The HVP contribution to a_μ from Lattice QCD

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Outline

1) Introduction + HPQCD method for analysing the hadronic vacuum polarisation contribution to a_μ

2) HPQCD results for different flavours of the connected HVP, total result + error budget.

→ R. van de Water, tomorrow. More details on u/d method and tests of robustness

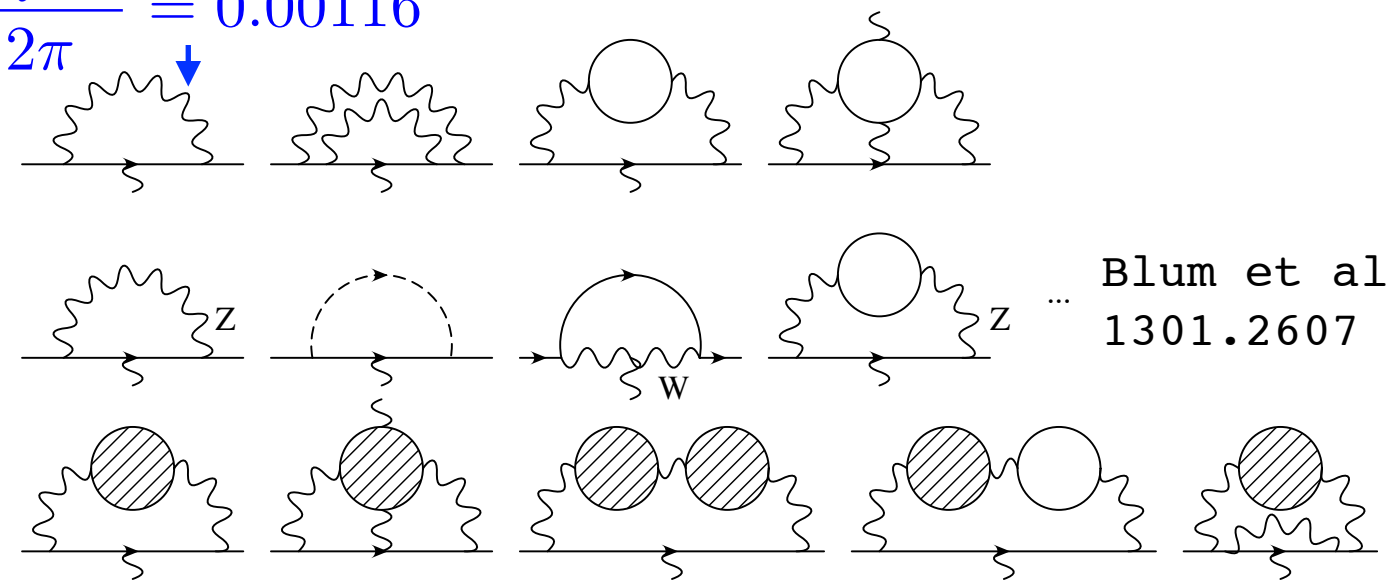
3) Future work with FNAL/MILC with some preliminary results on isospin-breaking and QED effects.

4) Conclusions

Standard Model theory expectations for muon anomalous magnetic moment $a_\mu = (g-2)/2$

Contributions from QED, EW and QCD interactions. QED dominates. QCD contribs start at α_{QED}^2

$\frac{\alpha_{QED}}{2\pi} = 0.00116$



Blum et al, 1301.2607

LO Hadronic vacuum polarisation (HVP) dominates uncertainty in SM result

$$a_\mu^{QED} = 11658471.885(4) \times 10^{-10}$$

$$a_\mu^{EW} = 15.4(2) \times 10^{-10}$$

$$a_\mu^{E821} = 11659208.9(6.3) \times 10^{-10}$$

Hadronic (and other) contributions = EXPT - QED - EW

$$a_\mu^{expt} - a_\mu^{QED} - a_\mu^{EW} = 721.7(6.3) \times 10^{-10}$$

$$= a_\mu^{HVP} + a_\mu^{HOHVP} + a_\mu^{HLBL} + a_\mu^{new\,physics}$$

Focus on lowest order hadronic vacuum polarisation, so assume:

$$a_\mu^{HLbL} = 10.5(2.6) \times 10^{-10}$$

$$a_\mu^{HOHVP} = -8.85(9) \times 10^{-10}$$

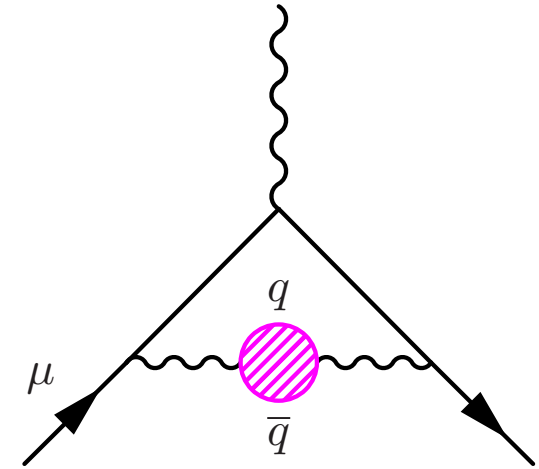
NLO+NNLO

Kurz et al, 1403.6400

$$a_\mu^{HVP,no\,new\,physics} = 719.8(6.8) \times 10^{-10}$$

Lattice calculation of HVP,LO

Analytically continue to Euclidean q^2.



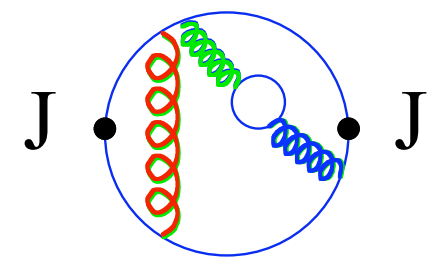
$$a_\mu^{HVP,i} = \frac{\alpha}{\pi}\int_0^\infty dq^2 f(q^2)(4\pi\alpha e_i^2)\hat{\Pi}_i(q^2)$$

connected contribution for flavour i

Blum, hep-lat/0212018

f(q^2) divergent function with scale set by m_μ

$\hat{\Pi}(q^2) = \Pi(q^2) - \Pi(0)$ vanishes at q^2=0



HPQCD method: time-moments of spatial vector JJ correlators give expansion around q^2=0

$$G_n \equiv \sum_{t,\vec{x}} t^n Z_V^2 \langle J^j(\vec{x},t) J^j(0)\rangle \qquad \hat{\Pi}(q^2) = \sum_{k=1}^{\infty} q^k \Pi_k$$

$n = 4, 6, 8, 10$

$$\Pi_k = (-1)^{k+1}\frac{G_{2k+2}}{(2k+2)!}$$

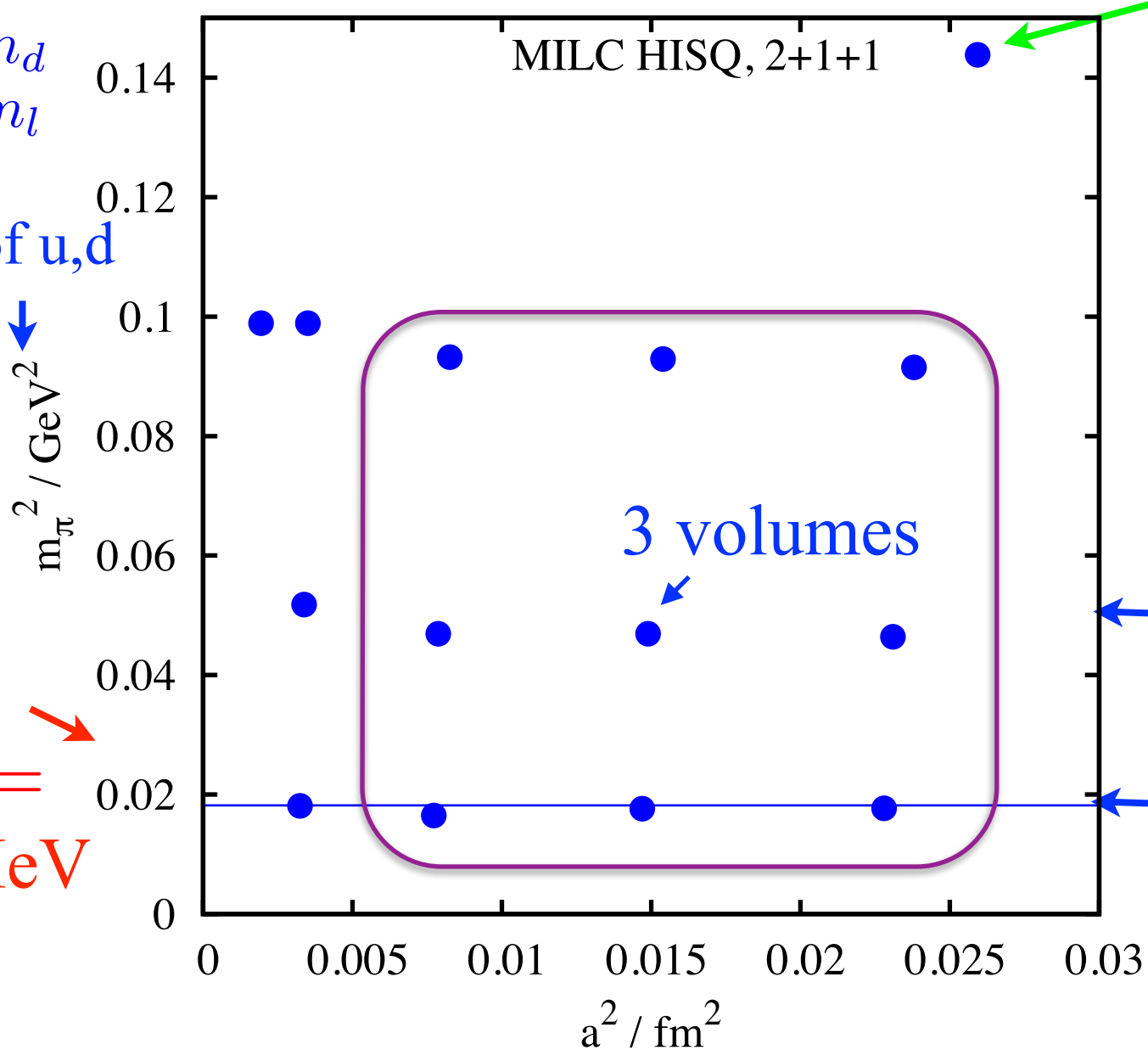
HPQCD, 1403.1778

replace with [2,2] Padé

Parameters for MILC gluon field configs on which quark propagators and hadron correlators are calculated

$m_u = m_d$
$= m_l$

mass of u,d quarks ↓



real world ↘ $m_{\pi^0} =$ 135 MeV

"2nd generation" lattices inc. c quarks in sea

HISQ = Highly improved staggered quarks - very accurate discretisation and numerically fast

E.Follana, et al, HPQCD, hep-lat/0610092.

$m_{u,d} \approx m_s/10$

$m_{u,d} \approx m_s/27$

Volume:

$m_\pi L > 3$

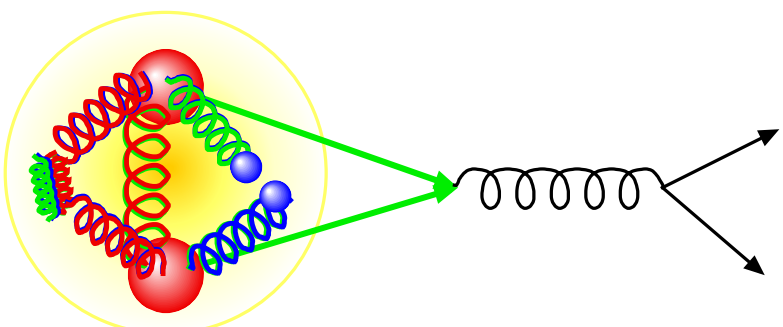
Hadron correlation functions ('2point functions') contain multiple states with decaying exponentials. Large times controlled by lightest masses.

$$\langle 0|H^\dagger(T)H(0)|0\rangle = \sum_n A_n e^{-m_n T} \xrightarrow[\text{large}]{T} A_0 e^{-m_0 T}$$

$\vec{p} = 0$

masses of all hadrons with quantum numbers of H

$$A_n = \frac{|\langle 0|H|n\rangle|^2}{2m_n} = \frac{f_n^2 m_n}{2}$$



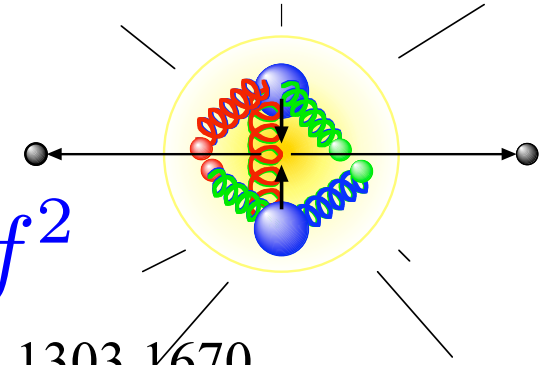
decay constant param. amplitude to annihilate. Relate to experimental decay rate ($\Gamma_{\ell^+\ell^-}$–for vector mesons) . Provides 'large time' test of correlators used

Need to fix QCD parameters: lattice spacing and quark masses

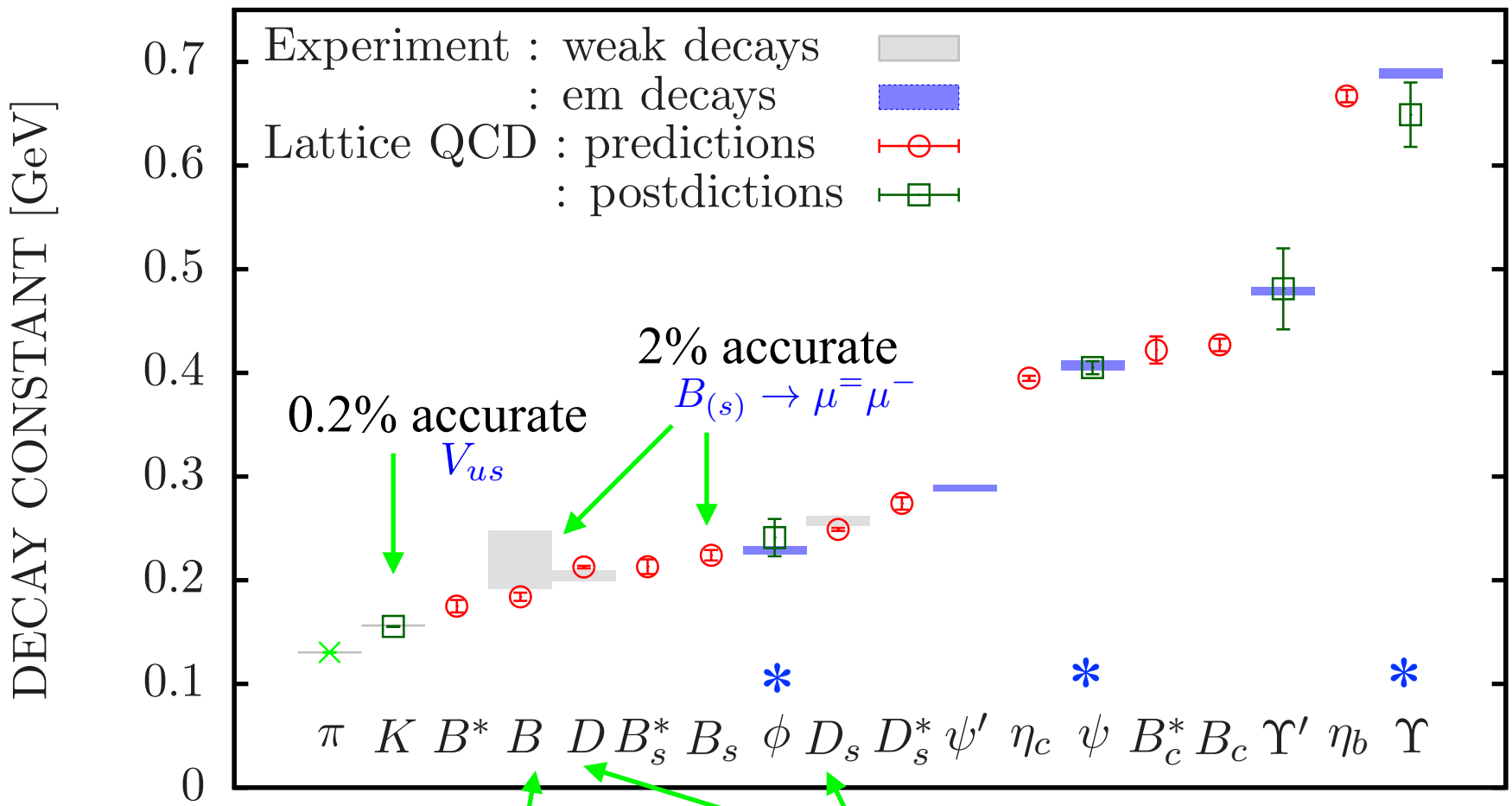
Meson decay constants

Parameterises hadronic information needed for annihilation rate to W or photon:

$$\Gamma \propto f^2$$



HPQCD 1503.05762, 1408.5768,1302.2644, 1303.1670



decay constants of vector mesons now being pinned down

2012

$B \to \tau\nu$

V_{ub}

0.5% accuracy from lattice QCD

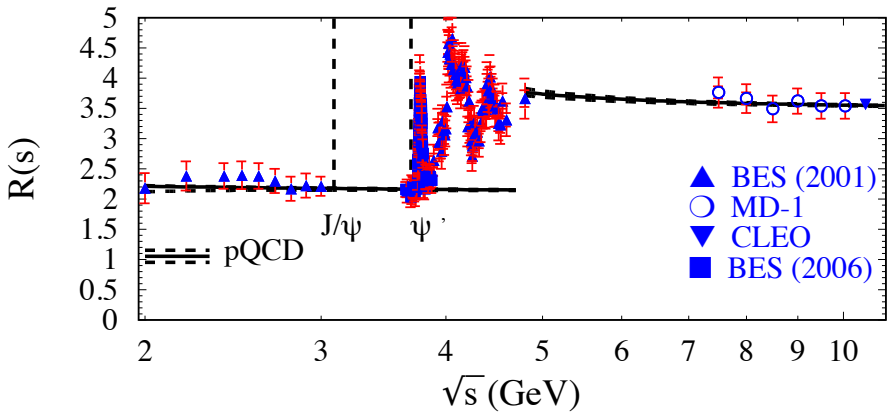
now : **FNAL/MILC** 1407.3772

BES will improve expt. V_{cd} V_{cs}

Testing correlator time-moments

Donald et al, HPQCD, 1208.2855

Charm contribution to $R = \sigma(e^+e^- \to \text{hadrons})/\sigma_{\text{pt}}$



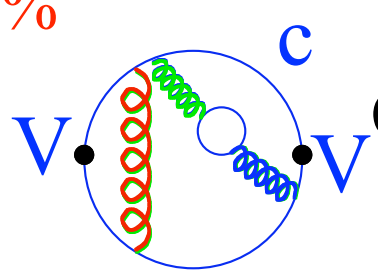
Subtract u,d,s with pert. th. to isolate charm contribution.

Calculate inverse-s-moment:

$$\mathcal{M}_n \equiv \int \frac{ds}{s^{n+1}} R_c(s)$$

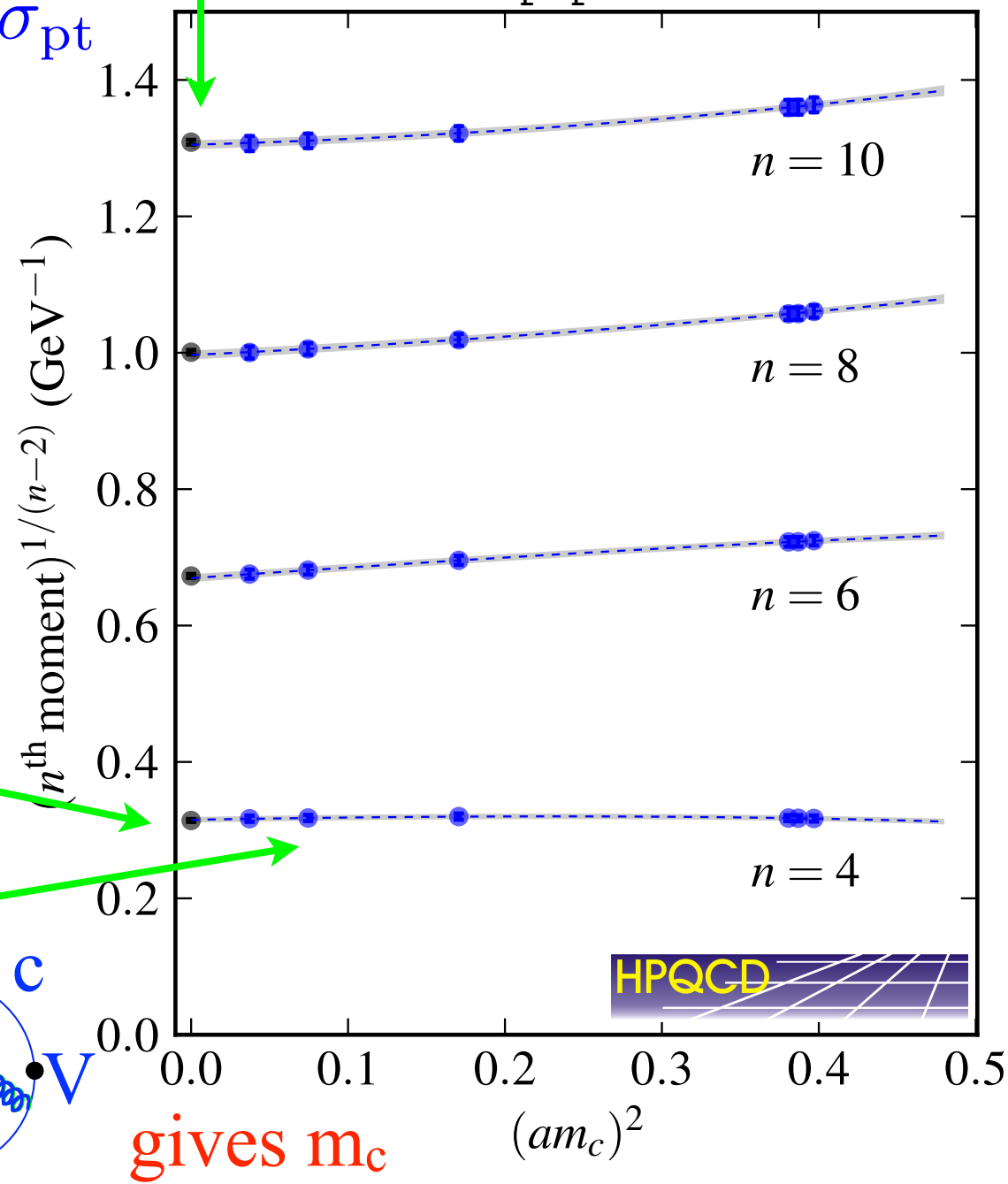
Lattice calcln:

$$M_n = \sum_t t^n G(t)$$



Agree to 1.5%

'expt' ← from J. Kuhn et al, hep-ph/0702103

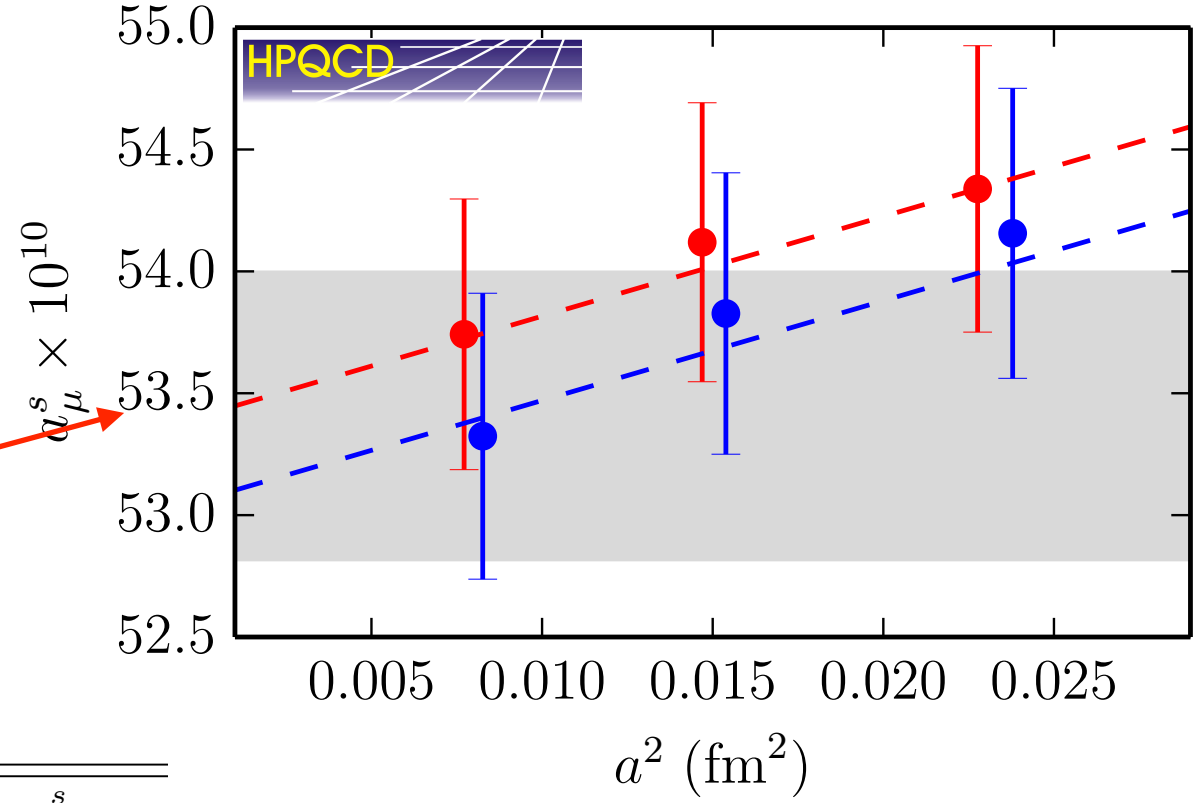


gives m_c

Use time-moments for first calc. of connected s quark contribution to a_μ

Chakraborty et al, HPQCD 1403.1778

HISQ valence quarks on MILC 2+1+1 HISQ configs, 9 ensembles. Local J_v - nonpert. Z_v. multiple a (fixed by w_0), m_l (inc. phys.), volumes. Tune s from η_s allowing for missing QED



	a_μ^s
Uncertainty in lattice spacing (w_0, r_1):	0.4%
Uncertainty in Z_V:	0.4%
Monte Carlo statistics:	0.1%
$a^2 \to 0$ extrapolation:	0.1%
QED corrections:	0.1%
Quark mass tuning:	0.4%
Finite lattice volume:	$< 0.1\%$
Padé approximants:	$< 0.1\%$
Total:	0.7%

update

$$a_\mu^{HVP,s} = 53.4(4) \times 10^{-10}$$

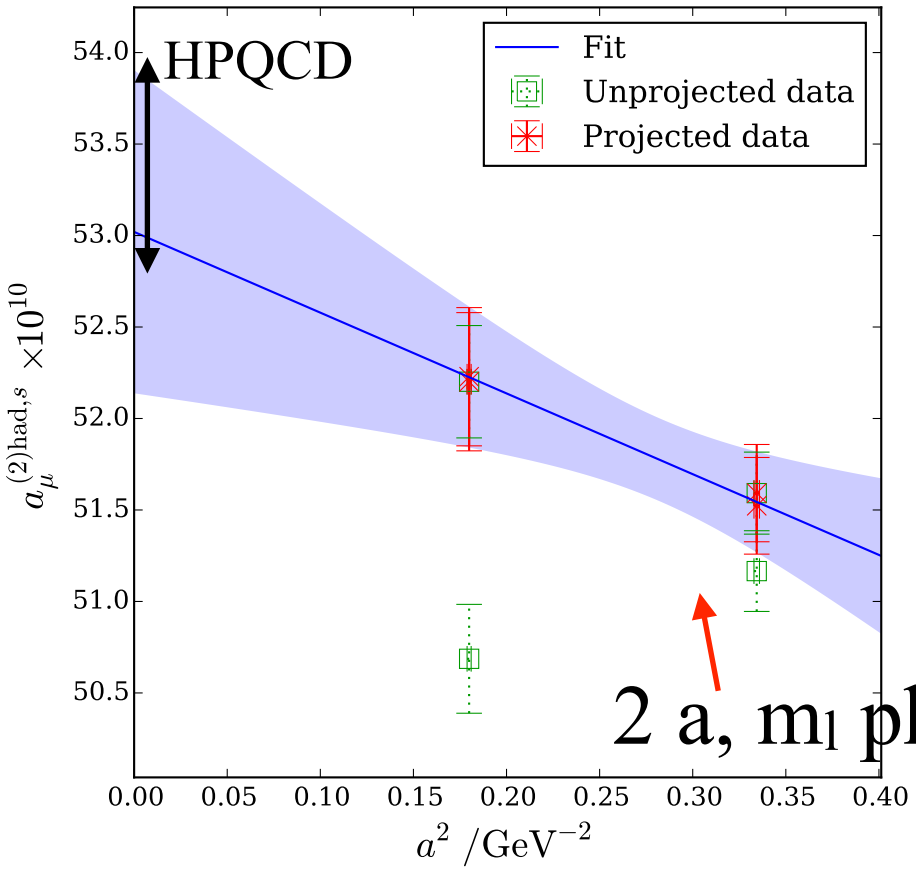
Also

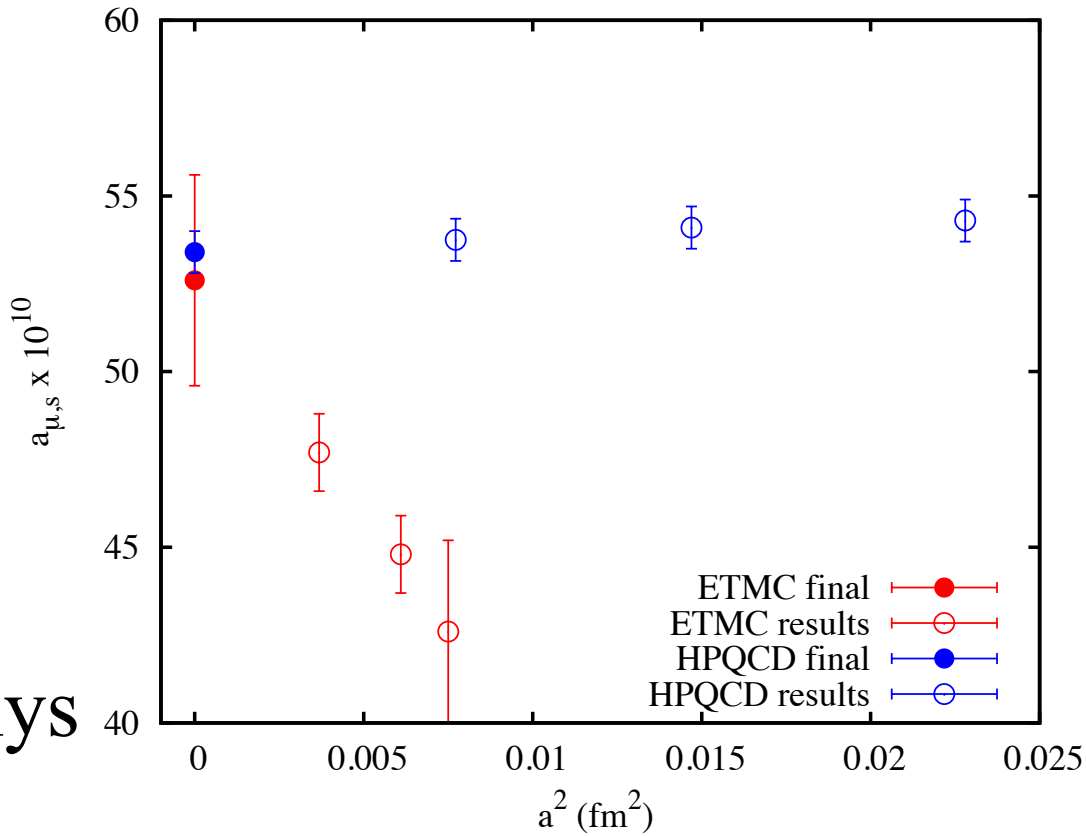
$$a_\mu^{HVP,c} = 14.4(4) \times 10^{-10}$$

$$a_\mu^{HVP,b} = 0.27(4) \times 10^{-10}$$

HPQCD 1408.5768 NRQCD

Later results from other formalisms provide good check





RBC/UKQCD domain wall

$$a_\mu^{HVP,s} = 53.1(9) \times 10^{-10}$$

1607.01767

ETMC twisted mass

$$a_\mu^{HVP,s} = 53.6(1.9) \times 10^{-10}$$

$$a_\mu^{HVP,c} = 14.2(6) \times 10^{-10}$$

1411.0705

From $R_{e^+e^-}$ we estimate $a_\mu^{HVP,s} <\approx 55 \times 10^{-10}$

UP/DOWN contribution $m_u = m_d$

Chakraborty et al, HPQCD 1601.03071

Much noisier and sensitive to u/d mass. Use ~16,000 2x2 matrix of correlators per ensemble (10 ensembles).

$$G(t) = \begin{cases} G_{\text{data}}(t) & \text{for } t \le t^* \\ G_{\text{fit}}(t) & \text{for } t > t^* \end{cases}$$

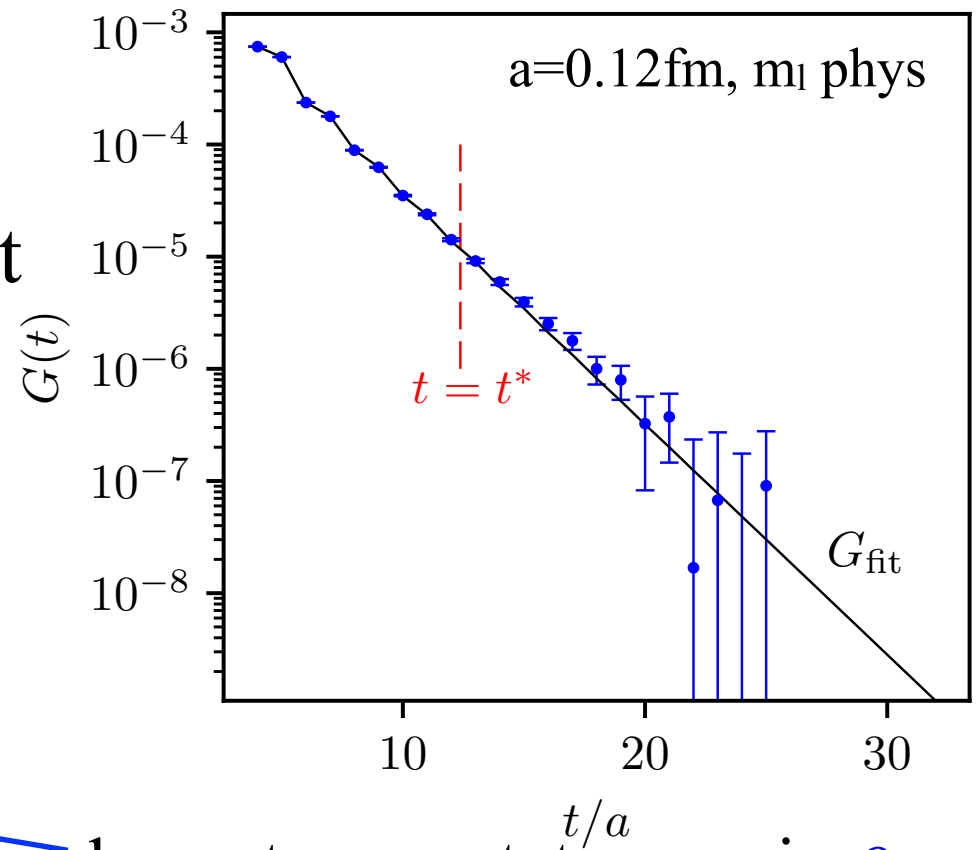
← from Monte Carlo

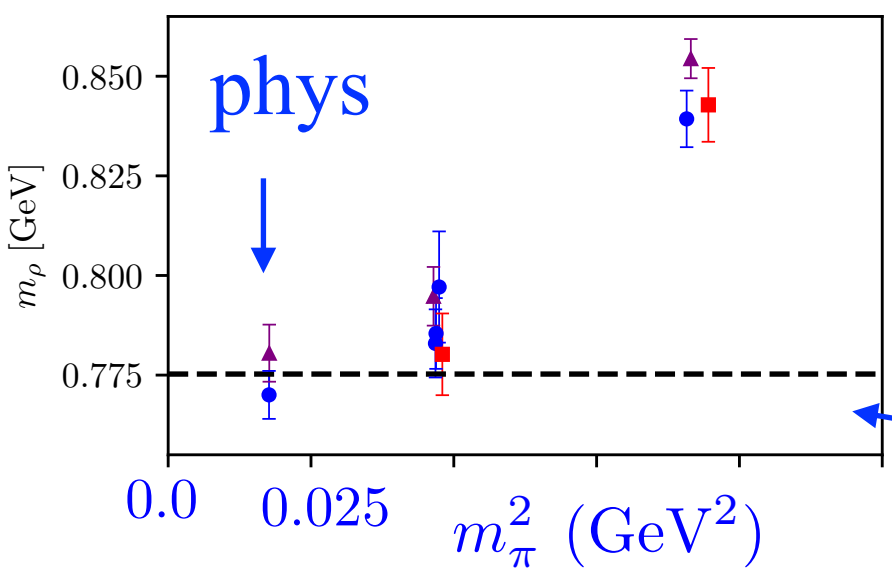
← from multi-exponential fit

$t^* = 1.5\text{fm} = 6/m_\rho$

so 80% of result from G_{data}

smeared sources improve fit accuracy



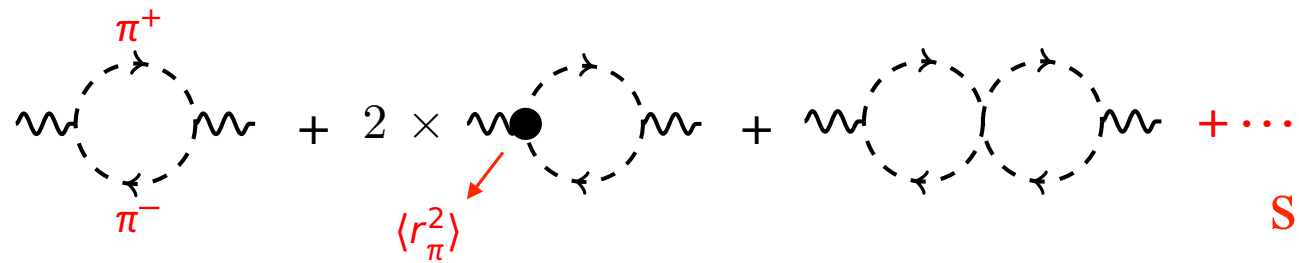


lowest mass state seen is ρ

test for lighter $\pi\pi$ by changing t*

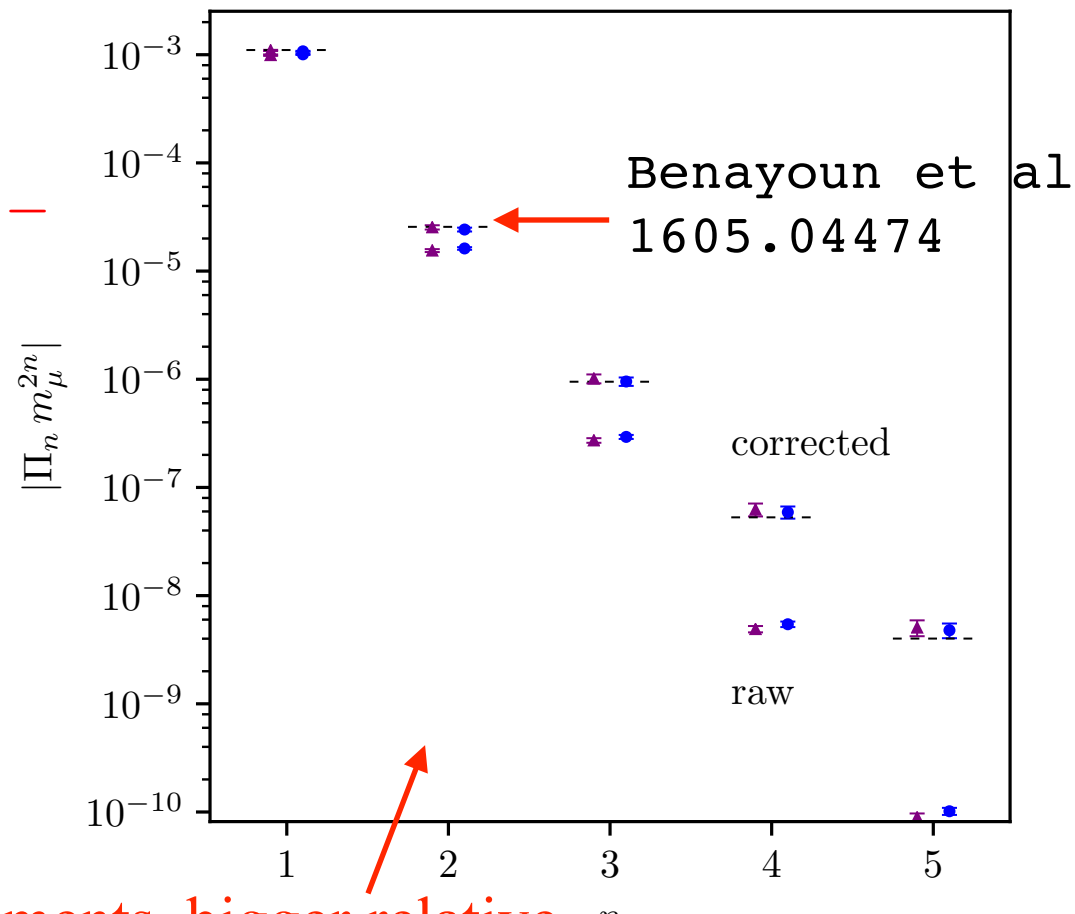
Must correct results for finite vol. + staggered pion effects using chiral perturbation theory (7% at physical point)

dominates

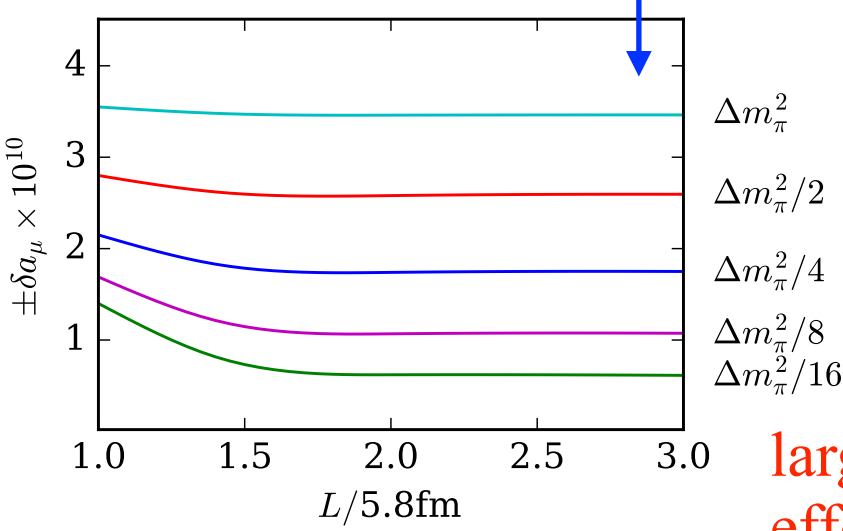


$\pi\pi$ scattering

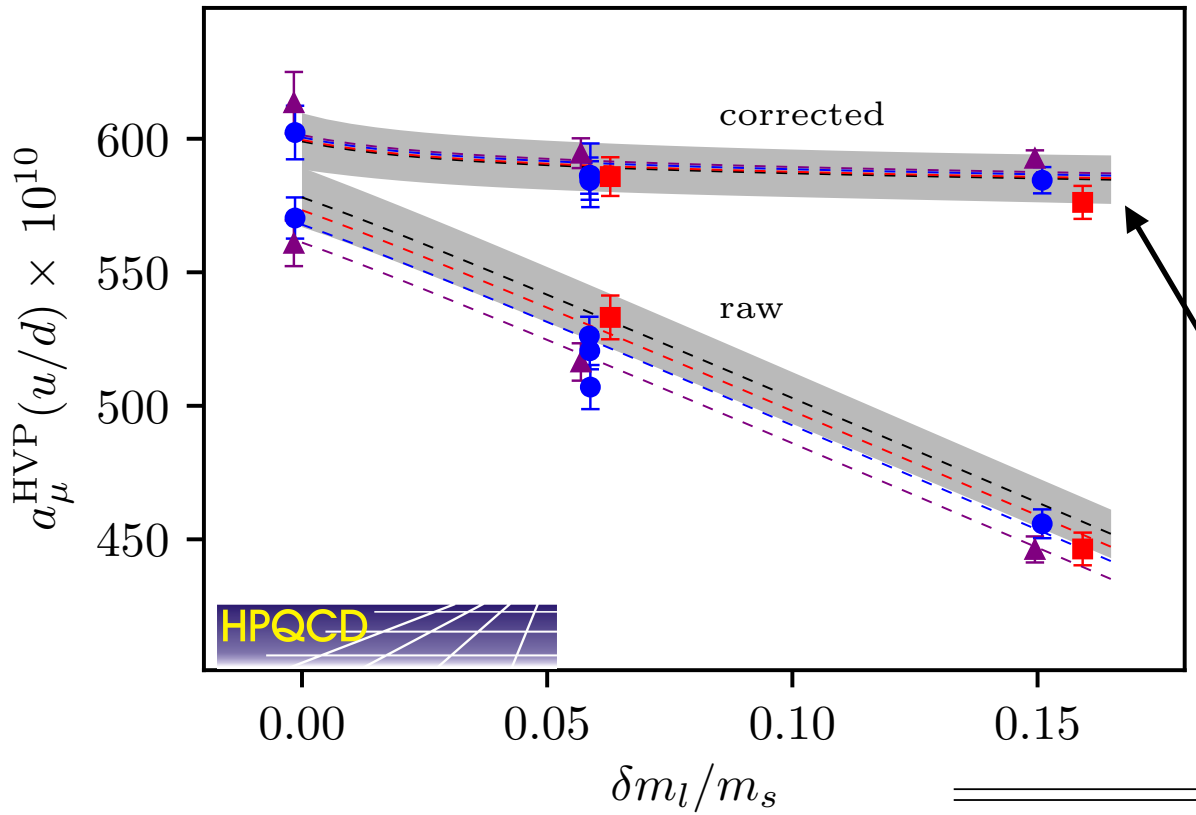
Check these, after the fact, against moments from $R_{e^+e^-}$



Need finer lattices to see finite volume effects in u/d



larger moments, bigger relative effect, smaller contribution



Rescale Π_j by $(m_\rho^{latt}/m_\rho^{expt})^{2j}$

much reduced dependence on m_l, a^2, volume - simple to fit.

Numerous tests of robustness …

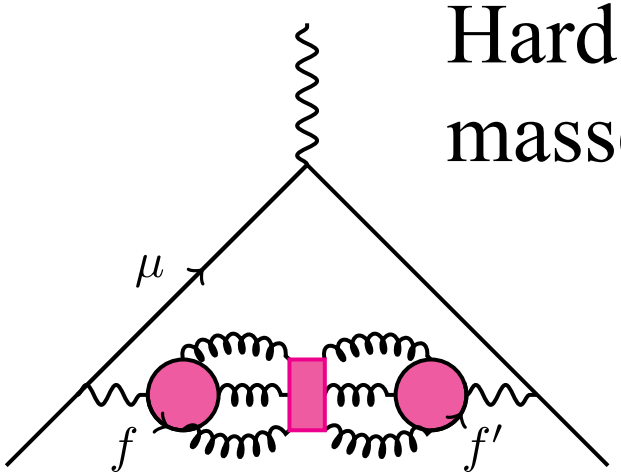
Error budget dominated by QED, isospin-breaking systematics, 1% each

	$a_\mu^{\rm HVP,LO}(u/d)$
QED corrections:	1.0 %
Isospin breaking corrections:	1.0 %
Staggered pions, finite volume:	0.7 %
Correlator fits (t^*):	0.5 %
m_ℓ extrapolation:	0.4 %
Monte Carlo statistics:	0.4 %
Padé approximants:	0.4 %
$a^2 \to 0$ extrapolation:	0.2 %
Z_V uncertainty:	0.2 %
Correlator fits:	0.2 %
Tuning sea-quark masses:	0.2 %
Lattice spacing uncertainty:	< 0.05 %
Total:	1.8 %

`Chakraborty et al, HPQCD 1703.05522 improves these`

Quark-line disconnected contribution

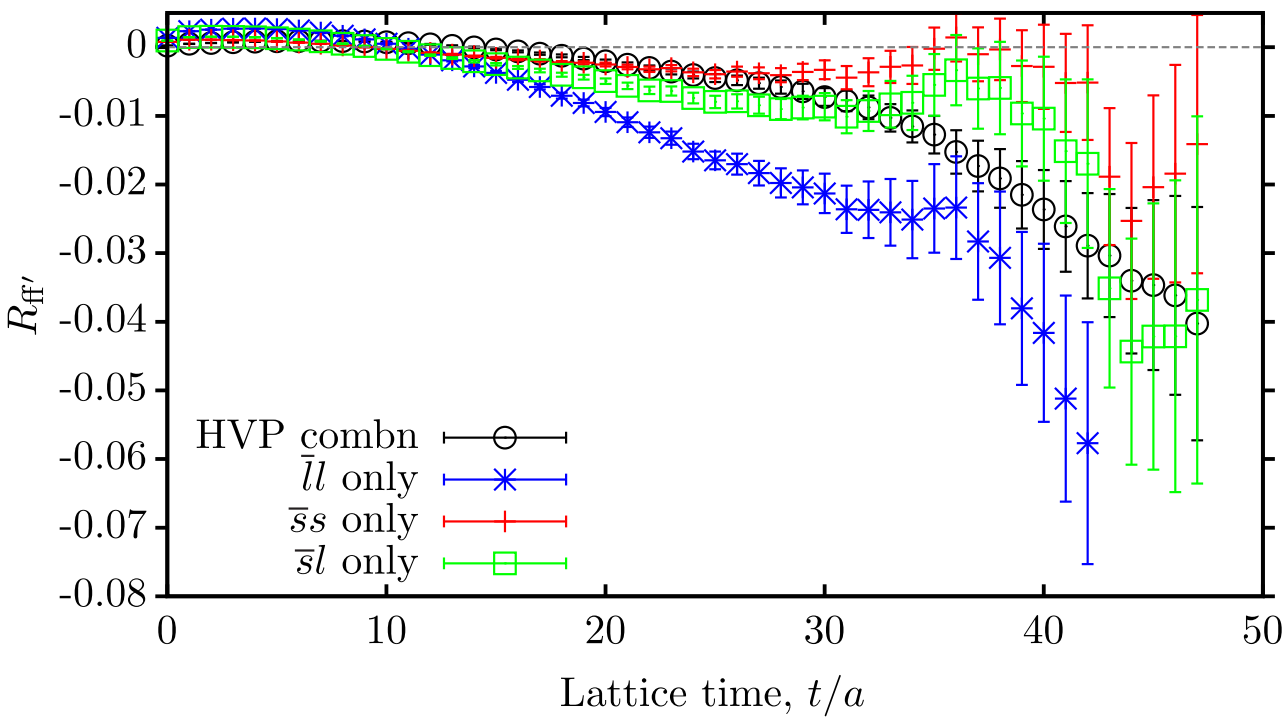
HPQCD/Hadspec 1512.03270



Hard to calculate but small. Suppressed by masses since $\sum_{u,d,s} Q_f = 0$

For u/d can estimate from ω/ρ

$$\frac{(\frac{1}{9}\Pi_j)_{disc}}{(\frac{5}{9}\Pi_j)_{conn}} = \frac{1}{10}\left[\frac{f_\omega^2/m_\omega^{2j+2}}{f_\rho^2/m_\rho^{2j+2}} - 1\right]$$



gives -1(1)%

$\pi\pi$ only, gives -1/10

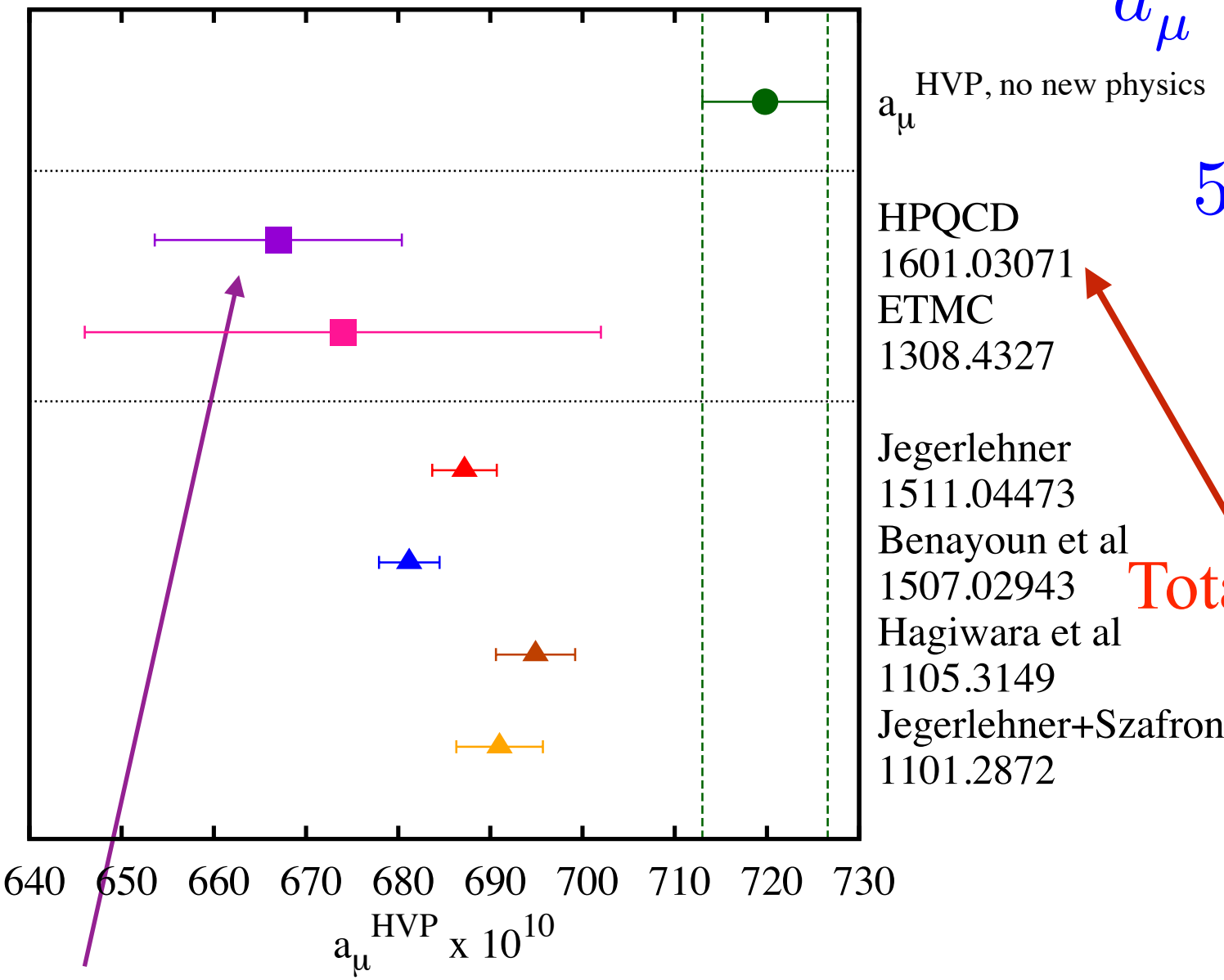
Estimate total (after fitting Hadspec data):

$$a_\mu^{HVP,disc} = 0(9) \times 10^{-10}$$

see also RBC/UKQCD 1512.09054:-9.6(4.0) x 10^{-10}

Combining numbers for a total

$$a_\mu^{\mathrm{HVP,LO}} \times 10^{-10}$$



$599(11)\ u/d$

$53.4(6)\ s$

$14.4(4)\ c$

$0.27(4)\ b$

Total $667(6)(12)$

add syst from disc. diags (1.5%) in quad with 1% QED, 1% isospin

3.5σ discrepancy with no new physics

Ongoing work planned with FNAL/MILC :

- Finer and higher stats physical point connected u/d correlators

- isospin-breaking and QED effects, both in valence sector (see below for preliminary results) + in sea.

- disconnected correlators using eigenvector deflation

Isospin-breaking - connected u/d

D. Hatton et al

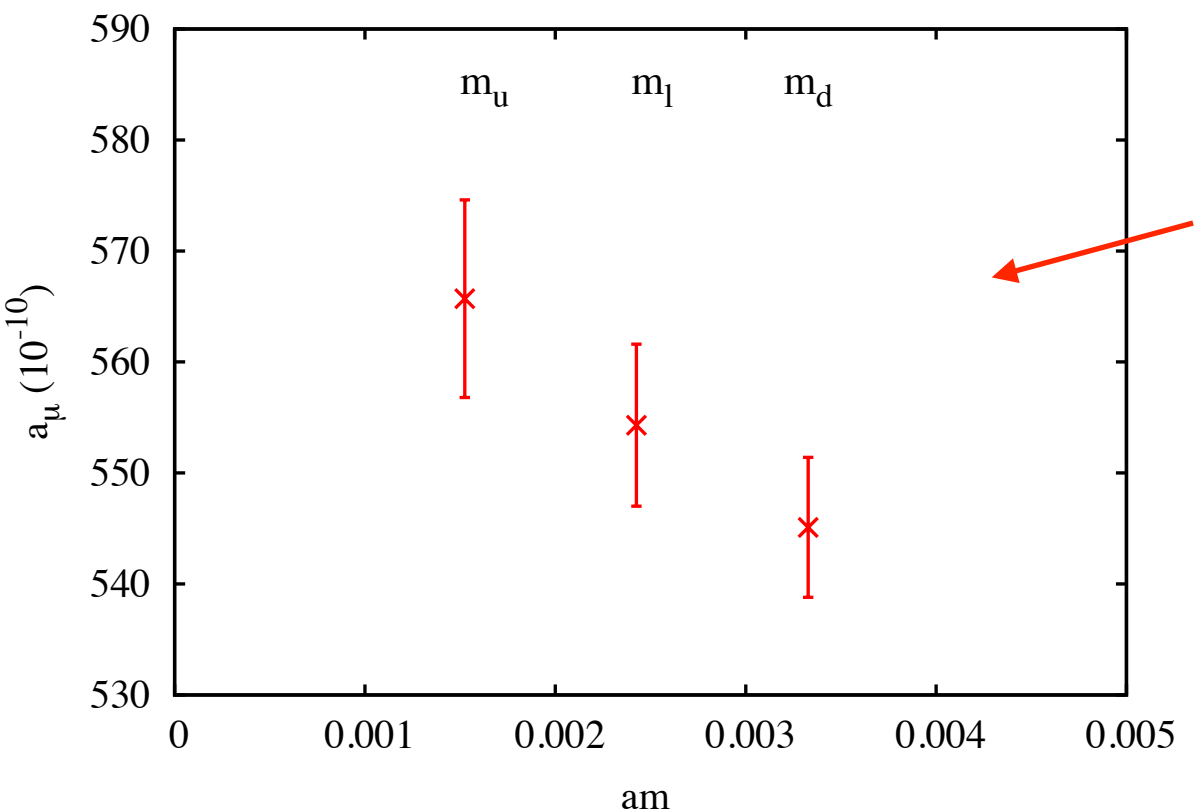
- a=0.15fm, physical m_l (tuned to m_{π^0}) new higher stats (2000x16 sources so far) ensemble.

RBC's truncated solver method reduces cost by factor 2.

3 quark masses: m_l, m_u, m_d with Fit and process simultaneously

$$\frac{m_u}{m_d} = 0.458$$

MILC 1606.01228



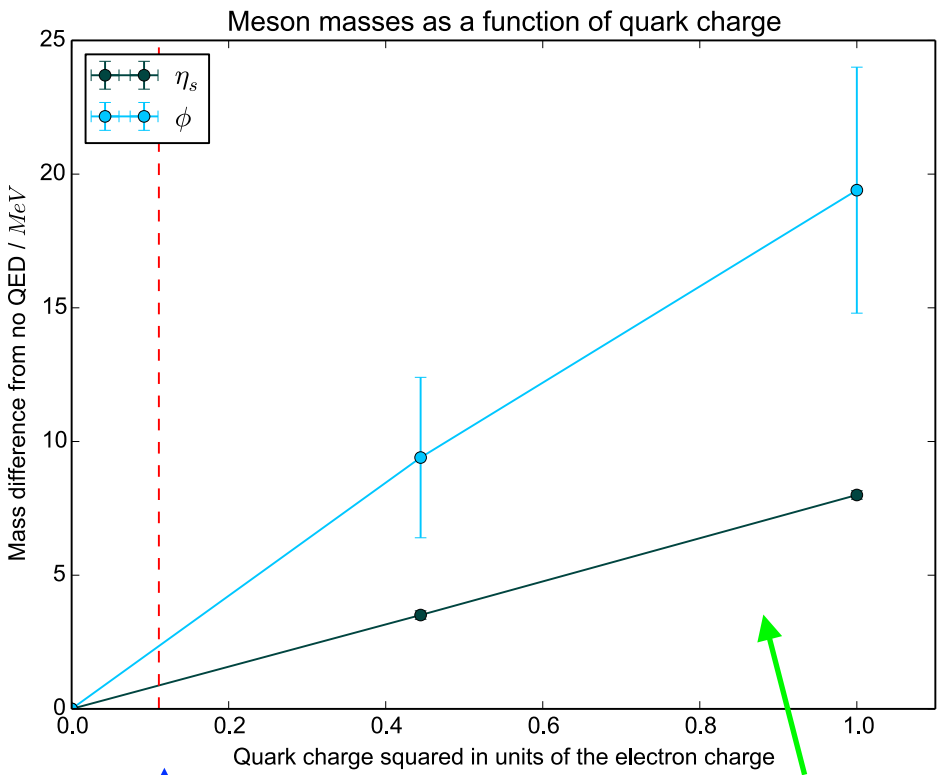
uncertainties are correlated, Q_f fixed - shows mass effects only

Result: +1.0(3)% effect without ρ rescaling.
0.3(3)% effect with ρ rescaling.

QED effects

D. Hatton, A. Lytle et al

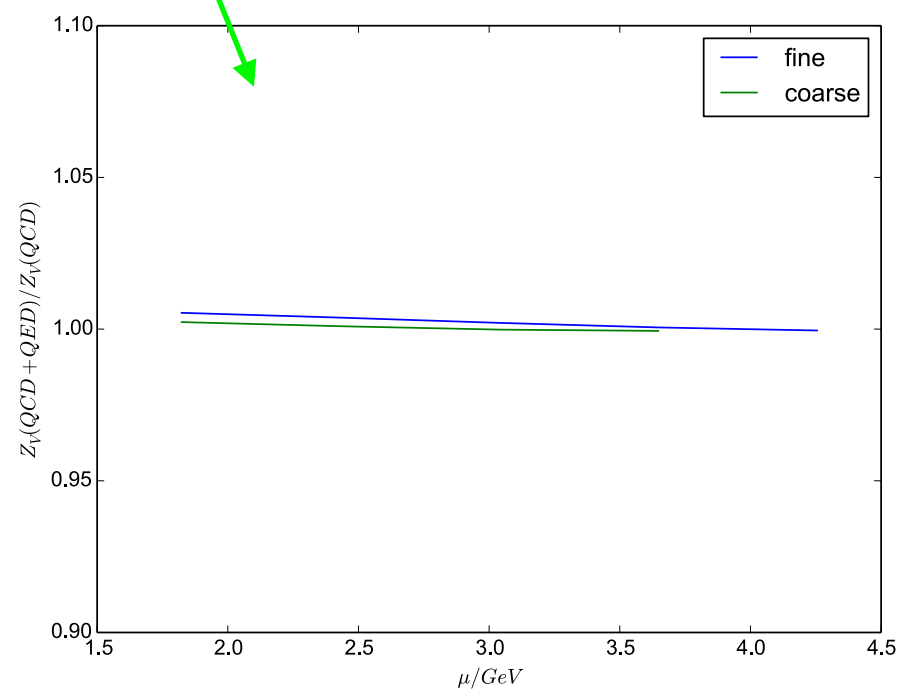
s quark case on a=0.12 fm lattices. Quenched QED using BMW's QED_{TL} scheme. Use unphysical e = 1 and 2/3 to amplify effect.



$$e = \frac{1}{3}$$

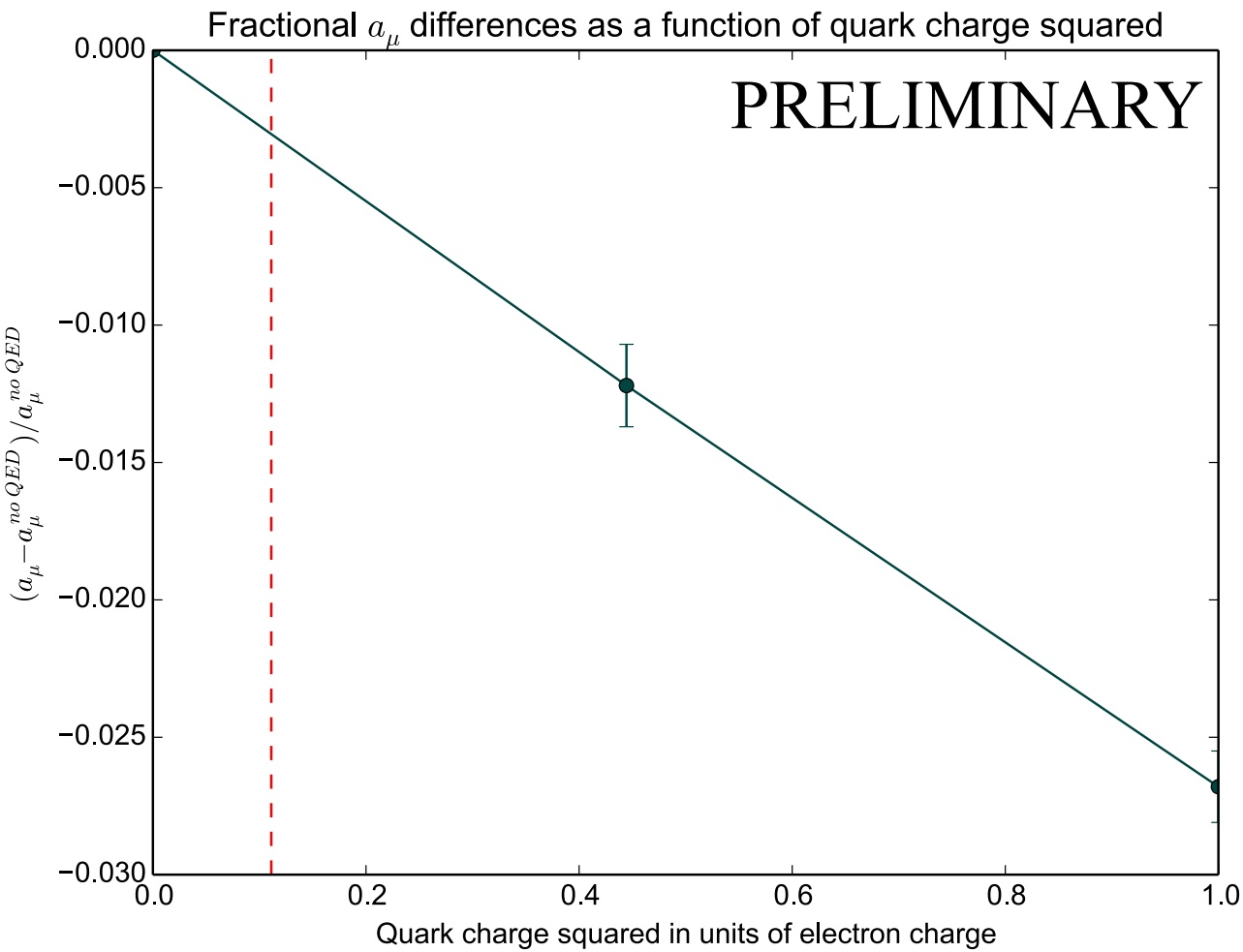
meson masses increase, vector more than pseudoscalar.

Inc. QED effects in Z_V using RI-SMOM scheme - tiny



Preliminary result : effect on $a_\mu^{HVP,s}$ is -0.3%

Need to check s-mass tuning, systs in Z_V, compare other lattice spacing, volume …



Conclusion

- HPQCD has pinned down 'connected' s, c and b connected contribution to the HVP, up to small QED corrections (in progress).

- HPQCD u/d 'connected' result yields a total LO HVP of 667(6)(12) x 10^{-10}, 3σ from 'no new physics' scenario.

- Dominated by systematics from u/d case: finite vol/ staggered pion corrections plus QED and isospin-breaking (in progress). Planned future work with FNAL/ MILC to improve.

- sub-1% uncertainties on lattice QCD calculations for HVP contribution to a_μ are within sight.