

Hadronic Vacuum Polarization with Twisted Mass Fermions

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Outline

- Motivation

- ▶ Twisted Mass fermions
- ▶ Setup for $N_f = 2$ and $N_f = 2 + 1 + 1$

- HVP @ ETMC

- ▶ results for $N_f = 2 + 1 + 1$
- ▶ initial results for $N_f = 2$ physical point ensemble
- ▶ current work for tm+clover at physical pion mass

- Summary & Outlook

- ▶ g-2 @ ETMC Project status
- ▶ Agenda

Motivation - Twisted Mass Lattice QCD

- various gauge field ensembles
 - ▶ $N_f = 2$ twisted mass
 - ▶ $N_f = 2 + 1 + 1$ twisted mass
 - ▶ $N_f = 2$ twisted mass + clover @ m_π
 - ▶ $N_f = 2 + 1 + 1$ twisted mass + clover @ m_π
- comprehensive software suite for solving Dirac equation and Wick contractions
 - ▶ tmLQCD (solvers, exact & inexact deflation)
[Jansen and Urbach, 2009, Abdel-Rehim et al., 2013]
 - ▶ DDAlphaAMG (adaptive multi-grid solver with tm support, port to GPUs)
[Alexandrou et al., 2016]
 - ▶ cvc code package (various Wick contractions)
- automatic $\mathcal{O}(a)$ improvement of physical observables in the continuum limit in particular renormalized vacuum polarization function $\Pi_R(Q^2)$ and a_μ^{hvp}
- parity and $SU(2)$ isospin symmetry breaking at non-zero lattice spacing $SU(2) \rightarrow U(1)_3$
 - ▶ $m_\pi^\pm \neq m_\pi^0$
 - ▶ $\langle J_\mu^{\text{up}} J_\nu^{\text{up}} \rangle \neq \langle J_\mu^{\text{dn}} J_\nu^{\text{dn}} \rangle$ by lattice artefacts

Motivation - Twisted Mass Lattice QCD

- (degenerate) light quark action ($N_f = 2$)

$$\begin{aligned} S_I &= \sum_x \bar{\chi}_I [D_W + m_q + i\mu_I \gamma_5 \tau^3] \chi_I(x) \\ D_W &= \frac{1}{2} \gamma_\mu (\nabla_\mu^f + \nabla_\mu^b) - \left[\frac{a}{2} \nabla_\mu^b \nabla_\mu^f - m_{cr} \right] \end{aligned} \quad (1)$$

$\mathcal{O}(a)$ improvement at *maximal twist*: $m_q = m_0 - m_{cr} \rightarrow 0$

- Noether current for J_μ^{em}

$$J_\mu(x) = \frac{1}{2} \left[\bar{\chi}_I(x) (\gamma_\mu - 1) U_\mu(x) \chi_I(x + a\hat{\mu}) + \bar{\chi}_I(x + a\hat{\mu}) (\gamma_\mu + 1) U_\mu^\dagger(x) \chi_I(x + a\hat{\mu}) \right]$$

$$Z_V = 1$$

$$0 = \partial_\mu^b \underbrace{\left[\langle J_\mu(x) J_\nu(y) \rangle + a^{-3} \delta_{x,y}^{(4)} \delta_{\mu\nu} \langle C_\nu(y) \rangle \right]}_{\Pi_{\mu\nu}(x,y)}$$

exact at non-zero lattice spacing

Motivation - Twisted Mass Lattice QCD

- (non-degenerate) heavy quark action ($\dots + 1 + 1$)

$$\mathcal{S}_h = \sum_x \bar{\chi}_h [D_W + m_q + i\mu_\sigma \gamma_5 \tau^1 + \mu_\delta \tau^3] \chi_h(x) \quad (2)$$

- $\mu_\sigma \tau^1$ and $\mu_\delta \tau^3$ break isospin symmetry completely
→ no conserved vector current for strange and charm
- Osterwalder-Seiler (= mixed action) setup [Frezzotti and Rossi, 2004]

$$\mathcal{S}_h^{\text{val}} = \sum_{f=s,c} \sum_x \bar{\psi}_f [D_W + m_q + i\mu_f \gamma_5 \tau^3] \psi_f(x)$$

- μ_s, μ_c tuned by physical value of $2m_K^2 - m_{PS}^2$ and m_D
- (valence) Noether currents $J_\mu^f \sim \bar{\psi}_f \psi_f$;
automatic $\mathcal{O}(a)$ remains valid

HVP from tmLQCD and $N_f = 2 + 1 + 1$

- consider hadronic leading-order $\Delta\alpha_{\text{QED}}(Q^2) \propto \Pi_\gamma(Q^2)$

$$\Delta\overline{\alpha}_{\text{QED}}^{\text{hvp}}(Q^2) = -4\pi\alpha_0 \left(\frac{5}{9} \Pi_{\text{R}}^{\text{ud}} \left(Q^2 \cdot \frac{H^2}{H_{\text{phys}}^2} \right) + \frac{1}{9} \Pi_{\text{R}}^{\text{s}} \left(Q^2 \right) + \frac{4}{9} \Pi_{\text{R}}^{\text{c}} \left(Q^2 \right) \right). \quad (3)$$

- inter-/extrapolation, $\Pi(0)$

$$\Pi_{\text{low}}^{\text{f}}(Q^2) = \sum_{i=1}^M \frac{g_i^2 m_i^2}{m_i^2 + Q^2} + \sum_{j=0}^{N-1} a_j (Q^2)^j$$

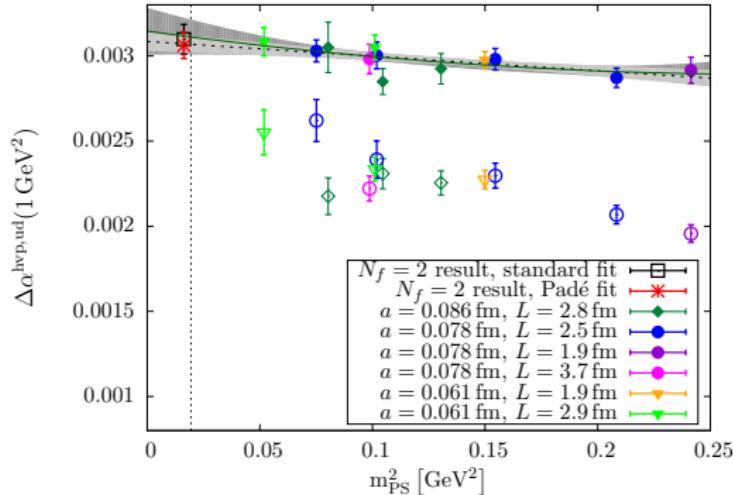
$$\Pi_{\text{high}}^{\text{f}}(Q^2) = \log(Q^2) \sum_{k=0}^{B-1} b_k (Q^2)^k + \sum_{l=0}^{C-1} c_l (Q^2)^l$$

$$\Pi^{\text{f}}(Q^2) = (1 - \Theta(Q^2 - Q_{\text{match}}^2)) \Pi_{\text{low}}^{\text{f}}(Q^2) + \Theta(Q^2 - Q_{\text{match}}^2) \Pi_{\text{high}}^{\text{f}}(Q^2)$$

- chiral and continuum extrapolation

$$\Delta\alpha_{\text{QED}}^{\text{hvp}}(Q^2)(m_{\text{PS}}, a) = A + B_1 m_{\text{PS}}^2 (+ \dots) + C a^2$$

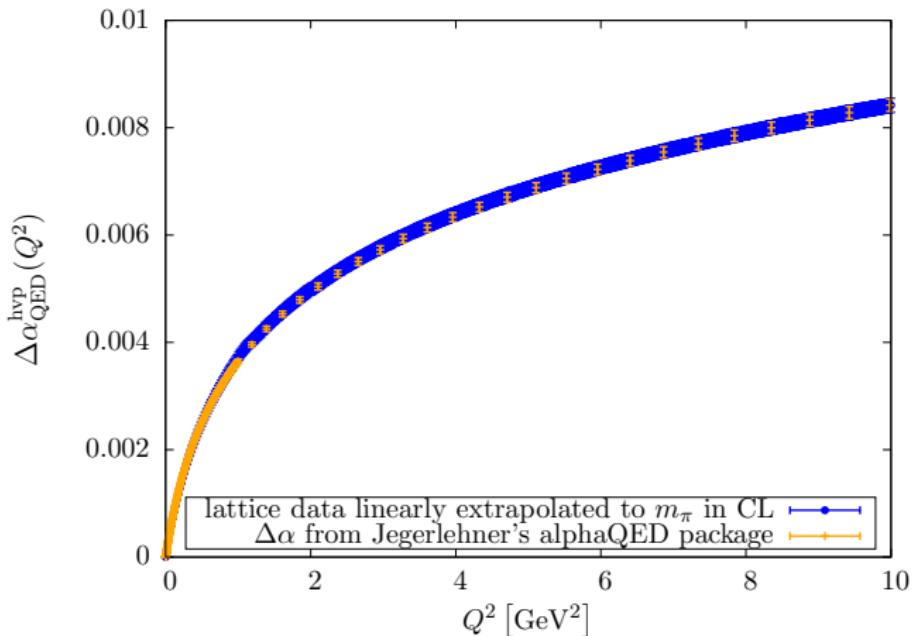
HVP from tmLQCD with $N_f = 2 + 1 + 1$



$a[\text{fm}]$	$m_{PS}[\text{MeV}]$	$L[\text{fm}]$	$m_{PS}L$
0.061	227	2.9	3.3
0.061	318	2.9	4.7
0.061	387	1.9	3.7
0.078	274	2.5	3.5
0.078	319	2.5	4.0
0.078	314	3.7	5.9
0.078	393	2.5	5.0
0.078	456	2.5	5.8
0.078	491	1.9	4.7
0.086	283	2.8	4.0
0.086	323	2.8	4.6
0.086	361	2.8	5.1

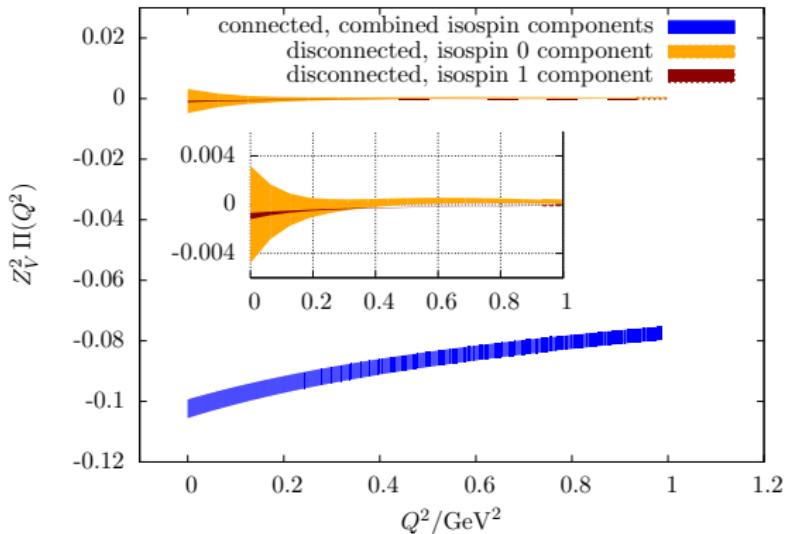
- statistics $\mathcal{O}(250/150/150)$ gauge configurations for up-down / strange / charm quark

HVP from tmLQCD with $N_f = 2 + 1 + 1$ and a_I^{hvp}



a_I^{hlo}	tmLQCD		disp. analyses
e	$1.782(64)(85) \cdot 10^{-12}$	$1.866(10)(05) \cdot 10^{-12}$	[Nomura and Teubner, 2013]
μ	$6.78(24)(16) \cdot 10^{-8}$	$6.91(01)(05) \cdot 10^{-8}$	[Jegerlehner and Szafron, 2011]
τ	$3.41(8)(6) \cdot 10^{-6}$	$3.38(4) \cdot 10^{-6}$	[Eidelman and Passera, 2007]

Quark-disconnected contribution



- isovector and isoscalar contribution

$$\Pi_{\mu\nu}^3(x, y) = \langle J_\mu^3(x) J_\nu^3(y) \rangle_{\text{disc}} \quad \text{tm only, with one-end trick}$$

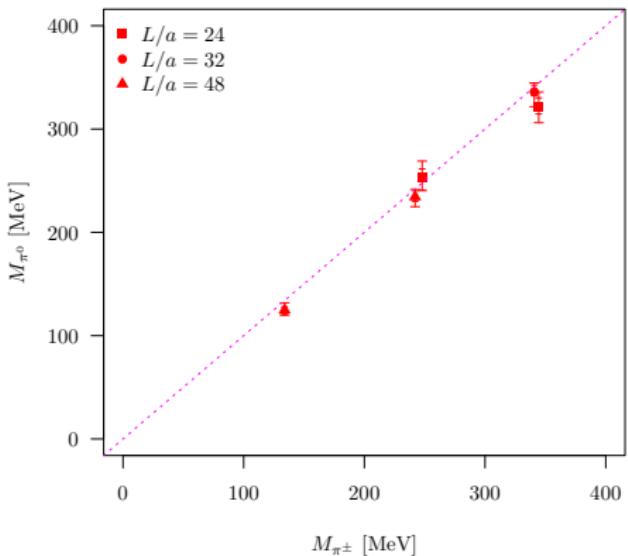
$$\Pi_{\mu\nu}^0(x, y) = \langle J_\mu^0(x) J_\nu^0(y) \rangle_{\text{disc}}$$

- $a = 0.078 \text{ fm}$, $m_\pi = 393 \text{ MeV}$, $L = 2.5 \text{ fm}$, $m_\pi L = 5.0$, up-down contribution
- $1548 \times 24 + 4996 \times 48$ gauge configurations \times stochastic volume sources

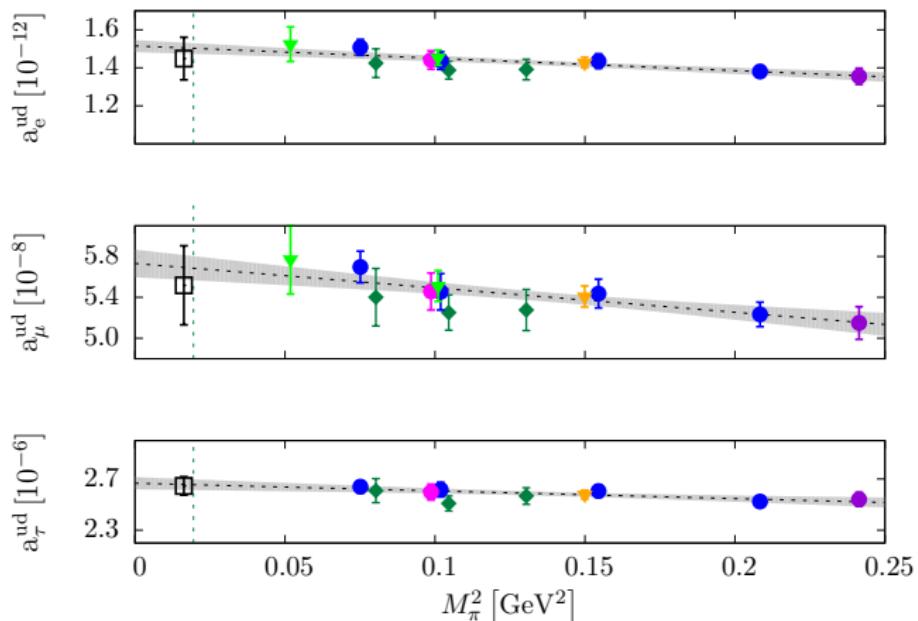
twisted mass + clover at $N_f = 2$ and physical pion mass

$$\mathcal{S}_I = \sum_x \bar{\chi}_I \left(D_W [U] + m_q + i\mu_I \gamma_5 \tau^3 + \frac{i}{4} c_{sw} \sigma_{\mu\nu} F_{\mu\nu} [U] \right) \chi_I(x) \quad (4)$$

- clover term *not* added for $\mathcal{O}(a)$ improvement
 - but to reduce the effects of isospin splitting
 - pion masses range from 130 to 350 MeV
 - 2 volumes at $m_{PS} = 130$ MeV, 4.4 fm and 5.8 fm
 - (single) lattice spacing
 $a = 0.0914(3)(15)$ fm
 - pion mass splitting compatible with zero
 \Rightarrow reduced finite size effects
 \Rightarrow reduces isospin splitting effects



twisted mass + clover at $N_f = 2$ and physical pion mass, $L = 4.4 \text{ fm}$



physical point	extr. $N_f = 2$	extr. $N_f = 2 + 1 + 1$
$a_e^{\text{hvp}} \cdot 10^{12}$	1.45(11)	1.51(04)
$a_\mu^{\text{hvp}} \cdot 10^8$	5.52(39)	5.72(16)
$a_\tau^{\text{hvp}} \cdot 10^6$	2.65(07)	2.65(02)

[Abdel-Rehim et al., 2015]

- focus on tmLQCD+clover at $N_f = 2$ and physical pion mass
- 1 lattice spacing ($a = 0.091$ fm), but 2 volumes (4.4 fm and 5.8 fm)
- twisted mass + clover $N_f = 2 + 1 + 1$ at physical pion mass in production
- more measurements and statistics

$$\Pi_{\mu\nu}(x, y) = \langle J_\mu(x) J_\nu(y) \rangle$$

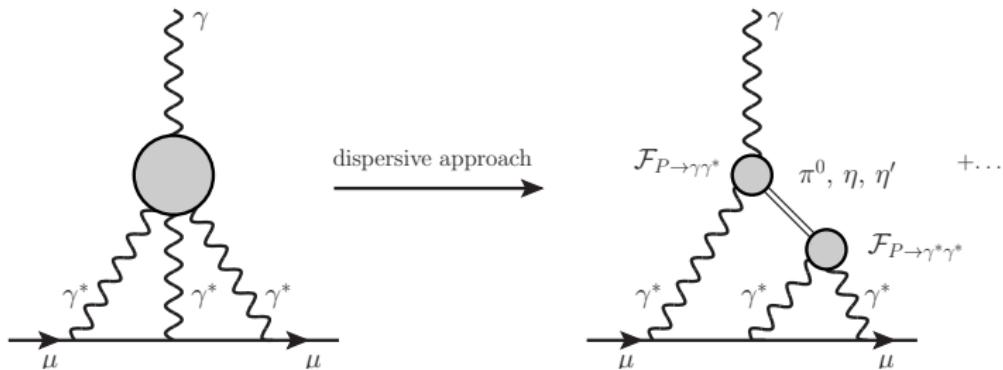
point-to-all, i.e. one \sim few, fixed source locations y per gauge configuration
⇒ little of information content per gauge configuration actually used

- signal in disconnected contribution

Restart 2017 - extended list of observables

- HVP and $\pi^0, \eta, \eta' \rightarrow \gamma\gamma$
(η, η' on upcoming $N_f = 2 + 1 + 1$ physical point gauge field ensemble)
- neutral pion decay on the lattice [Ji and Jung, 2001, Cohen et al., 2008, Shintani et al., 2009, Feng et al., 2011, Feng et al., 2012]
- dispersive approach to $g - 2$ HLbL
[Colangelo et al., 2014b, Colangelo et al., 2014a, Pauk and Vanderhaeghen, 2014]

$$\Pi_{\mu\nu\lambda\sigma}^{\text{HLbL}} = \Pi_{\mu\nu\lambda\sigma}^{\pi^0} + \Pi_{\mu\nu\lambda\sigma}^{\text{FsQED}} + \Pi_{\mu\nu\lambda\sigma}^{\eta} + \Pi_{\mu\nu\lambda\sigma}^{\pi\pi} + \Pi_{\mu\nu\lambda\sigma}^{\eta'} + \dots$$



recent calculation by Mainz group [Nyffeler, 2016, Antoine et al., 2016]

Restart 2017 - from point-to-all towards all-to-all

- use even-odd preconditioned exact low mode + stochastic high mode contributions to build correlators [Blum et al., 2016]
- quark propagator = inverse twisted mass Dirac matrix

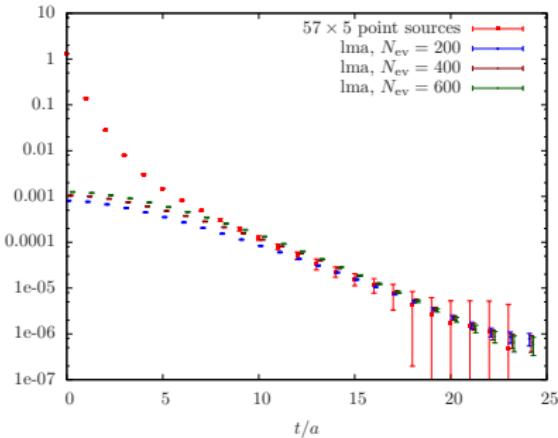
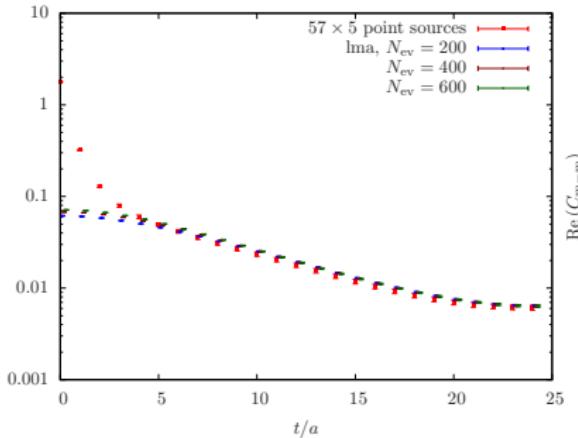
$$D_{\text{tm}}^{-1} = \begin{pmatrix} M_{ee} & M_{eo} \\ M_{oe} & M_{oo} \end{pmatrix}^{-1} = \begin{pmatrix} \mathbb{1} & -M_{ee}^{-1} M_{eo} C^{-1} \\ 0 & C^{-1} \end{pmatrix} \times \begin{pmatrix} M_{ee}^{-1} & 0 \\ -\gamma_5 M_{oe} M_{ee}^{-1} & \gamma_5 \end{pmatrix}$$

- exact inverse M_{ee}^{-1} available
- computationally intensive part: inversion of $C \rightarrow C^{-1}$
- construct subspace from N_{ev} eigenvectors to lowest-lying eigenvalues of CC^\dagger ,
 $CC^\dagger V = V \Lambda$
- decomposition of odd sub-lattice with orthogonal projectors $\mathbb{1} = P_V + P_V^\perp$,
 $P_V = V V^\dagger$
- inversion on P_V becomes trivial
- inversion on P_V^\perp becomes cheap (exact deflation of lowest eigenmodes)

$$D_{\text{tm}}^{-1} = \begin{pmatrix} \mathbb{1} & -M_{ee}^{-1} M_{eo} C^{-1} \\ 0 & C^{-1} \end{pmatrix} \begin{pmatrix} \mathbb{1} & 0 \\ 0 & P_V + P_V^\perp E[\xi \xi^\dagger] \end{pmatrix} \begin{pmatrix} M_{ee}^{-1} & 0 \\ -\gamma_5 M_{oe} M_{ee}^{-1} & \gamma_5 \end{pmatrix}$$

Convergence test of exact low modes

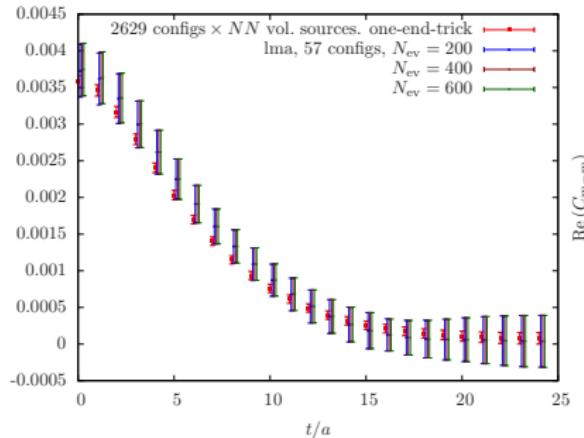
Re (C_{m-m})



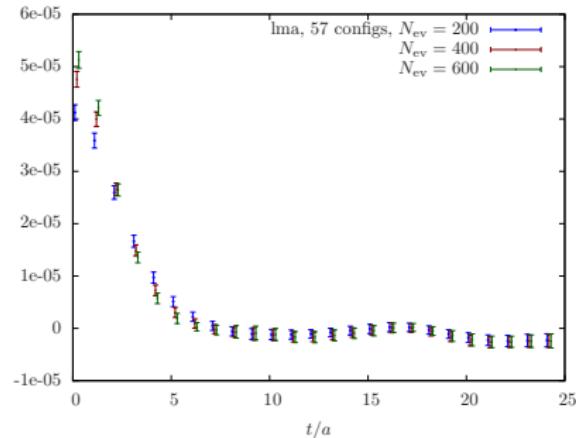
- connected 2-point correlation functions
- charged pion 2-point function (left) and connected (local) vector current 2-point function (right)
- $N_f = 2 + 1 + 1$, $a = 0.086$ fm, $m_\pi = 320$ MeV, $L = 2$ fm

Convergence test of exact low modes

Re(C_{m-m})



Re(C_{m-m})



- disconnected 2-point correlators
- disconnected $\eta - \eta$ 2-point function (left) and (isoscalar) vector 2-point function (right)
- $N_f = 2 + 1 + 1$, $a = 0.086$ fm, $m_\pi = 320$ MeV, $L = 2$ fm

2- and 3-point functions

- HVP

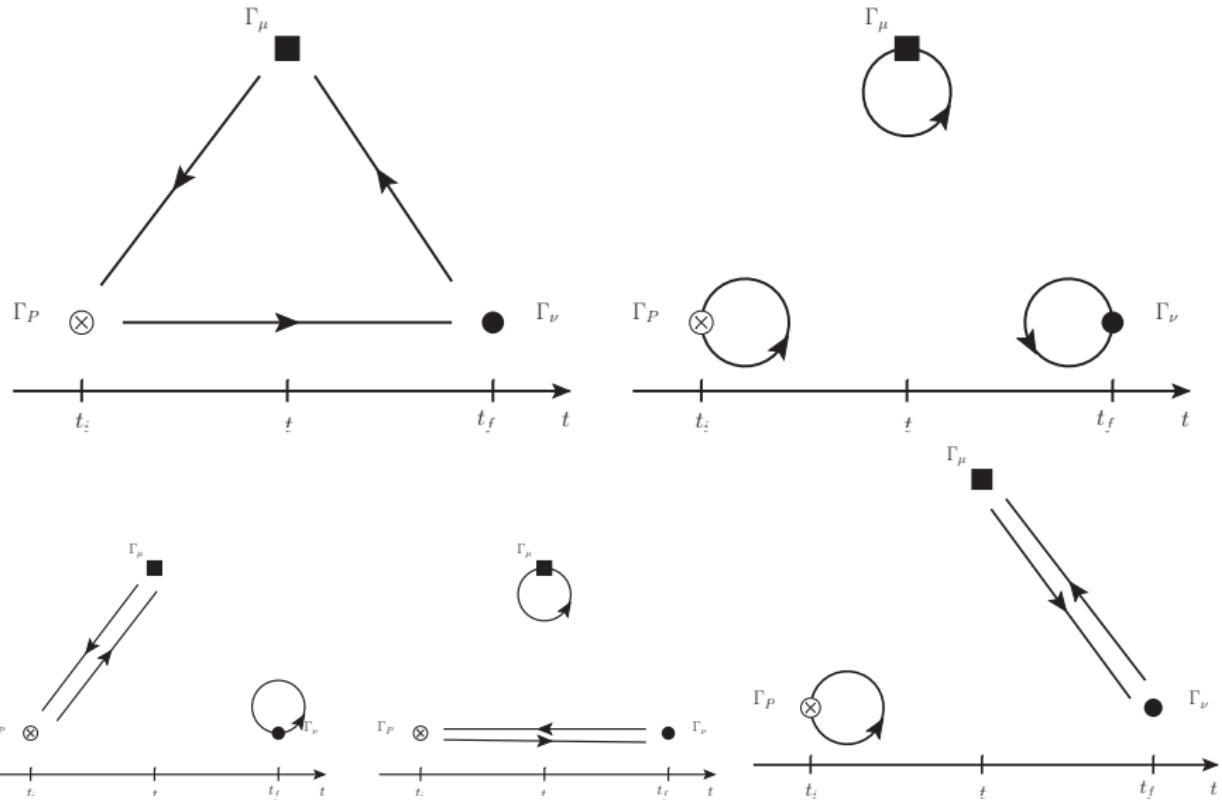
$$\langle J_\mu(x) J_\nu(y) \rangle$$

- $P \rightarrow \gamma\gamma$

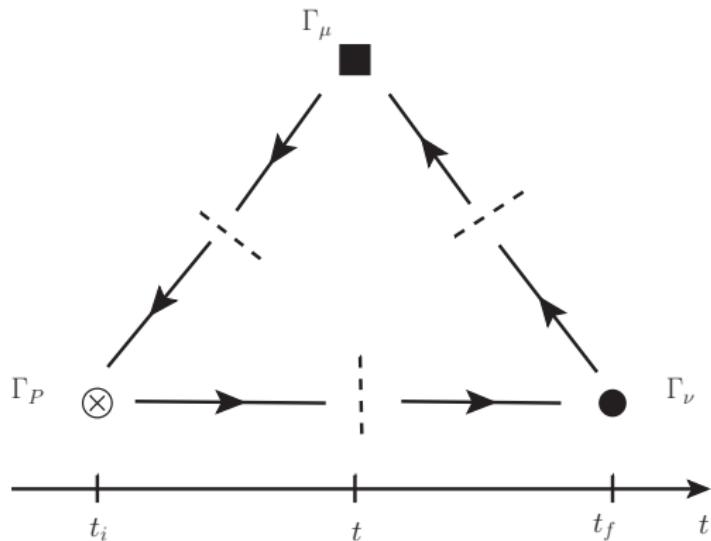
$$\langle P(x) J_\mu(y) J_\nu(z) \rangle$$

- J_μ conserved vector current
- P pseudoscalar (π^0, η) interpolating field

Connected and disconnected diagrams



Exact low modes + stochastic high modes - diagram factorization



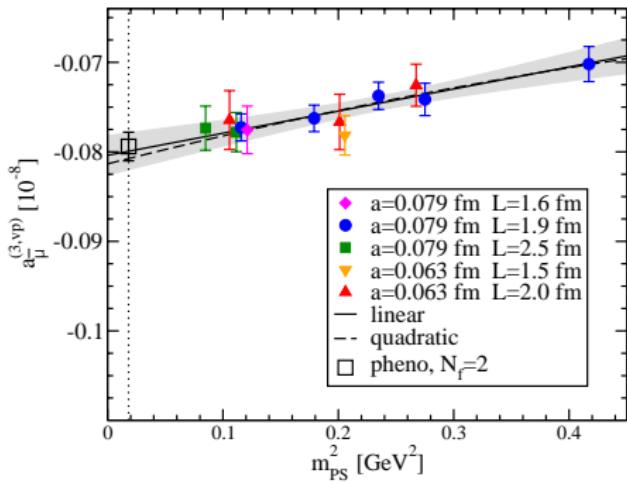
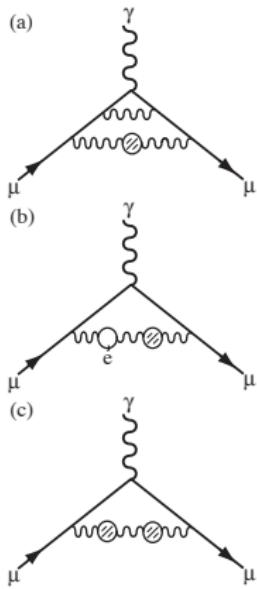
- exact low / stochastic high mode decomposition allows factorization of diagrams
- sufficient to calculate building blocks of type $V^\dagger \Gamma W$
- build also 4-point function $\rightarrow \langle J_\mu(x_1) J_\nu(x_2) J_\lambda(x_3) J_\sigma(x_4) \rangle$

Summary - project status

- Wick contractions for building blocks are implemented
- accumulation of measurements for $N_f = 2$ tm+clover physical point ensemble with $L = 4.4$ fm (48^3)
- first measurements for $L = 5.8$ fm (III/2017)
- measurements for $N_f = 2 + 1 + 1$ tm+clover physical point ensemble with $L = 5.2$ fm (64^3) (end of 2017)
- generalization to inexact deflation, combination with multi-grid method
- on-going work on leading isospin-breaking corrections by collaboration in Rome (RM123 method):
path integral expansion in small parameters $m_u - m_d$ and α_{QED}

Also on the agenda

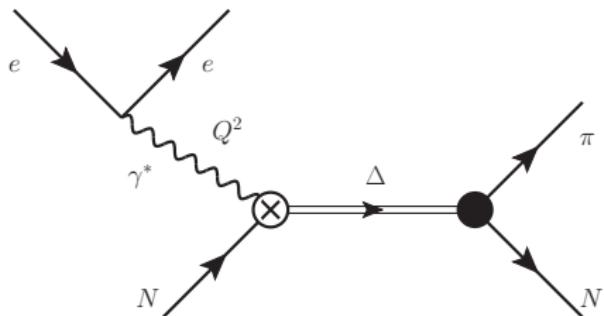
- update on HVP NLO contribution with $N_f = 2 + 1 + 1$, $N_f = 2$ tm+clover



ETMC result for $N_f = 2$ twisted mass
from Lattice 2011

Also on the agenda

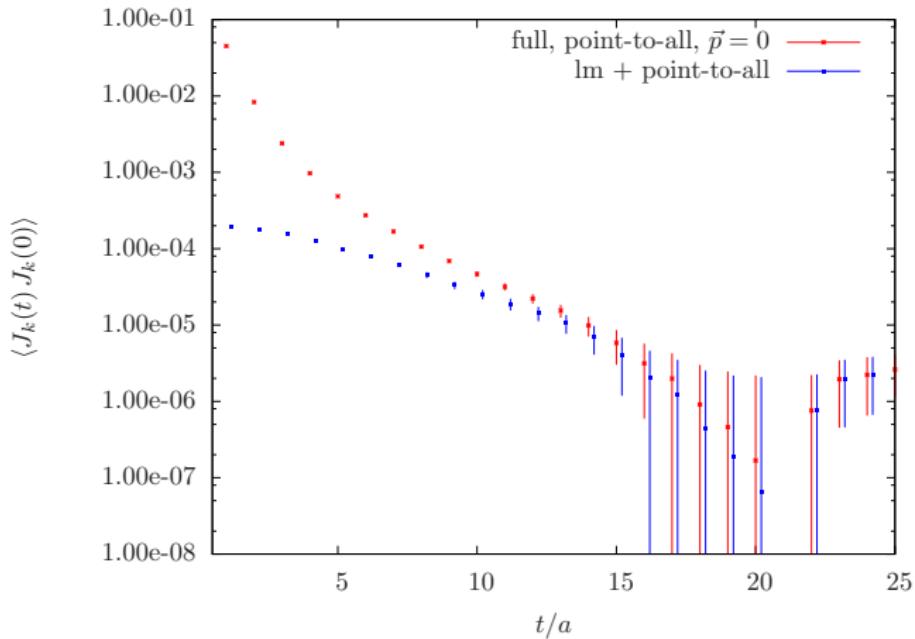
- since May 2017 pion-nucleon scattering project



- on $N_f = 2$ tm+clover physical point lattices (4.4 fm and 5.8 fm)
- includes measurements for pion-pion $I = 1$ scattering (FV spectrum, phase shift)

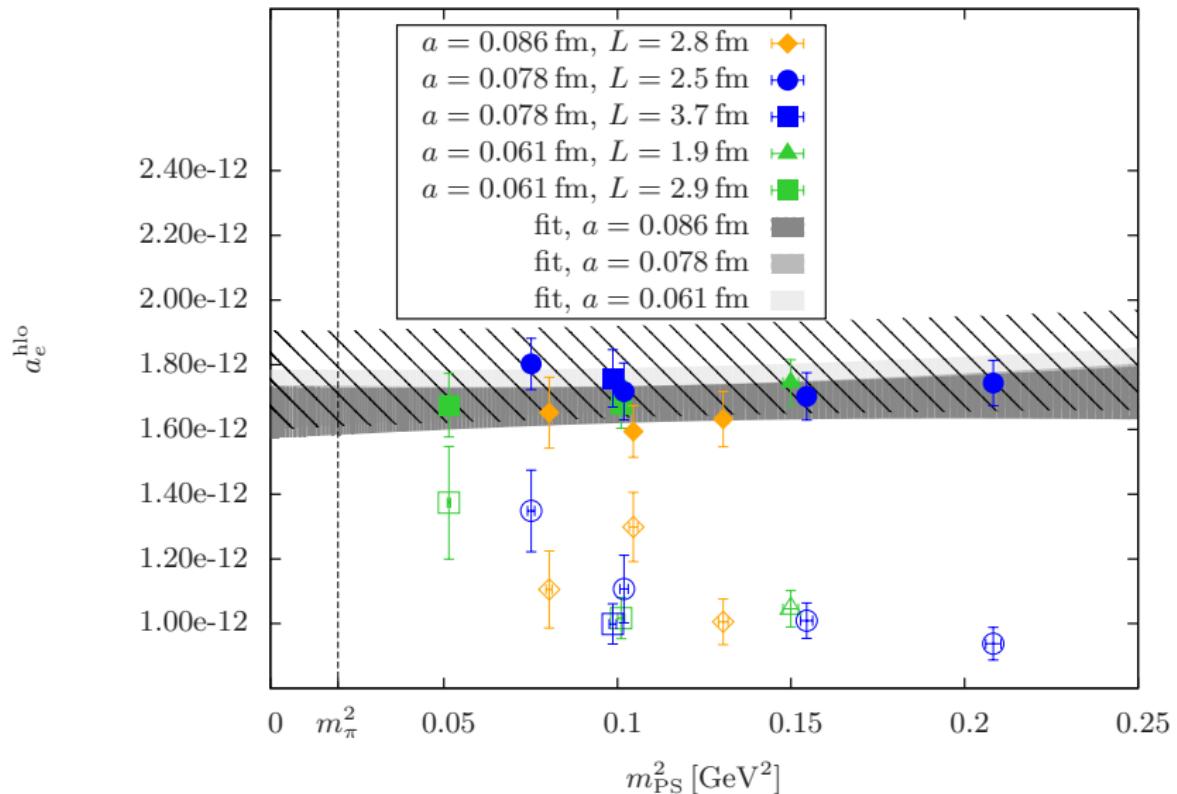
Thank you very much for your attention.

Comparison of point-to-all and Im+point-to-all mixed current correlator

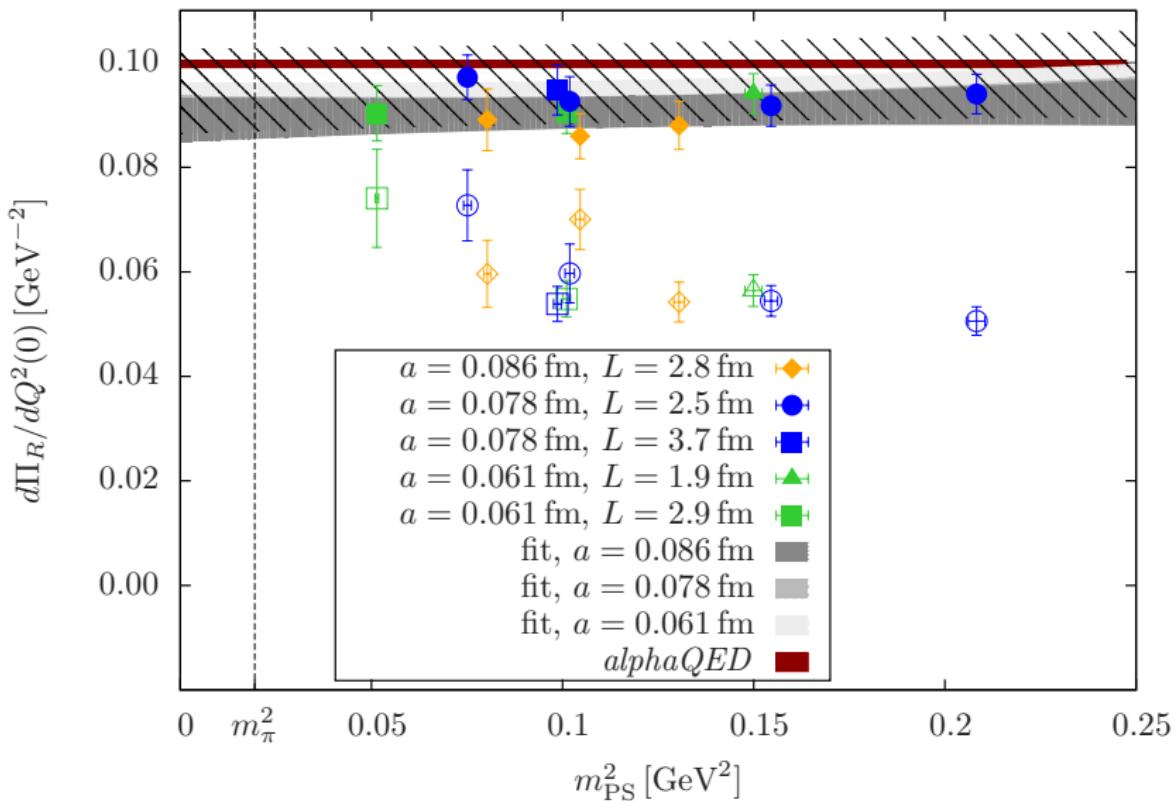


- full, point-to-all result from 110×16 measurements
- low-mode + point-to-all with $N_{\text{ev}} = 1200$ eigenvectors

a_e^{hvp} with TMR



$d\Pi_R/dQ^2(0)$ with TMR



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