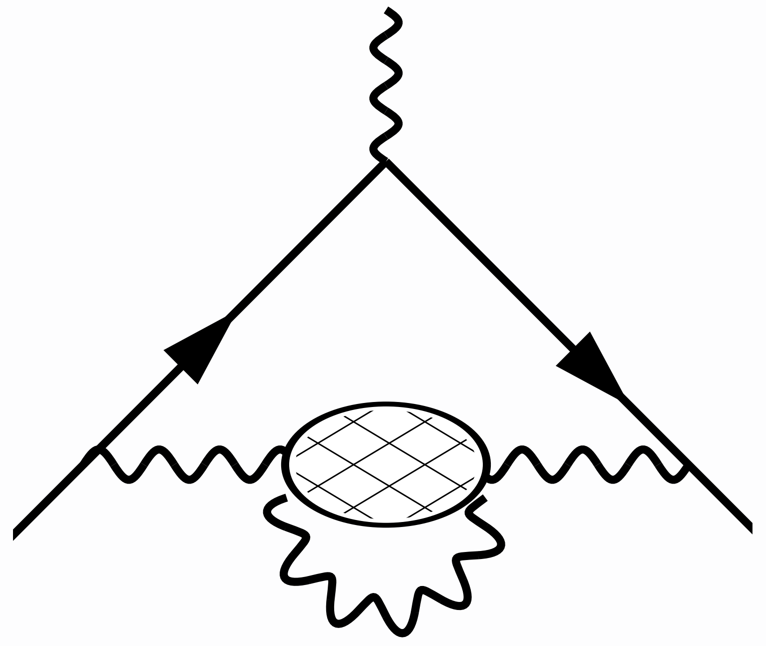
COTTINGHAM-TYPE FORMULA
FOR THE
LbL CONTRIBUTION TO HVP



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with
F. Hagelstein, J. Green, H. Meyer, A. Gerardin, …

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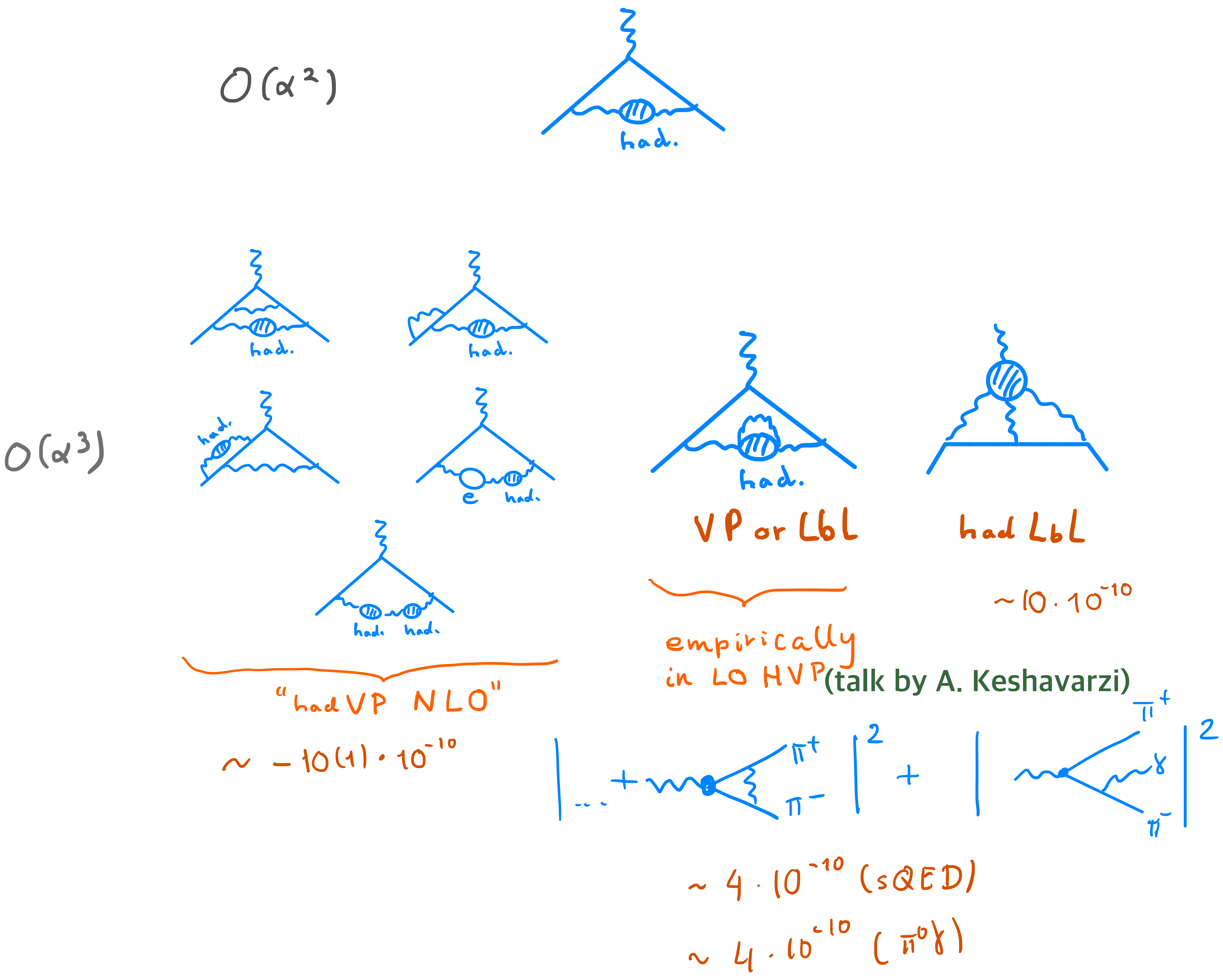
JG|U

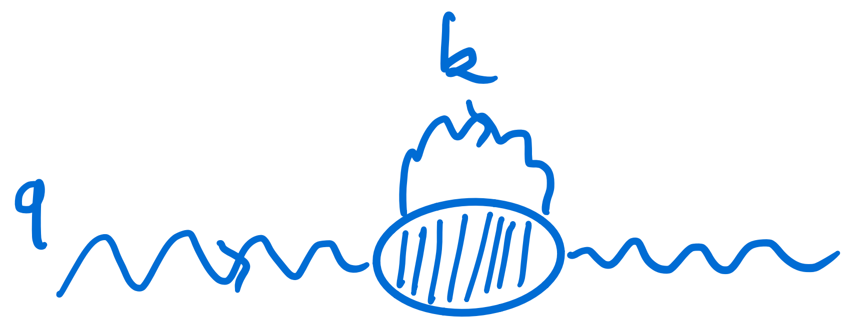
SFB 1044
THE LOW-ENERGY FRONTIER OF THE STANDARD MODEL

@ 1st Workshop on
Muon g-2 Theory Initiative
Fermilab, USA
June 2-6, 2017

Hadronic Contributions to g-2

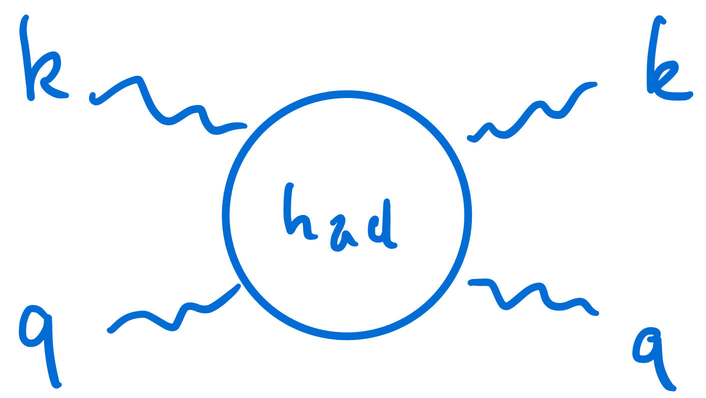


Cottingham-type formula



$\equiv \Pi_1(q^2)$

Hadronic LbL contribution to VP



$= \mathcal{M}(\underbrace{k\cdot q}_{\nu},\ k^2,\ q^2)$

$$\Pi_1(q^2) = -\frac{1}{3q^2}\int \frac{d^4k}{(2\pi)^4 i}\,\frac{1}{k^2}\sum_{\substack{\lambda,\lambda' \\ = \pm, 0}} \mathcal{M}_{\lambda\lambda\lambda'\lambda'}(k\cdot q,\ k^2,\ q^2)$$

$$= -\frac{1}{3Q^4(2\pi)^3}\int_0^\infty \mathrm{d}K^2\,\frac{1}{K^2}\int_0^{K^2Q^2} \mathrm{d}\nu^2 \left(\frac{K^2Q^2}{\nu^2}-1\right)^{1/2}\mathcal{M}(\nu^2,\ K^2,\ Q^2)$$

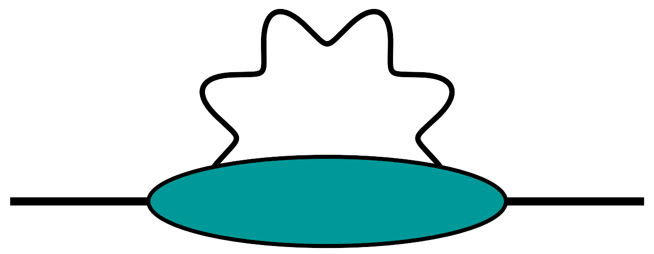
Euclidean LbL amplitude from Lattice QCD
(talks by H. Meyer, A. Gerardin at LATTICE'17)

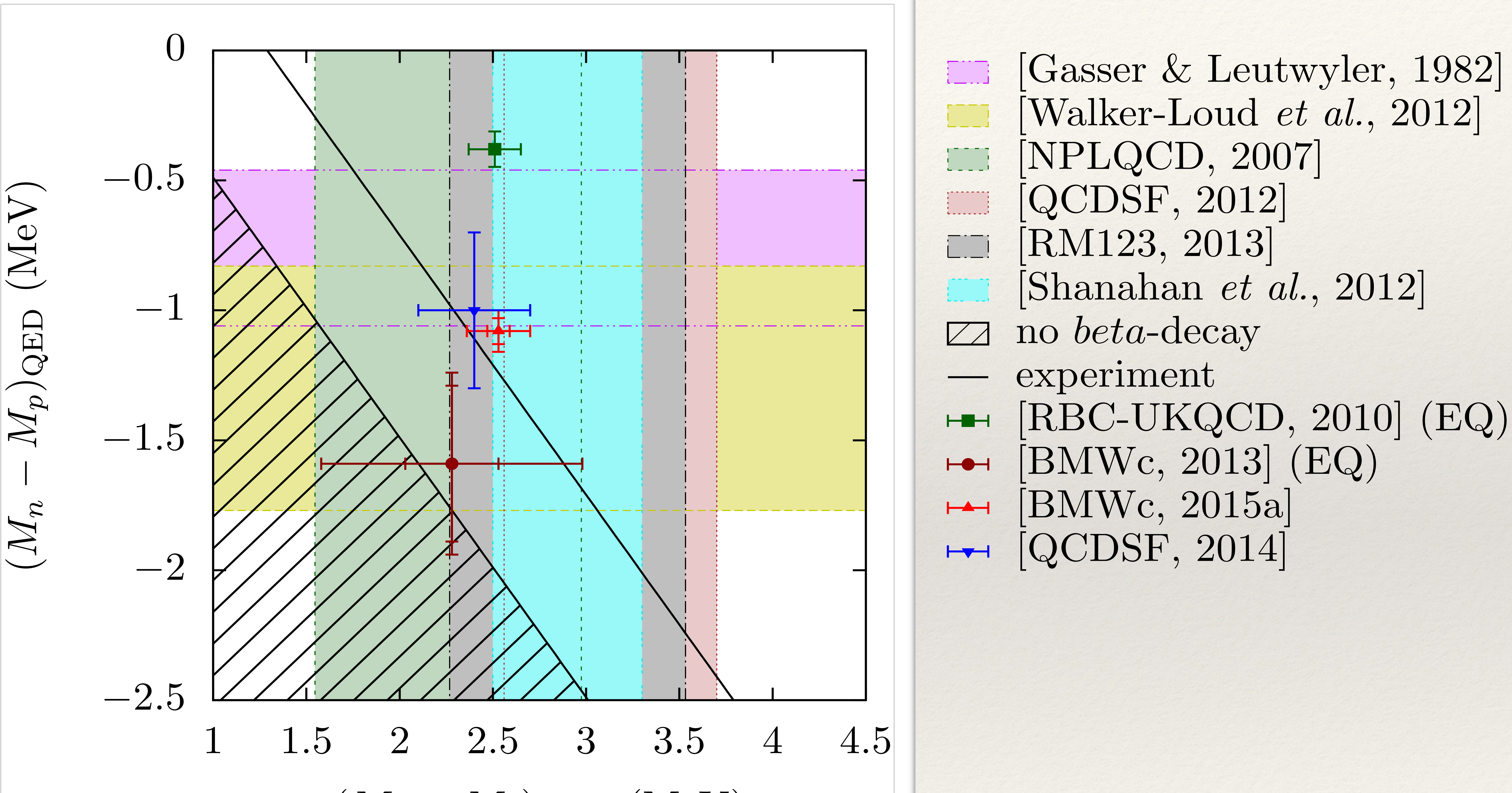
Isospin breaking of the nucleon mass

$$M_n - M_p = 1.2933322(4)\ \text{MeV}$$

	up	down
Mass (MeV)	$2.3(^{+0.7}_{-0.5})$	$4.8(^{+0.5}_{-0.3})$
Charge (e)	2/3	-1/3

source: [PDG, 2013]

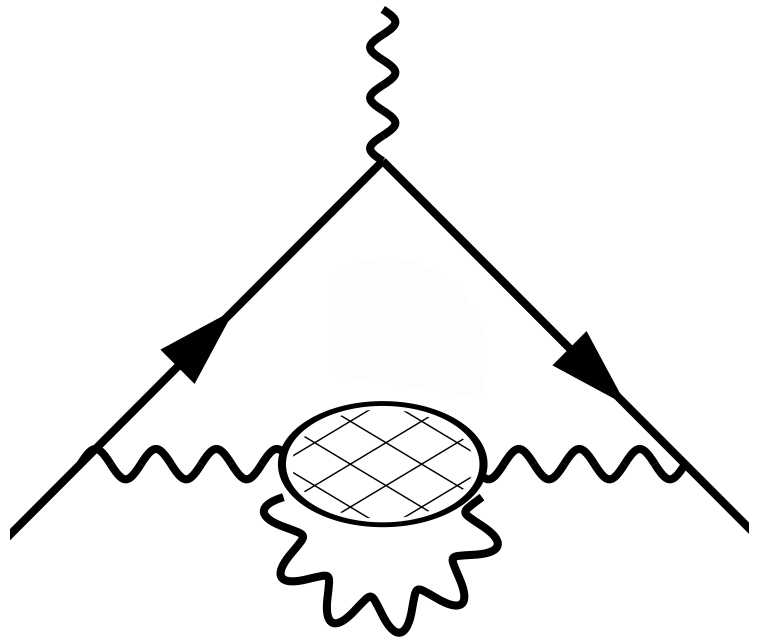




from A.J. Portelli, talk at CD15

Forward LbL contribution to g-2

$$\Pi_1(Q^2) = -\frac{1}{3Q^4(2\pi)^3}\int_0^\infty dK^2\,\frac{1}{K^2}\int_0^{K^2Q^2} d\nu^2\left(\frac{K^2Q^2}{\nu^2}-1\right)^{1/2}\mathcal{M}(\nu^2,K^2,Q^2)$$

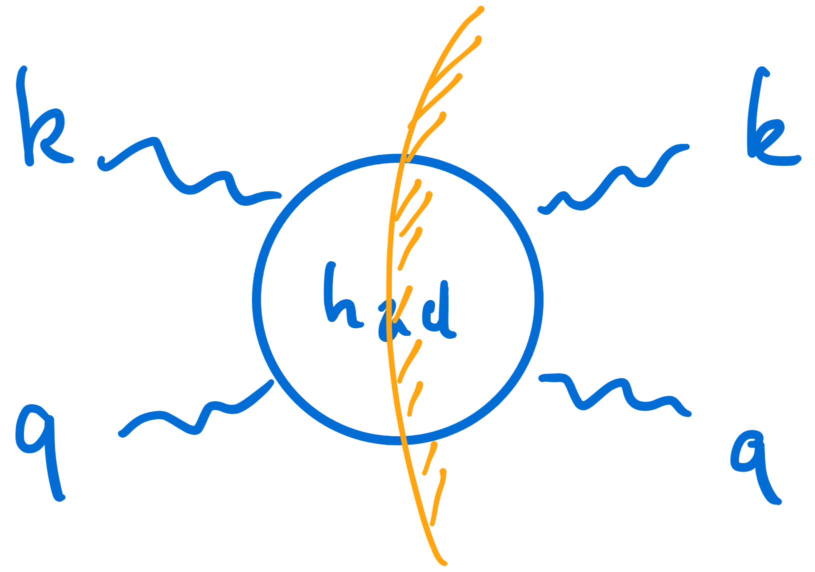


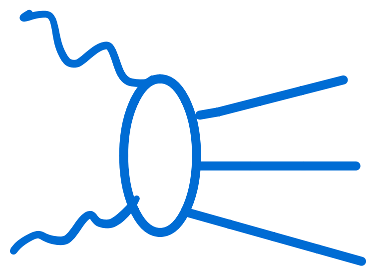
$$= \frac{\alpha}{\pi}\int_0^\infty dQ^2\,\mathcal{K}(Q^2)\,\overline{\Pi}_1(Q^2)$$

$$\mathcal{K}(Q^2) = \frac{1}{2m_\mu^2}\frac{(v-1)^3}{2v(v+1)}, \qquad v = \sqrt{1+\frac{4m_\mu^2}{Q^2}},$$

$$\overline{\Pi}(q^2) = \Pi(q^2) - \Pi(0)$$

Dispersion relations for light-by-light



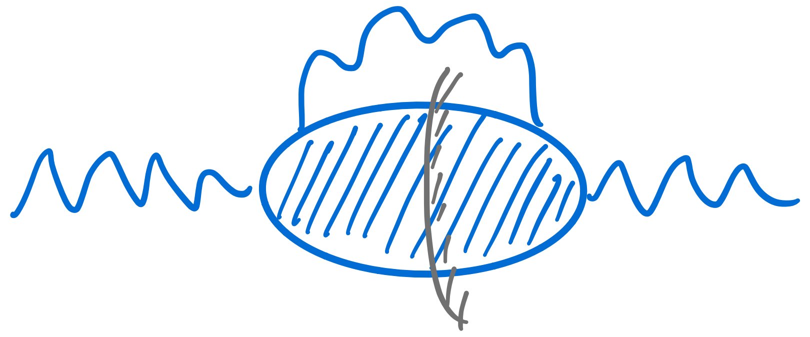
$=$ $\Bigg|$  $\Bigg|^2$ $\sim \sigma(\gamma^* \gamma^* \to \text{hadrons})$

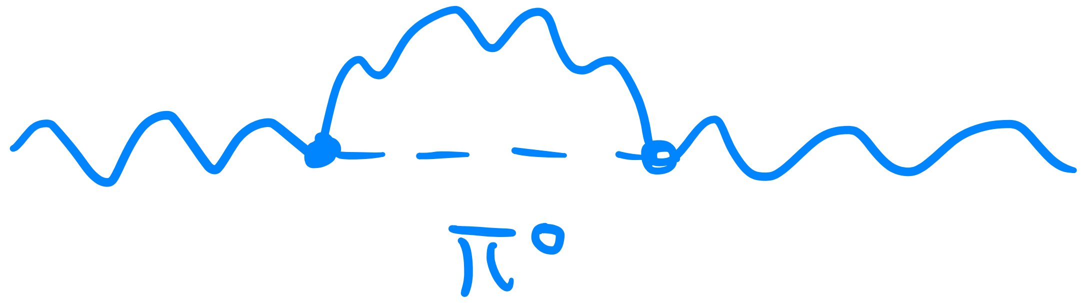
(talks by Danilkin, Dai, Redmer)

$$\mathcal{M}(\nu, K^2, Q^2) = \mathcal{M}(0, K^2, Q^2) + \frac{4\nu^2}{\pi} \int_{\nu_0}^{\infty} d\nu' \, \frac{\sqrt{\nu'^2 - K^2 Q^2}}{\nu'(\nu'^2 - \nu^2)} \left[4\sigma_{TT} + 2(\sigma_{TL} + \sigma_{LT}) + \sigma_{LL} \right](\nu', K^2, Q^2)$$

Two ways to do dispersion relations

1) 

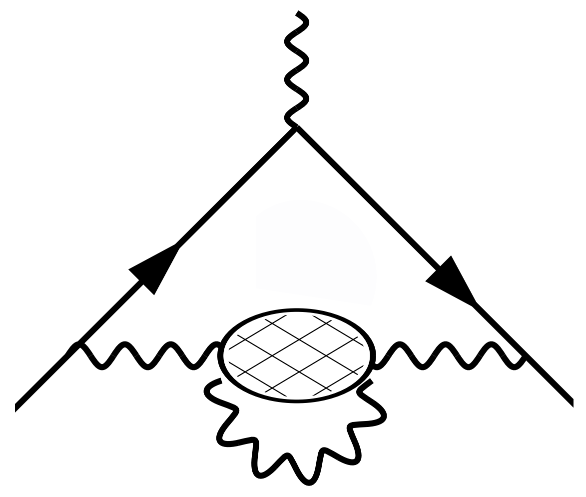
2) 

E.g.: 

Timelike TFF: $R(s) \sim \left| \mathcal{F}_{\pi^0\gamma\gamma^*}(s, 0) \right|^2$

Spacelike TFF: $\sigma_{\gamma^*\gamma^* \to \pi^0}(\nu, K^2, Q^2) \sim \mathcal{F}^2_{\pi^0\gamma^*\gamma^*}(K^2, Q^2)\, \delta(\nu - \nu_\pi)$

Concl. :



1) empirically included in LO VP (FSR + rad. corr.), albeit approximately.

2) Lattice

2a) QCD + QED VP (talks by C. Lehner, M. Marinkovic)

2b) QCD LbL contrib. to VP (talks by H. Meyer, L. Jin)

3) Dispersion theory

3a) $\gamma^* \longrightarrow$ hadrons $+ \gamma^{(*)}$

3b) $\gamma^* \gamma^* \rightarrow$ hadrons

<u>Testing ground for LbL calculations</u>