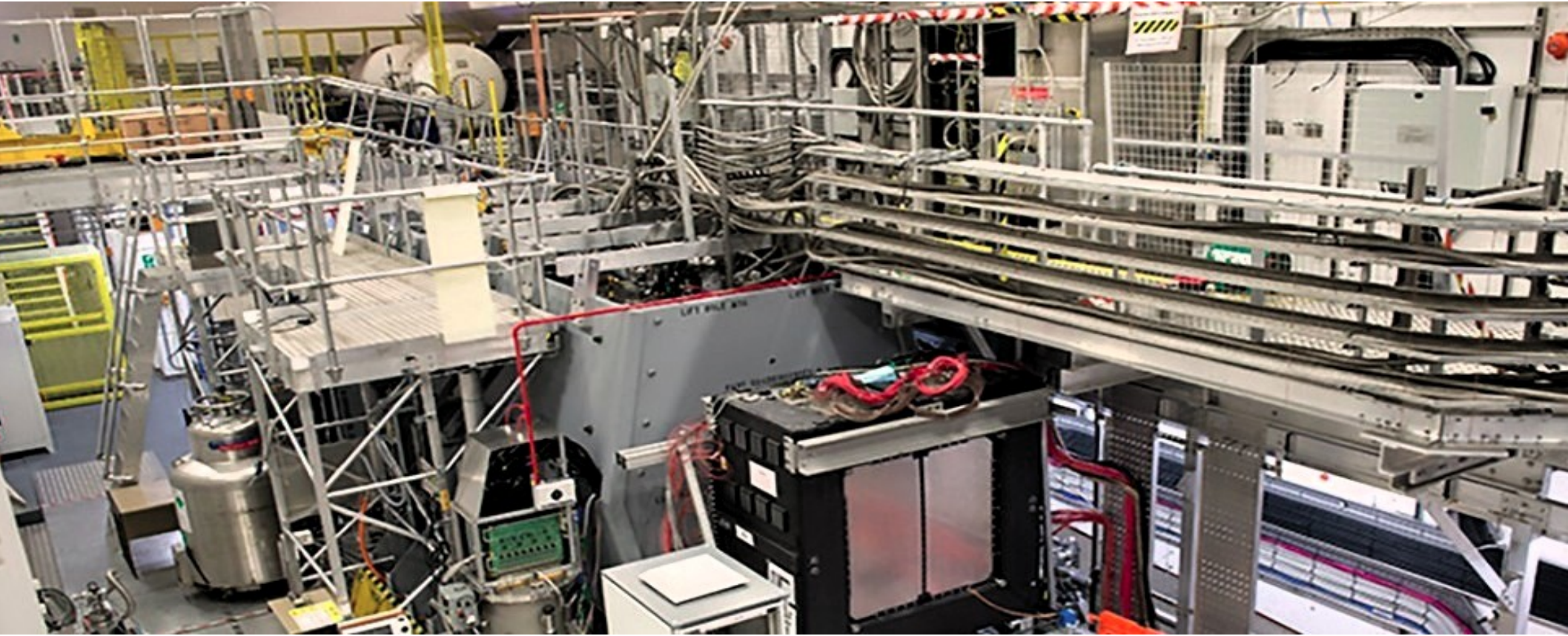
Novel Implementation of Density Estimation in Muon Cooling

Tanaz Angelina Mohayai, for the MICE Collaboration

New Perspectives 2017, Fermilab



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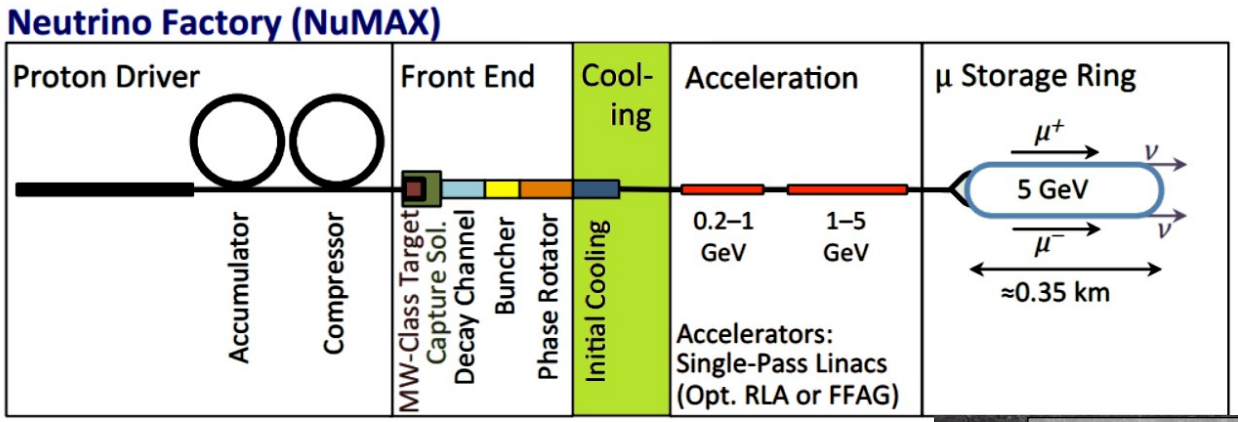
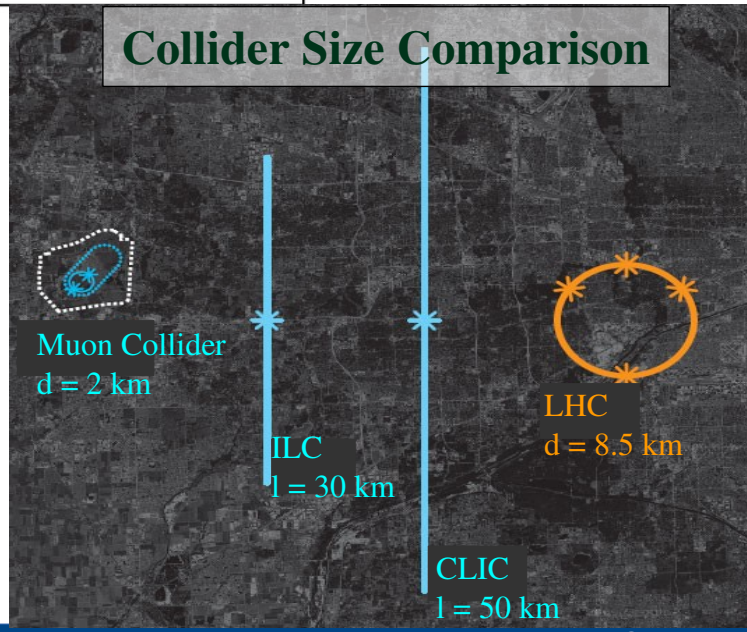
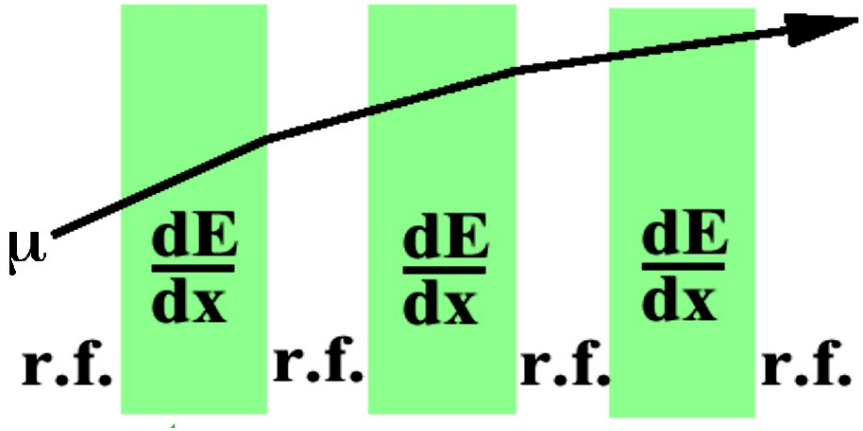
Motivation

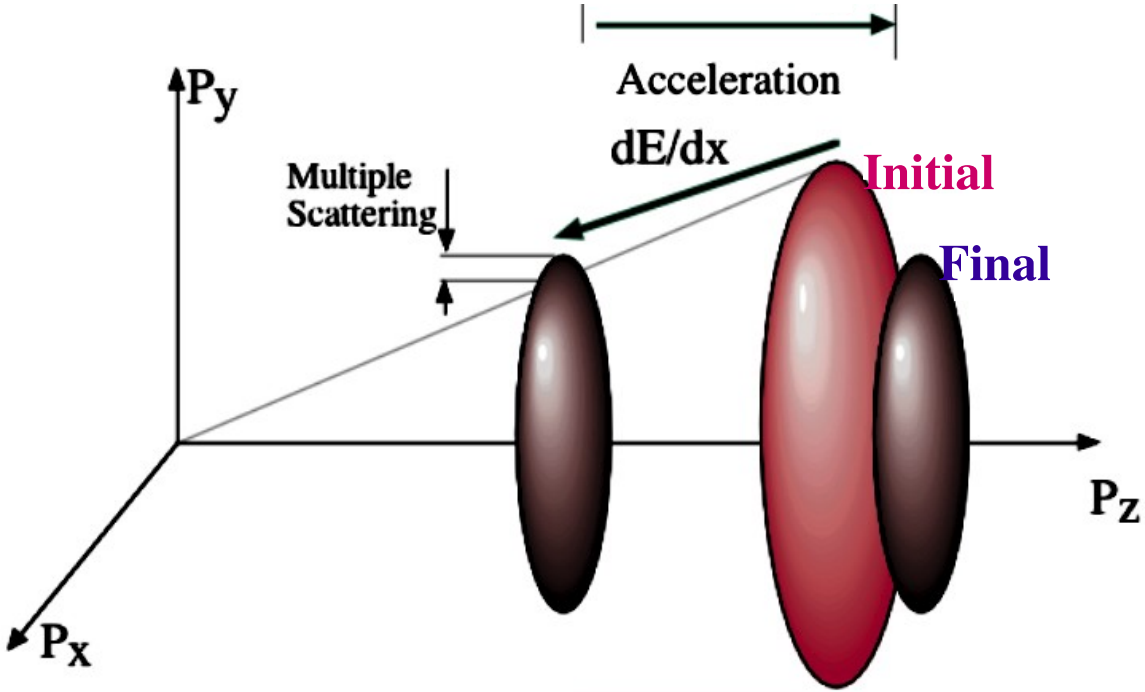


- Purpose:
  - Neutrino Factory: best neutrino oscillation sensitivity via intense, pure ν_e/ν_μ beams from $\mu^{+/-}$ decay.
  - Muon Collider: clean multi-TeV collisions with compact facility.
- Challenge:
  - Large phase-space volume of muons and their short lifetime.
- Solution:
  - Rapid beam cooling via ionization energy loss.
- Test:
  - Muon Ionization Cooling Experiment (MICE).



Muon Ionization Cooling

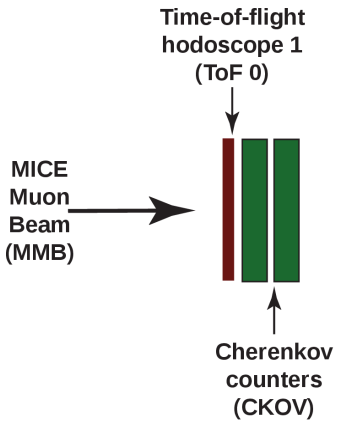


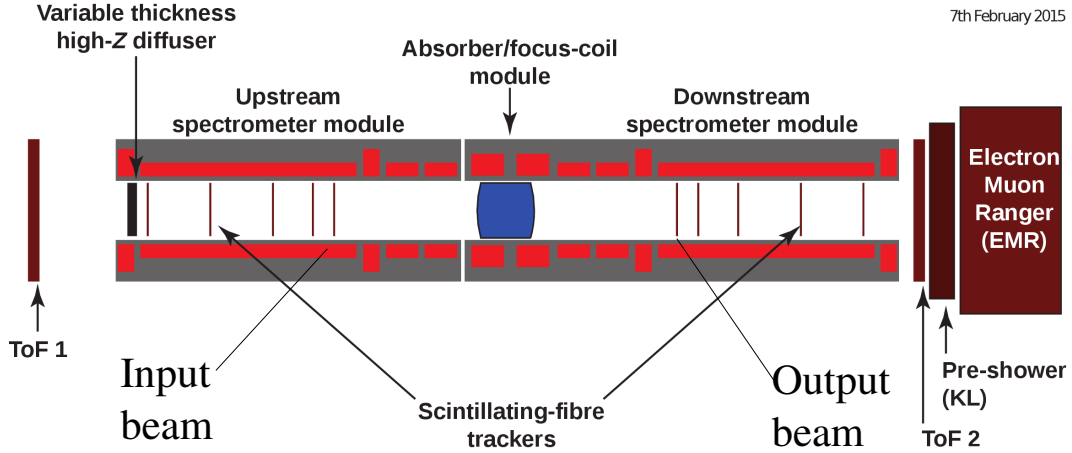


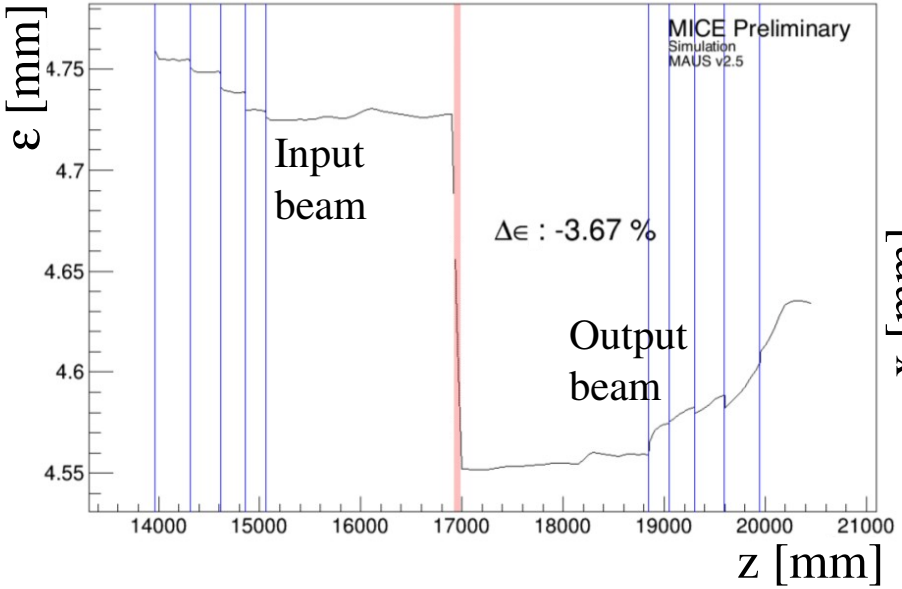
- Cooling by ionization energy loss.
- Heating by multiple (Coulomb) scattering.
- Measures of muon beam cooling:
  - ☆ Reductions: Phase-space volume, emittance (RMS measure of beam size).
  - ☆ Increase: Phase-space density.

MICE Cooling Channel

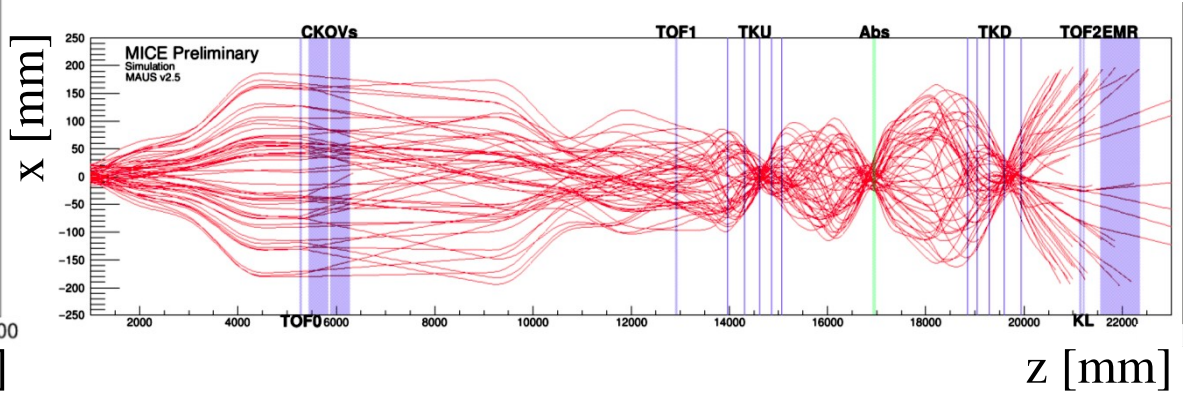
- Particle ID with Time-of-flight, Cherenkov counters, calorimetry (μ^+ beam contaminated with e, π^+).





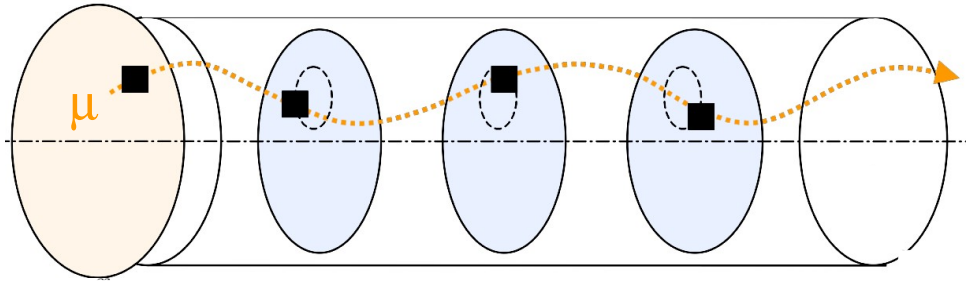


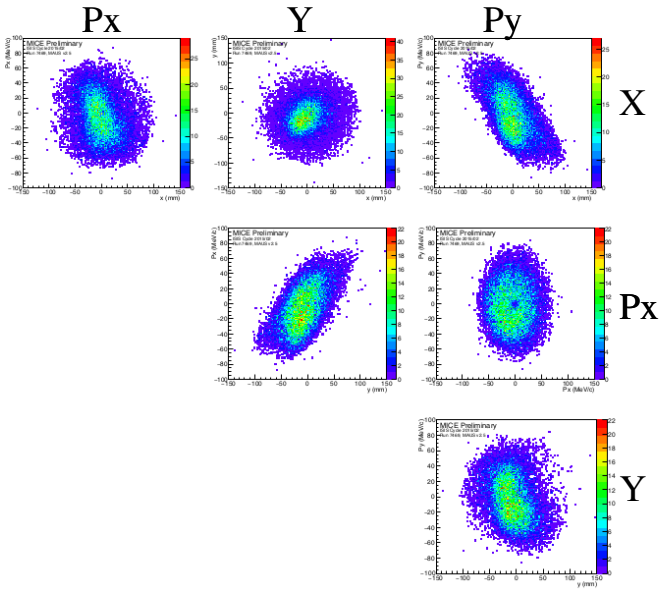
- Muons measured **one by one**:
  - ☆ Changes in Density, volume, emittance (ε) before (input) and after (output) an absorber.

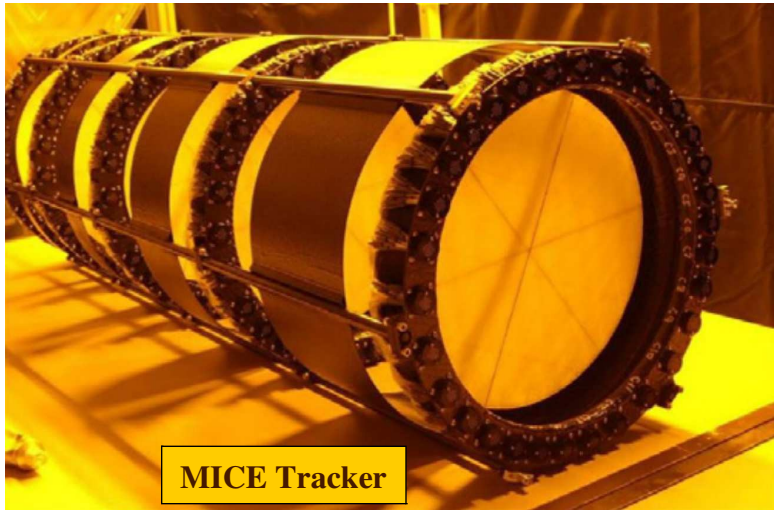


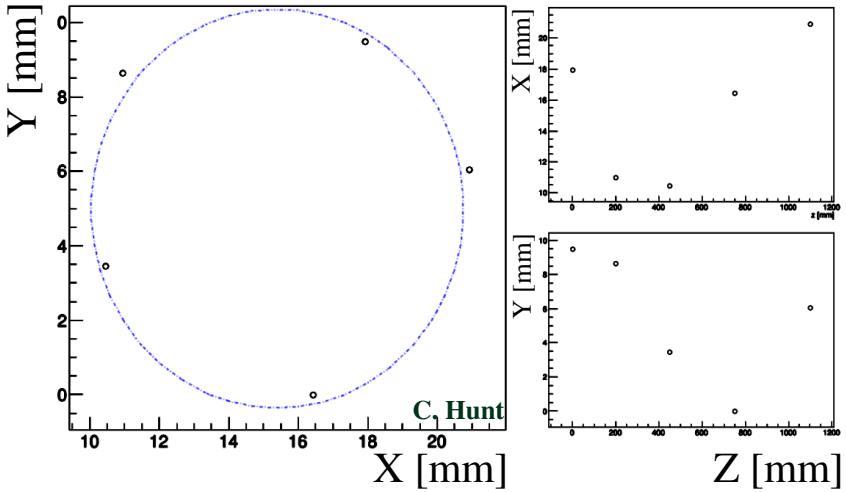
Tracker Reconstruction

- Helical tracks formed in spectrometer solenoids:
  - ★ Phase-space coordinates reconstructed in trackers.





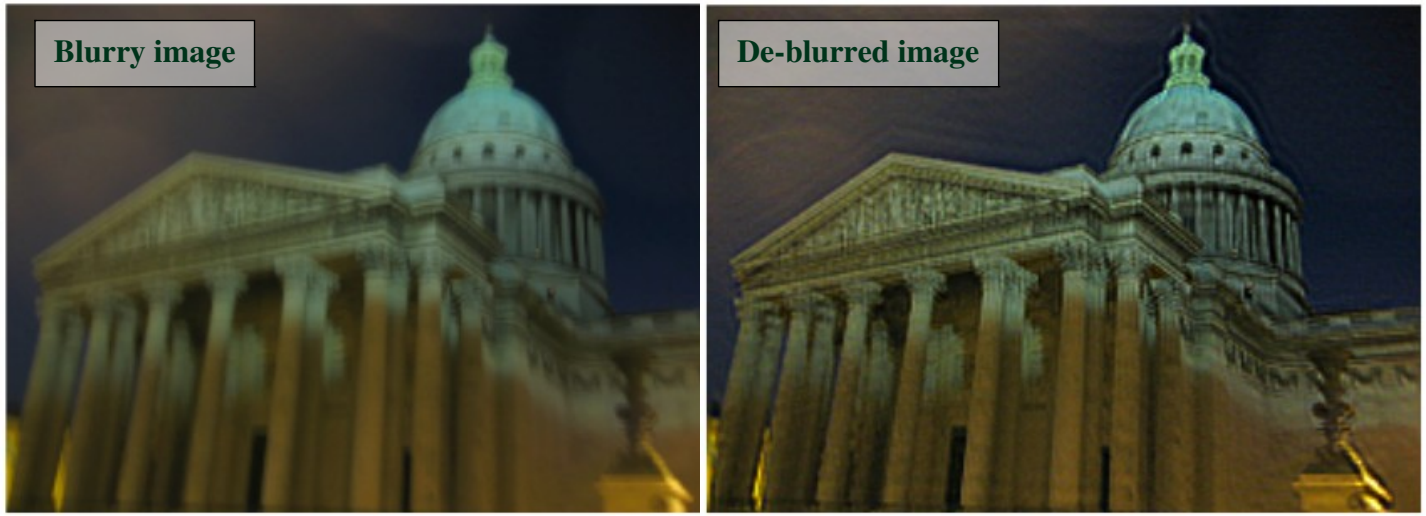




Kernel Density Estimation (KDE)

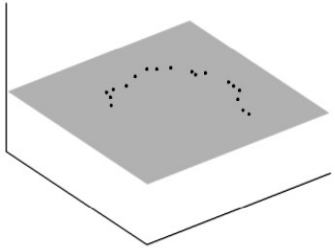
- Estimates the unknown probability density function (PDF) or density using kernels (smooth weight functions of certain widths).

Image processing with KDE

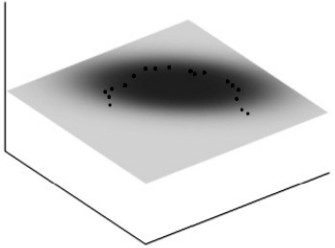


D. Krishnan et al., "Blind Deconvolution Using a Normalized Sparsity Measure", DOI: 10.1109/CVPR.2011.5995521

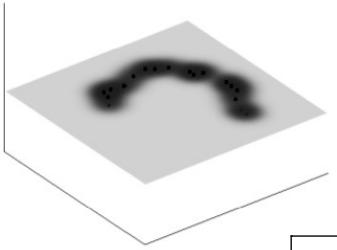
Actual distribution



Gaussian density



Kernel density estimation



M. Rousson et al., "Efficient Kernel Density Estimation of Shape and Intensity Priors for Level Set Segmentation", DOI:10.1007/978-0-387-68343-0_13

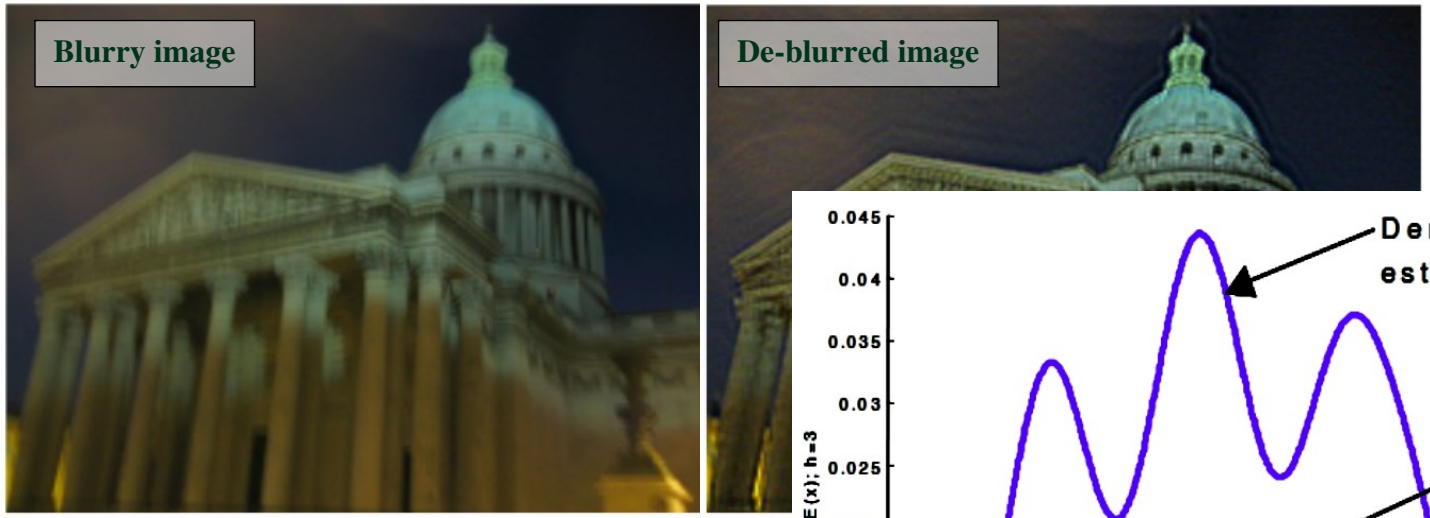
- Kernel functions at each data point: powerful single muon measurement tool for MICE.

Power of KDE

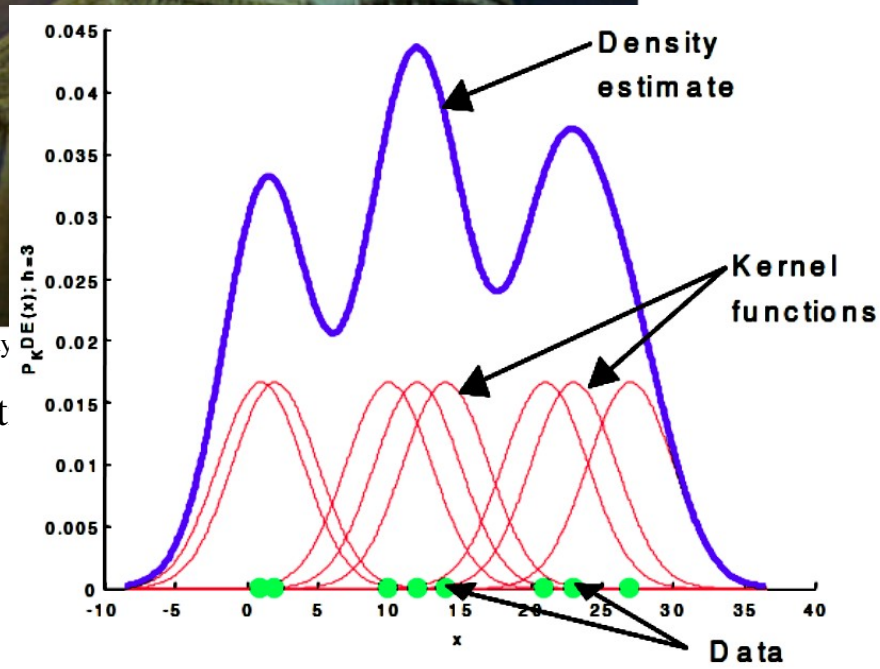
Kernel Density Estimation (KDE)

- Estimates the unknown probability density function (PDF) or density using kernels (smooth weight functions of certain widths).

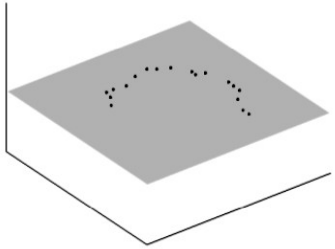
Image processing with KDE



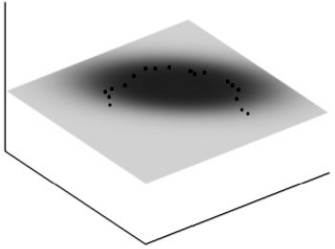
D. Krishnan et al., "Blind Deconvolution Using a Normalized Sparsity



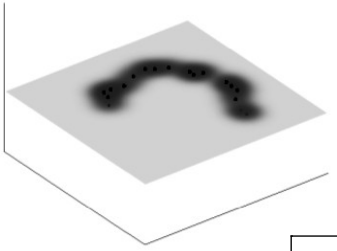
Actual distribution



Gaussian density



Kernel density estimat



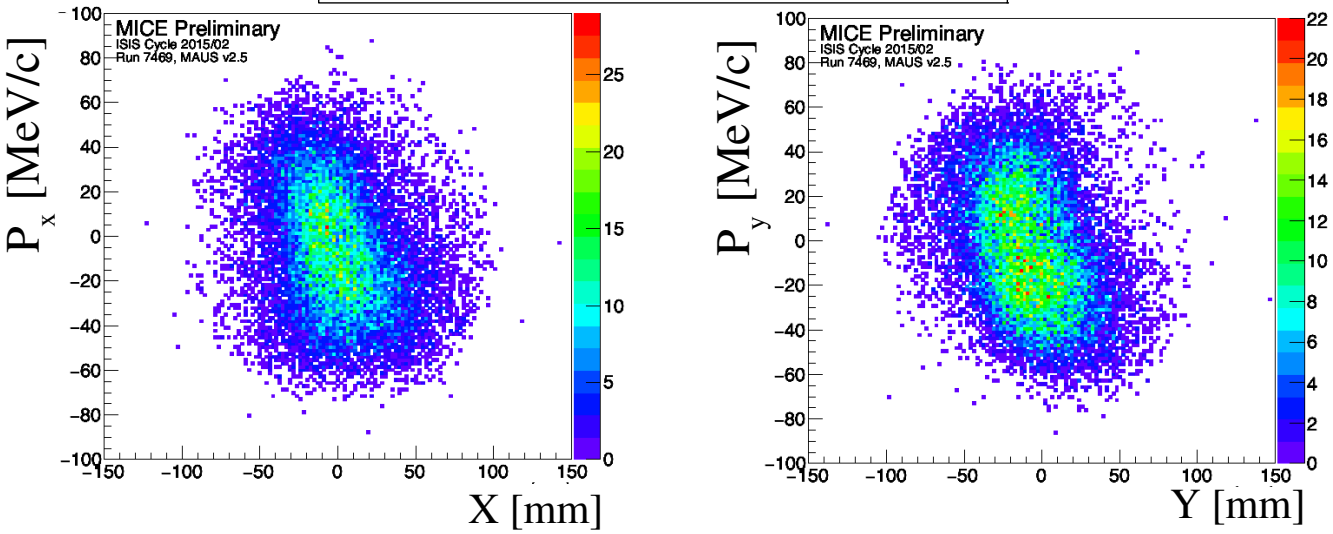
M. Rousson et al., "Efficient Kernel Density Estimation of Shape and Intensity Priors for Level Set Segmentation", DOI:10.1007/978-0-387-68343-0_13

Power of KDE

R. Gutierrez Osuna, "Kernel density estimation", CSCE 666 Pattern Analysis, Texas AM University.

Kernel Density Estimation (KDE) in MICE

Measured Muon Beam Phase Space

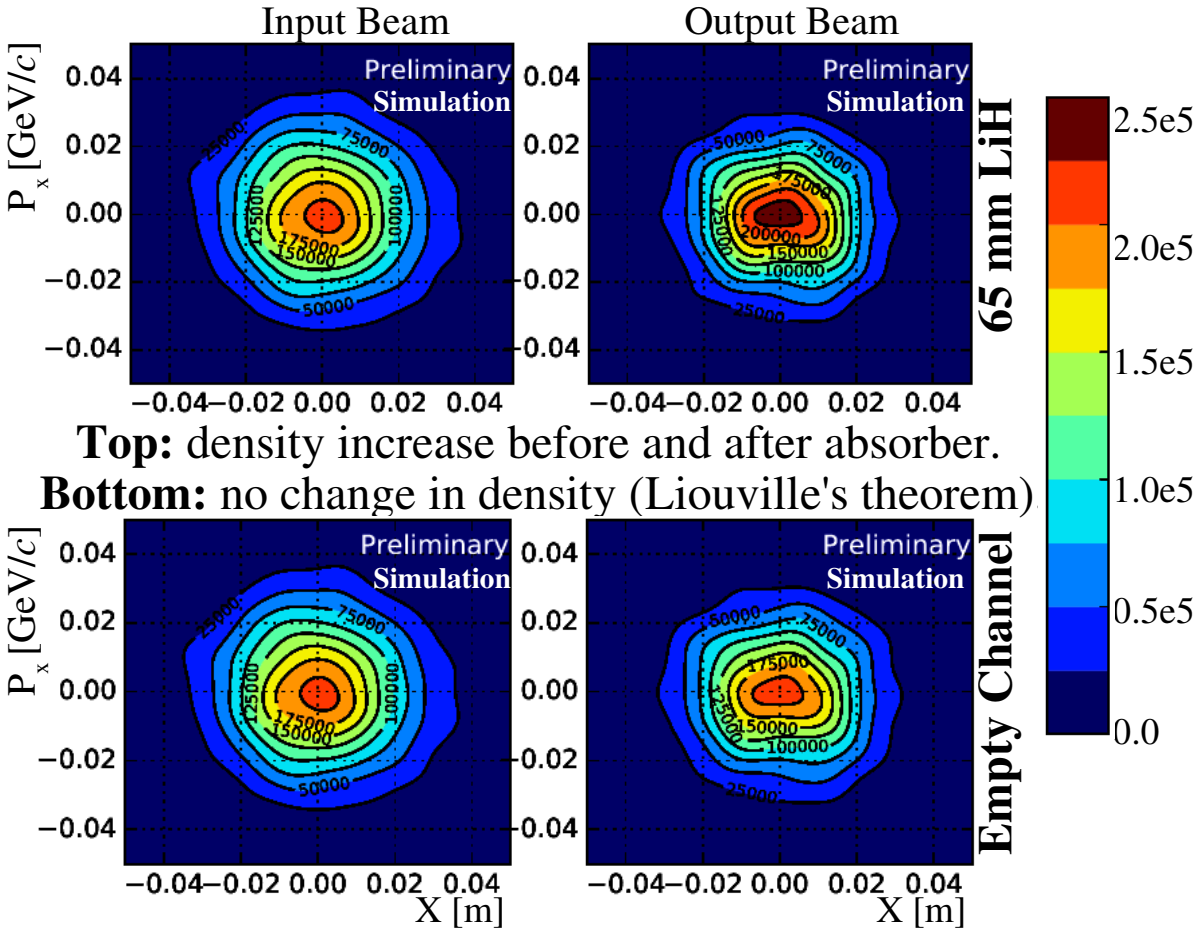


- Real-life particle beam is non-Gaussian (chromatic and non-linear effects).
- KDE:
  - ☆ Estimates PDF or density with few assumptions about the underlying distribution.
  - ☆ Gives detailed diagnostics of the particles in a cooling channel.

Density and Volume Measurements – Method

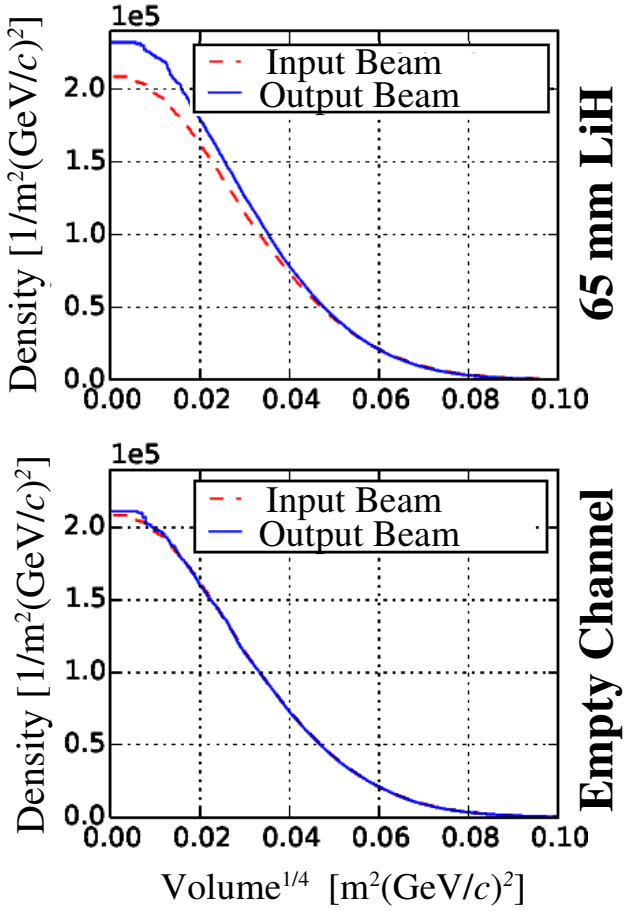
Phase-space density: assign Gaussian kernel functions at each muon in 4-dimensions, then sum each contribution.

Contour volume: generate Monte Carlo (MC) points inside a bounding box.

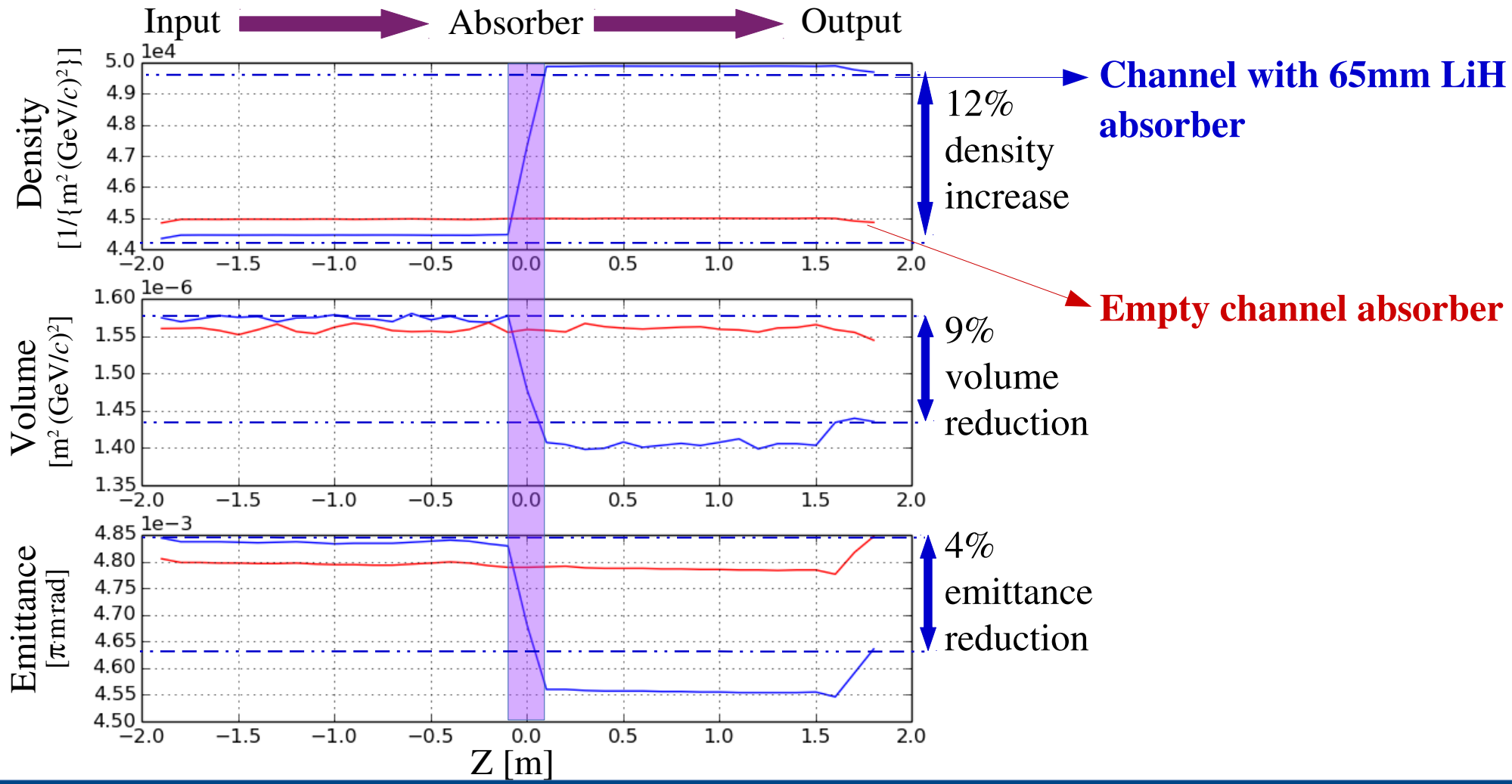


Top: density increase before and after absorber.

Bottom: no change in density (Liouville's theorem)



Preliminary Simulation Results



Conclusion and Future Prospects

- **Better** beam cooling estimation with KDE-based density, and volume as compared with the conventional emittance measurement.
- **First** application of KDE to beam cooling (as far as I know).
- Other applications:
  - Image processing in time projection chambers.
  - Data mining, machine learning in particle physics.
- MICE currently taking **experimental data**:
  - KDE application in progress.

Acknowledgements

Thank You!

- MICE is supported by DOE, INFN, and STFC.
- Research work presented here is supported by DOE Office of Science Graduate Student Research, SCGSR under contract No. DE–AC05–06OR23100.

References

1. D. Neuffer, “Principles and applications of muon cooling”, Part. Accel., Vol. 1, Gordon and Breach, Science Publishing (1983).
2. T. A. Mohayai, et al., “Novel Application of Kernel Density Estimation in MICE”, MICE-Note-506 (2016).
3. T. A. Mohayai, et al., “Simulated Measurements of Beam Cooling in Muon Ionization Cooling Experiment”, Proc. NA-PAC’16, NA-PAC-2016-WEPOA36 (2016).
4. T. A. Mohayai, et al., “Simulated Measurements of Cooling in Muon Ionization Cooling Experiment”, Proc. IPAC’16, IPAC-2016-TUPMY011 (2016).
5. Tanaz Angelina Mohayai, “Novel Application of Kernel Density Estimation in MICE”, MICE-Note - 506 (2017).
6. T. A. Mohayai, et al., “Novel Implementation Of Non-Parametric Density Estimation in MICE”, Pre-press, IPAC’17, IPAC-2017-WEPAB135 (2017).
7. Berg, J. S., “Phase space density as a measure of cooling performance for the international muon ionization cooling experiment”, Proc. IPAC’15, IPAC-2015-WEPJE025 (2015).
8. B. W. Silverman, “Density Estimation for Statistics and Data Analysis”, Monographs on Statistics and Applied Probability, Chapman and Hall(1986).

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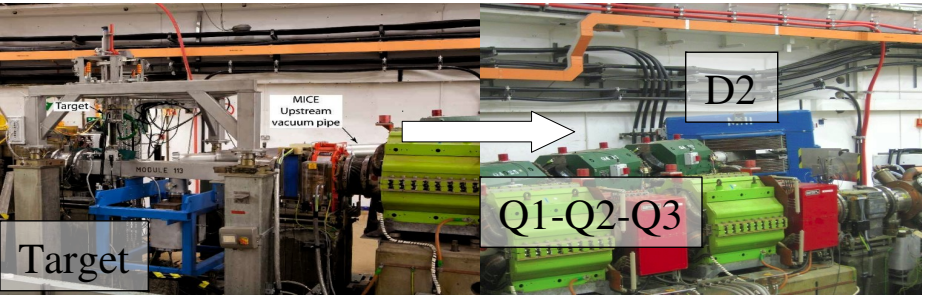
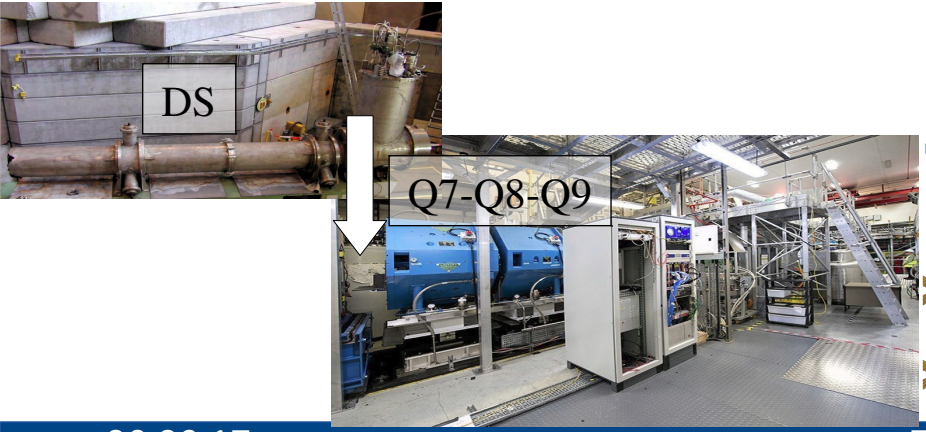
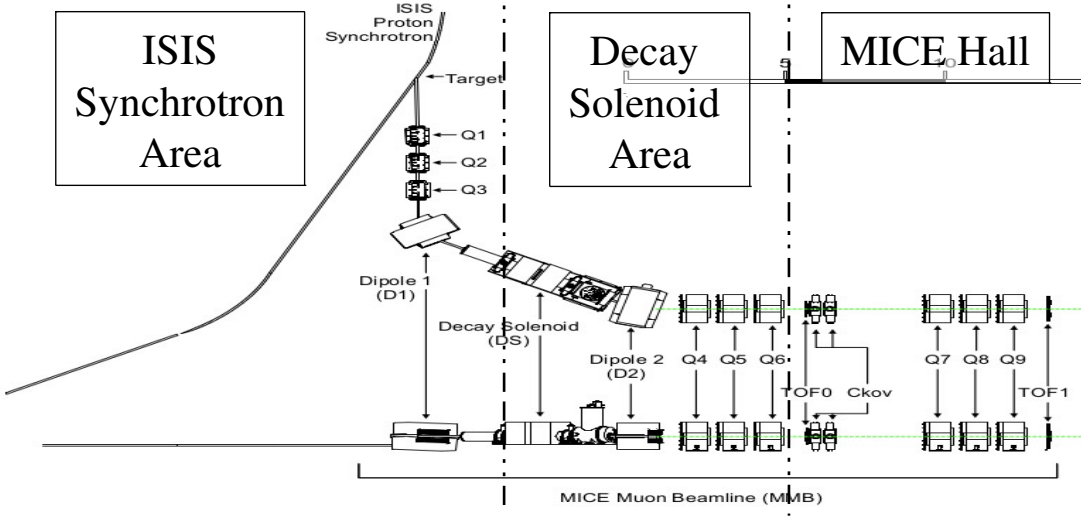
MICE Beam Line



- Pions produced via target-ISIS proton beam interactions: $p+p \rightarrow p+n+\pi^+$
  - Focused by *1*st quadrupole triplet magnets, Q1-Q2-Q3
  - Momentum selected and steered by *1*st dipole magnet, D1.



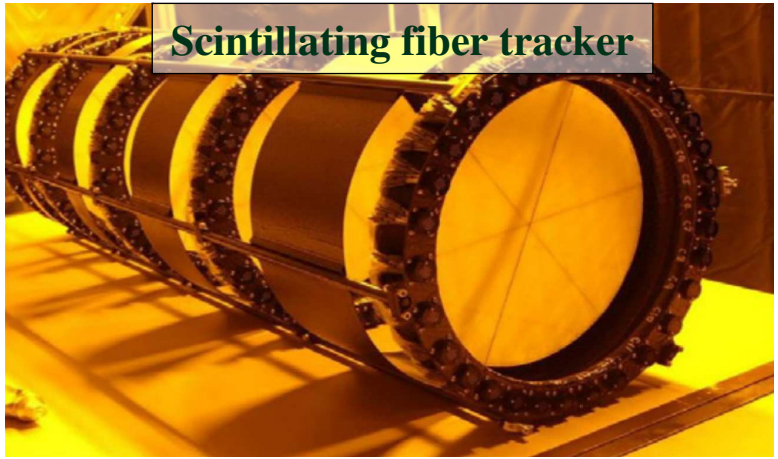
- Protons produced in ISIS proton synchrotron:
  - H^- bunches accelerated to *70* MeV in the Linac, transported to Al_2O_3 foil at the entrance of the synchrotron. H^+ bunched accelerated to *800* MeV in the synchrotron.

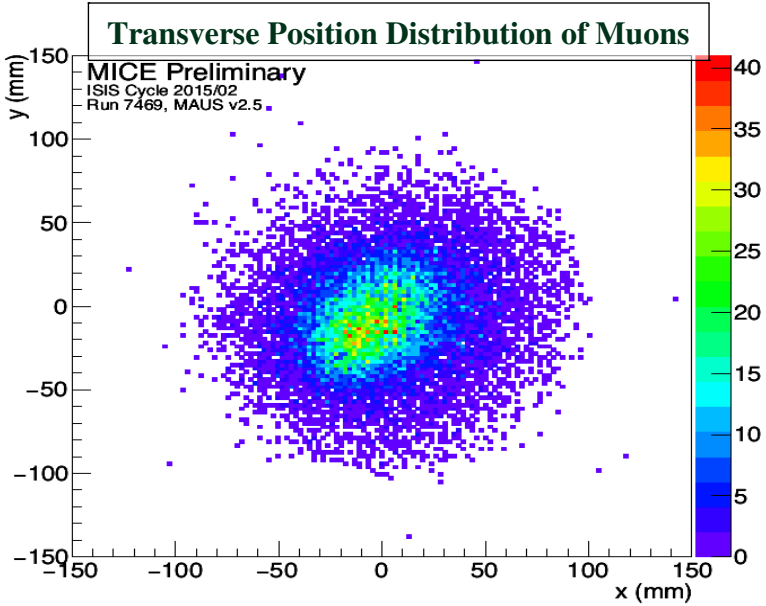


- Muons produced via pions decay in Decay Solenoid, DS: $\pi^+ \rightarrow \mu^+ + \nu_\mu$
  - Momentum selected and steered by *2*nd dipole magnet, D2.
  - Focused by pairs of quadrupole triplet magnets, Q4-Q5-Q6, Q7-Q8-Q9.

Trackers and Spectrometer Solenoids

- Spectrometer Solenoids:
  - 5 superconducting coil windings (Match1, Match2, End1, Center, End2)
- Trackers:
  - 5 tracker stations per tracker
  - 3 scintillating fiber planes per station
  - 214 scintillating fibers per plane





MICE Cooling Channel – Tracker Reconstruction

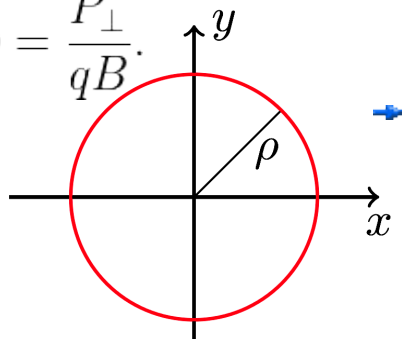
- **Helical track** reconstruction in solenoidal field:
  - Take z along beam axis (longitudinal) and x, y along a plane ⊥ to z (transverse).
  - Record at least 5 transverse points for each particle track & fit a circle to them.

$$x = -\rho \cos\left(\frac{qBt}{m}\right)$$

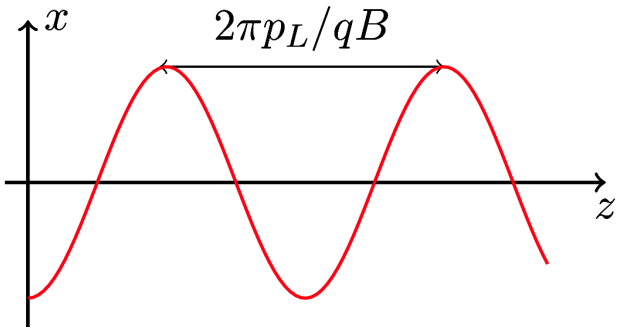
$$y = \rho \sin\left(\frac{qBt}{m}\right)$$

$$z = p_{\parallel}\left(\frac{t}{m}\right)$$

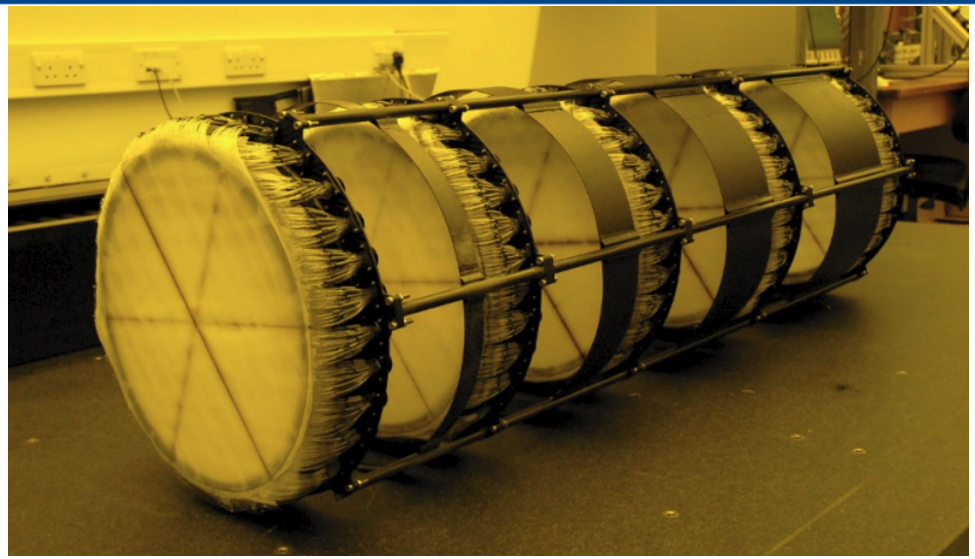
$$\rho = \frac{P_{\perp}}{qB}.$$

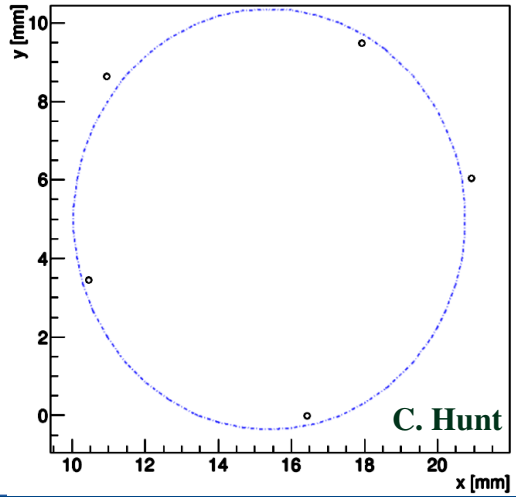


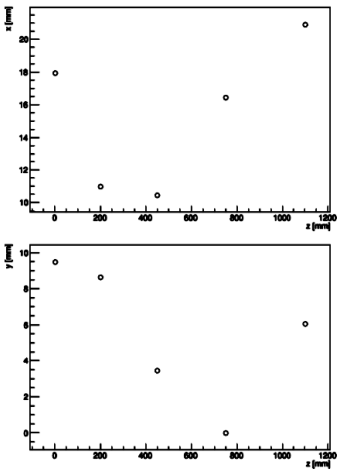
  - Use fitted circle to find radius ρ and $p_{\perp}$.



  - Take derivate of arc length with respect to z to find $p_{|}$.







Emittance Measurement

- Emittance measurement:
  - ☆ Reconstruct position and momentum coordinates using trackers.
  - ☆ Construct covariance matrix,

$$\Sigma = \begin{pmatrix} \sigma_{xx} & \sigma_{p_x x} & \sigma_{yx} & \sigma_{p_y x} \\ \sigma_{xp_x} & \sigma_{p_x p_x} & \sigma_{yp_x} & \sigma_{p_y p_x} \\ \sigma_{xy} & \sigma_{p_x y} & \sigma_{yy} & \sigma_{p_y y} \\ \sigma_{xp_y} & \sigma_{p_x p_y} & \sigma_{yp_y} & \sigma_{p_y p_y} \end{pmatrix}$$

$$\sigma_{xp_x} = \langle x p_x \rangle - \langle x \rangle \langle p_x \rangle$$

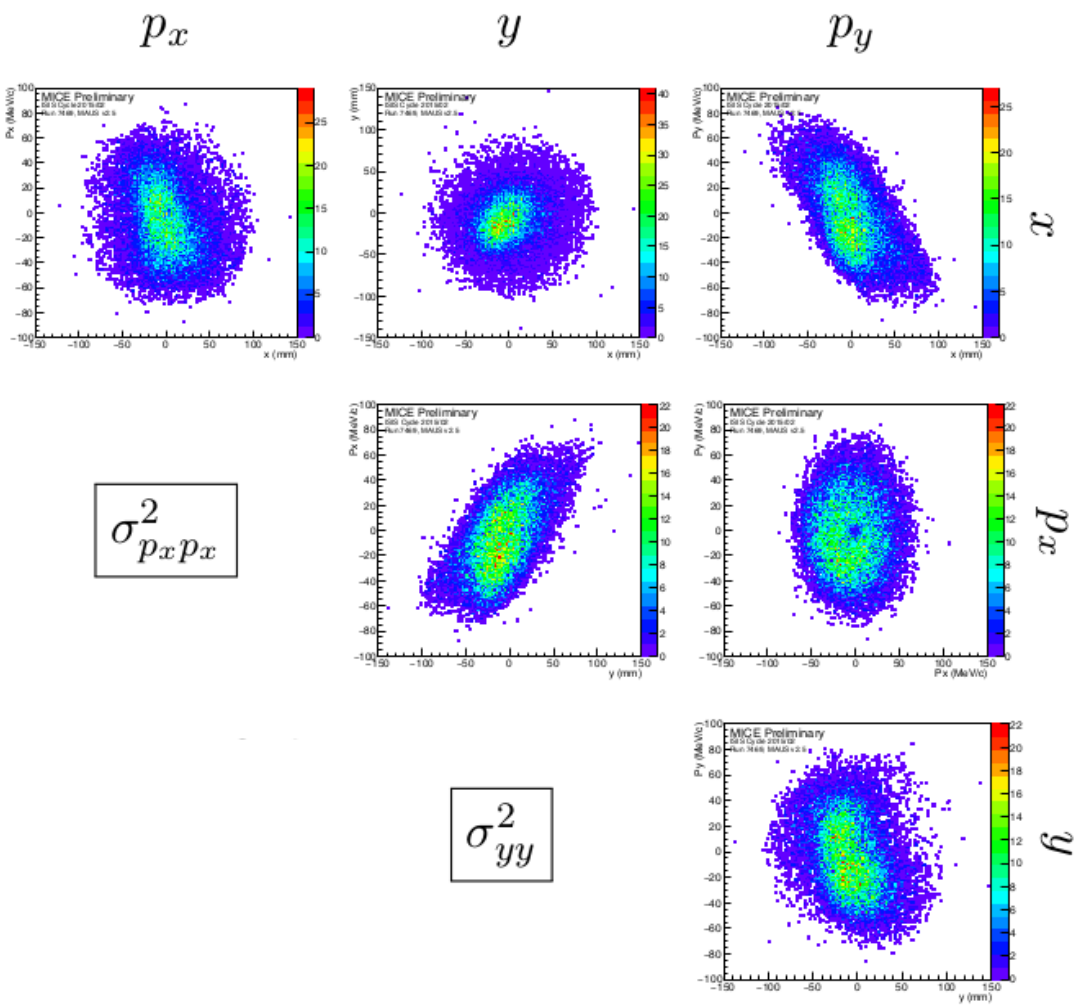
  - ☆ Compute transverse normalized RMS emittance,

$$\varepsilon_n = 1/m \, |\Sigma|^{\frac{1}{4}}$$

σ^2_{xx}

$\sigma^2_{p_x p_x}$

σ^2_{yy}

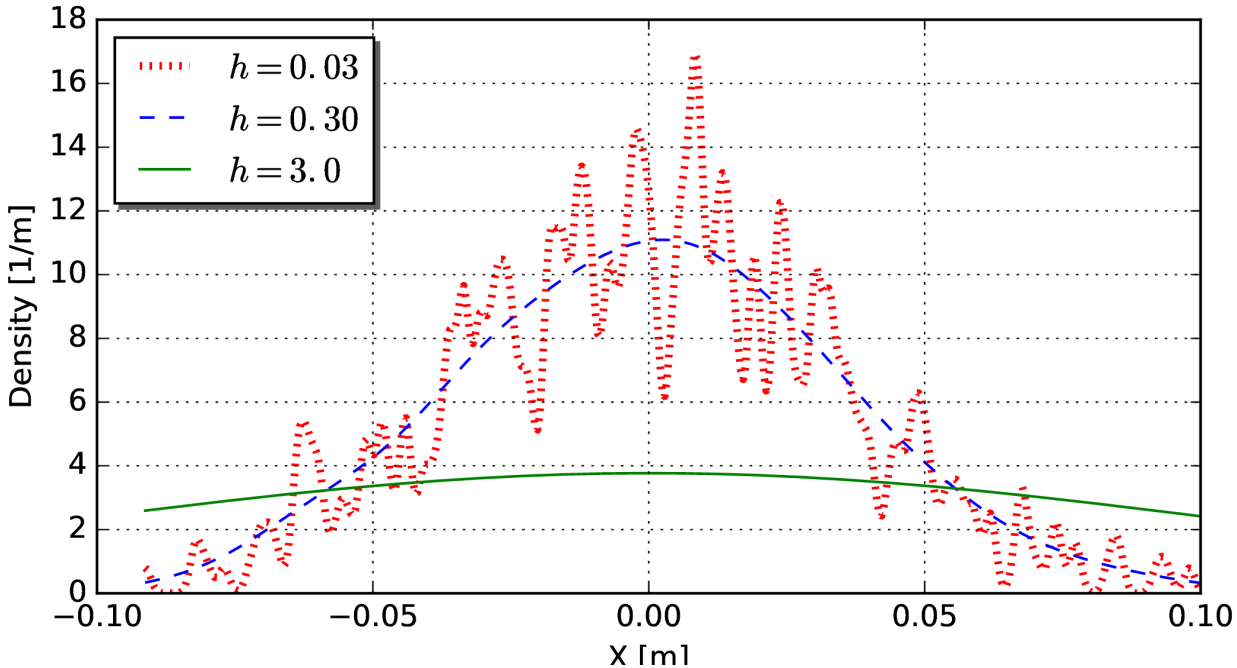


Kernel Width (Bandwidth)

- To get small bias (systematic error), bandwidth, h should be small, but small bandwidth leads to large variance (random error) and vice versa.
- Changing bandwidth affects estimated PDF: large h over-smooths density, while small h under-smooths it.

- x coordinates of a subsample of *500* muons at the entrance of MICE's upstream tracker.
- True PDF approximately Gaussian.
- h = *0.3* reveals a Gaussian.
- h = *3.0* over-smooths the PDF.
- h = *0.03* reveals a noisier PDF.

Bandwidth Effect on the Distribution



Beam Cooling in MICE

A good understanding of energy loss and multiple scattering of muons essential for studies of muon beam cooling.

1. Cooling via ionization energy loss

2. Heating via multiple Coulomb scattering

$$\frac{d\varepsilon_n}{ds} \simeq -\frac{\varepsilon_n}{\beta^2 E_\mu}\left\langle\frac{dE}{ds}\right\rangle + \frac{\beta_\perp\,(13.6\mathrm{MeV}/c)^2}{2\beta^3 E_\mu m_\mu X_0}$$

$$\varepsilon_\perp \cong \frac{\beta_\perp(13.6\mathrm{MeV})^2}{2X_0\beta m_\mu}\left\langle\frac{dE}{ds}\right\rangle^{-1}.$$

$\varepsilon_\perp$=transverse beam emittance, βc=muon velocity, E_μ=muon energy, dE/ds=muon ionization energy loss in material, $\beta_\perp$=transverse beta function, m_μ=muon mass, X_0=radiation length of absorber

- Equilibrium emittance obtained when cooling and heating rates are equal.
- Smaller equilibrium emittance obtained with minimized $\beta_\perp$ (experimentally with focusing coil windings) and maximized energy loss per X_0 (with low-Z absorber).