Theory status after Glasgow

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First Workshop of the Muon $g - 2$ Theory Initiative
FERMILAB June 2017

$g_\mu - 2$ Meetings since Glasgow

in which I have participated

- Muon Magnetic Moment Workshop
  Glasgow, October 2007
- INT Workshop on Hadronic Light-by-Light Contribution to the Muon Anomaly
  Seattle, March 2011
- High-precision QCD at low energy
  Benasque, August 2015

Topics I have chosen to discuss in this talk

- Determination of α from $g_e - 2$
- Status of $g_\mu - 2$ Theory versus Experiment
- Comments about the HLbyL contribution
- Comments about the HVP contribution

Electron Anomaly: *Recent update from Stefano Laporta'17*

$$a_e(\text{exp.}) = 1\ 159\ 652\ 180.73\ (0.28) \times 10^{-12}\ \ [0.24\ \text{ppb}]$$

Harvard group: *Gabrielse et al '08 '11*

$$a_e(\text{QED} - \text{massless}) = \sum_n a^{(2n)} \left(\frac{\alpha}{\pi}\right)^n$$

$$a^{(2)} = +0.5 \qquad \textit{Schwinger '48}$$

$$a^{(4)} = -0.328\ 478\ 965\ 579\ 193 \cdots \qquad \textit{Peterman, Sommerfield '58}$$

$$a^{(6)} = +1.181\ 241\ 456 \cdots \qquad \textit{Laporta and Remiddi '96}$$

$$a^{(8)} = -1.91298\ (84) \quad [891\ \text{Feynman diagrams}] \qquad \textit{Kinoshita et al '07 '08 '15}$$

$$a^{(8)} = -1.9122457649264455741526\ldots [110\ \text{digits}] \qquad \textit{Laporta '17}$$

$$a^{(10)} = +7.795(336) \quad [12672\ \text{Feynman diagrams}] \qquad \textit{Kinoshita et al '15}$$

With $\frac{m_e}{m_\mu}$ and $\frac{m_e}{m_\tau}$ corrections incorporated
as well as HVP ($\sim 2 \times 10^{-12}$), HLbyL ($\sim 3 \times 10^{-14}$) and EW ($\sim 3 \times 10^{-14}$) corrections:

$$a_e(\text{SM}) = 1\ 159\ 652\ 181.664\ \underbrace{(23)}_{\text{tenth}}\ \underbrace{(17)}_{\text{H-EW}}\ \underbrace{(763)}_{\alpha^{-1}(\text{Rb})} \times 10^{-12}$$

Fine Structure Constant

From the latest theoretical evaluation of a_e in the SM
and the Harvard measurement:

Reference Value of alpha for all other QED observables (a_μ in particular)

$$\alpha^{-1}(a_e) = 137.035\,999\,1596\ \underbrace{(27)}_{\text{tenth}}\ \underbrace{(18)}_{\text{H}-\text{EW}}\ \underbrace{(331)}_{\text{Harvard}}\quad [0.24\ \text{ppb}]$$

This is a fantastic achievement!

Comment on the lowest-order HVP Contribution to the Electron $g_e - 2$

M. Davier '10

$$a_e^{(\mathrm{HVP-lo})} = (1.875 \pm 0.017) \times 10^{-12}$$

J.S. Bell-deR '69

Upper Bound

$$a_e^{(\mathrm{HVP-lo})} \leq \frac{\alpha}{\pi}\frac{1}{3}\frac{m_e^2}{4m_\pi^2}\underbrace{\int_{4m_\pi^2}^{\infty}\frac{dt}{t}\,\frac{4m_\pi^2}{t}\,\frac{1}{\pi}\mathrm{Im}\Pi(t)}_{\mathcal{M}(0)} = \frac{\alpha}{\pi}\frac{1}{3}\frac{m_e^2}{4m_\pi^2}\underbrace{\left(-4m_\pi^2\frac{\partial}{\partial Q^2}\Pi(Q^2)\right)_{Q^2=0}}_{\mathrm{LQCD}}$$

This upper bound is practically the calculation for $a_e^{(\mathrm{HVP-lo})}$

Importance of making a Precise LQCD Determination of $\mathcal{M}(0)$
and compare it with the Experimental Results

Muon Anomaly

$a_\mu(\mathrm{E821-BNL}) = 116\,592\,089(54)_{\mathrm{stat}}(33)_{\mathrm{syst}} \times 10^{-11}[0.54\mathrm{ppm}]$

White Paper '13: *T. Blum, A. Denig, I. Logashenko, E. de Rafael, B. Lee Roberts, Th. Teubner, G. Venanzoni*

Future Experiments:

Fermilab with ± 0.14 ppm overall uncertainty

J-PARC with similar uncertainty but very different technique

QED Contributions (Leptons) $\{\alpha^{-1} = 137.035\,999\,1596\,(333)\,[0.24\,\mathrm{ppb}]\}$

Contribution	Result in Powers of $\frac{\alpha}{\pi}$	Numerical Value in 10^{-11} Units
$a_\mu^{(2)}$	$0.5\,\left(\frac{\alpha}{\pi}\right)$	116 140 973.22 (0.03)
$a_\mu^{(4)}(\mathrm{total})$	$0.765\,857\,425\,(17)\,\left(\frac{\alpha}{\pi}\right)^2$	413 217.63 (0.01)
$a_\mu^{(6)}(\mathrm{total})$	$24.050\,509\,96\,(32)\,\left(\frac{\alpha}{\pi}\right)^3$	30 141.90 (0.00)
$a_\mu^{(8)}(\mathrm{total})$	$130.879\,6\,(63)\,\left(\frac{\alpha}{\pi}\right)^4$	*Kinoshita et al '12* 381.01 (0.02)
$a_\mu^{(10)}(\mathrm{total})$	$753.29\,(1.04)\,\left(\frac{\alpha}{\pi}\right)^5$	*Kinoshita et al '12* 5.09 (0.01)
$a_\mu^{(2+4+6+8+10)}(\mathrm{QED})$		116 584 718.85 (0.02)(0.03)

Muon Anomaly

Standard Model Contributions

Contribution	Result in 10^{-11} units
QED (leptons)	$116\,584\,718.85 \pm 0.04$
HVP(lo)[e^+e^-] *Davier et al*	$6\,926 \pm 33$
HVP(lo)[e^+e^-] *Hagiwara et al*	$6\,949 \pm 43$
HVP(ho)	-98.4 ± 0.7
HLxL *P-deR-V "Glasgow-consensus"*	105 ± 26
EW	154 ± 1
Total SM (*Davier et al*)	$116\,591\,805 \pm 42$
Total SM (*Hagiwara et al*)	$116\,591\,828 \pm 50$

This is a 3.2σ to 3.7σ discrepancy
between SM theory and Experiment

Benayoun et al (BHLS-Model) $\Rightarrow 4.1\sigma$ to 4.7σ

Comments on the "Glasgow Consensus"

Reference Value for Models and LQCD

$$a_\mu^{\mathrm{HLbL}} = (10.5 \pm 2.6) \times 10^{-10}$$

Prades-de Rafael-Vainshtein '10

Since Glasgow there has been progress on various fronts

- Progress on Off-Shell Pion Form Factors (data and models)
- Dressed Pion Loop
- Scalar contributions and Axial-Vector Contributions
- **Dispersive Approach from the BERNE Group**

This meeting may provide an "improved consensus value" for a_μ^{HLbL}

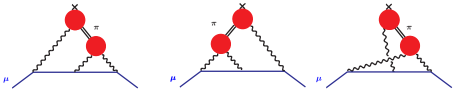
The HLbyL Contribution is known in a *Theoretical Limit*

Sponteneous Chiral Symmetry Breaking in QCD

- Implies a spectrum with *GOLDSTONE PARTICLES (pions)* and a *MASS GAP M* to the other hadronic states.
- The HLbyL contribution to a_μ in the limit where $m_{u,d,s} \to 0$ and *LARGE MASS GAP M* is known from the point-like WZW coupling:

HLbyL Contribution to the Muon Anomaly in Chiral Limit with *M* Large

$$a_\mu^{(\mathrm{HLbyL})} = \underbrace{\left(\frac{\alpha}{\pi}\right)^3 \mathrm{N_c}^2 \frac{m_\mu^2}{16\pi^2 f_\pi^2}\left[\frac{1}{3}\log^2\frac{M}{m_\pi} + \mathcal{O}\left(\log\frac{M}{m_\pi}\right) + \mathcal{O}(1)\right]}_{Knecht-Nyffeler-Perrottet-de\ \ Rafael'\ 02} + \mathcal{O}\left(\left(\frac{\alpha}{\pi}\right)^3 \mathrm{N_c} \frac{m_\mu^2}{M^2}\right)$$



HOWEVER: Comments and Questions

HLbyL Contribution to the Muon Anomaly in Chiral Limit with $M \to \infty$

$$a_\mu^{(\mathrm{HLbyL})} = \underbrace{\left(\frac{\alpha}{\pi}\right)^3 \mathrm{N_c}^2 \frac{m_\mu^2}{16\pi^2 f_\pi^2} \left[\frac{1}{3}\log^2\frac{M}{m_\pi}\right.}_{95\times 10^{-11}\ \text{for}\ M=M_\rho} + \mathcal{O}\left(\log\frac{M}{m_\pi}\right) + \mathcal{O}(1)\Big] + \mathcal{O}\left(\left(\frac{\alpha}{\pi}\right)^3 \mathrm{N_c}\frac{m_\mu^2}{M^2}\right)$$

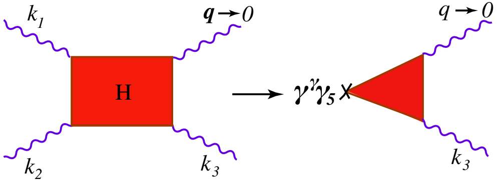
- Clearly, *in the M-Large limit*, the $\log^2 \frac{M}{m_\pi}$ term dominates.
- Once m_μ^2 factored out, the pion mass is the *infrared cut-off*.

However, in our World

- The mass gap of the hadronic spectrum $M = M_\rho$ (*is not that large*) and m_π is *bigger* than m_μ.
- Therefore, in practice one has to worry about $\mathcal{O}\left(\log\frac{M}{m_\pi}\right)$, $\mathcal{O}(1)$, $\mathcal{O}\left(\mathrm{N_c}\frac{m_\mu^2}{M^2}\right)$ corrections and $\frac{m_\mu}{m_\pi}$ dependence.
- Furthermore, sub-leading corrections in $1/\mathrm{N_c}$ (*pion-loop contribution*), will likely become relevant at the wanted level of accuracy.

Short-Distance Constraint

- There is also a Short–Distance constraint from the OPE in QCD (*Melnikov and Vainshtein '04*):



When $k_1^2 \approx k_2^2 \gg k_3^2$, and $k_1^2 \approx k_2^2 \gg m_\rho^2$

$$\int d^4x_1 \int d^4x_2\, \mathrm{e}^{-ik_1\cdot x_1 - ik_2\cdot x_2} J_\nu(x_1) J_\rho(x_2) = \frac{2\epsilon_{\nu\rho\delta\gamma}\hat{k}^\delta}{\hat{k}^2} \int d^4z\, \mathrm{e}^{-ik_3\cdot z} J_5^\gamma(z) + \mathcal{O}\left(\frac{1}{\hat{k}^3}\right)$$

  - At large $k_{1,2}$ Pseudoscalar (and Axial-Vector) exchanges dominate.
  - The AVV limit implies that the $\mathcal{F}_{\pi^0\gamma^*\gamma^*}(k^2, k^2)$ form factor must fall as $1/k^2$.

These QCD constraints are, however, not sufficient for a full model independent evaluation of $a_\mu^{(\mathrm{HLbyL})}$

Hadronic Vacuum Polarization

The Game nowadays is between "Improvement from Experiments" and "Improvement in LQCD Calculations"

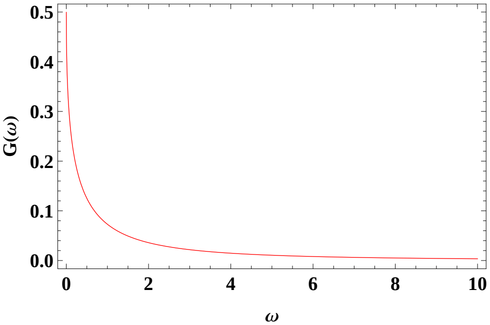
No room left for "models" at the $\leq 0.5\%$ *level of accuracy*

Theorists may, however, help in providing tools for a good interpolation of LQCD determinations

Comment on Lattice QCD (LQCD) Evaluations

LQCD uses $\omega \equiv \frac{Q^2}{m_\mu^2} = \frac{x^2}{1-x}$ instead of x-Feynman (*T. Blum '03*):

$$a_\mu^{\rm HVP} = \frac{\alpha}{\pi}\int_0^\infty \frac{d\omega}{\omega} \underbrace{\frac{1}{4}\left[(2+\omega)\left(2+\omega-\sqrt{\omega}\sqrt{4+\omega}\right)-2\right]}_{G(\omega)} \underbrace{\left(-\omega\frac{d}{d\omega}\Pi\left(\omega m_\mu^2\right)\right)}_{\text{Adler Function}}$$



LQCD evaluations *-at a few* ω *points-* need extrapolations.
This has been made with the help of *Padé Approximants* at low ω-values

Golterman-Peris-et al '12,'14,'16

CONCLUSIONS

- There has been progress
  both in Theory and Experiment since Glasgow

- This Meeting is an excellent initiative
  to make Further Progress

- Thanks to the organizers
  for creating this opportunity to make further progress !