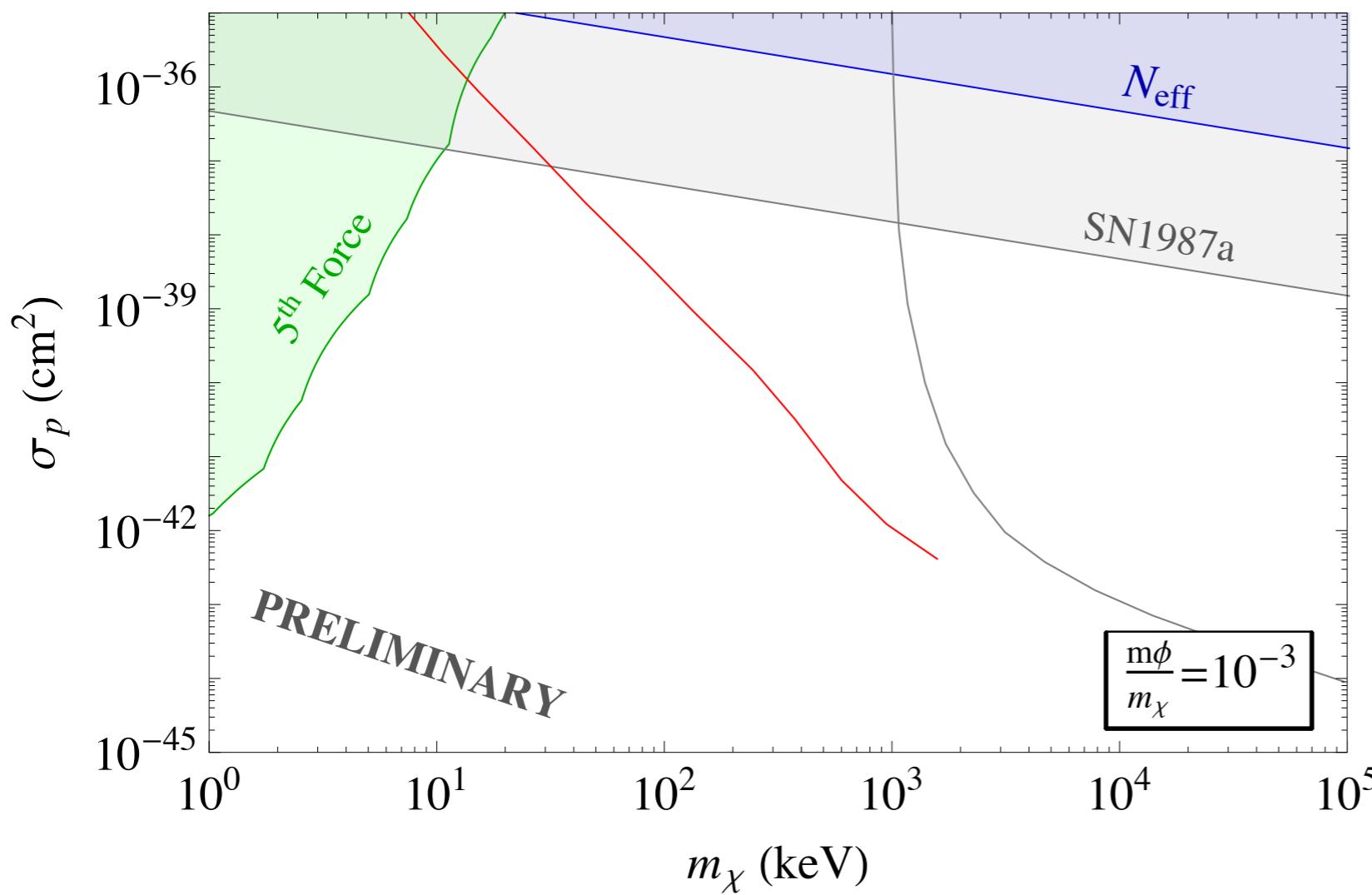
A simple benchmark model

Scalar mediator coupling to nuclei



Complimentary with superconductors

Example



Reach for heavy mediator a bit better,
but existing constraints stronger

(Reach for dark photon mediators not great)

Conclusions

We can estimate the rate for DM scattering in superfluid Helium

- decent agreement with state-of-the-art many body theory, reproduce momentum dependence
- extrapolation needed for low momenta

More theory work needed to compute differential rates, multi-phonon production, phonon propagation etc

Conclusions

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Bonus: Learn about superfluid Helium!

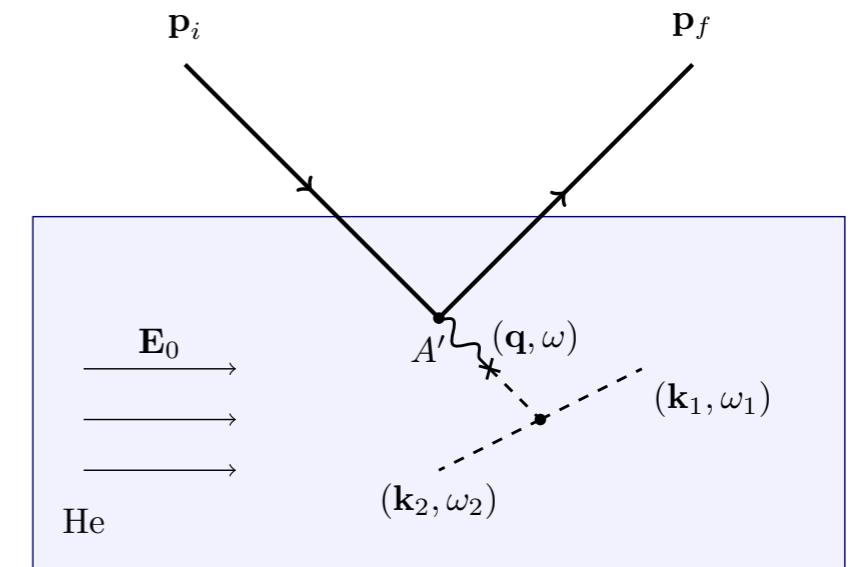
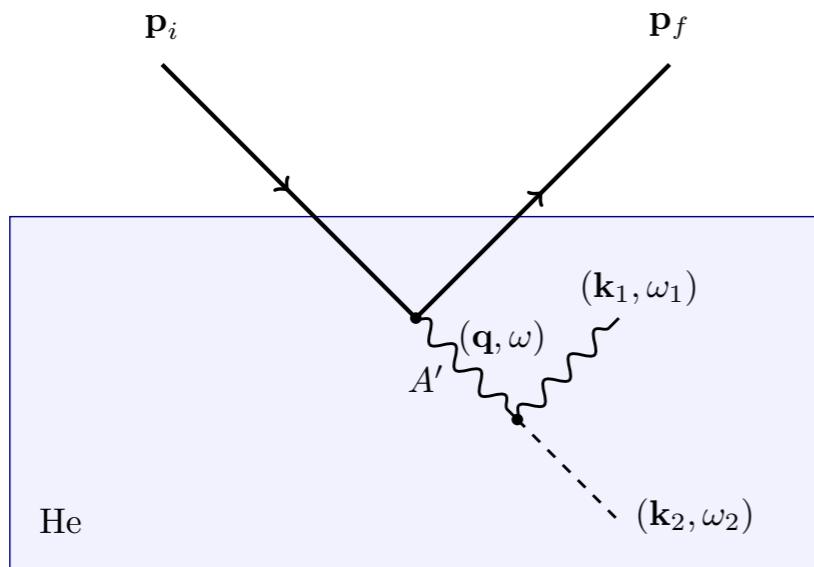
Back-up slides

Dark photon processes

Couple to polarization (Brioullin scattering)

Fetter (1972)

$$H = -\alpha \int d^3r n(\mathbf{r}) \mathbf{E}(\mathbf{r}) \cdot \mathbf{E}(\mathbf{r}) \xrightarrow{\kappa F^{\mu\nu} F'_{\mu\nu}} H = -\kappa \alpha \int d^3r n(\mathbf{r}) \mathbf{E}(\mathbf{r}) \cdot \mathbf{E}'(\mathbf{r})$$

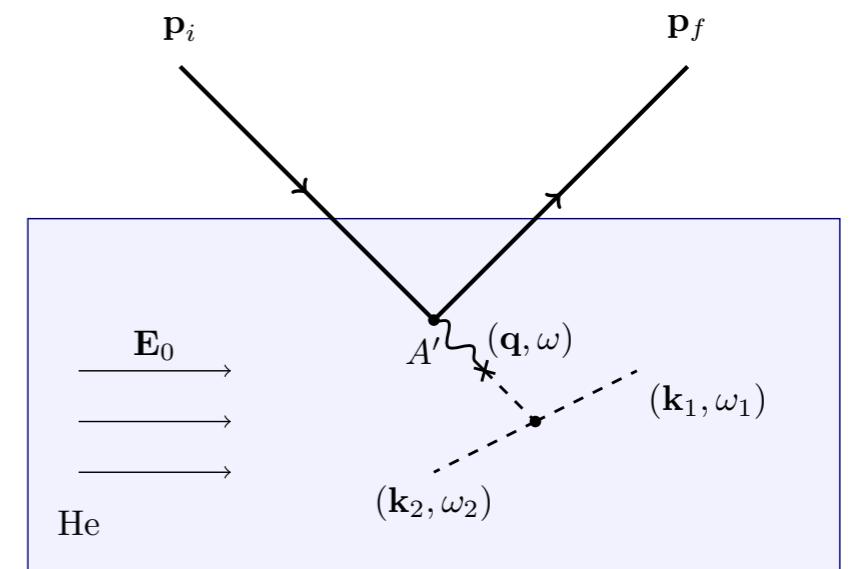
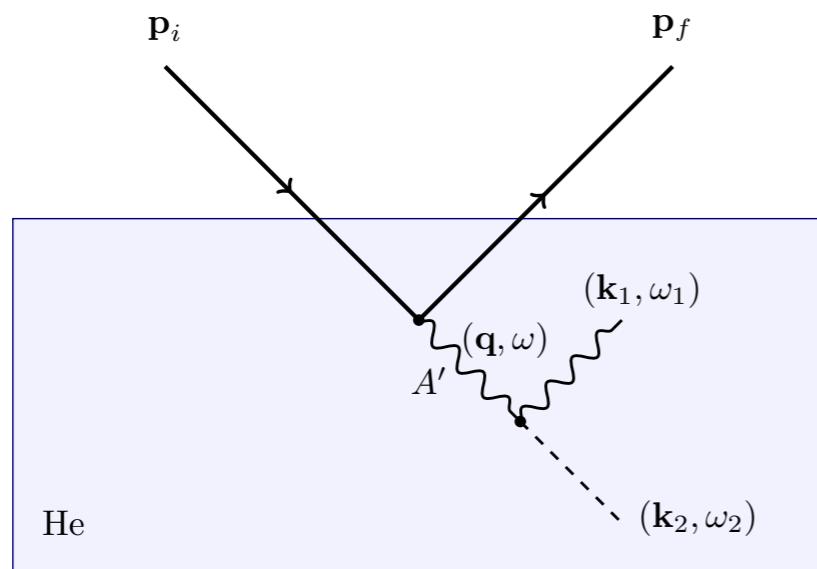


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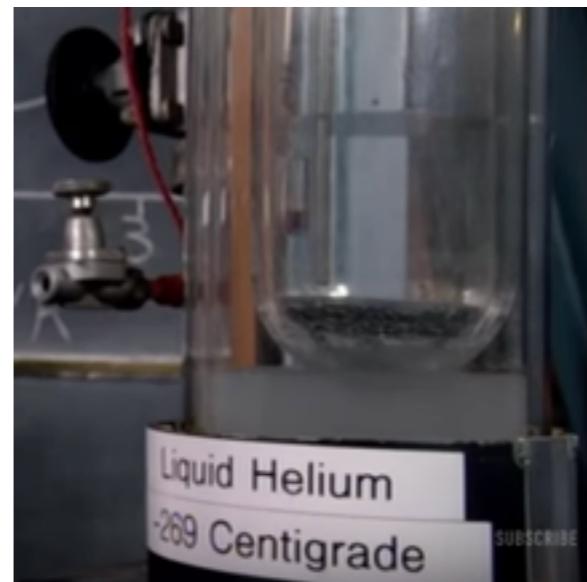
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The reach is not great, because

- small polarizability
- momentum suppression

Helium is very transparent



Hamiltonian

Two equivalent pictures

Quantum hydrodynamics

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(+ continuity equation)

Microscopic formulation

$$H = \sum_i \left(-\frac{\nabla_i^2}{2m_{\text{He}}} \right) + \mathcal{V}(\{\mathbf{r}_i\})$$

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Answer

$${}^0\langle \mathbf{q} - \mathbf{k}, \mathbf{k} | H - E_0 | \mathbf{q} \rangle = \frac{\mathbf{q} \cdot (\mathbf{q} - \mathbf{k}) S(\mathbf{k}) + \mathbf{q} \cdot \mathbf{k} S(\mathbf{q} - \mathbf{k})}{2m_{\text{He}} \sqrt{N} \sqrt{S(\mathbf{q} - \mathbf{k}) S(\mathbf{k}) S(\mathbf{q})}}$$

Matrix element

Overlap term cannot be computed without knowledge of the potential,
but must satisfy consistency conditions

In convolution approximation

$${}^0\langle \mathbf{q} - \mathbf{k}, \mathbf{k} | \mathbf{q} \rangle = \frac{\sqrt{S(\mathbf{q} - \mathbf{k})S(\mathbf{k})S(\mathbf{q})}}{\sqrt{N}}$$

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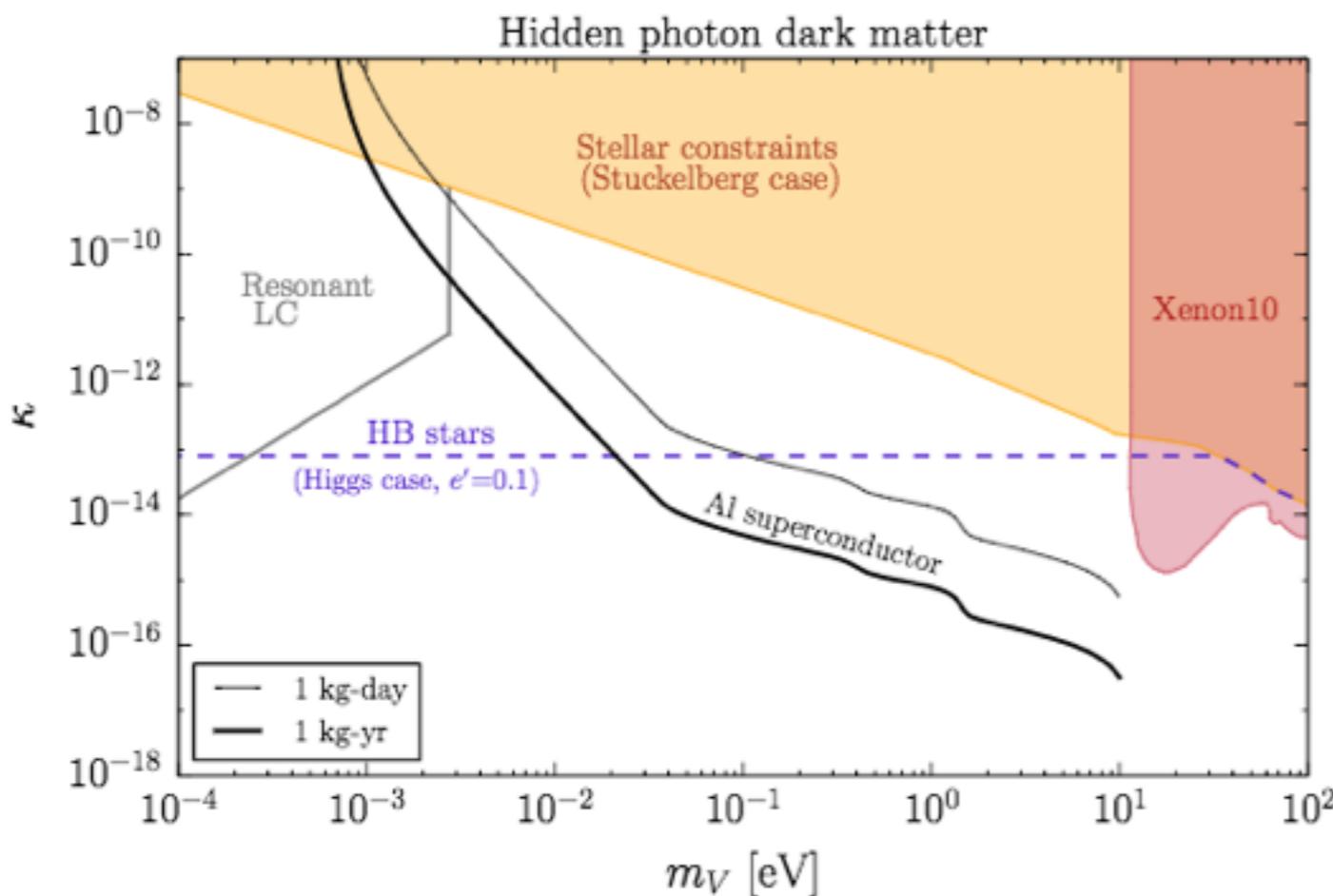
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$$S(\mathbf{q}, \omega) \approx \frac{1}{16\pi^2} \frac{\mathbf{q}^4}{n_0 m_{\text{He}}^2 \omega^2} \sum_i \tilde{\mathbf{k}}_i^2 (1 - S(\tilde{\mathbf{k}}_i))^2 \quad \epsilon_0(\tilde{k}_i) = \omega/2$$

Power law reproduced in state-of-the-art calculations

Dark photon reach



Reach for κ

	no E field	with E field
Scattering	3×10^{-7}	2×10^{-9}
Absorption	3×10^{-8}	unknown, but very small

1604.06800: Y. Hogberg, T. Lin, K. Zurek

Convolution approximation

$$\begin{aligned} {}^0\langle \mathbf{q} - \mathbf{k}, \mathbf{k} | \mathbf{q} \rangle = & \frac{1}{\sqrt{N S(\mathbf{q} - \mathbf{k}) S(\mathbf{q}) S(\mathbf{k})}} \left[-2 + S(\mathbf{q}) + S(\mathbf{k}) + S(\mathbf{q} - \mathbf{k}) \right. \\ & \left. + \frac{1}{N} \int d^3 \mathbf{r}_1 d^3 \mathbf{r}_2 d^3 \mathbf{r}_3 e^{i \mathbf{q} \cdot \mathbf{r}_{12} + i \mathbf{k} \cdot \mathbf{r}_{23}} p_3(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3) \right] \end{aligned}$$

With

$$p_3(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3) \equiv N(N-1)(N-2) \int d^3 \mathbf{r}_4 \dots d^3 \mathbf{r}_N \psi_0^2$$

Which must obey

$$\frac{1}{N-1} \int d^3 \mathbf{r}_2 p_2(\mathbf{r}_1, \mathbf{r}_2) = p_1(\mathbf{r}_1) = n_0$$

$$\frac{1}{N-2} \int d^3 \mathbf{r}_3 p_3(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3) = p_2(\mathbf{r}_1, \mathbf{r}_2)$$

Convolution approximation (II)

Decompose as

$$p_3(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3) = n_0^3 \left[1 + h(\mathbf{r}_{12}) + h(\mathbf{r}_{23}) + h(\mathbf{r}_{13}) + h(\mathbf{r}_{12})h(\mathbf{r}_{23}) + h(\mathbf{r}_{23})h(\mathbf{r}_{31}) + h(\mathbf{r}_{31})h(\mathbf{r}_{12}) \right] + \delta p_3(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3)$$

Take the ansatz

$$h(\mathbf{r}_{12}) = \frac{1}{n_0^2} p_2(\mathbf{r}_1, \mathbf{r}_2) - 1$$

$$\delta p_3(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3) = n_0^4 \int h(\mathbf{r}_{14})h(\mathbf{r}_{24})h(\mathbf{r}_{34}) d^3\mathbf{r}_4$$

Put everything back together

$${}^0\langle \mathbf{q} - \mathbf{k}, \mathbf{k} | \mathbf{q} \rangle = \frac{\sqrt{S(\mathbf{q} - \mathbf{k})S(\mathbf{k})S(\mathbf{q})}}{\sqrt{N}}$$