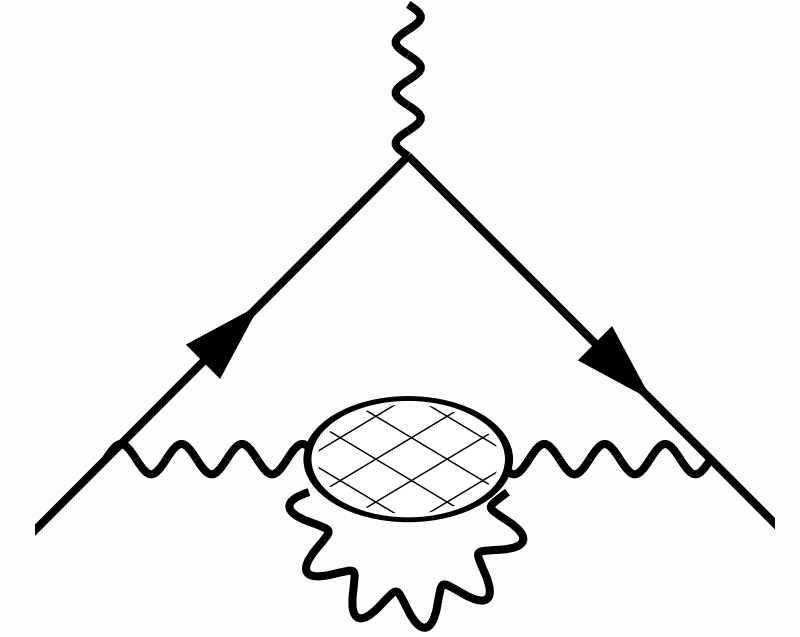


COTTINGHAM-TYPE FORMULA FOR THE **LbL CONTRIBUTION TO HVP**



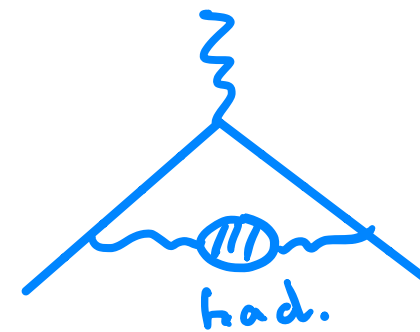
Vladimir Pascalutsa
with
F. Hagelstein, J. Green, H. Meyer, A. Gerardin, ...

**Institute for Nuclear Physics & Cluster of Excellence PRISMA
University of Mainz, Germany**

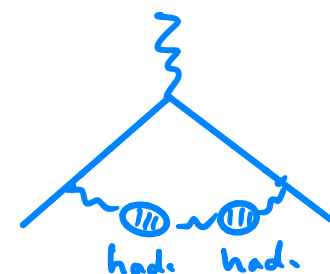
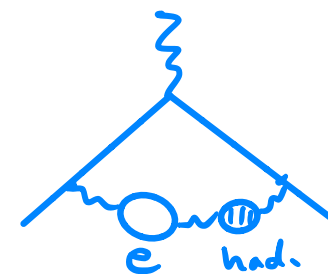
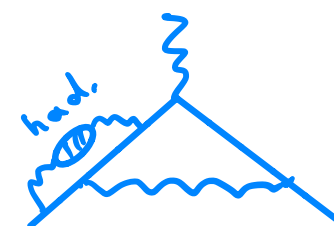
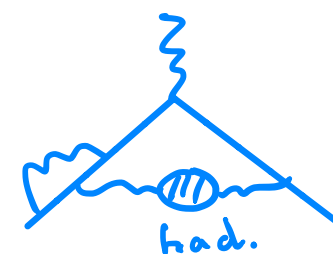


Hadronic Contributions to g-2

$O(\alpha^2)$



$O(\alpha^3)$

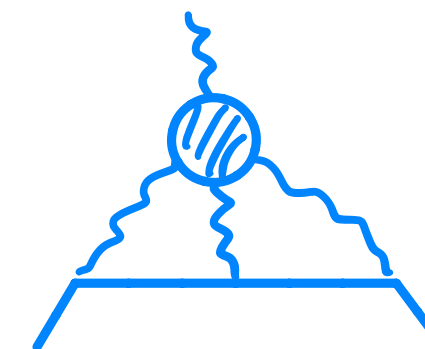


"had VP NLO"

$$\sim -10(1) \cdot 10^{-10}$$



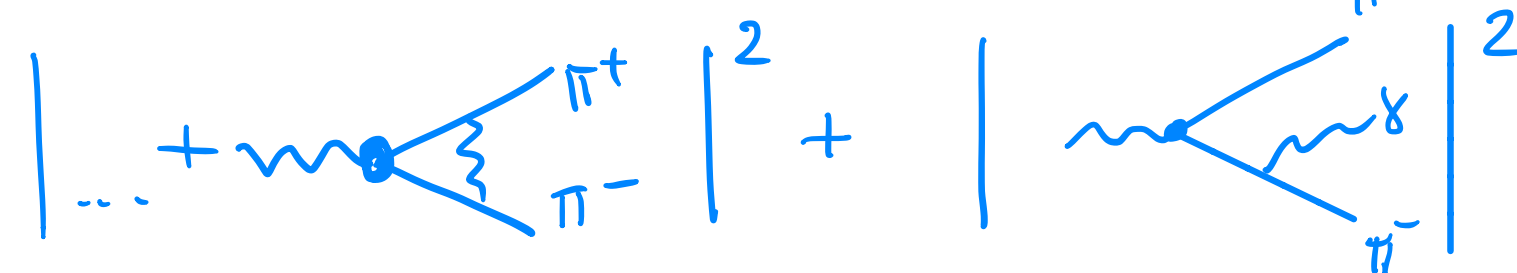
VP or LbL



had LbL

$$\sim 10 \cdot 10^{-10}$$

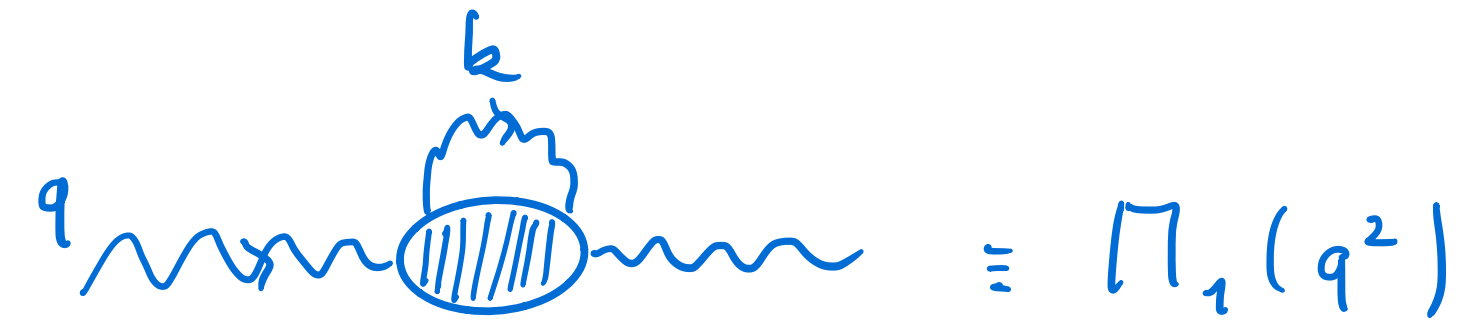
empirically
in LO HVP (talk by A. Keshavarzi)



$$\sim 4 \cdot 10^{-10} (\text{sQED})$$

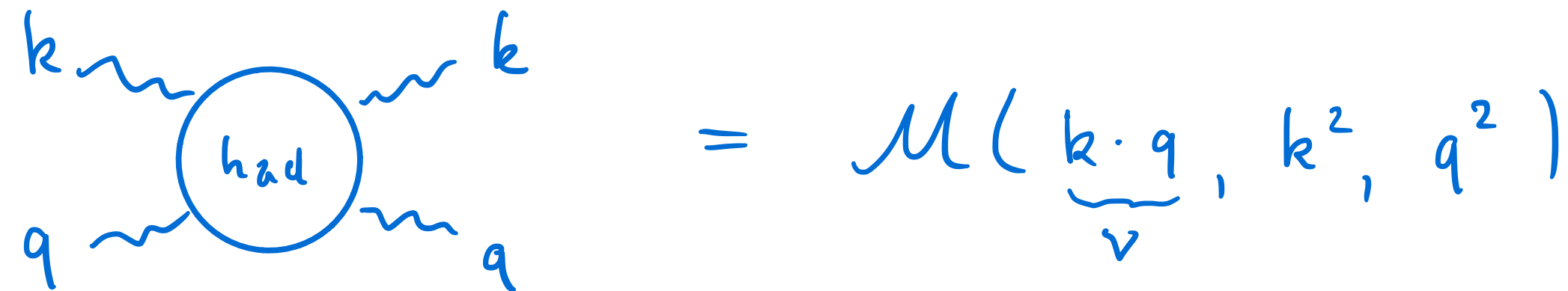
$$\sim 4 \cdot 10^{-10} (\pi^0 \gamma)$$

Cottingham-type formula



$$q \text{ wavy line} \text{ --- } \text{shaded circle with } k \text{ --- } \text{wavy line} \equiv \Pi_1(q^2)$$

Hadronic LbL contribution to VP



$$\text{hadronic LbL diagram} = \mathcal{M}(\underbrace{k \cdot q}_v, k^2, q^2)$$

$$\Pi_1(q^2) = -\frac{1}{3q^2} \int \frac{d^4k}{(2\pi)^4 i} \frac{1}{k^2} \sum_{\substack{\lambda, \lambda' \\ = \pm, 0}} \mathcal{M}_{\lambda\lambda\lambda'\lambda'}(k \cdot q, k^2, q^2)$$

$$= -\frac{1}{3Q^4(2\pi)^3} \int_0^\infty dK^2 \frac{1}{K^2} \int_0^{K^2 Q^2} d\nu^2 \left(\frac{K^2 Q^2}{\nu^2} - 1 \right)^{1/2} \mathcal{M}(\nu^2, K^2, Q^2)$$

Euclidean LbL amplitude
from Lattice QCD

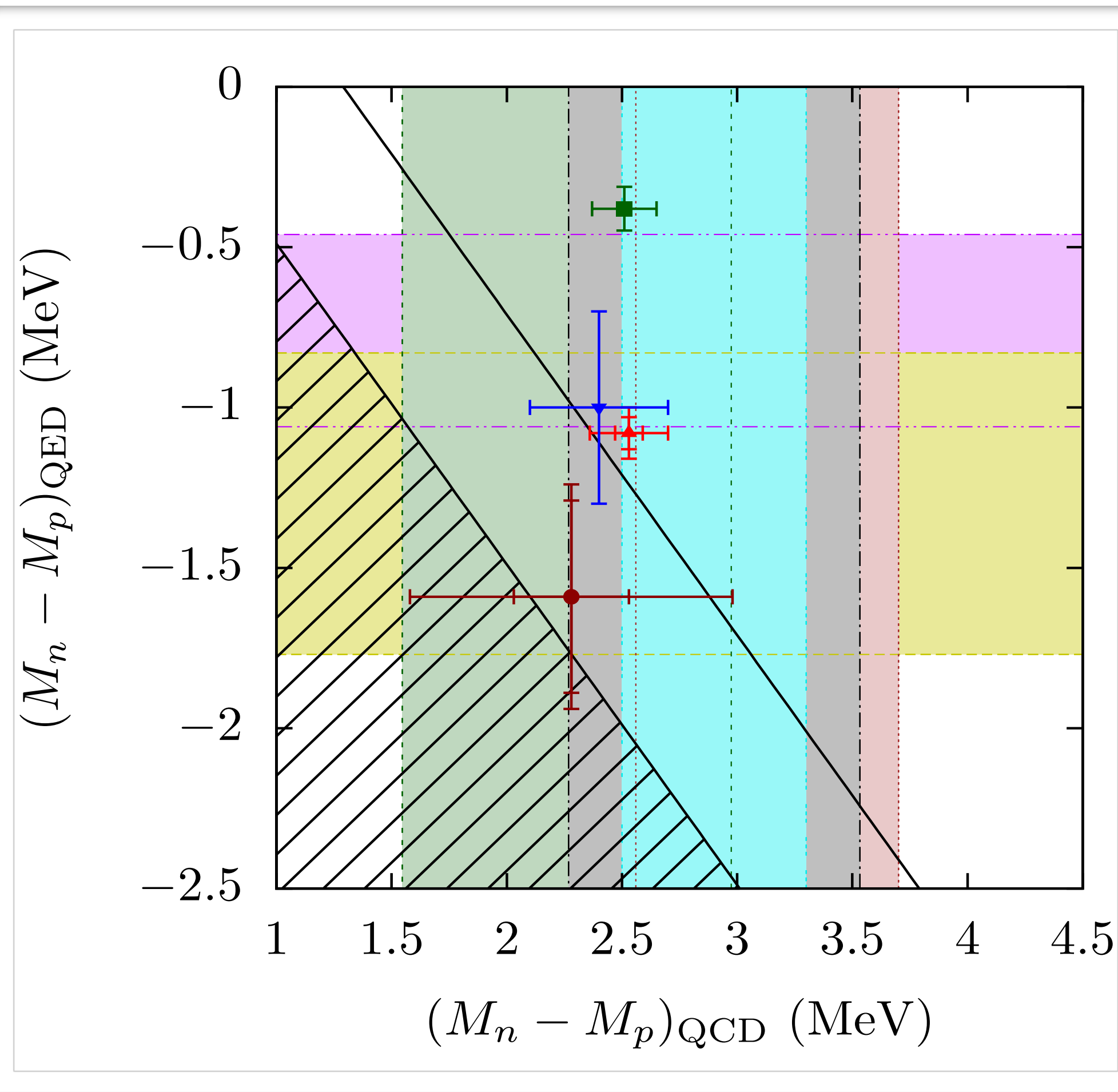
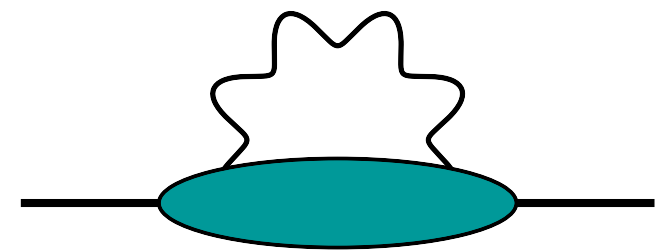
(talks by H. Meyer,
A. Gerardin at LATTICE'17)



Isospin breaking of the nucleon mass

$$M_n - M_p = 1.2933322(4) \text{ MeV}$$

	up	down	source: [PDG, 2013]
Mass (MeV)	$2.3^{(+0.7)}_{(-0.5)}$	$4.8^{(+0.5)}_{(-0.3)}$	
Charge (e)	2/3	-1/3	

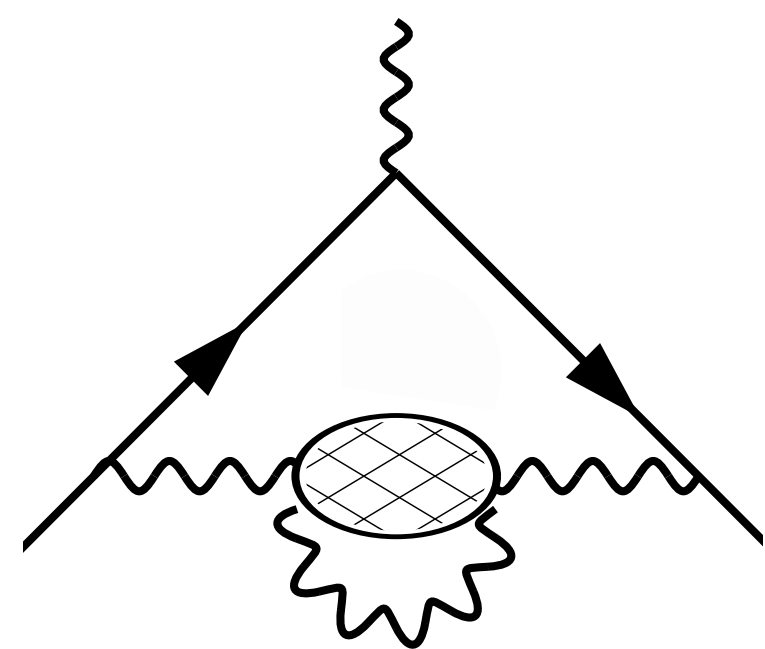


- [Gasser & Leutwyler, 1982]
- [Walker-Loud *et al.*, 2012]
- [NPLQCD, 2007]
- [QCDSF, 2012]
- [RM123, 2013]
- [Shanahan *et al.*, 2012]
- no *beta*-decay
- experiment
- [RBC-UKQCD, 2010] (EQ)
- [BMWc, 2013] (EQ)
- [BMWc, 2015a]
- [QCDSF, 2014]

from A.J. Portelli, talk at CD15

Forward LbL contribution to g-2

$$\Pi_1(Q^2) = -\frac{1}{3Q^4(2\pi)^3} \int_0^\infty dK^2 \frac{1}{K^2} \int_0^{K^2 Q^2} dv^2 \left(\frac{K^2 Q^2}{v^2} - 1 \right)^{1/2} \mathcal{M}(v^2, K^2, Q^2)$$

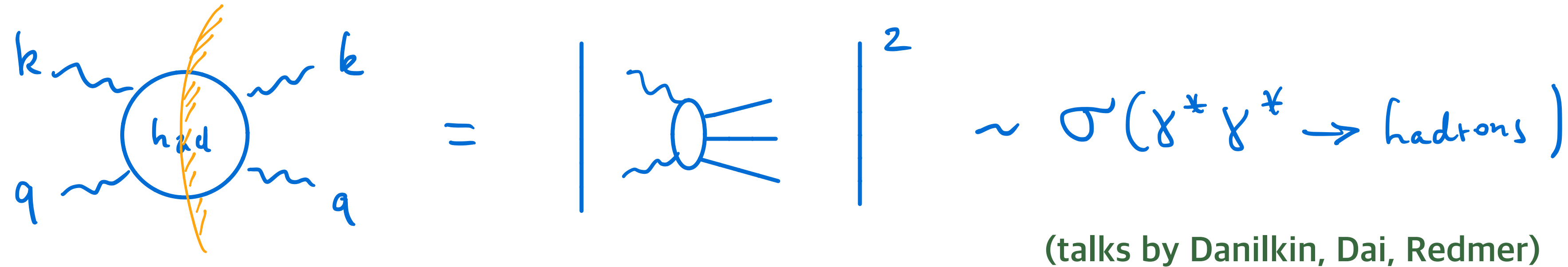


$$= \frac{\alpha}{\pi} \int_0^\infty dQ^2 \mathcal{K}(Q^2) \bar{\Pi}_1(Q^2)$$

$$\mathcal{K}(Q^2) = \frac{1}{2m_\mu^2} \frac{(v-1)^3}{2v(v+1)}, \quad v = \sqrt{1 + \frac{4m_\mu^2}{Q^2}}$$

$$\bar{\Pi}(q^2) = \Pi(q^2) - \Pi(0)$$

Dispersion relations for light-by-light

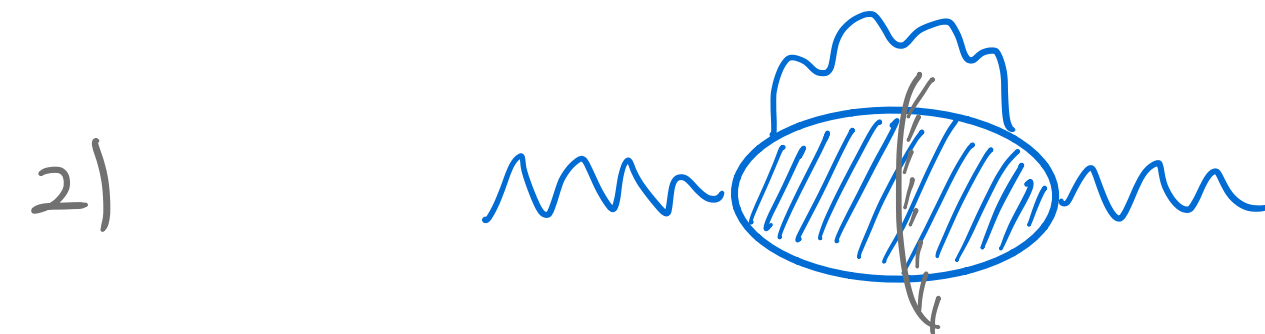
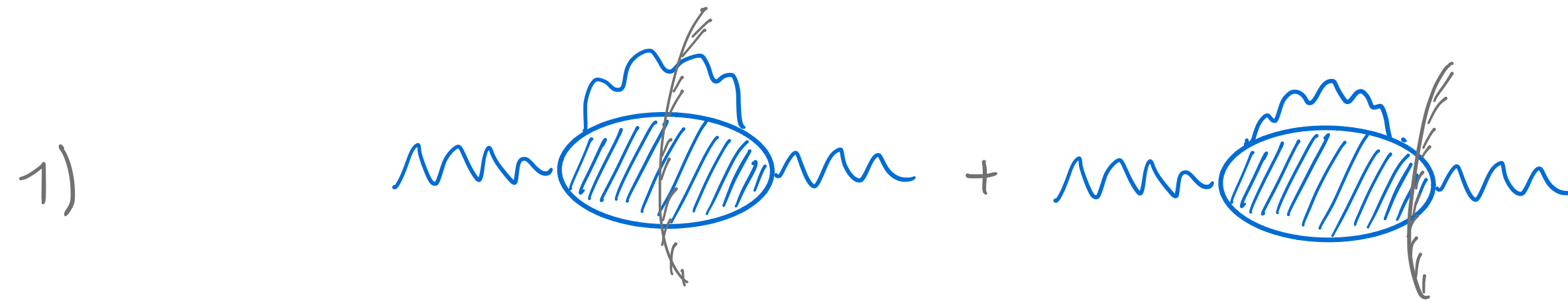


The diagram on the left shows a circle labeled 'had' with two incoming wavy lines labeled 'k' and 'q' from the left, and two outgoing wavy lines labeled 'k' and 'q' to the right. A vertical orange dashed line is drawn through the circle. This is followed by an equals sign and a diagram of a vertical line with a small circle in the middle. The top of the vertical line has a wavy line, and the bottom has a wavy line. From the right side of the small circle, three lines extend outwards. This is followed by a superscript '2' and a tilde symbol, then the expression $\sigma(\gamma^* \gamma^* \rightarrow \text{hadrons})$.

(talks by Danilkin, Dai, Redmer)

$$\mathcal{M}(\nu, K^2, Q^2) = \mathcal{M}(0, K^2, Q^2) + \frac{4\nu^2}{\pi} \int_{\nu_0}^{\infty} d\nu' \frac{\sqrt{\nu'^2 - K^2 Q^2}}{\nu'(\nu'^2 - \nu^2)} [4\sigma_{TT} + 2(\sigma_{TL} + \sigma_{LT}) + \sigma_{LL}](\nu', K^2, Q^2)$$

Two ways to do dispersion relations



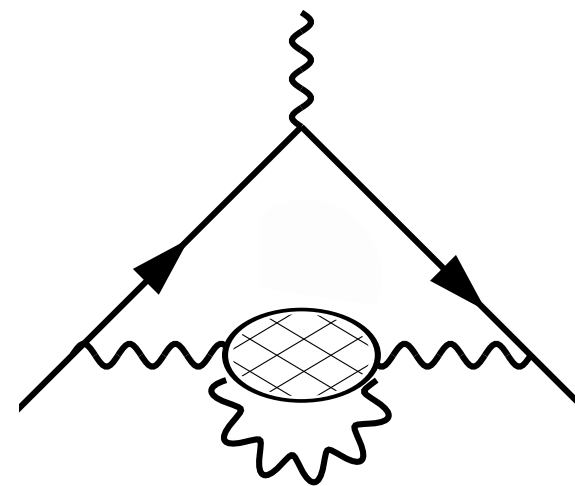
E.g.:



Timelike TFF: $R(s) \sim |F_{\pi^0 \gamma \gamma^*}(s, 0)|^2$

Spacelike TFF: $\sigma_{\gamma^* \gamma^* \rightarrow \pi^0}(\nu, k^2, Q^2) \sim F_{\pi^0 \gamma \gamma^*}^2(k^2, Q^2) \delta(\nu - \nu_\pi)$

Concl. :



1) empirically included in LO VP (FSR + rad. corr.), albeit approximately.

2) Lattice

2a) QCD+QED VP (talks by C. Lehner, M. Marinkovic)

2b) QCD LbL contrib. to VP (talks by H. Meyer, L. Jin)

3) Dispersion theory

3a) $\gamma^* \rightarrow \text{hadrons} + \gamma^{(*)}$

3b) $\gamma^* \gamma^* \rightarrow \text{hadrons}$

Testing ground for LbL calculations