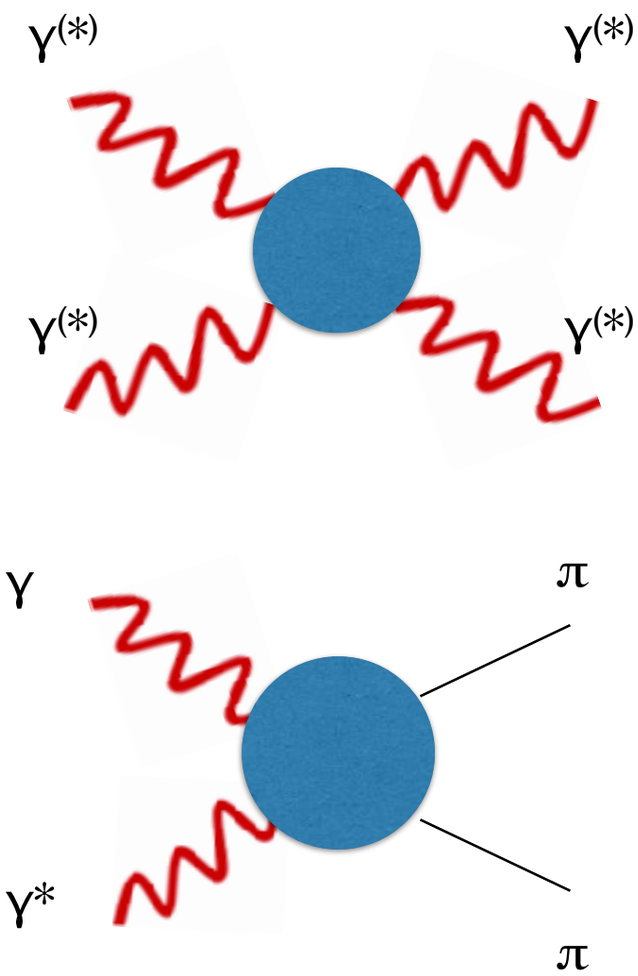


Light-by-light sum rules and dispersive analysis of $\gamma\gamma^* \to \pi\pi$

Igor Danilkin

in coll. with Marc Vanderhaeghen

June 5, 2017

PRISMA

JOHANNES GUTENBERG UNIVERSITÄT MAINZ

JG|U

(g-2) theory vs exp

Experiment:

$$a_\mu^{exp} = (11\,659\,208.9 \pm 6.3) \times 10^{-10}$$

BNL, (2006)
PRD 73 072003

Theory:

$$a_\mu^{SM} = (11\,659\,182.8 \pm 4.9) \times 10^{-10}$$

HLMNT, (2011)
J. Ph. G 38, 085003

$$a_\mu^{exp} - a_\mu^{SM} =$$
$$(26.1 \pm 4.9_{th} \pm 6.3_{exp}) \times 10^{-10}$$

? 1.6_{exp}

3 - 4 σ
deviation !

FNAL, J-PARC
experiments

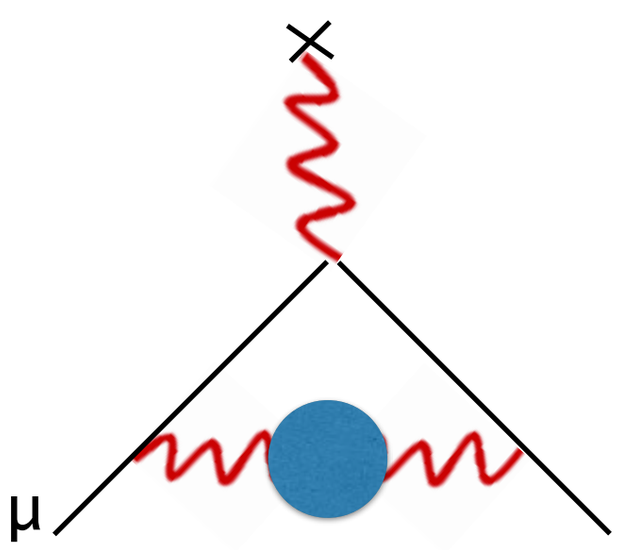
QCD contribution to (g-2)

$$a_\mu^{QCD} = (695.6 \pm 4.9) \times 10^{-10}$$

Hagiwara (2011)
Jegerlehnner (2015)

Hadronic vacuum polarization

$$a_\mu^{QCD,\,VP[LO]} = (694.9 \pm 4.3) \times 10^{-10}$$
$$a_\mu^{QCD,\,VP[HO]} = (-9.8 \pm 0.1) \times 10^{-10}$$

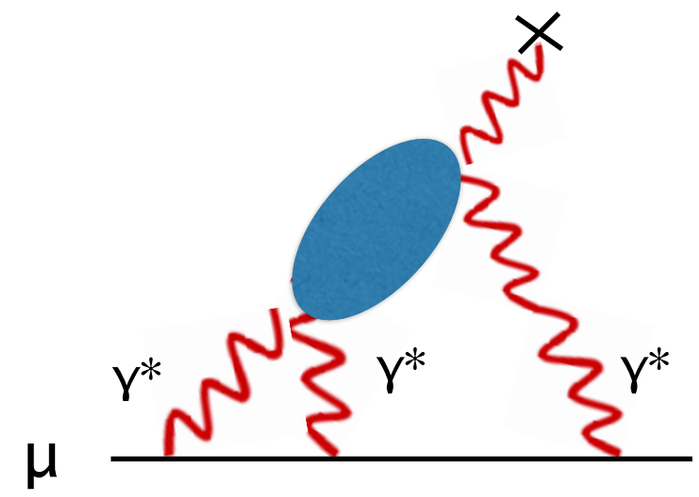


relies on experiment $e^+e^- \rightarrow$ hadrons through unitarity

$$\sigma(s)_{e^+e^- \rightarrow hadrons}$$

Hadronic light-by-light scattering

$$a_\mu^{QCD,\,LbL} = (10.5 \pm 2.6) \times 10^{-10}$$
$$= (10.2 \pm 3.9) \times 10^{-10}$$



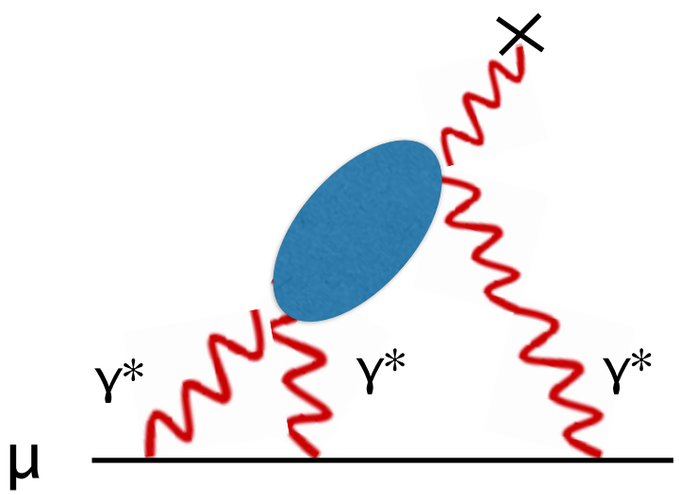
relies on measurements of **TFF** to reduce model dependence

$$\pi^0\gamma^*\gamma^{(*)}, \eta\gamma^*\gamma^{(*)}, \ldots$$
$$f_1\gamma^*\gamma^{(*)}, f_2\gamma^*\gamma^{(*)}, \ldots$$

QCD contribution to (g-2)

Hadronic light-by-light scattering

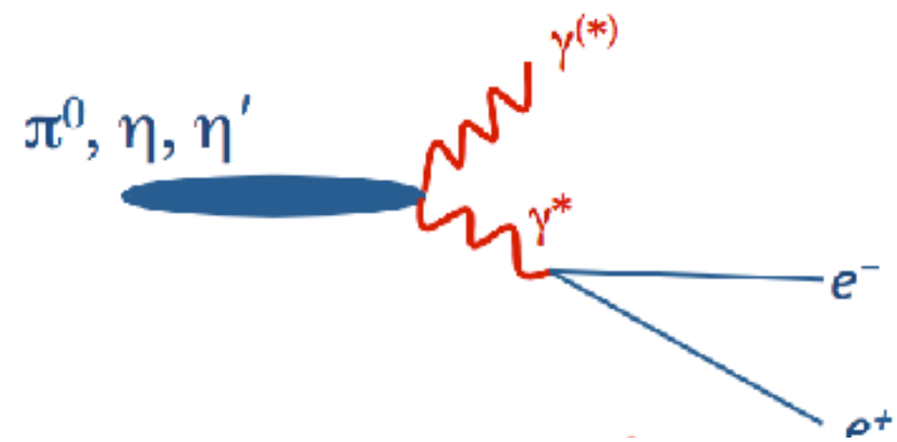
$$a_\mu^{QCD,\,LbL} = (10.5 \pm 2.6) \times 10^{-10}$$
$$= (10.2 \pm 3.9) \times 10^{-10}$$



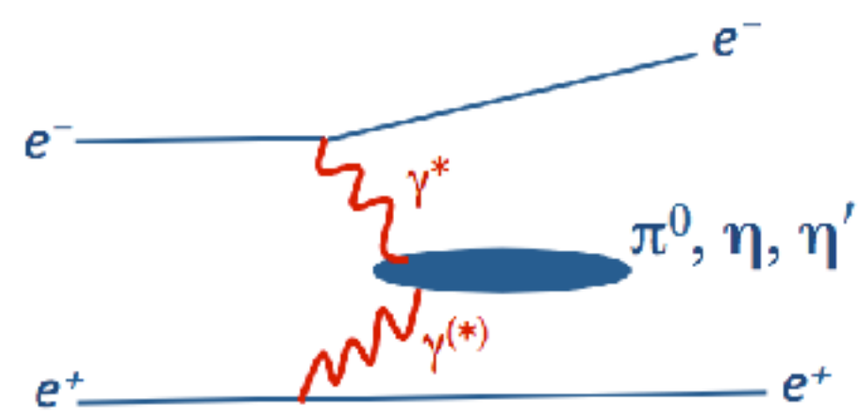
relies on measurements of **TFF** to reduce model dependence

$$\pi^0\gamma^*\gamma^{(*)}, \eta\gamma^*\gamma^{(*)}, \ldots$$
$$f_1\gamma^*\gamma^{(*)}, f_2\gamma^*\gamma^{(*)}, \ldots$$

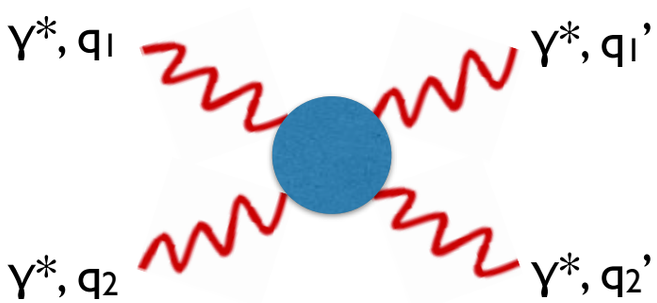
Timelike: KLOE, MAMI/A2, NA62



Spacelike: CLEO, BaBar, Belle, BESIII



Light by light scattering



$$\lambda_i = \pm 1, 0$$
$$q_i^2 = -Q_i^2$$

Helicity amplitudes

$$M_{\lambda_1' \lambda_2' \lambda_1 \lambda_2} = M^{\mu\nu\alpha\beta} \epsilon_\mu^*(\lambda_1') \, \epsilon_\nu^*(\lambda_2') \, \epsilon_\alpha(\lambda_1) \, \epsilon_\beta(\lambda_2)$$

Forward scattering $q_1 = q_1', \; q_2 = q_2'$

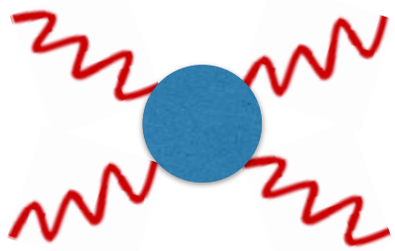
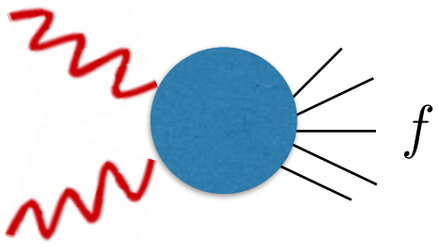
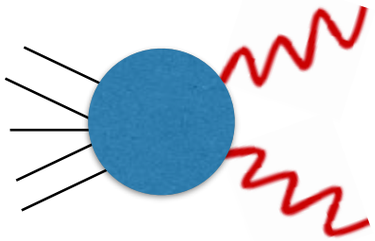
$$s = (q_1 + q_2)^2$$
$$t = (q_1 - q_1')^2 = 0$$

P and T symmetry: **81** → **8** independent amplitudes

$$M_{++,++}, \; M_{+-,+-}, \; M_{++,--}$$
$$M_{00,00}, \; M_{+0,+0}, \; M_{0+,0+}$$
$$M_{++,00}, \; M_{0+,-0}$$

Light by light scattering

Unitarity

2 Im  $= \sum_f \int d\Pi_f$  f f 

For the forward scattering (optical theorem)

$$\mathrm{Im}\, M_{++,++} = 2\sqrt{X}\,\sigma_0$$

$$\mathrm{Im}\, M_{+-,+-} = 2\sqrt{X}\,\sigma_2$$

$$\mathrm{Im}\, M_{++,--} = 2\sqrt{X}\,(\sigma_{\parallel} - \sigma_{\perp})$$

$$\mathrm{Im}\, M_{00,00} = 2\sqrt{X}\,\sigma_{LL}$$

$$\ldots$$

X - flux factor

Observables in the experiment: $e^+e^- \to e^-e^+f$

Light by light scattering

Unitarity

$$\mathrm{Im}\, M_{++,--} = 2\sqrt{X}\,(\sigma_{\parallel} - \sigma_{\perp})$$

$$\ldots$$

Analyticity (fixed *t* dispersion relation)

$$M_{++,--}(\nu) = \int_{\nu_0}^{\infty} \frac{d\nu'}{\pi} \frac{2\,\nu'\,\mathrm{Im} M_{++,--}(\nu')}{\nu'^2 - \nu^2 + i\,0}\,, \quad \nu = \frac{s-u}{4}$$

$$\ldots$$

(modulo subtractions)

Matching around $\nu = 0$ to the LbL Lagrangian

$$\mathcal{L} = c_1 (F_{\mu\nu} F^{\mu\nu})^2 + c_2 (F_{\mu\nu} \tilde{F}^{\mu\nu})^2 + \ldots$$

yield a number of **constraints** for Im M,
and thus **on cross section**

LbL sum rules

Three super convergence relations

Gerasimov, Moulin (1975), Brodsky, Schmidt (1995)

Pascalutsa, Vanderhaeghen (2010)

$$0 = \int_{s_0}^{\infty} ds \frac{1}{(s+Q_1^2)} \left[\sigma_2 - \sigma_0\right]_{Q_2^2=0}$$

$$0 = \int_{s_0}^{\infty} ds \frac{1}{(s+Q_1^2)^2} \left[\sigma_{\parallel} + \sigma_{LT} + \frac{(s+Q_1^2)}{Q_1 Q_2} \tau_{TL}^a\right]_{Q_2^2=0}$$

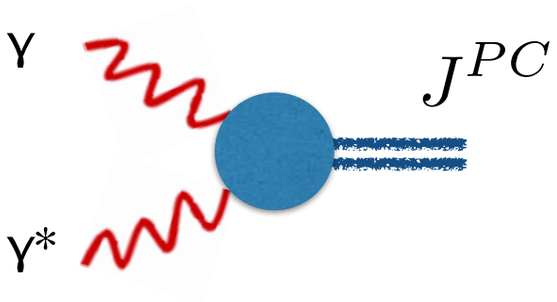
$$0 = \int_{s_0}^{\infty} ds \left[\frac{\tau_{TL}(s, Q_1^2, Q_2^2)}{Q_1 Q_2}\right]_{Q_2^2=0}$$

Pascalutsa, Pauk Vanderhaeghen (2012), (2014)

These sum rules have been tested in perturbative QFT both at tree-level and one loop level:

- ✓ scalar QED
- ✓ spinor QED
- ✓ ϕ^4 theory
- ✓ ϕ^4 theory + resum.

Meson production



C=+1: $J^{PC}=0^{-+}, 0^{++}, 1^{++}, 2^{++}, \ldots$

$Q^2 \neq 0$

Landau-Yang theorem

Narrow width approximation

$$\sigma\left(\gamma^*\gamma \to J^P(\Lambda)\right) = \delta(s-m^2)\, 8\pi^2 \frac{(2J+1)\,\Gamma_{\gamma\gamma}\left(J^P\right)}{m}\left(1+\frac{Q^2}{m^2}\right)\left[\mathrm{T}^{(\Lambda)}\left(Q^2\right)\right]^2$$

Sum rules will relate 2γ width or TFFs:

$$0 = \int_{s_0}^{\infty} ds \frac{1}{(s+Q^2)}\left[\sigma_2 - \sigma_0\right]$$

$Q^2 = 0$ →

$$0 = -\sum_{\mathcal{P}} 16\pi^2 \frac{\Gamma_{\gamma\gamma}(\mathcal{P})}{m_{\mathcal{P}}^3} - \sum_{\mathcal{S}} \ldots$$

$Q^2 \neq 0$ →

$$0 = -\sum_{\mathcal{P}} 16\pi^2 \frac{\Gamma_{\gamma\gamma}(\mathcal{P})}{m_{\mathcal{P}}^3}\left[T(Q^2)\right]^2 - \sum_{\mathcal{S}} \ldots$$

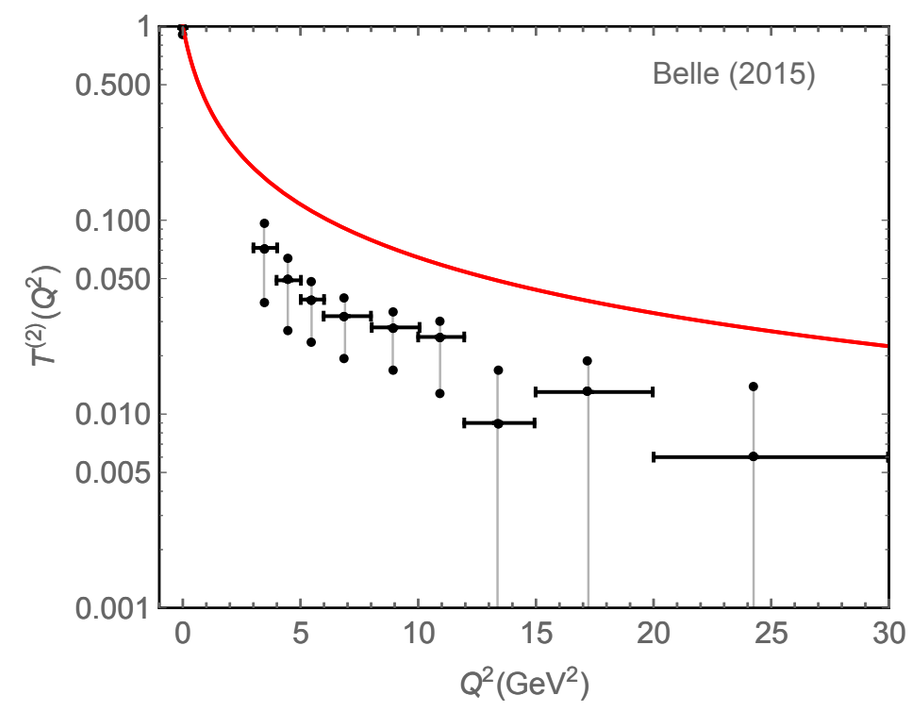
Sum rule I (Isospin=0)

$$0 = \int_{s_0}^{\infty} ds \frac{1}{(s+Q^2)} \left[\sigma_2 - \sigma_0\right]$$

$$0 = -\sum_{\mathcal{P}} 16\pi^2 \frac{\Gamma_{\gamma\gamma}(\mathcal{P})}{m_{\mathcal{P}}^3} \left[T_{\mathcal{P}}(Q^2)\right]^2 - \sum_{\mathcal{S},\mathcal{A}} \ldots$$
$$+ \sum_{\mathcal{T}} 16\pi^2 \frac{\Gamma_{\gamma\gamma}(\mathcal{T})}{m_{\mathcal{T}}^3} \left(\left[T_{\mathcal{T}}^{(\Lambda=2)}(Q^2)\right]^2 - \left[T_{\mathcal{T}}^{(\Lambda=0)}(Q^2)\right]^2 \right)$$

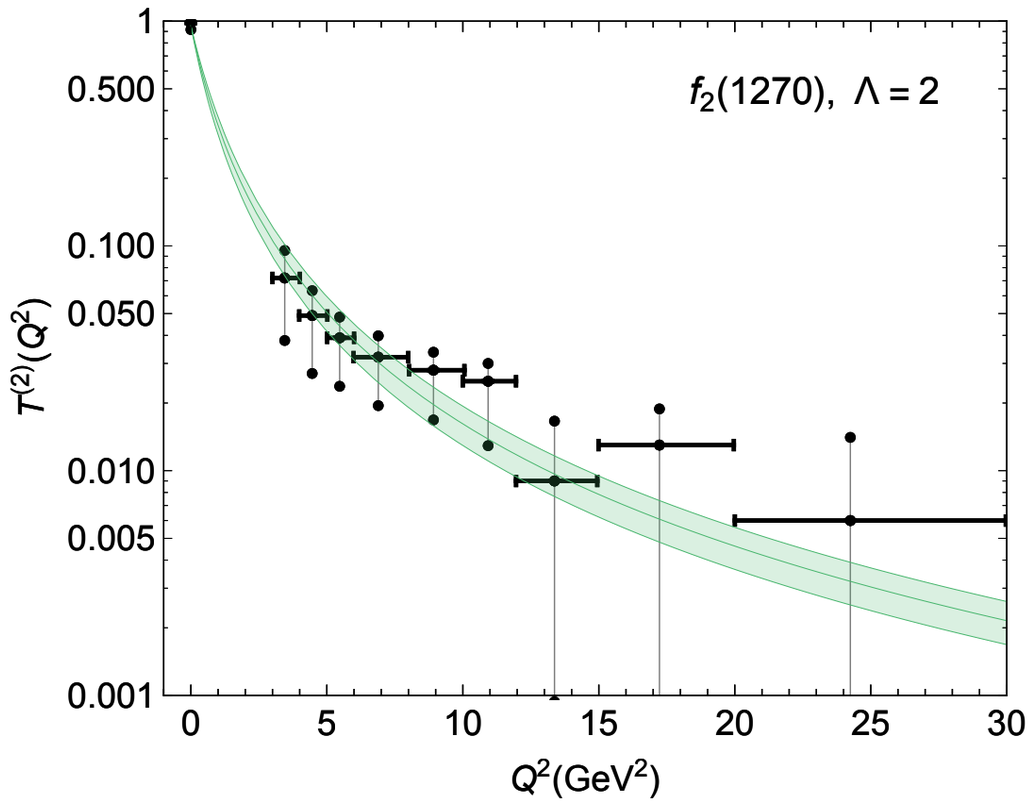
Dominant contributions

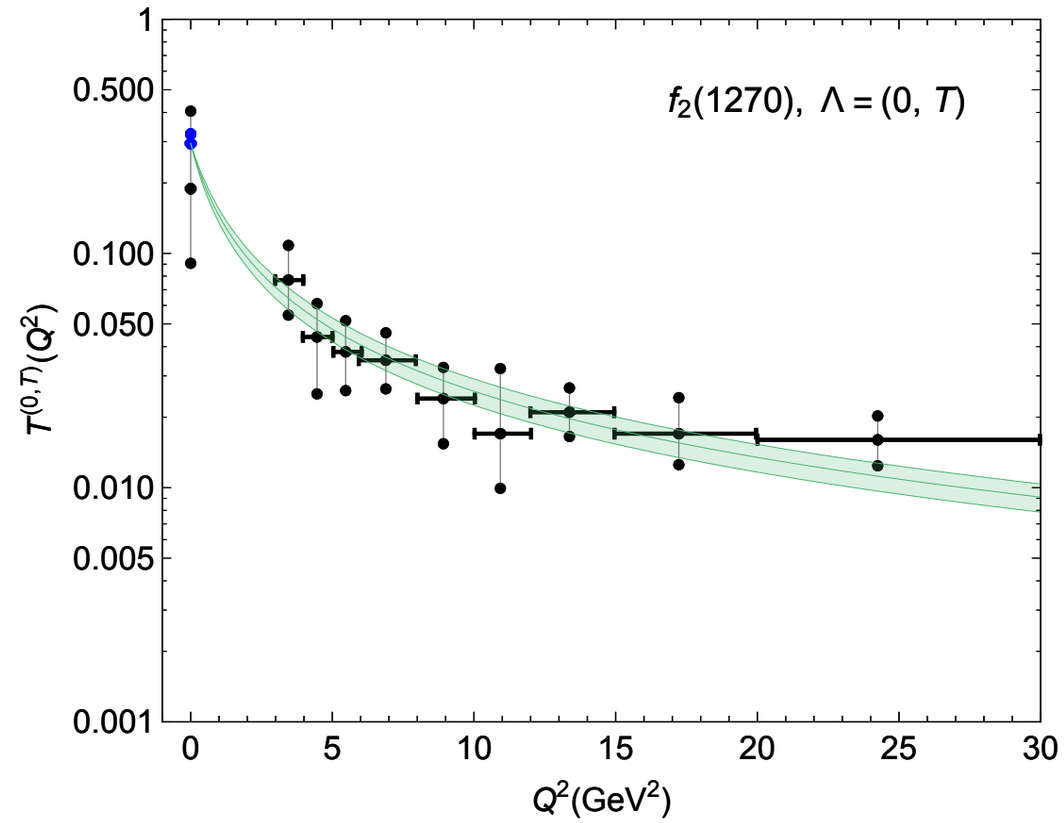
State	m (MeV)	$\Gamma_{\gamma\gamma}$(keV)		SR_1 $(Q^2=0)$ (nb)
η	547.862±0.017	0.516±0.020		-193±7
η'	957±0.06	4.35±0.25		-304±17
$f_2(1270)$	1275.5±0.8	2.93±0.40	(Λ=2)	434±60
			(Λ=0)	≈0
$f_2(1565)$	1562±13	0.70±0.14		56±11
.....				
sum				-7±64



Pascalutsa, Pauk Vanderhaeghen (2012)

Belle (2015)





$$T^{(\Lambda)}(Q^2) = Factor(Q^2) \times \frac{1}{\left(1 + Q^2/\lambda^2_{(\Lambda)}\right)^2}$$

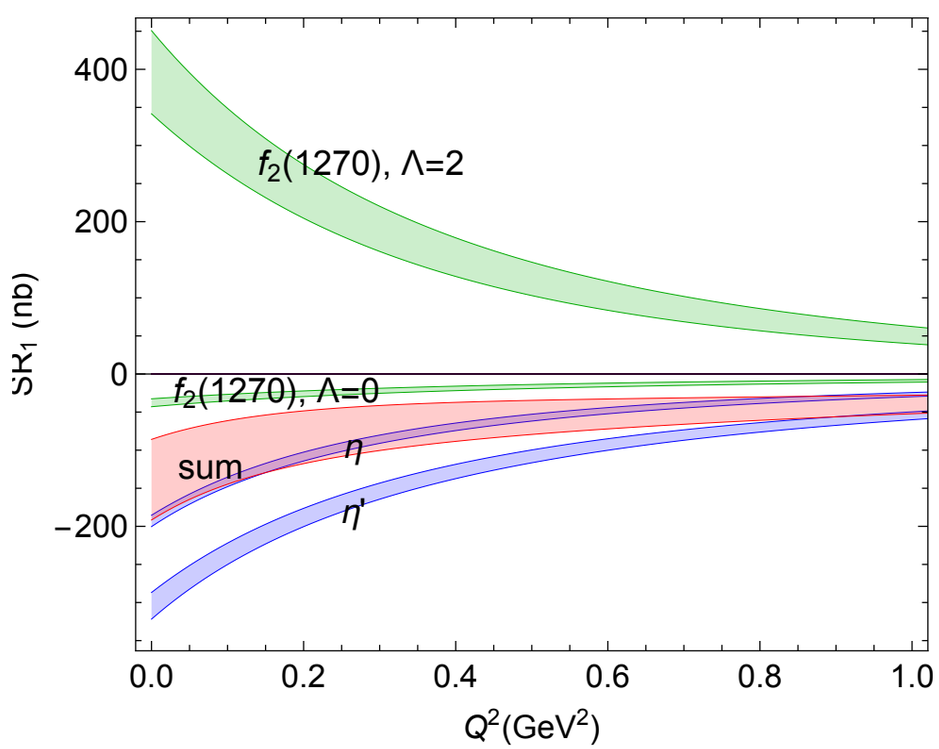
$$\lambda_{\Lambda=2} = 1222 \pm 66\,\mathrm{MeV}$$

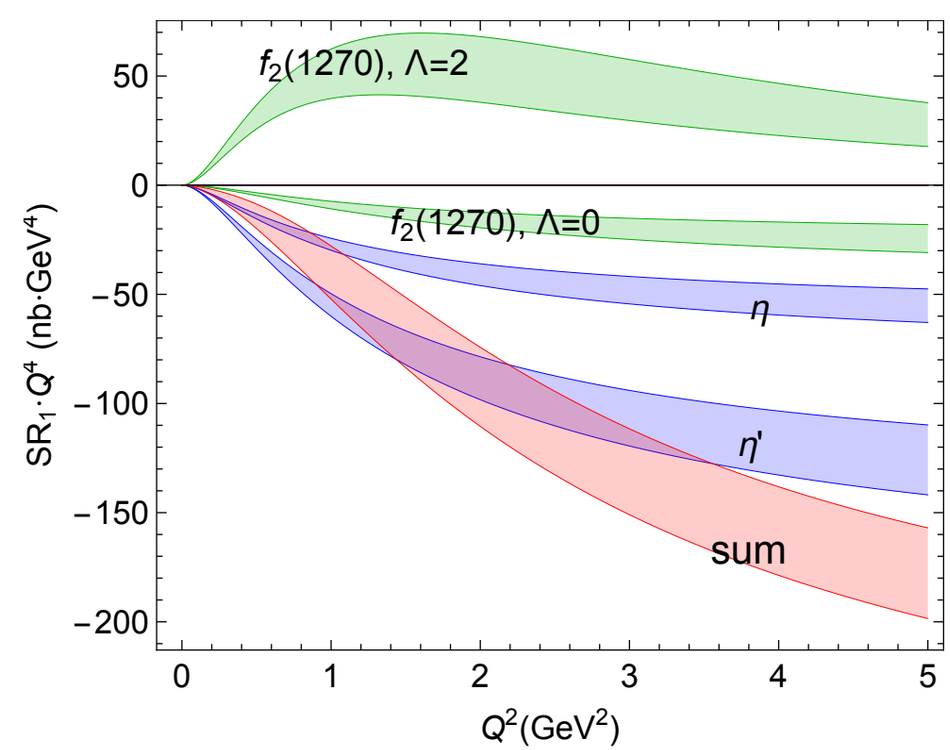
$$\lambda_{\Lambda=(0,T)} = 1051 \pm 36\,\mathrm{MeV}$$

Sum rule I (Isospin=0)

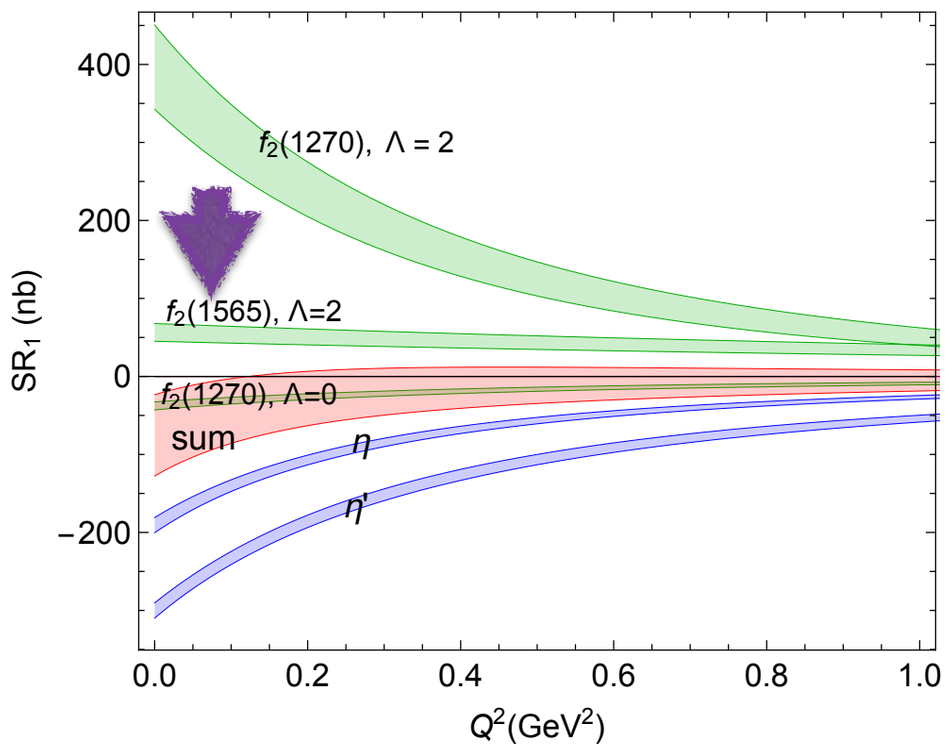
State	m (MeV)	$\Gamma_{\gamma\gamma}$(keV)		SR_1 $(Q^2=0)$ (nb)
η	547.862±0.017	0.516±0.020		-193±7
η'	957±0.06	4.35±0.25		-304±17
$f_2(1270)$	1275.5±0.8	2.93±0.40	(Λ=2)	396±54
			(Λ=0)	-38±5
$f_2(1565)$	1562±13	0.70±0.14		56±11
.....				
sum				-82±54

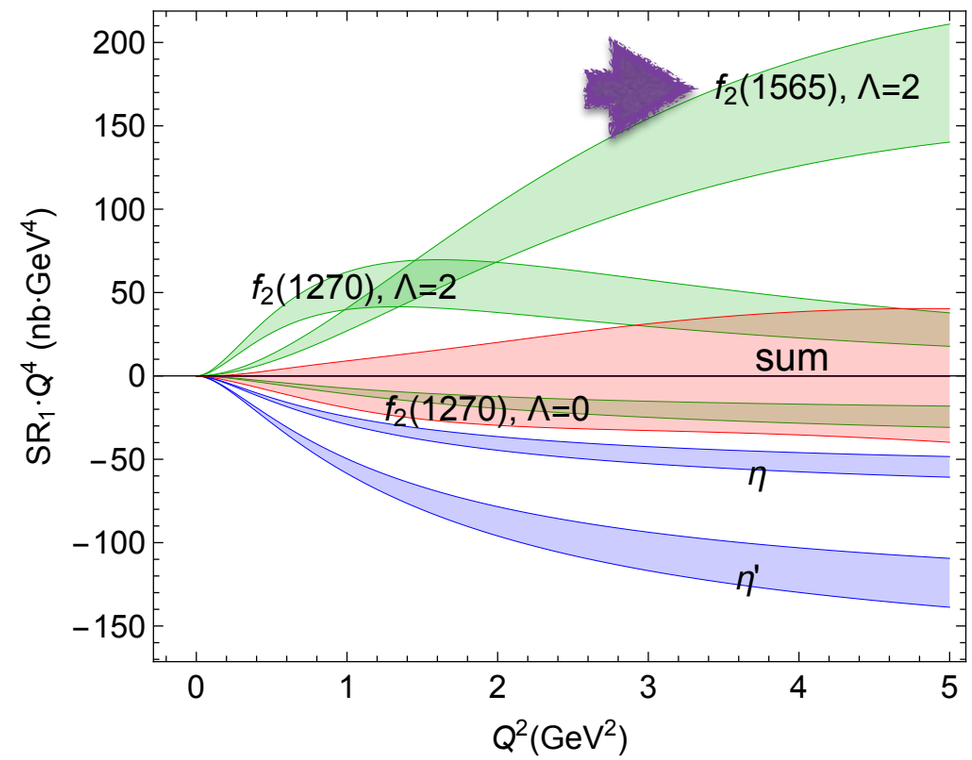
$$0 = \int_{s_0}^{\infty} ds \frac{1}{(s+Q^2)} \left[\sigma_2 - \sigma_0\right]$$

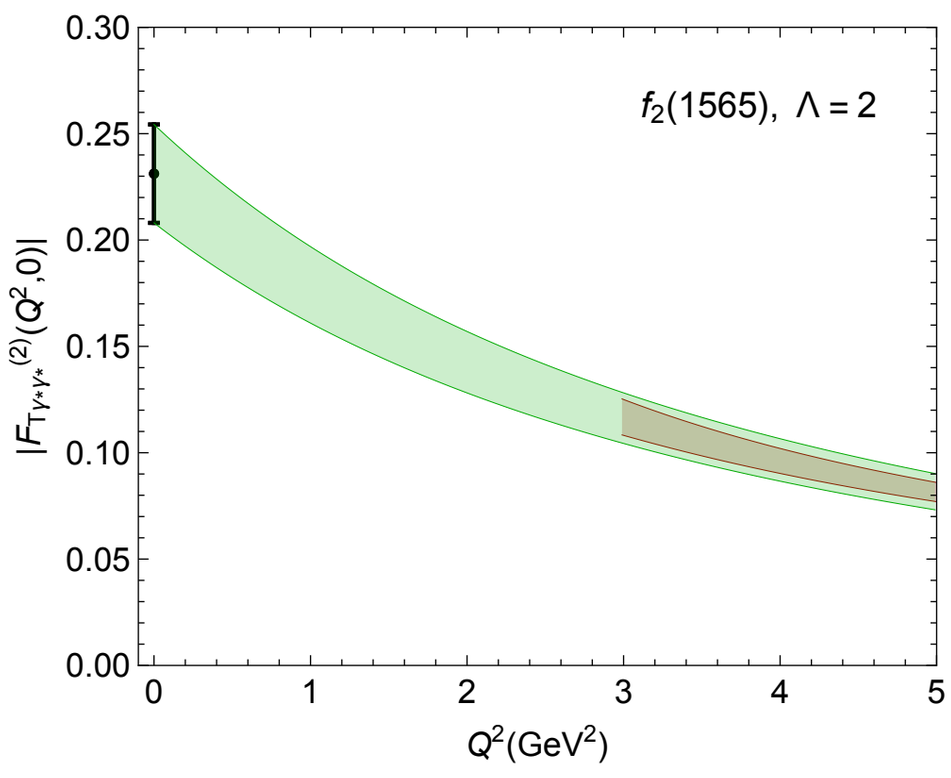




Sum rule I (Isospin=0)







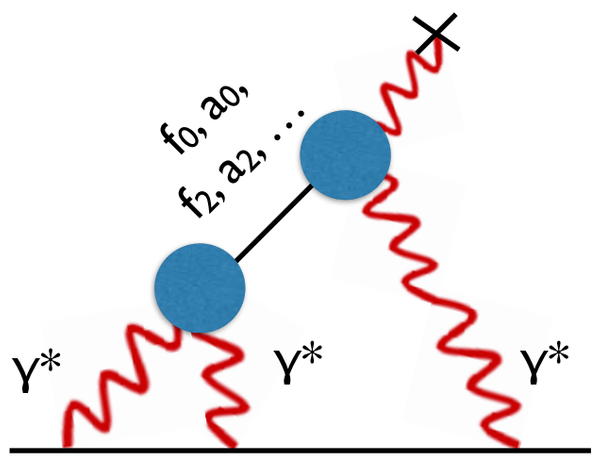
Prediction:

$$f_2(1565)$$
$$\lambda_{\Lambda=2} = 2719 \pm 53\,\mathrm{MeV}$$

I.D., Vanderhaeghen (2016)

future Belle data

Meson contributions to (g-2)



Lepton tensor: well known

Hadron tensor: requires input from **TFFs**

$$a_\mu^{LbL} = \lim_{k\to 0} ie^6 \int \frac{d^4q_1}{(2\pi)^4} \int \frac{d^4q_2}{(2\pi)^4} T^{\mu\nu\lambda\sigma}(q_1, k-q_1-q_2, q_2)\, \Pi_{\mu\nu\lambda\sigma}(q_1, k-q_1-q_2, q_2)$$

$$\frac{1}{q_1^2}\frac{1}{q_2^2}\frac{1}{(k-q_1-q_2)^2}\frac{1}{(p+q_1)^2-m^2}\frac{1}{(p'-q_2)^2-m^2}$$

Results (excluding low energy region):

$$\begin{aligned} a_\mu[f_0(980), a_0(980)] &= (-0.03 \pm 0.01)\times 10^{-10} \\ a_\mu[f_2(1270), f_2(1565), a_2(1320), a_2(1700)] &= (0.1 \pm 0.01)\times 10^{-10} \end{aligned}$$

$$T_{a_i}(Q^2) \approx T_{f_i}(Q^2)$$

New evaluation of axial vector contributions (satisfying Landau-Yang theorem)

$$\begin{aligned} a_\mu[f_1(1285), f_1(1420)] &= (0.64 \pm 0.20)\times 10^{-10} \\ &= (0.75 \pm 0.27)\times 10^{-10} \end{aligned}$$

Pauk, Vdh (2013)
Jegerlehner (2015)

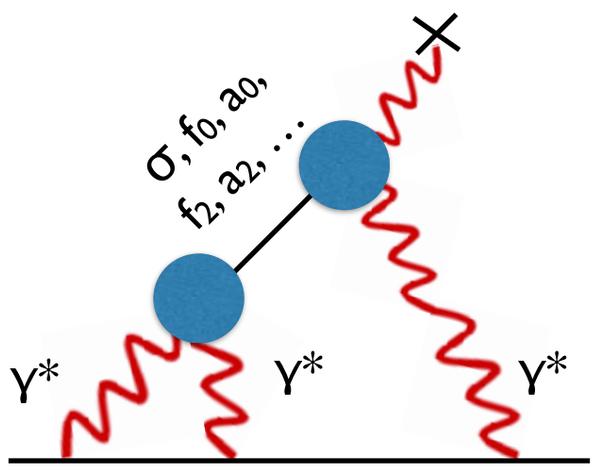
Compared to (1.5 ± 1.0) 10^{-10}
(which enters the Glasgow consensus)

$$\delta a_\mu^{exp} = 1.6 \times 10^{-10}$$

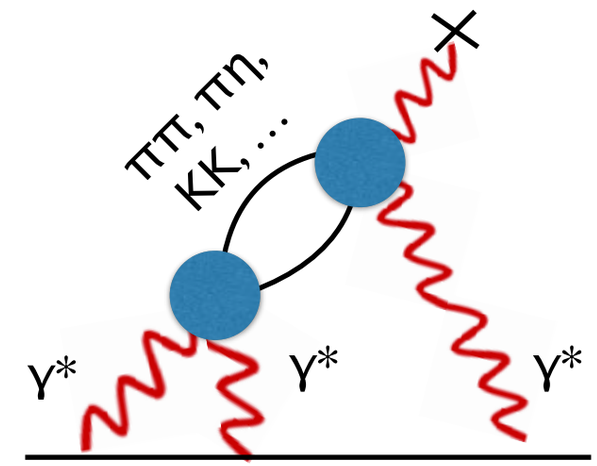
FNAL, J-PARC experiments

Improvements: Multi-meson production

Important contributions beyond pseudo-scalar poles



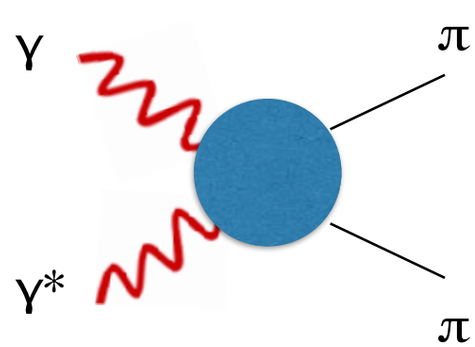
dispersive analysis for
ππ, πη, … loops



Pauk, Vanderhaeghen, (2014)

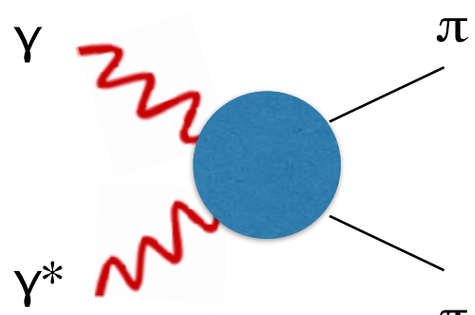
Colangelo, Hoferichter, Procura, Stoffer, (2014, 2015)

Important ingredient: $\gamma\gamma^* \to \pi\pi, \pi\eta, \ldots$



$\gamma\gamma \to \pi\pi$, KK, ηη, πη (Belle: 07,08, 09,10, ..)
$\gamma\gamma^* \to \pi\pi, \pi\eta$ (BESIII in progress)

Cross section



$$q_1^2 = 0$$
$$q_2^2 = -Q^2$$

C=+1: $J^{PC}=0^{++}, 2^{++}, 1^{-+}, \ldots$

$Q^2 \neq 0$

Landau-Yang theorem

Helicity amplitudes

$$H_{\lambda_1\lambda_2} = H^{\mu\nu}\epsilon_\mu(\lambda_1)\,\epsilon_\nu(\lambda_2), \quad \lambda_1 = \pm 1,\, \lambda_2 = \pm 1, 0$$

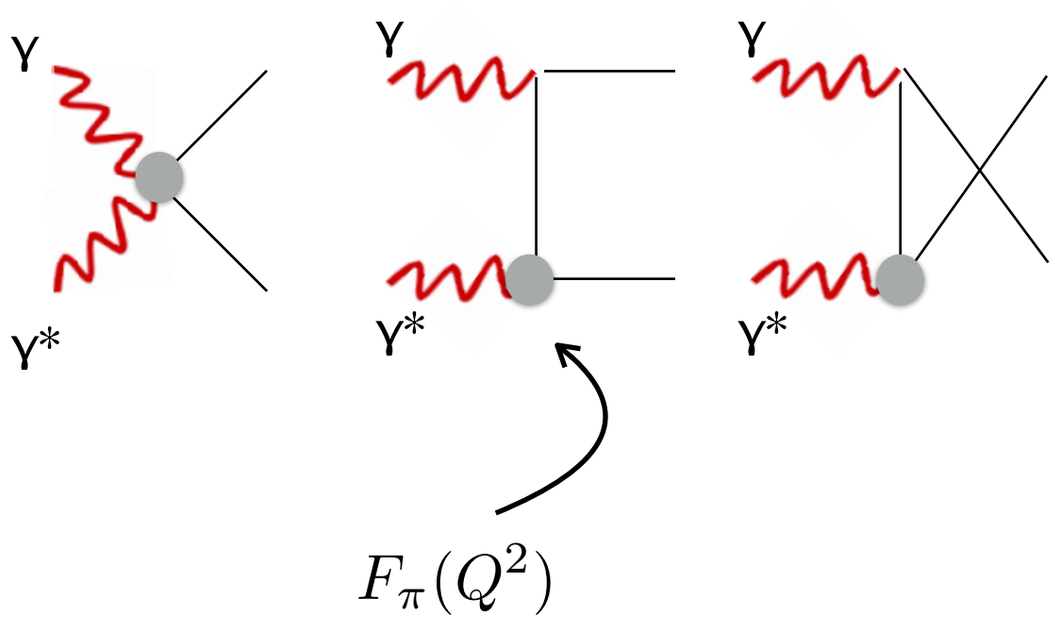
P symmetry: **6** → **3** independent amplitudes

$$H_{++}, H_{+-}, H_{+0}$$

Differential cross section

$$\frac{d\sigma}{d\,\cos\theta} = \pi\alpha^2 \frac{\rho(s)}{4\,(s+Q^2)}\,\left(|H_{++}|^2 + |H_{+-}|^2 + |H_{+0}|^2\right)$$

Born amplitudes ($Q^2 \neq 0$)



Vertex ππγ*

$$\langle \pi^+ | j_\mu(0) | \pi^+(p') \rangle = (p + p')_\mu \, F_\pi(Q^2)$$

Space-like region

$$F_\pi(Q^2) = \frac{1}{1 + Q^2/M_\rho^2}$$

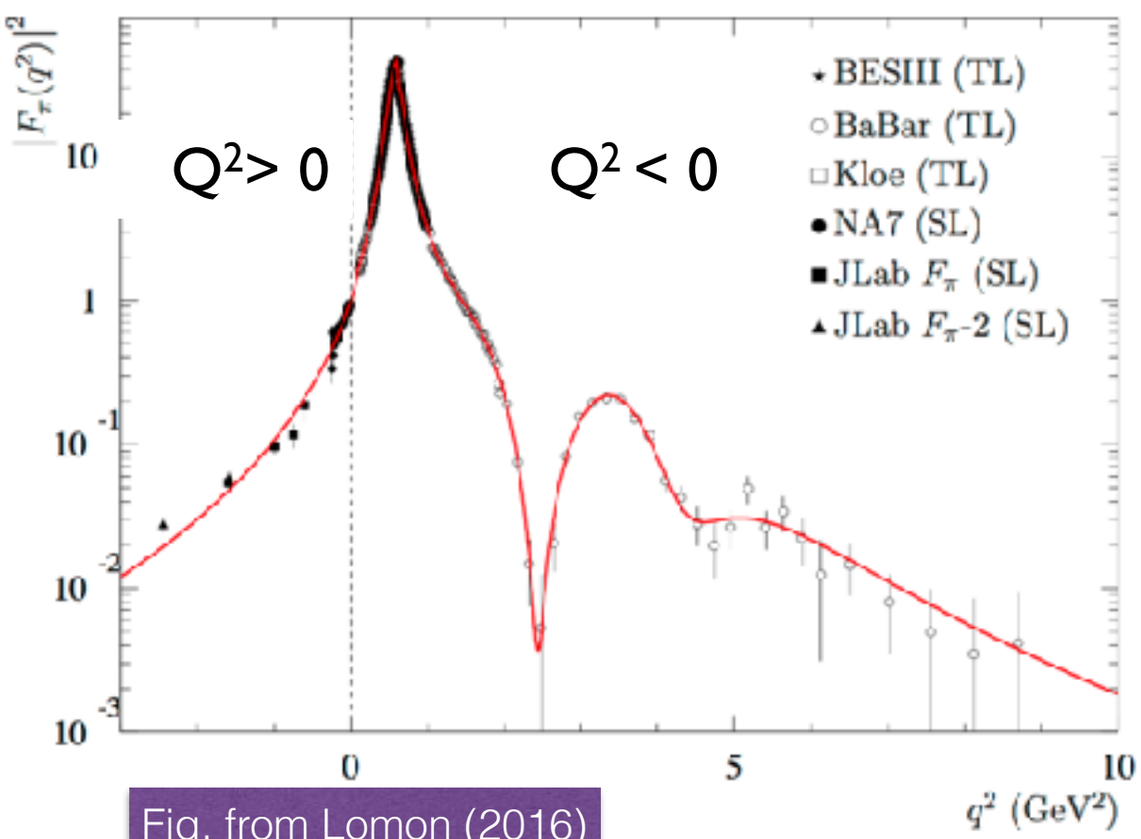
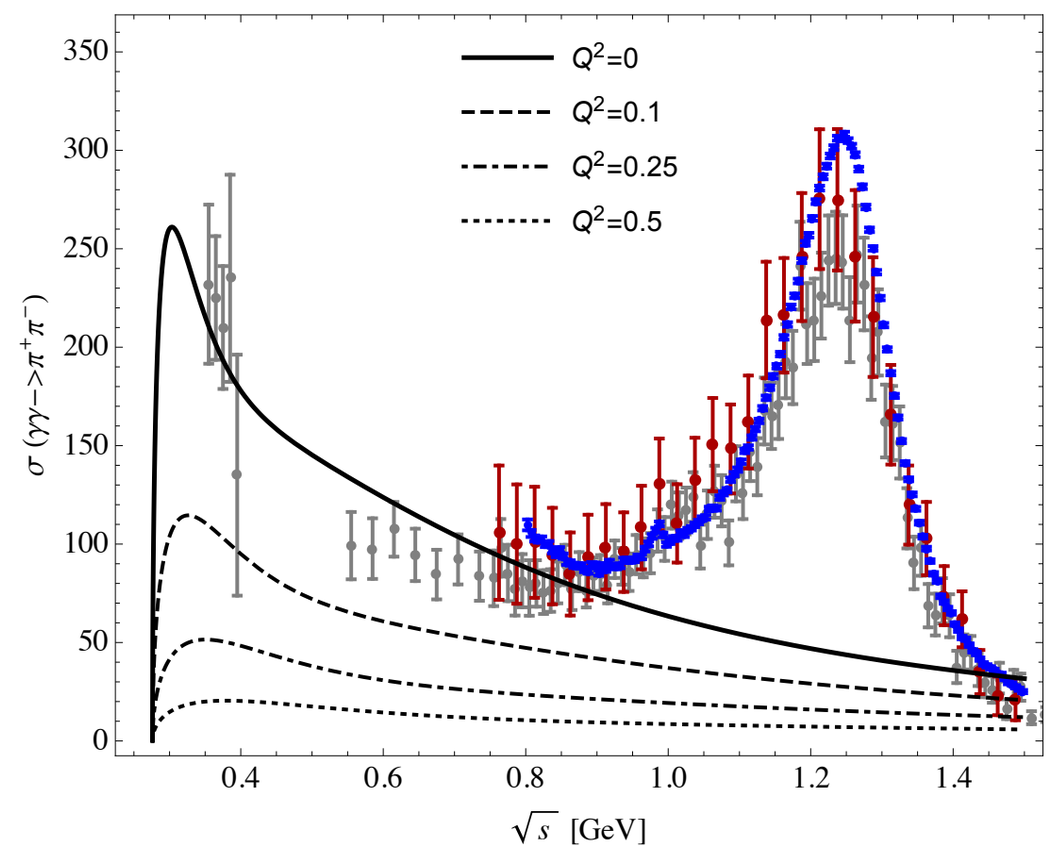


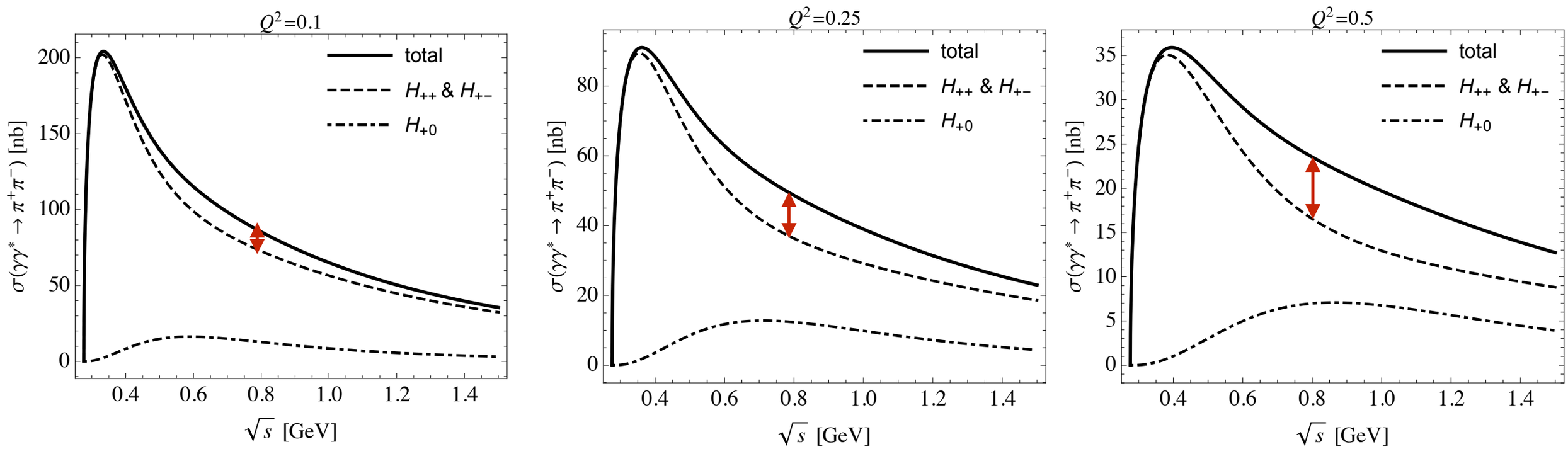
Fig. from Lomon (2016)



Born amplitudes ($Q^2 \neq 0$)

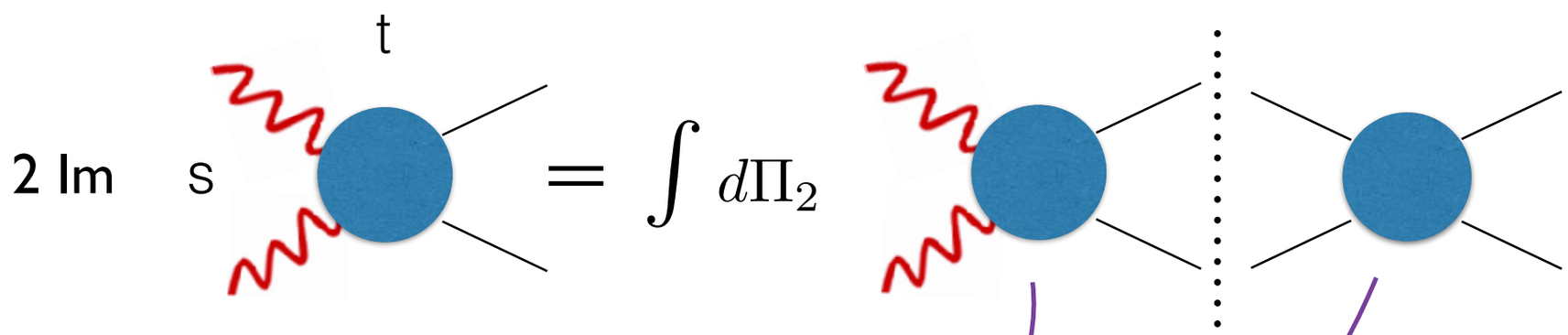
Differential cross section

$$\frac{d\sigma}{d\,\cos\theta} = \pi\alpha^2 \frac{\rho(s)}{4\,(s+Q^2)}\,\left(|H_{++}|^2 + |H_{+-}|^2 + |H_{+0}|^2\right)$$



$\sqrt{s}$ =0.8 GeV: 15% 25% 30%

Unitarity



Partial wave expansion

$$H_{\lambda_1\lambda_2}(s,t) = \sum_{J=0}^{\overset{J_{max}=2}{\cancel{\infty}}} (2J+1)\, h_{J,\lambda_1\lambda_2}(s)\, d^J_{\lambda_1-\lambda_2,0}(\theta)$$

$$T(s,t) = \sum_{J=0}^{\overset{J_{max}=2}{\cancel{\infty}}} (2J+1)\, t_J(s)\, P_J(\theta)$$

These "diagonalise unitarity" and contain resonance information

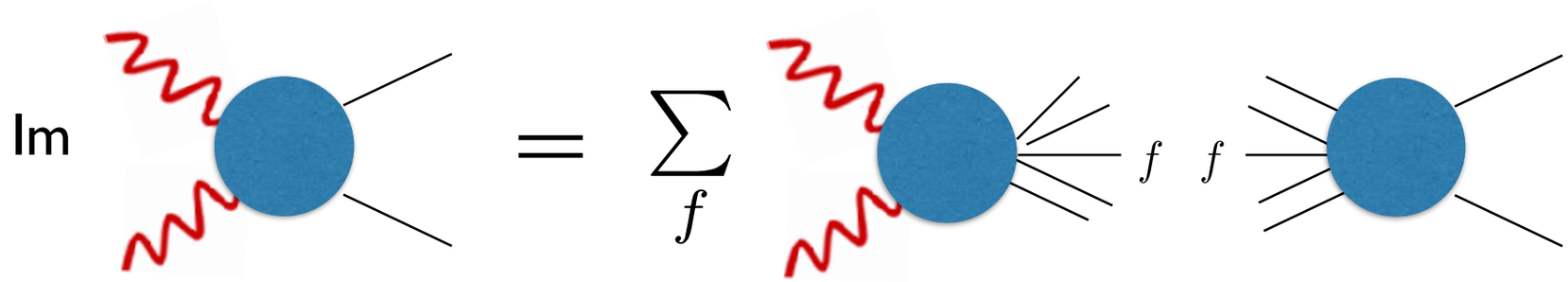
Definite: J, λ_1, λ_2

$$\mathrm{Im}\, h_{\gamma\gamma^*\to\pi\pi}(s) = h_{\gamma\gamma^*\to\pi\pi}(s)\, \rho_{\pi\pi}(s)\, t^*_{\pi\pi\to\pi\pi}(s)$$

Coupled channel Unitarity

Coupled-channel unitarity

Definite: J, λ_1, λ_2



$$\mathrm{Im}\, h_{\gamma\gamma^*,b}(s) = \sum_f h_{\gamma\gamma^*,f}(s)\, \rho_f(s)\, t^*_{fb}(s)$$

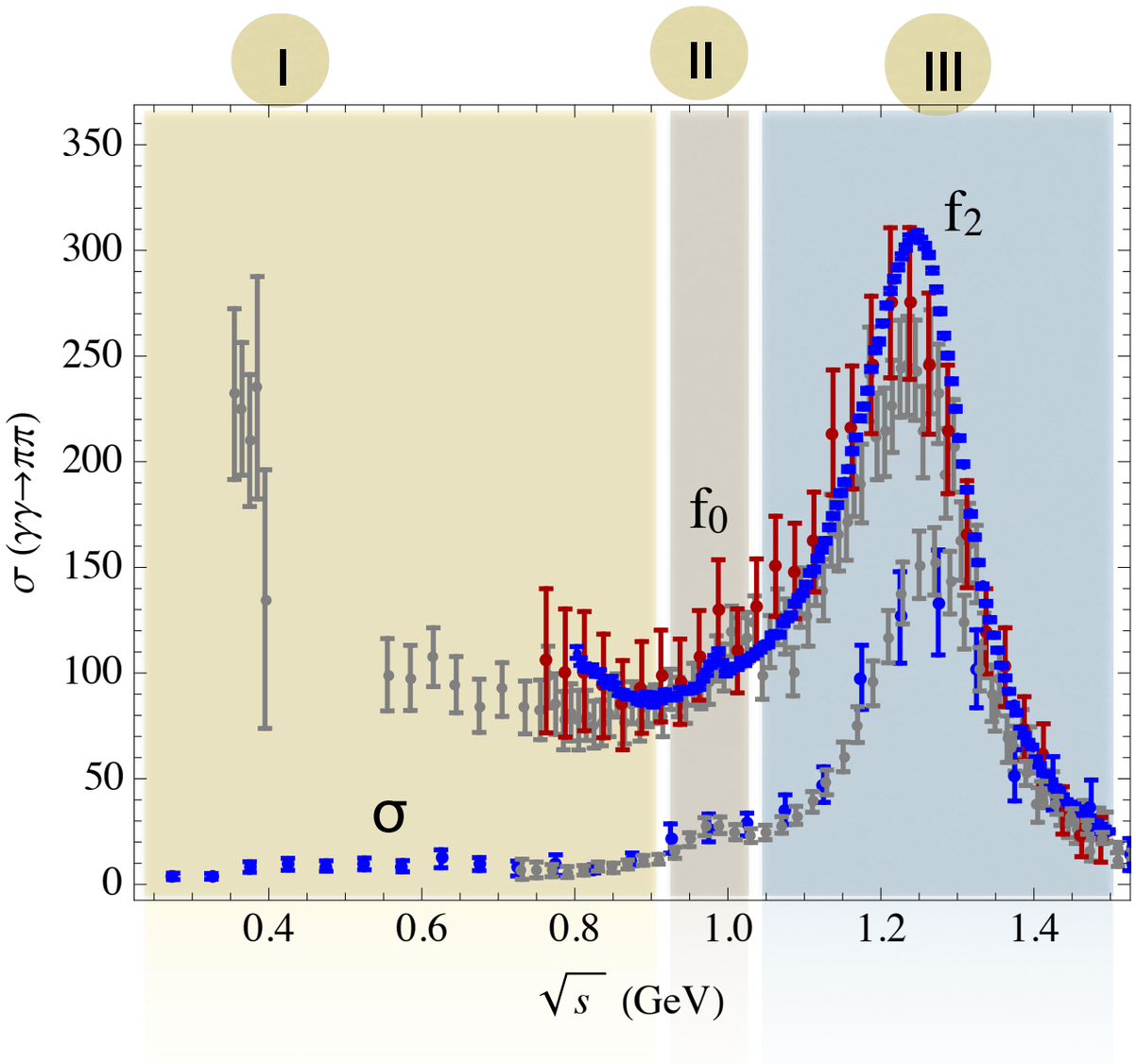
$$\begin{aligned}
\mathrm{Im}\, h_{\gamma\gamma^*,1}(s) &= \rho_1\, h_{\gamma\gamma^*,1}\, t^*_{11} + \rho_2\, h_{\gamma\gamma^*,2}\, t^*_{21} \\
\mathrm{Im}\, h_{\gamma\gamma^*,2}(s) &= \rho_1\, h_{\gamma\gamma^*,1}\, t^*_{12} + \rho_2\, h_{\gamma\gamma^*,2}\, t^*_{22}
\end{aligned}$$

1 = $\pi\pi$
2 = KK

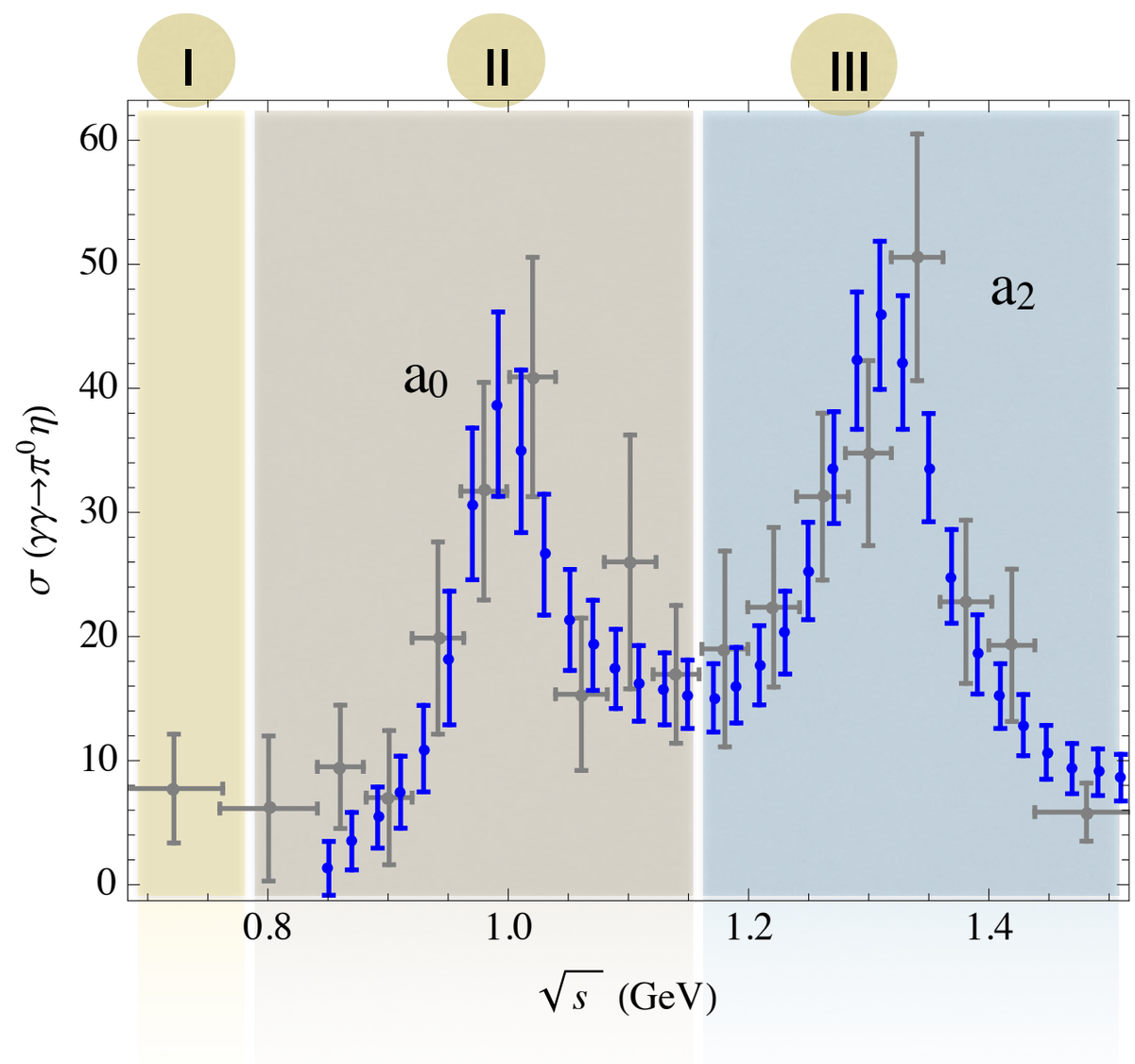
Entire dynamical information that does not depend on the underlying theory (e.g. QCD) comes from **unitarity**

Experimental data

$\gamma\gamma \to \pi^+\pi^-,\ \pi^0\pi^0$



$\gamma\gamma \to \pi^0\eta$

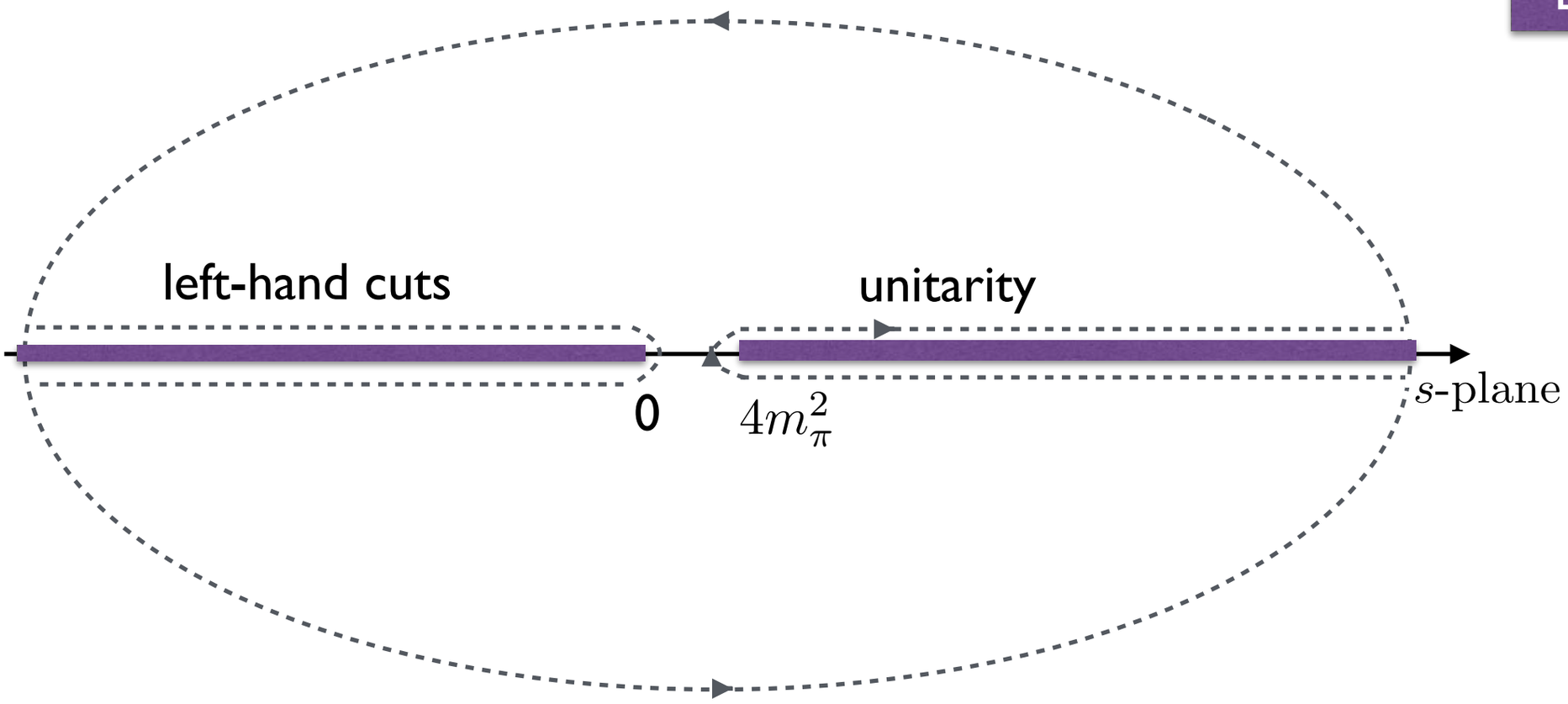


$\gamma\gamma \to \pi^+\pi^-$: Mark II ('90), CELLO ('92), Belle ('07)
$\gamma\gamma \to \pi^0\pi^0$: Crystal Ball ('90), Belle ('09)
$\gamma\gamma \to \pi^0\eta$: Crystal Ball ('86), Belle ('09)

$\gamma\gamma \to \eta\eta$: Belle ('10)
$\gamma\gamma \to KK$: ARGUS ('90), TASSO ('85), CELLO ('89), Belle ('13)

Dispersion relation

Definite: J, λ_1, λ_2



$$h(s) = \frac{1}{2\pi i}\int_C ds' \frac{h(s')}{s'-s} = \int_{-\infty}^{0} \frac{ds'}{\pi}\frac{\mathrm{Im}\, h(s')}{s'-s} + \int_{4m_\pi^2}^{\infty} \frac{ds'}{\pi}\frac{\mathrm{Im}\, h(s')}{s'-s}$$

analyticity relates scattering amplitude at different energies

Dispersion relation

Left and right-hand cuts

Definite: J, λ_1, λ_2

$$h(s) = \int_{-\infty}^{0} \frac{ds'}{\pi} \frac{\mathrm{Im}\, h(s')}{s' - s} + \int_{4m_\pi^2}^{\infty} \frac{ds'}{\pi} \frac{\mathrm{Im}\, h(s')}{s' - s}$$

Looking for a solution in the form (N/D technique)

$$h(s) = h^{Born}(s) + \Omega(s)\, N(s)$$

$$s > 4m_\pi^2$$

$$\mathrm{Im}\, \Omega(s) = \Omega(s)\, \rho(s)\, t^*(s)$$

$$\mathrm{Im}\, h(s) = h(s)\, \rho(s)\, t^*(s)$$

Omnes (1958)

Dispersive integral for J=0

$$h(s) = h^{Born}(s) + \Omega(s) \Big(a + b\, s + \frac{s^2}{\pi} \int_{-\infty}^{-s_L} \frac{ds'}{s'^2} \frac{\mathrm{Im}(h(s'))\Omega^{-1}(s)}{s' - s} - \frac{s^2}{\pi} \int_{4m_\pi^2}^{\infty} \frac{ds'}{s'^2} \frac{h^{Born}(s')\, \mathrm{Im}(\Omega^{-1}(s'))}{s' - s} \Big)$$

Q^2 - dependent

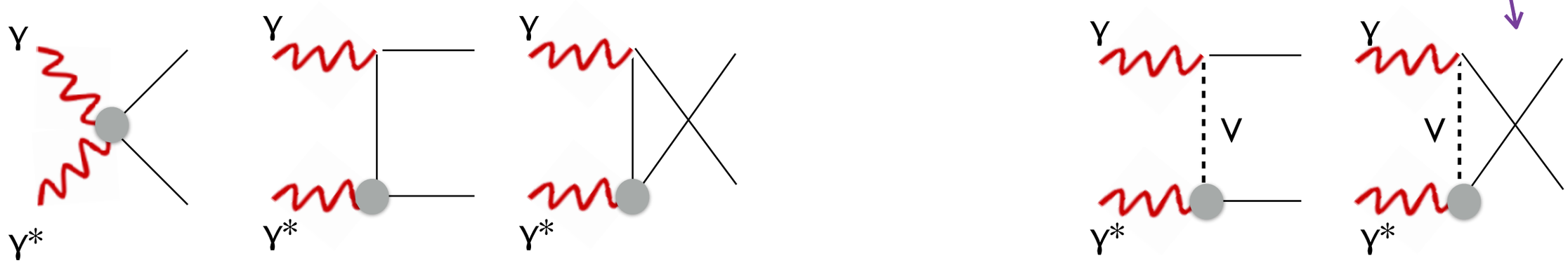
see also Moussallam (2013)

similar eq. for coupled-channel (ππ,KK)

Left-hand cuts

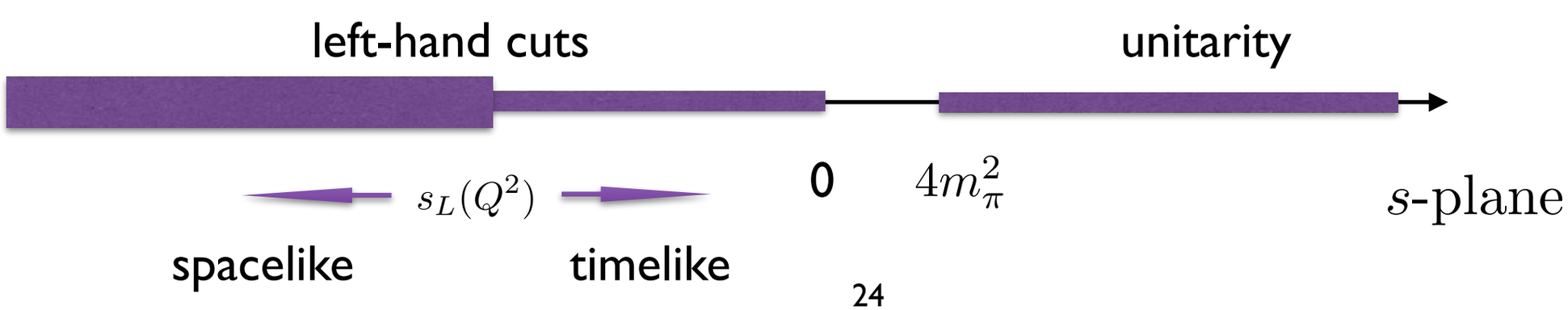
Dispersive integral for J=0

$$h(s) = h^{Born}(s) + \Omega(s)\Big(a + b\,s + \frac{s^2}{\pi}\int_{-\infty}^{-s_L} \frac{ds'}{s'^2}\frac{\mathrm{Im}(h(s'))\Omega^{-1}(s)}{s'-s} - \frac{s^2}{\pi}\int_{4m_\pi^2}^{\infty}\frac{ds'}{s'^2}\frac{h^{Born}(s')\,\mathrm{Im}(\Omega^{-1}(s'))}{s'-s}\Big)$$



Singularities Born: Left-hand cut $s = [-\infty,\, 0]$ and pole $s = -Q^2$

V-exch: Left-hand cut $s = [-\infty,\, s_L(Q^2)]$



Omnes functions

Dispersive integral for J=0

$$h(s) = h^{Born}(s) + \Omega(s)\Big(a + b\,s + \frac{s^2}{\pi}\int_{-\infty}^{-s_L}\frac{ds'}{s'^2}\frac{\mathrm{Im}(h^V(s'))\Omega^{-1}(s)}{s'-s} - \frac{s^2}{\pi}\int_{4m_\pi^2}^{\infty}\frac{ds'}{s'^2}\frac{h^{Born}(s')\,\mathrm{Im}(\Omega^{-1}(s'))}{s'-s}\Big)$$

Coupled channel Omnes

$$\Omega(s) = \begin{pmatrix} \Omega_{\pi\pi\to\pi\pi} & \Omega_{\pi\pi\to K\bar{K}} \\ \Omega_{K\bar{K}\to\pi\pi} & \Omega_{K\bar{K}\to K\bar{K}} \end{pmatrix}$$

Solve p.w. dispersion relation using N/D technique using model-independent form for the left-hand cuts

$$T(s) = U(s) + \frac{s}{\pi}\int_R \frac{ds'}{s'}\frac{\rho(s')|T(s')|^2}{s'-s}$$

$$\sum_k C_k\,\xi(s)^k$$

conformal mapping expansion

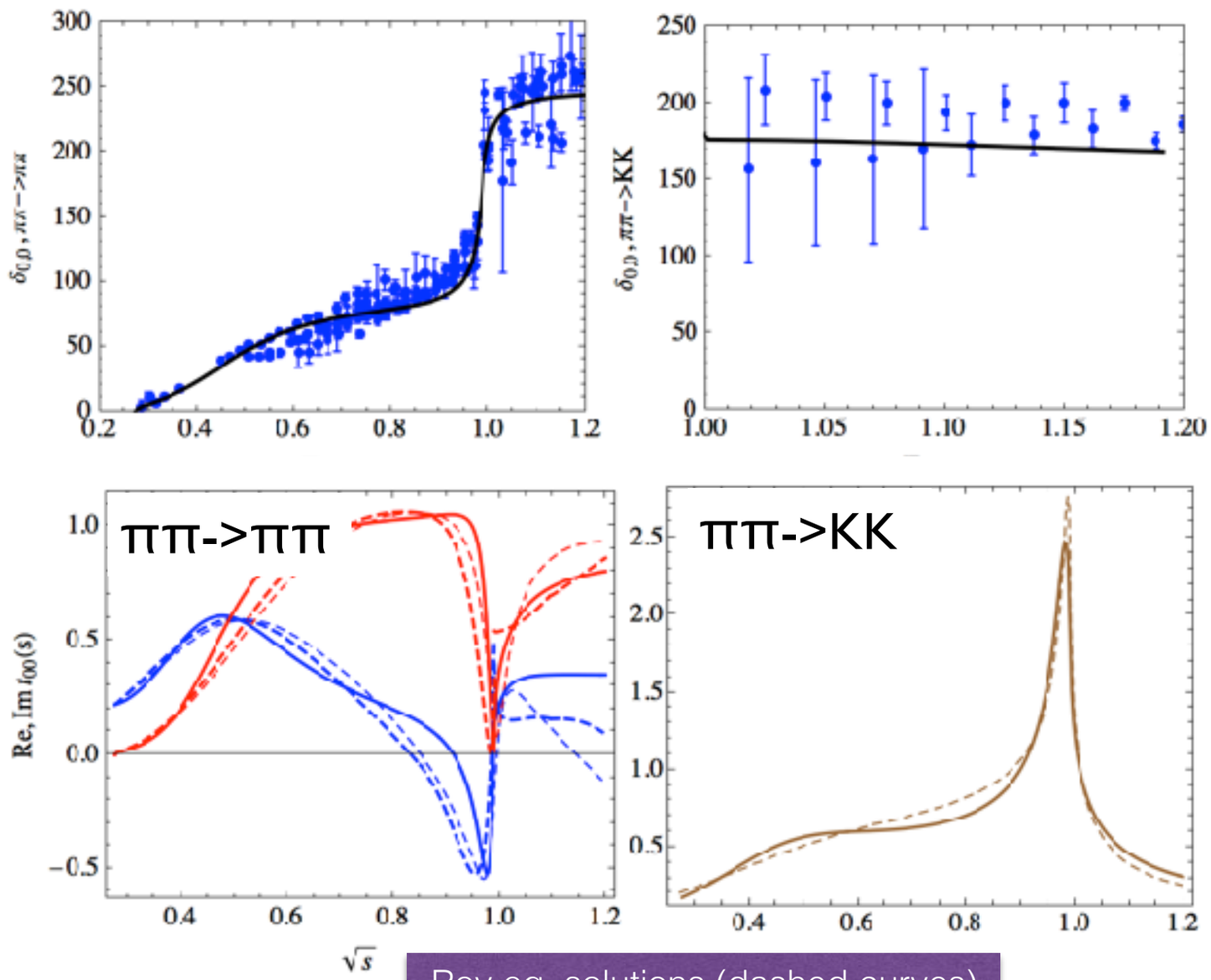
C_k fitted to exp data and Roy eq. solutions

Chew, Mandelstam
Lutz, Gasparyan

Omnes functions

Coupled channel Omnes

$$\Omega(s) = \begin{pmatrix} \Omega_{\pi\pi\to\pi\pi} & \Omega_{\pi\pi\to K\bar{K}} \\ \Omega_{K\bar{K}\to\pi\pi} & \Omega_{K\bar{K}\to K\bar{K}} \end{pmatrix}$$



Roy eq. solutions (dashed curves)
Coupled channel N/D (solid curves)

Bounded p.w. amplitudes and Omnes at large energies

$$T(s) = \frac{N(s)}{D(s)} = \Omega(s)\, N(s)$$

$$N(s) = U(s) + \frac{s}{\pi}\int_R \frac{ds'}{s'}\frac{\rho(s')N(s')(U(s)-U(s'))}{s'-s}$$

$$D(s) = 1 - \frac{s}{\pi}\int_R \frac{ds'}{s'}\frac{\rho(s')N(s')}{s'-s}$$

$$U(s) = \sum_k C_k\, \xi(s)^k$$

$f_2(1270)$ contribution

Watson theorem (for elastic unitarity) J=2:

$$\phi(\gamma\gamma \to \pi\pi) = \phi(\pi\pi \to \pi\pi) = \delta(\pi\pi \to \pi\pi)$$

$$\Omega(s) = \exp\left(\frac{s}{\pi}\int_{4m_\pi^2}^{\infty}\frac{ds'}{s'}\frac{\phi_{\gamma\gamma\to\pi\pi}(s)}{s'-s}\right)$$

Roy analysis (2011)
R. Garcia-Martin
at.al.

When there are **no VM**, it is not possible to describe J=2 partial wave using Omnes functions and we parametrize it with the Breit Wigner + Background

$$h_{J=2}^{f_2} = \frac{C_{f_2\to\gamma\gamma}C_{f_2\to\pi\pi}}{10\sqrt{6}}\frac{s(s+Q^2)\beta(s)}{s-M^2+i\,M\,\Gamma(s)}T_{f_2}^{(\Lambda=2)}(Q^2)$$

$$h_{J=2} = B_{D2} + h_{J=2}^{f_2}\,e^{i\phi_0} = |h_{J=2}|e^{i\delta(\pi\pi\to\pi\pi)}$$

Background: Born

Relative phase: unitarization

Subtraction constants

Dispersive integral for J=0

$$h(s) = h^{Born}(s) + \Omega(s)\Big(a + b\,s + \frac{s^2}{\pi}\int_{-\infty}^{-s_L}\frac{ds'}{s'^2}\frac{\mathrm{Im}(h^V(s'))\Omega^{-1}(s)}{s'-s} - \frac{s^2}{\pi}\int_{4m_\pi^2}^{\infty}\frac{ds'}{s'^2}\frac{h^{Born}(s')\,\mathrm{Im}(\Omega^{-1}(s'))}{s'-s}\Big)$$

NO Vector mesons
fix: **a**

With Vector mesons
fix: **a**, **b**

Soft photon limit (q_l=0)

$$H_{\lambda_1\lambda_2} \to H^{Born}_{\lambda_1\lambda_2}$$
$$s = -Q^2,\ t = u = m_\pi^2$$

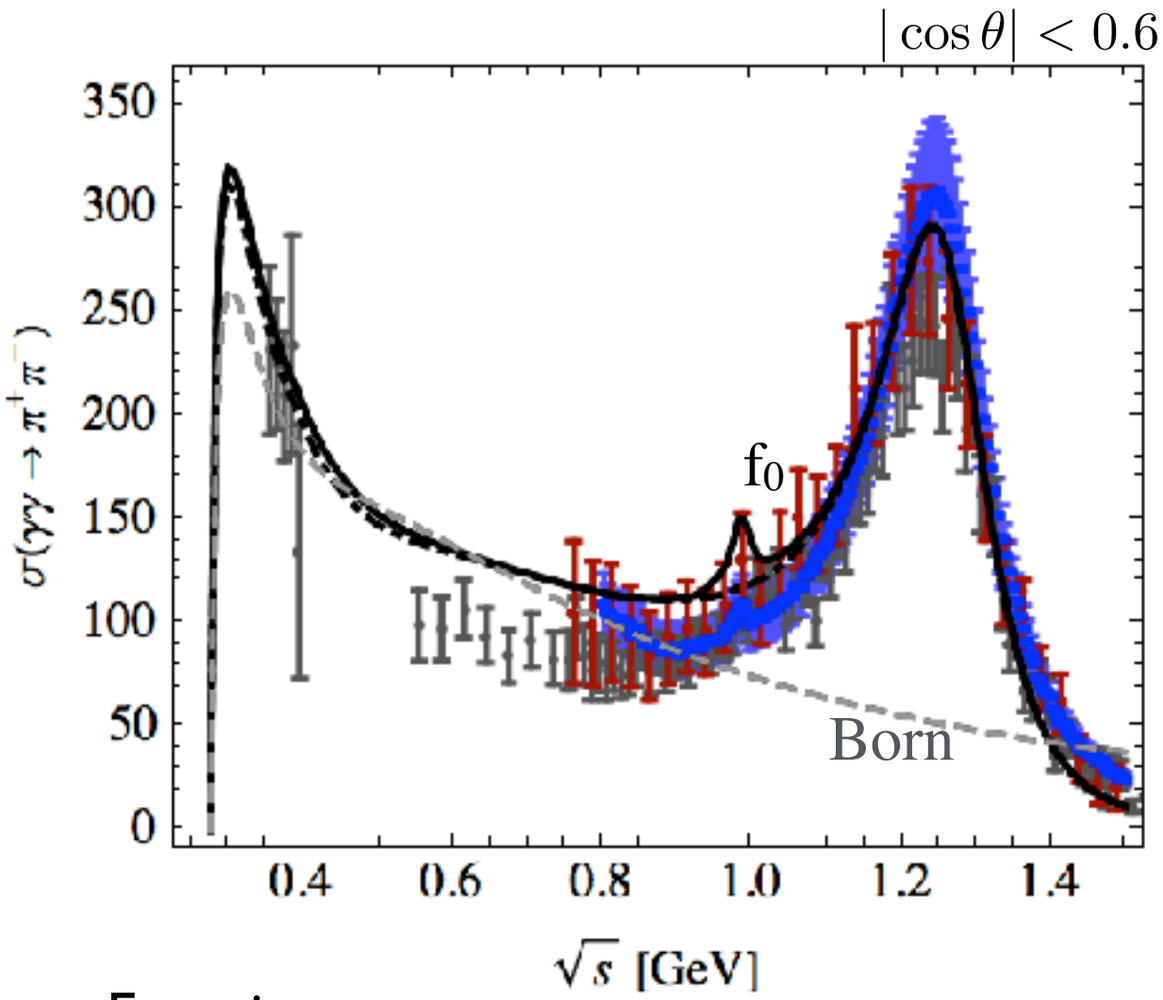
prediction for **b**: generalised polarizabilities

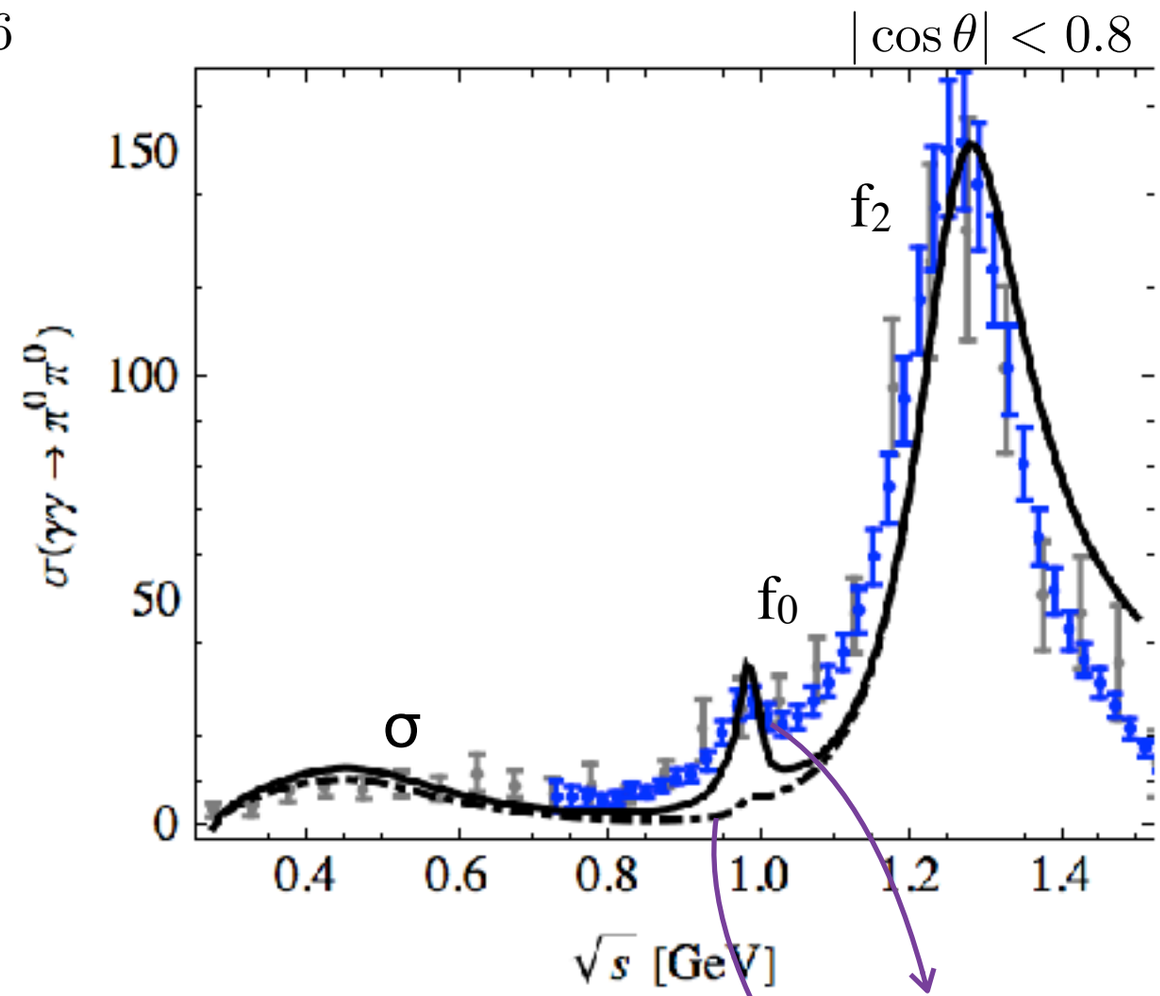
For space like photons: generalized polarizabilities

$$\pm\frac{2\alpha}{m_\pi}\frac{H^n_{+\pm}}{s+Q^2} = (\alpha_1 \mp \beta_1)_{\pi^0} + \ldots$$
$$\pm\frac{2\alpha}{m_\pi}\frac{(H^c_{+\pm} - H^{Born}_{+\pm})}{s+Q^2} = (\alpha_1 \mp \beta_1)_{\pi^+} + \ldots$$

more realistic l.h.cut: **fix b** from **ChPT** and **COMPASS**

No VM ($Q^2=0$)





Coupled channel

Single channel

Experiment:

γγ→π⁺π⁻: Mark II ('90), CELLO ('92), Belle ('07)

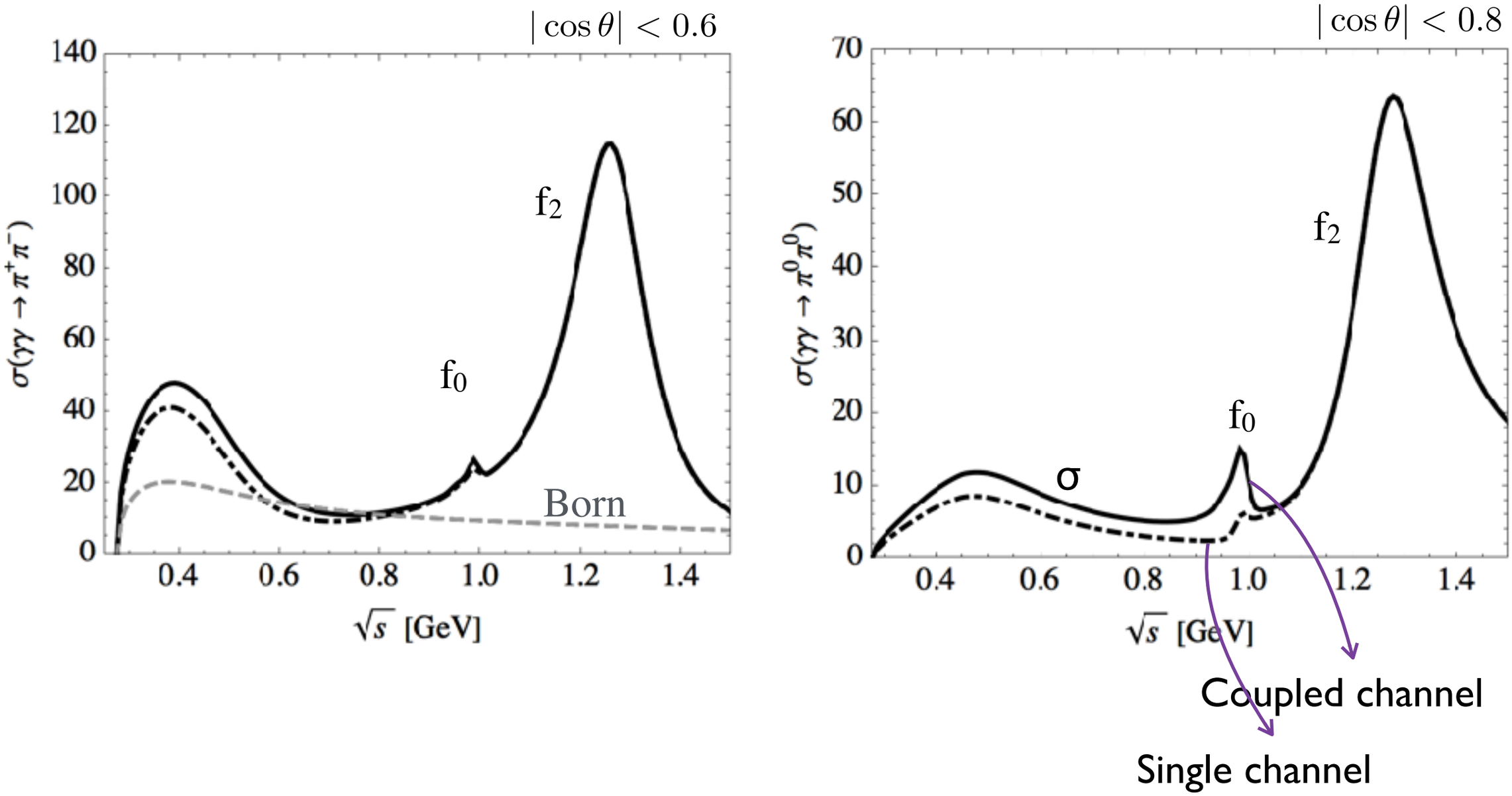
γγ→π⁰π⁰: Crystal Ball ('90), Belle ('09)

$$C^{exp}_{f_2\to\gamma\gamma} = 0.20 \pm 0.02$$
$$C^{eff}_{f_2\to\gamma\gamma} = 0.22$$

I.D., Vanderhaeghen (work in progress)

see also Pennington ('14), Hoferichter ('11), Garcia-Martin et. al ('10)

No VM (Q^2=0.5)

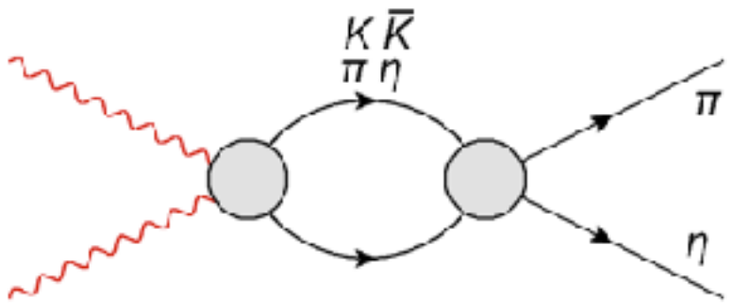


Results **with VM** and fully **dispersive f_2(1270)** contribution are on their way ….

Ongoing experiment: BES III

I.D., Vanderhaeghen (work in progress)

$\gamma\gamma \to \pi\eta$ ($Q^2=0$)

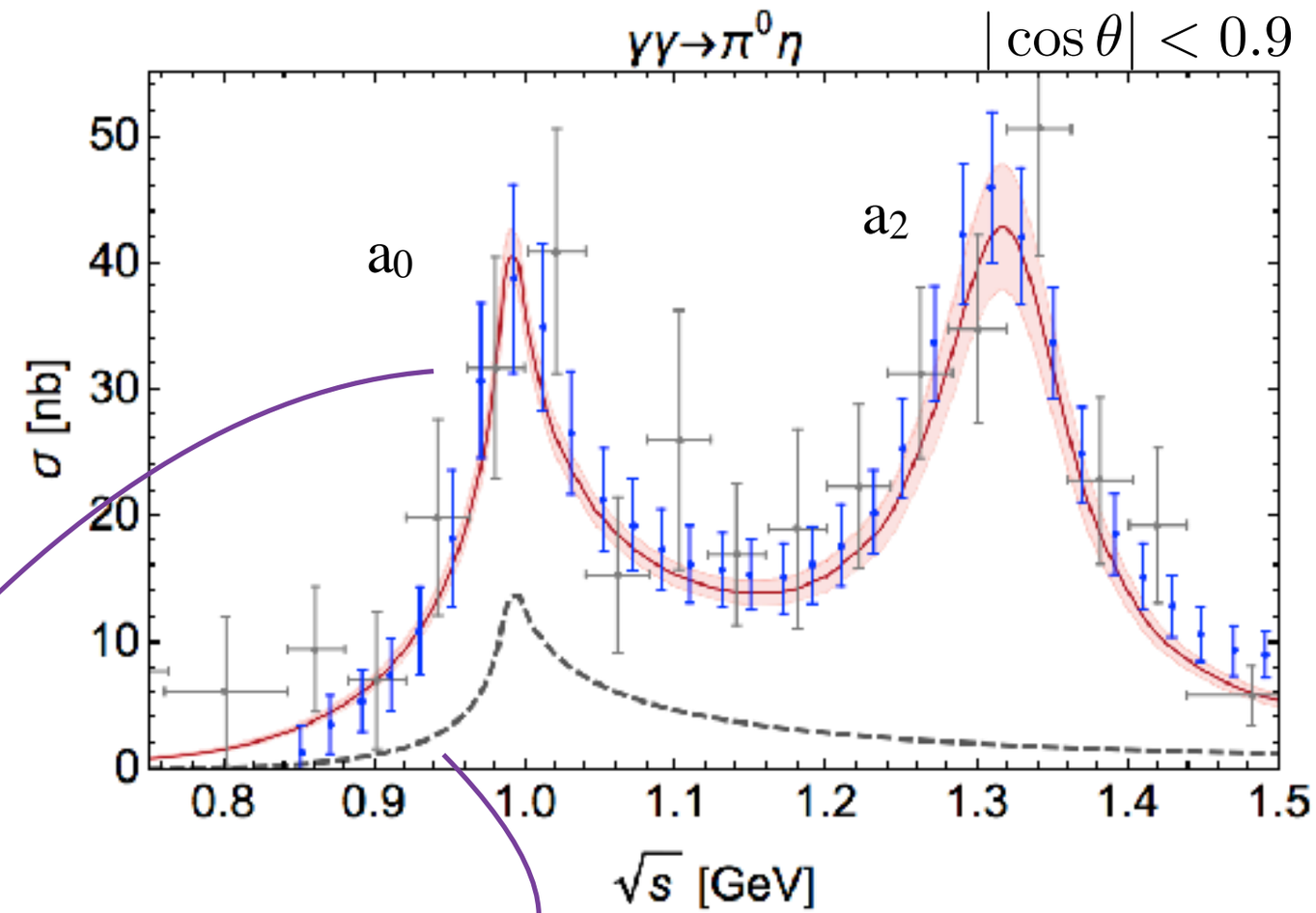


$$\Omega(s) = \begin{pmatrix} \Omega_{\pi\eta\to\pi\eta} & \Omega_{\pi\eta\to K\bar{K}} \\ \Omega_{K\bar{K}\to\pi\eta} & \Omega_{K\bar{K}\to K\bar{K}} \end{pmatrix}$$

Danilkin, Gil, Lutz (2011)

Coupled-channel dispersive treatment for J=0 is **crucial**

$a_2(1230)$ described as a Breit Wigner resonance



Coupled channel: **with VM**

Coupled channel: **no VM**

I.D., Deineka, Vanderhaeghen (work in progress)

Summary and Outlook

- In light of the **new Belle data (2015)** for $f_2(1270)$ TFFs and using LbL sum rules we **predicted** (Λ=2) TFF for $f_2(1565)$

- **Update for meson contributions to (g-2) LbL**
  Tensor mesons contributions found to be small compared to anticipated exp. uncertainty $1.6*10^{-10}$

  Axial vector mesons contributions (satisfying Landau-Yang theorem constraint) evaluated by 2 groups and found to be between $(0.64 - 0.75 \pm 0.27)10^{-10}$

- **Next steps?**
  Need to take into account $f_0(500)$ and non resonant contributions in a dispersive approach

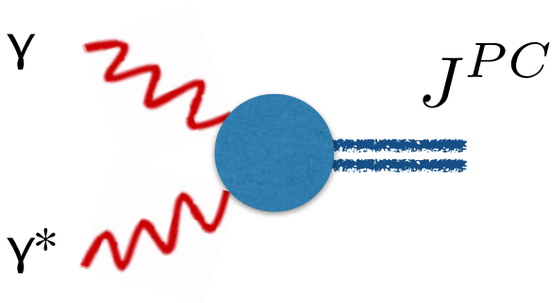
- Main ingredients: **$\gamma\gamma^* \to \pi\pi$, $\pi\eta$,**… (work in progress). Can be used in **different** (g-2) dispersive approaches.

  It is important to **validate** dispersive treatment of **$\gamma\gamma^* \to \pi\pi$, $\pi\eta$,**… with upcoming BES III data

Thank you!

Extra slides

Sum rules II and III



C=+1: $J^{PC}=0^{-+}, 0^{++}, 1^{++}, 2^{++}, \ldots$

$Q^2 \neq 0$

Landau-Yang theorem

$$0 = \int_{s_0}^{\infty} ds \, \frac{1}{(s+Q_1^2)^2} \left[\sigma_{\parallel} + \sigma_{LT} + \frac{(s+Q_1^2)}{Q_1 Q_2} \tau_{TL}^a \right]_{Q_2^2=0}$$

$$0 = \int_{s_0}^{\infty} ds \left[\frac{\tau_{TL}(s, Q_1^2, Q_2^2)}{Q_1 Q_2} \right]_{Q_2^2=0}$$

Axial-vector mesons 1^{++} are **allowed** if one of the photons is virtual: interplay between $\mathcal{A}, \mathcal{T}$

Equivalent 2γ width: $\tilde{\Gamma}_{\gamma\gamma}(\mathcal{A}) \equiv \lim_{Q_1^2 \to 0} \frac{m_{\mathcal{A}}^2}{Q_1^2} \frac{1}{2} \Gamma\left(\mathcal{A} \to \gamma_L^* \gamma_T\right)$

TFFs $\gamma^*\gamma \to f_1(1285), f_1(1420)$ were measured

L3 Collaboration (2002), (2007)

Sum rules II and III ($Q^2 \approx 0$)

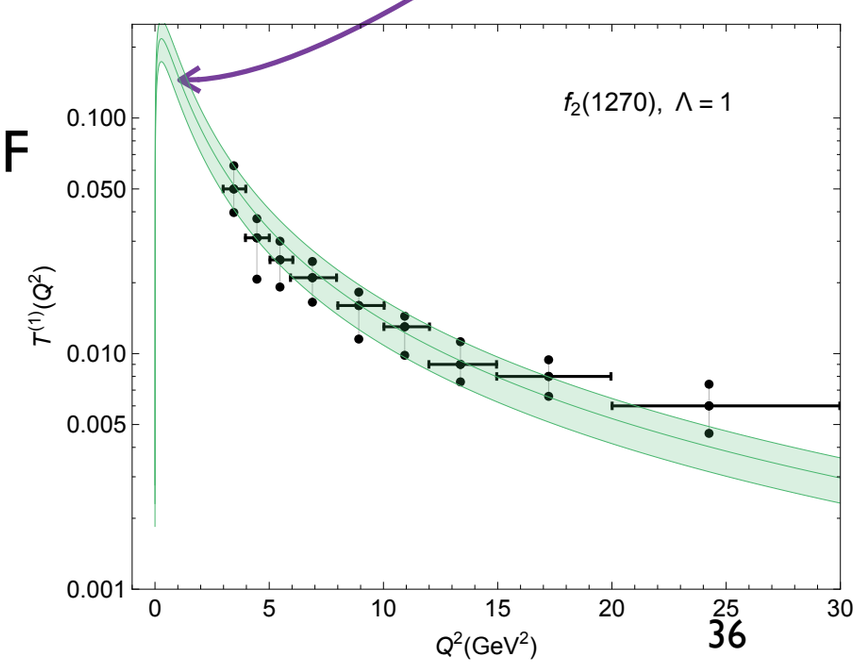
	m	$\Gamma_{\gamma\gamma}$	$\int ds \left[\frac{1}{s^2}\sigma_{\parallel} + \frac{1}{s}\frac{\tau^a_{TL}}{Q_1 Q_2}\right]_{Q_i^2=0}$	$\int_{s_0}^{\infty} ds \left[\frac{\tau_{TL}(s,Q_1^2,Q_2^2)}{Q_1 Q_2}\right]_{Q_i^2=0}$
	[MeV]	[keV]	[nb / GeV^2]	[nb]
$f_1(1285)$	1281.8 ± 0.6	3.5 ± 0.8	-93 ± 21	$+153 \pm 35$
$f_1(1420)$	1426.4 ± 0.9	3.2 ± 0.9	-50 ± 14	$+102 \pm 29$
$f_0(980)$	990 ± 20	0.31 ± 0.05	$+40 \pm 13$	$+19 \pm 10$
$f_2(1270)$	1275.5 ± 0.8	2.93 ± 0.40		
$\Lambda = 2$			$+122 \pm 17$	-
$\Lambda = (0, T)$			$+23 \pm 3$	-
$\Lambda = (0, L)$			??	??
$\Lambda = 1$			??	??
Sum				
$f_2(1565)$	1562 ± 13	0.70 ± 0.14		
$\Lambda = 2$			$+12 \pm 2$	-
Sum			≈ 0 (def.)	≈ 0 (def.)

Sum rules II and III ($Q^2 \approx 0$)

	m [MeV]	$\Gamma_{\gamma\gamma}$ [keV]	$\int ds \left[\frac{1}{s^2}\sigma_{\parallel} + \frac{1}{s}\frac{\tau^c_{TL}}{Q_1 Q_2}\right]_{Q_i^2=0}$ [nb / GeV^2]	$\int_{s_0}^{\infty} ds \left[\frac{\tau_{TL}(s,Q_1^2,Q_2^2)}{Q_1 Q_2}\right]_{Q_i^2=0}$ [nb]
$f_1(1285)$	1281.8 ± 0.6	3.5 ± 0.8	-93 ± 21	$+153 \pm 35$
$f_1(1420)$	1426.4 ± 0.9	3.2 ± 0.9	-50 ± 14	$+102 \pm 29$
$f_0(980)$	990 ± 20	0.31 ± 0.05	$+40 \pm 13$	$+19 \pm 10$
$f_2(1270)$	1275.5 ± 0.8	2.93 ± 0.40		
$\Lambda = 2$			$+122 \pm 17$	-
$\Lambda = (0,T)$			$+23 \pm 3$	-
$\Lambda = (0,L)$			-111 ± 15	-180 ± 43
$\Lambda = 1$			$+58 \pm 24$	-94 ± 40
Sum			$+92 \pm 26$	-274 ± 53
$f_2(1565)$	1562 ± 13	0.70 ± 0.14		
$\Lambda = 2$			$+12 \pm 2$	-
Sum			≈ 0 (def.)	≈ 0 (def.)

Constrain low Q^2 region of (Λ=1) TFF

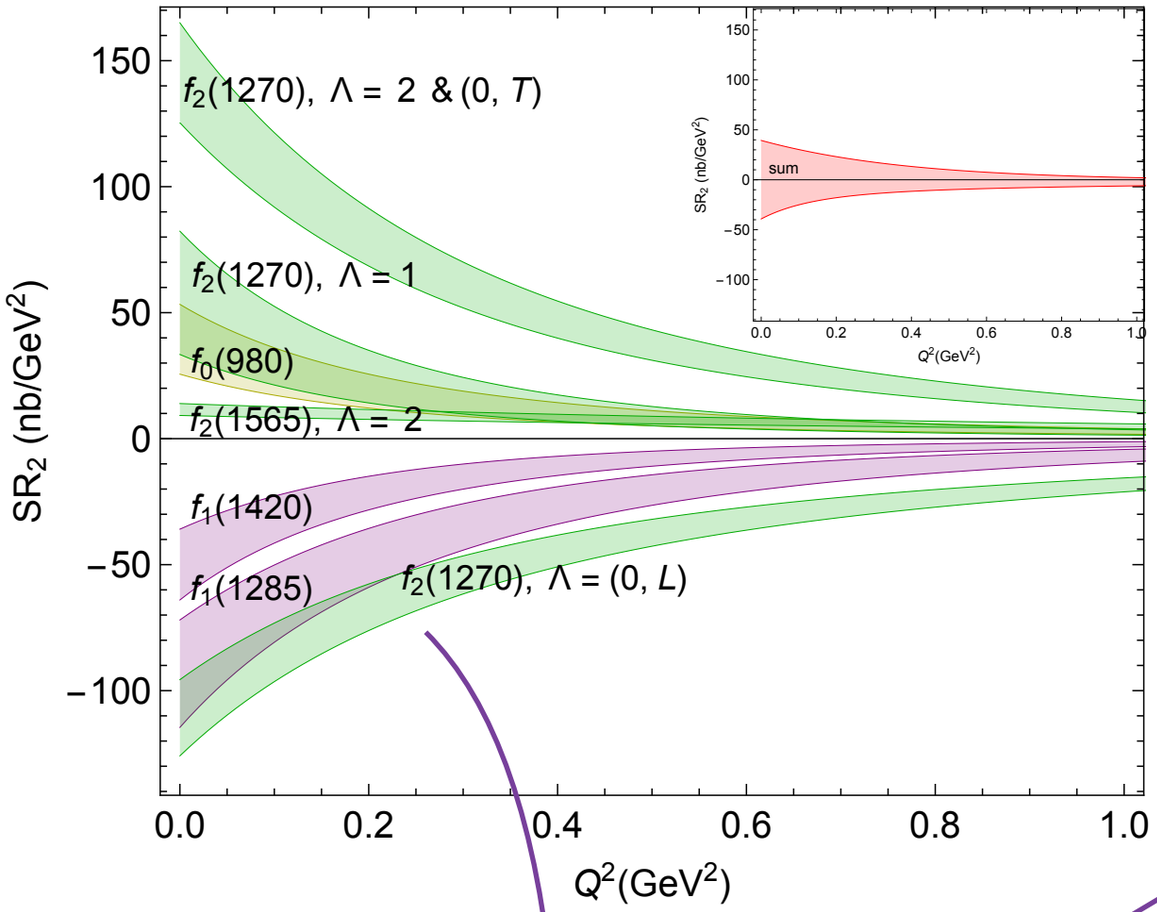
future BES III

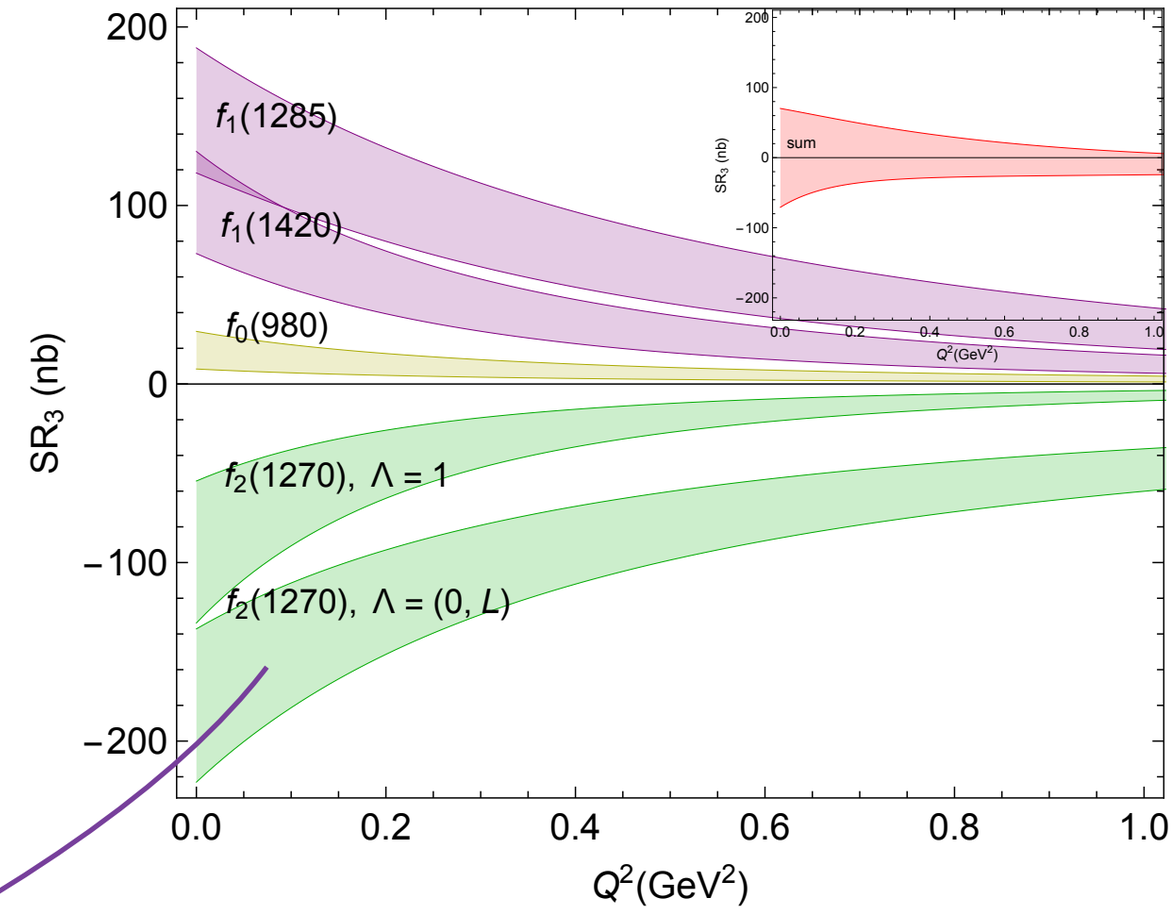


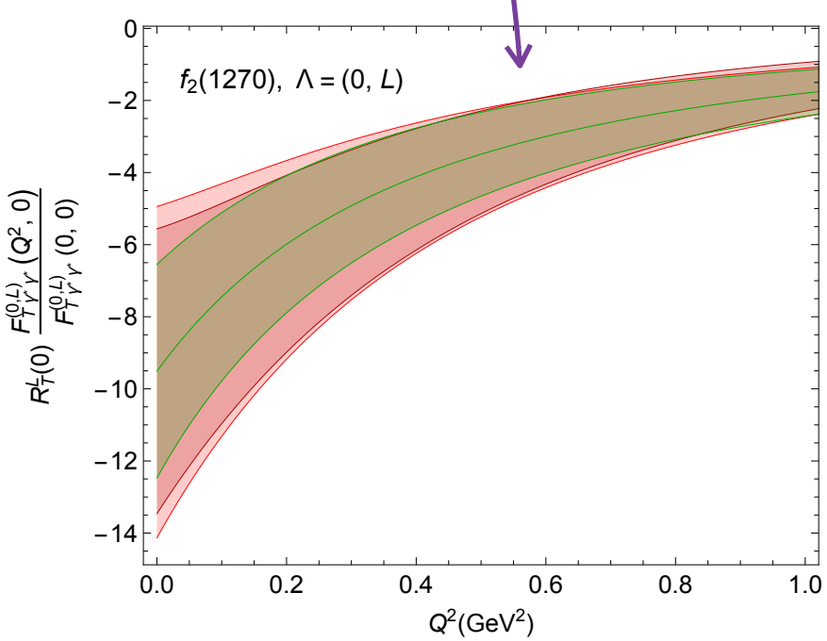
Constrain longitudinal coupling ratio for $Q^2 \to 0$

$$R_T^L(Q^2) \equiv \frac{T^{(0,L)}(Q^2)}{T^{(0,T)}(Q^2)}$$

Sum rules II and III (Q^2>0)







Prediction:

$$f_2(1270)$$
$$\lambda_{\Lambda=(0,L)} = 977 \pm 66\,\mathrm{MeV}$$

Danilkin, Vanderhaeghen, (2016)

Conformal mapping

Solve p.w. dispersion relation using N/D technique using model-independent form for the left-hand cuts

$$T(s) = U(s) + \frac{s}{\pi}\int_R \frac{ds'}{s'}\frac{\rho(s')|T(s')|^2}{s'-s}$$

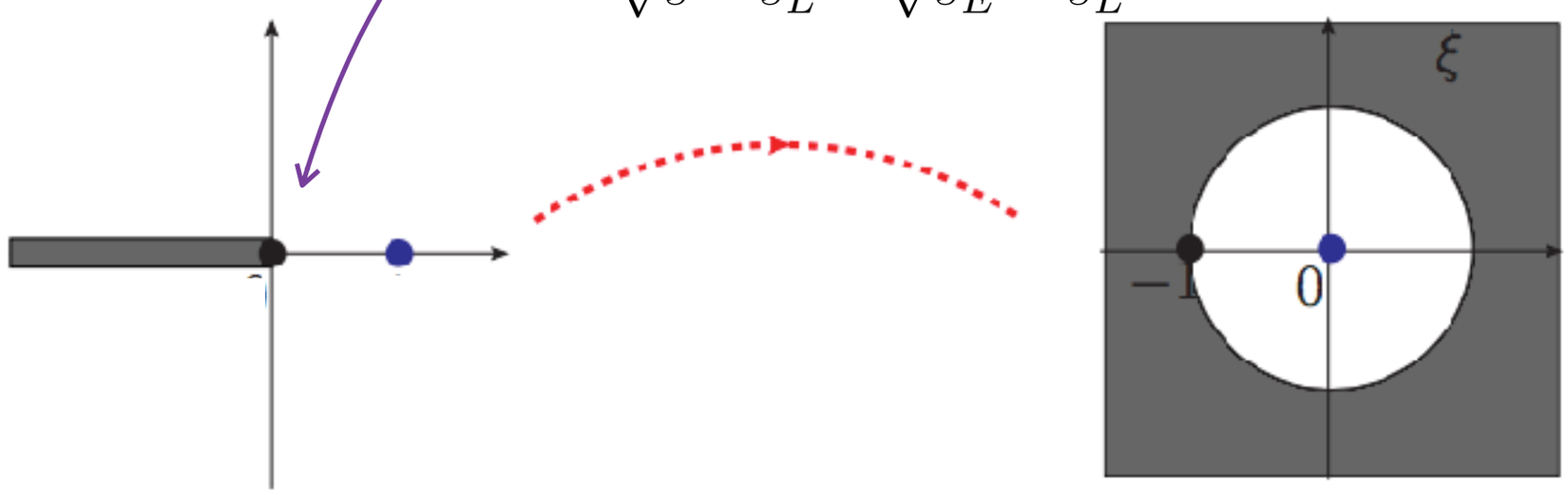
$$\sum_k C_k\,\xi(s)^k$$

conformal mapping expansion

C_k fitted to exp data and Roy solutions

Chew, Mandelstam
Lutz, Gasparyan

$$\xi(s) = \frac{\sqrt{s-s_L}-\sqrt{s_E-s_L}}{\sqrt{s-s_L}-\sqrt{s_E-s_L}}$$

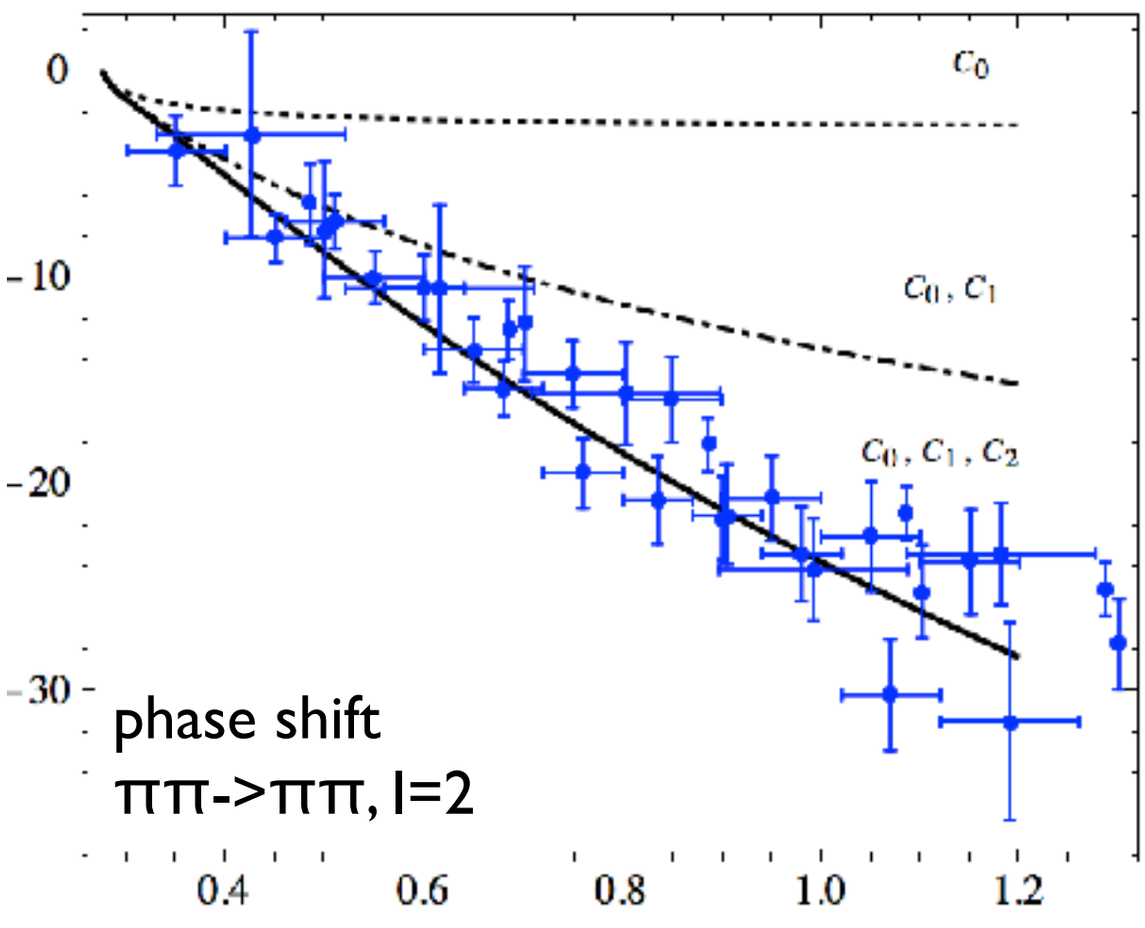


Example: I=2

Solve p.w. dispersion relation using N/D technique using model-independent form for the left-hand cuts

$$T(s) = U(s) + \frac{s}{\pi}\int_R \frac{ds'}{s'}\frac{\rho(s')|T(s')|^2}{s'-s}$$

$$\sum_k C_k\,\xi(s)^k = C_0 + C_1\xi(s) + C_2\xi(s)^2 + \ldots$$



fixed from the threshold parameter **a_2** (scattering length)

fixed from the threshold parameters **a_2, b_2**

$$\frac{1}{m_\pi}\mathrm{Re}\left(T_{\pi\pi\to\pi\pi}/16\pi\right) = a + b\,p_{\mathrm{cm}}^2 + \ldots$$

one parameter fit

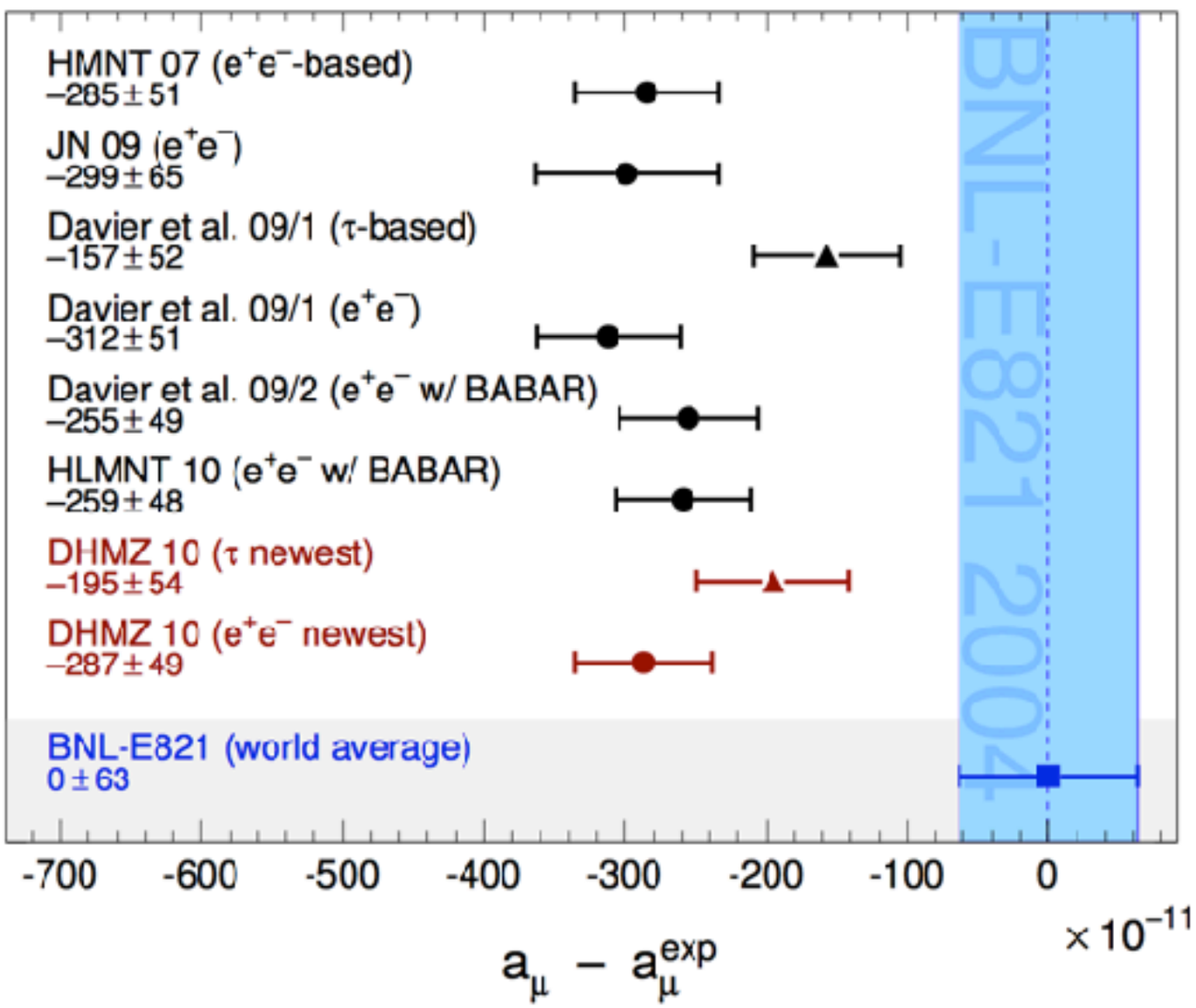
$$N(s) = U(s) + \frac{s}{\pi}\int_R \frac{ds'}{s'}\frac{\rho(s')N(s')(U(s)-U(s'))}{s'-s}$$

$$D(s) = 1 - \frac{s}{\pi}\int_R \frac{ds'}{s'}\frac{\rho(s')N(s')}{s'-s}$$

(g-2)

- Anomalous magnetic moment of the muon

$$a_\mu = \frac{(g-2)_\mu}{2}$$



$$a_\mu^{exp} - a_\mu^{SM} =$$
$$(26.1 \pm 4.9_{th} \pm 6.3_{exp}) \times 10^{-10}$$

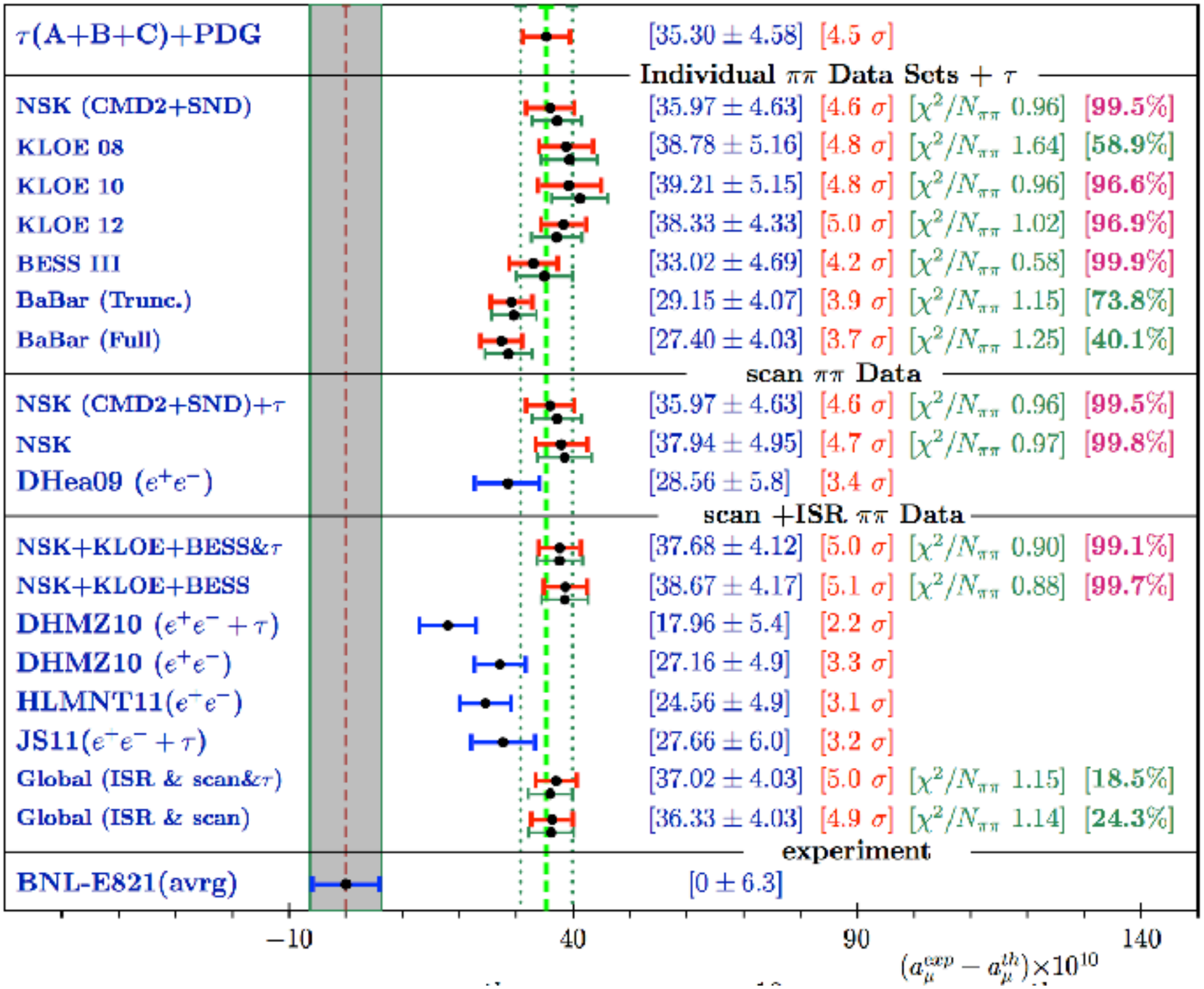
3 - 4 σ
deviation !

DHMZ, 2011
Eur. Ph. J. C 71 1515

(g-2)

- Anomalous magnetic moment of the muon

Jegerlehner et. al. (2015)



(g-2)

- Anomalous magnetic moment of the muon

Total HLbL [a_μ in units 10^{-11}]

Contribution	HKS	BPP	KN	MV	PdRV	N/JN
π^0, η, η'	82.7±6.4	85±13	83±12	114±10	114±13	99±16
π, K loops	−4.5±8.1	−19±13	–	0±10	−19±19	−19±13
axial vectors	1.7±1.7	2.5±1.0	–	22± 5	15±10	22± 5
scalars	–	−6.8±2.0	–	–	−7± 7	−7± 2
quark loops	9.7±11.1	21± 3	–	–	2.3	21± 3
total	89.6±15.4	83±32	80±40	136±25	105±26	116±39

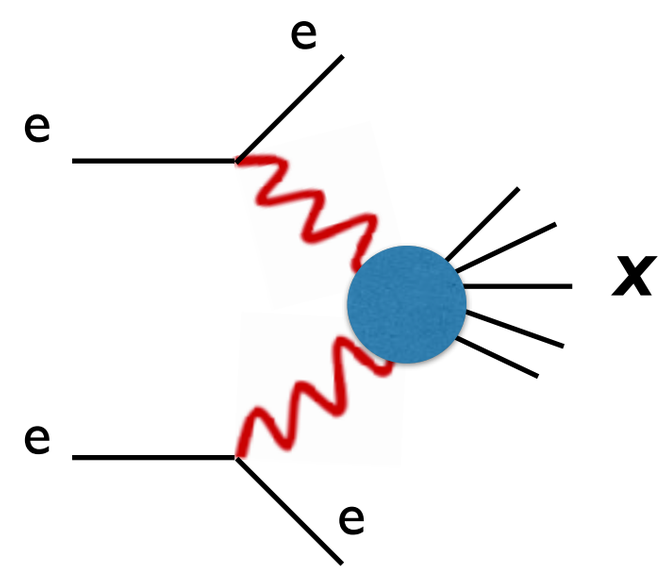
Jegerlehner (2015)

→ 7.5 ± 2.7

→ 102 ± 39

Light by light scattering

Observables in experiment $e^+e^- \to e^-e^+X$



$$
\begin{aligned}
d\sigma \;=\;& \frac{\alpha^2}{16\pi^4\, Q_1^2\, Q_2^2}\, \frac{2\sqrt{X}}{s(1-4m^2/s)} \cdot \frac{d^3\vec{p}_1{}'}{E_1'} \cdot \frac{d^3\vec{p}_2{}'}{E_2'} \\
\times\;& \Big\{ 4\,\rho_1^{++}\,\rho_2^{++}\,\sigma_{TT} + \rho_1^{00}\,\rho_2^{00}\,\sigma_{LL} + 2\,\rho_1^{++}\,\rho_2^{00}\,\sigma_{TL} + 2\,\rho_1^{00}\,\rho_2^{++}\,\sigma_{LT} \\
& +2\,\left(\rho_1^{++}-1\right)\,\left(\rho_2^{++}-1\right)\,\left(\cos 2\tilde{\phi}\right)\,\tau_{TT} + 8\left[\frac{\left(\rho_1^{00}+1\right)\,\left(\rho_2^{00}+1\right)}{\left(\rho_1^{++}-1\right)\,\left(\rho_2^{++}-1\right)}\right]^{1/2}\left(\cos\tilde{\phi}\right)\tau_{TL} \\
& +h_1 h_2\, 4\left[\left(\rho_1^{00}+1\right)\,\left(\rho_2^{00}+1\right)\right]^{1/2}\tau_{TT}^a + h_1 h_2\, 8\left[\left(\rho_1^{++}-1\right)\,\left(\rho_2^{++}-1\right)\right]^{1/2}\left(\cos\tilde{\phi}\right)\tau_{TL}^a \Big\},
\end{aligned}
$$