Hadronic vacuum polarization from lattice QCD: RBC/UKQCD

Christoph Lehner (BNL)

June 3, 2017 – Muon $g-2$ Theory Initiative

Collaborators

RBC/UKQCD $g-2$ effort

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James Harrison (Southampton)
Taku Izubuchi (BNL/RBRC)
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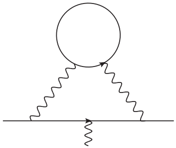
Luchang Jin (Columbia)
Antonin Portelli (Edinburgh)
Matt Spraggs (Southampton)

Theory status – summary

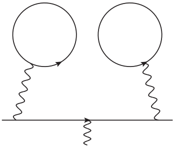
Contribution	Value $\times 10^{10}$	Uncertainty $\times 10^{10}$
QED (5 loops)	11 658 471.895	0.008
EW	15.4	0.1
HVP LO	692.3	**4.2**
HVP NLO	-9.84	0.06
HVP NNLO	1.24	0.01
Hadronic light-by-light	10.5	**2.6**
Total SM prediction	11 659 181.5	4.9
BNL E821 result	11 659 209.1	6.3
FNAL E989/J-PARC E34 goal		$\approx$ **1.6**

A reduction of uncertainty for HVP and HLbL is needed. A systematically improvable first-principles calculation is desired.

First-principles approach to HVP LO

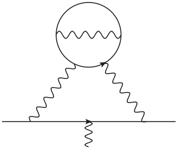


Quark-connected piece with by far dominant part from up and down quark loops, $\mathcal{O}(700 \times 10^{-10})$

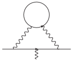


Quark-disconnected piece, $-9.6(4.0) \times 10^{-10}$

Phys.Rev.Lett. 116 (2016) 232002



QED corrections, $\mathcal{O}(10 \times 10^{-10})$

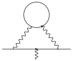


HVP quark-connected contribution

Biggest challenge to direct calculation at physical pion masses is to control statistics and potentially large finite-volume errors.

Statistics: for strange and charm solved issue, for up and down quarks existing methodology less effective

Finite-volume errors are exponentially suppressed in the simulation volume but may be sizeable



HVP quark-connected contribution

Starting from the vector current

$$J_\mu(x) = i \sum_f Q_f \overline{\Psi}_f(x) \gamma_\mu \Psi_f(x)$$

we may write

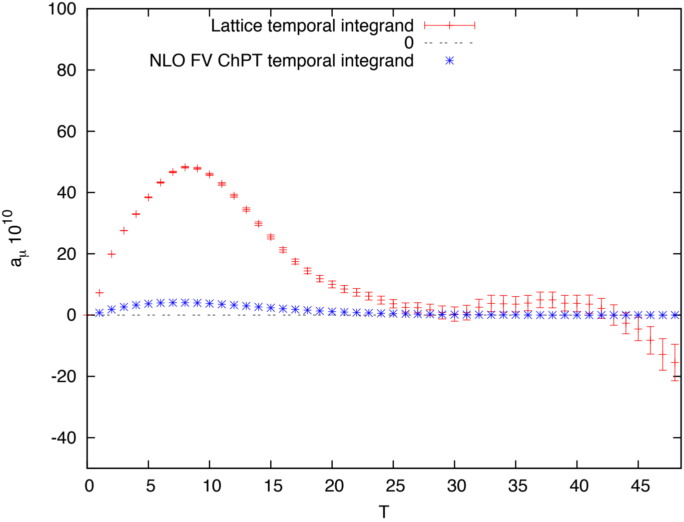
$$a_\mu^{\mathrm{HVP}} = \sum_{t=0}^{\infty} w_t C(t)$$

with

$$C(t) = \frac{1}{3} \sum_{\vec{x}} \sum_{j=0,1,2} \langle J_j(\vec{x}, t) J_j(0) \rangle$$

and w_t capturing the photon and muon part of the diagram (Bernecker-Meyer 2011).

Integrand $w_T C(T)$ for the light-quark connected contribution:



$m_\pi = 140$ MeV, $a = 0.11$ fm (RBC/UKQCD 48^3 ensemble)

Statistical noise from long-distance region

Addressing the long-distance noise problem

There are two general classes of solutions to the long-distance noise problem

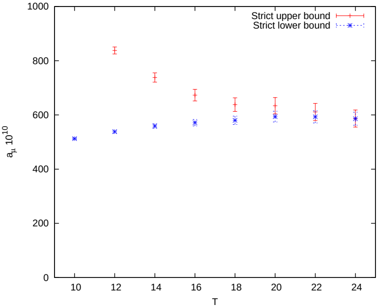
- Statistics $\rightarrow$ Systematics: One can reduce statistical uncertainty at the cost of introducing an additional systematic uncertainty that then needs to be controlled; This requires additional care in estimating a potential systematic bias but may be overall beneficial.
- Statistics $\uparrow$: One can devise improved statistical estimators without additional systematic uncertainties

Concrete recent proposals:

- Replace $C(t)$ for large t with model, say multi-exponentials for $t \geq t^*$ HPQCD arXiv:1601.03071 (Statistics $\rightarrow$ Systematics)

- Define stochastic estimator for strict upper and lower bounds of a_μ which have reduced statistical fluctuations RBC/UKQCD 2015, BMWc arXiv:1612.02364 (Statistics $\uparrow$)

More details, e.g., talk C.L. at Rutgers 2015



Bound $C_l(t) \leq C(t) \leq C_u(t)$ with

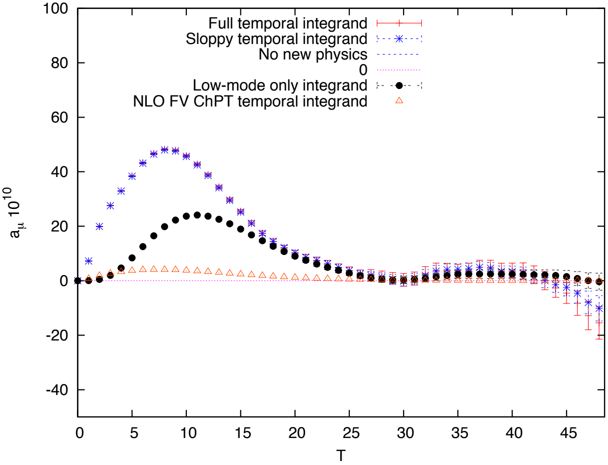
$$C_{l/u}(t) = \begin{cases} C(t) & t < T\,, \\ C(T)e^{-(t-T)\bar{E}_{l/u}} & t \geq T \end{cases}$$

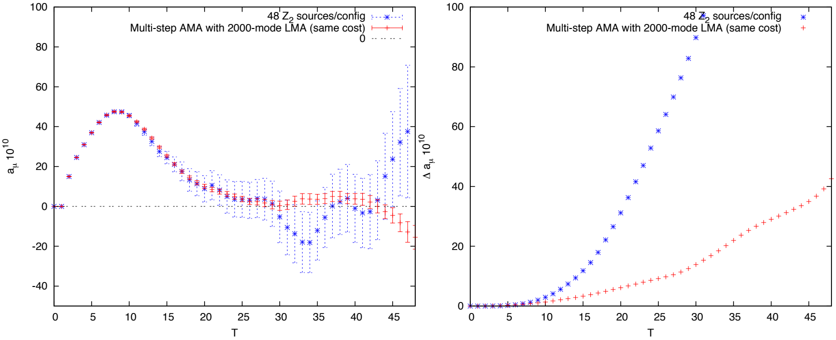
with $\bar{E}_u$ being the ground state of the VV correlator and

$$\bar{E}_l = \log(C(T)/C(T+1))\,.$$

Concrete recent proposals (continued):

- RBC/UKQCD 2015 Improved stochastic estimator; hierarchical approximations including exact treatment of low-mode space DeGrand & Schäfer 2004: (Statistics ↑):





Calculation of deflation space originally required a significant amount of disk space (36TB for a single $64^3 \times 128$ configuration).

New method: Multi-Grid Lanczos utilizing local coherence of eigenvectors yields $10\times$ reduction in memory (Poster by C.L. at Lattice 2017)

Concrete recent proposals (continued):

- Phase reweighting (Savage et al.) (Statistics $\to$ Systematics)

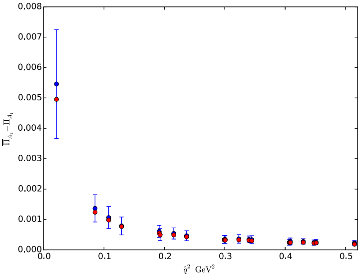
$$C(t) \to C(t)\,\mathrm{Sign}[C(t-\Delta)]$$

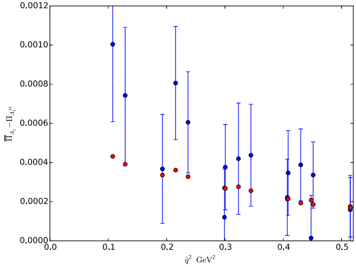
  extrapolate to $\Delta \to \infty$

- Multi-level gauge field generation (Ce/Giusti/Schafer) (Statistics $\uparrow$)
  - Action is local $\Rightarrow$ independent evolution of gauge fields in sub-domains possible
  - Recombination of independent samples over all subdomains may lead to exponential reduction of noise
  - We are currently investigating this method for the HVP (M. Bruno for RBC/UKQCD)

Addressing the finite-volume problem

From Aubin et al. 2015 (arXiv:1512.07555v2)



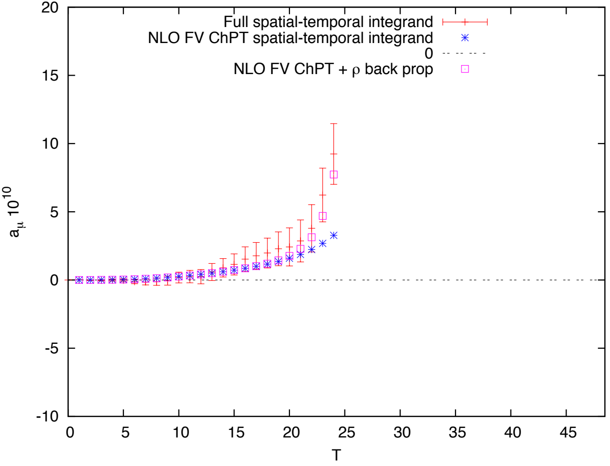


MILC lattice data with $m_\pi L = 4.2$, $m_\pi \approx 220$ MeV; Plot difference of $\Pi(q^2)$ from different irreps of 90-degree rotation symmetry of spatial components versus NLO FV ChPT prediction (red dots)

While the absolute value of a_μ is poorly described by the two-pion contribution, the volume dependence may be described sufficiently well to use ChPT to control FV errors at the 1% level; this needs further scrutiny

Aubin et al. find an $O(10\%)$ finite-volume error for $m_\pi L = 4.2$ based on the $A_1 - A_1^{44}$ difference (right-hand plot)

Compare difference of integrand of $48 \times 48 \times 96 \times 48$ (spatial) and $48 \times 48 \times 48 \times 96$ (temporal) geometries with NLO FV ChPT $(A_1 - A_1^{44})$:



$m_\pi = 140$ MeV, $p^2 = m_\pi^2/(4\pi f_\pi)^2 \approx 0.7\%$

Our efforts to control the finite-volume error:

- We have generated three additional lattices with physical pion mass and $L = 4.8$fm, 6.4fm, and 9.6fm; we have started first measurements on these lattices.

- We are currently tuning our new Multi-Grid Lanczos method on the largest volumes to continue to use our noise-reduction techniques for these studies. For these ensembles the improved Multi-Grid Lanczos is critical.

Complete first-principles analysis

- Currently the statistical uncertainty for a pure first-principles analysis in the continuum limit is at the $\Delta a_\mu \approx 15 \times 10^{-10}$ level (bounding method)

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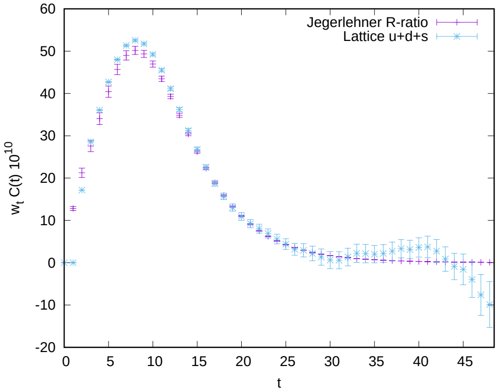
Fermilab E989 target		$\approx$ **1.6**

- Sub-percent statistical error achievable with a few more months of running
- While we are waiting for more statistics ...

Combined lattice and dispersive analysis

We can use the dispersion relation to overlay experimental e^+e^- scattering data (Bernecker, Meyer 2011). Below the experimental result is taken from Jegerlehner 2016:



The lattice data here includes finite-volume corrections based on NLO FV ChPT. Will study different individual datasets: BaBar, KLOE.

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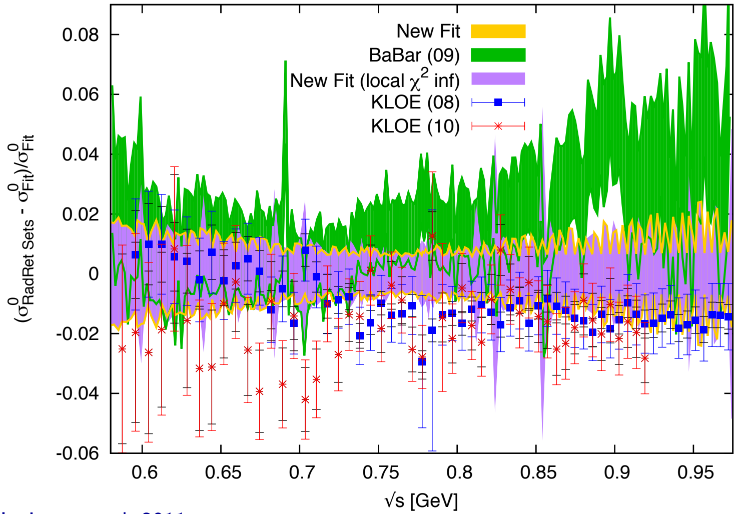

Hagiwara et al. 2011:

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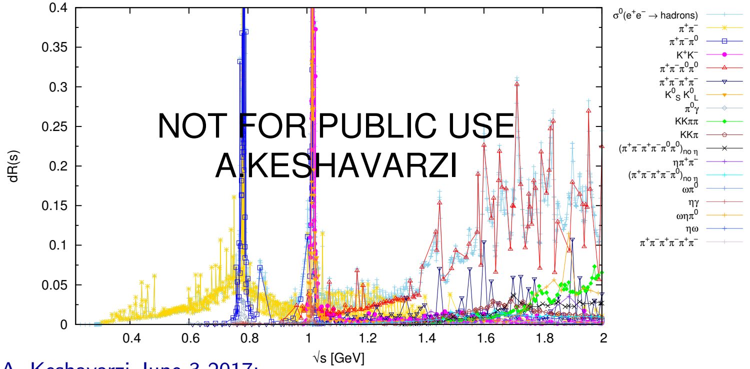
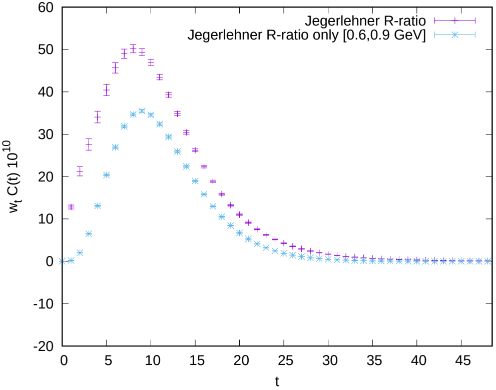


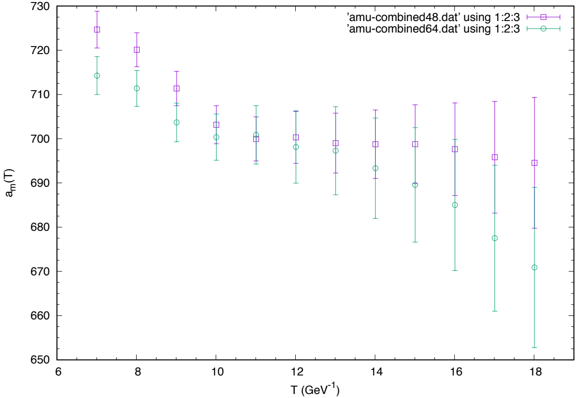
A. Keshavarzi June 3 2017:

The lattice data here includes finite-volume corrections based on NLO FV ChPT. Will study different individual datasets: BaBar, KLOE.



Problematic experimental region can readily be replaced by precise lattice data. Lattice also can be arbiter regarding different experimental data sets.

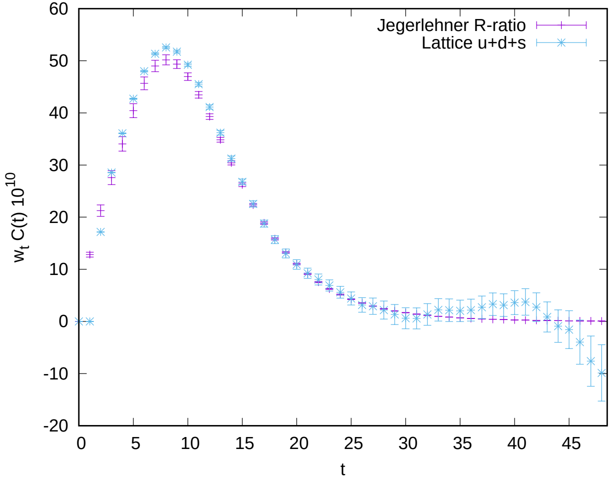
The lattice data is precise at shorter distances and the experimental data is precise at longer distances. We can do a combined analysis with lattice and experimental data:
$a_\mu = \sum_{t=0}^{T} w_t C^{\mathrm{lattice}}(t) + \sum_{t=T+1}^{\infty} w_t C^{\mathrm{exp}}(t)$

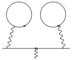


Errors range from ~ 0.5 to 1.2 % for T $\lesssim 12$ (GeV^{-1})

This is a promising way to reduce the overall uncertainty on a short time-scale.

The R-ratio data can also help to control long-distance modeling





HVP quark-disconnected contribution

First results at physical pion mass with a statistical signal
Phys.Rev.Lett. 116 (2016) 232002

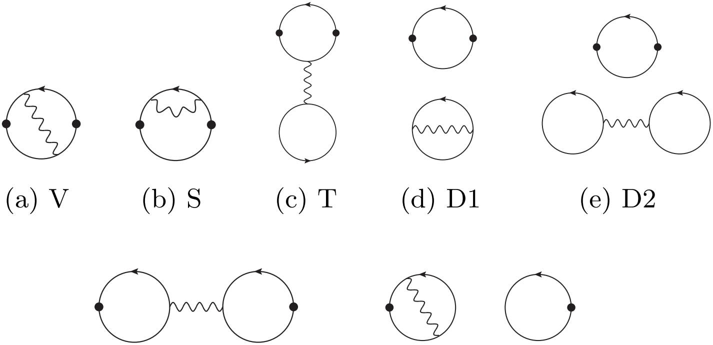
Statistics is clearly the bottleneck; calculation was a potential road-block of a first-principles calculation for a long time; due to very large pion-mass dependence calculation at physical pion mass is crucial.

New stochastic estimator allowed us to get result

$$a_\mu^{\mathrm{HVP\ (LO)\ DISC}} = -9.6(3.3)_{\mathrm{stat}}(2.3)_{\mathrm{sys}} \times 10^{-10}$$

from a modest computational investment ($\approx$ 1M core hours).

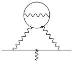
HVP QED contribution



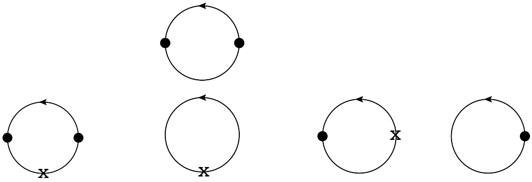
(a) V (b) S (c) T (d) D1 (e) D2

(f) F (g) D3

New method: use importance sampling in position space and local vector currents

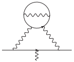


HVP strong IB contribution



(a) M (b) R (c) O

Calculate strong IB effects via insertions of mass corrections in an expansion around isospin symmetric point



HVP QED+strong IB contributions

Strategy

1. Re-tune parameters for QCD+QED simulation
   (m_u, m_d, m_s, a)

2. Verify simple observables ($m_{\pi^+} - m_{\pi^0}$, ...)

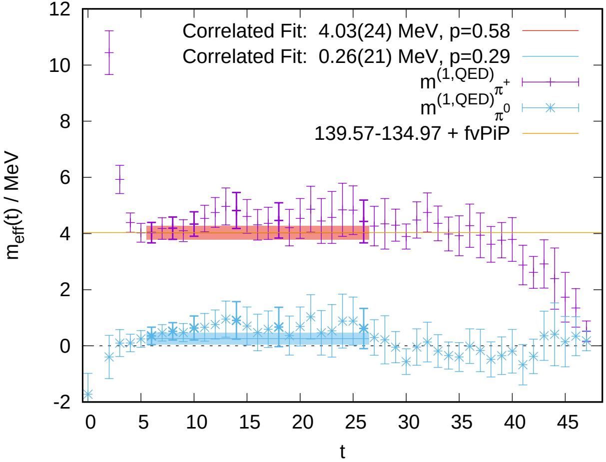
3. Calculate QED and strong IB corrections to HVP LO

All results shown below are preliminary! For now focus on diagrams S, V, F; preliminary study below does not yet include re-tuning of a.

HVP QED+strong IB contributions

RBC 2017

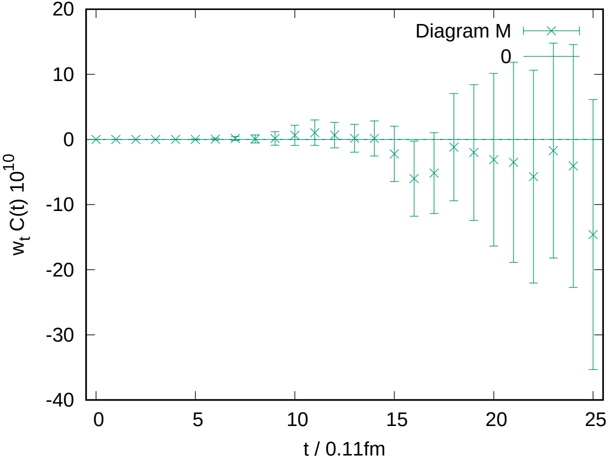
Diagrams S, V for pion mass:

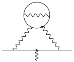


HVP QED+strong IB contributions

RBC 2017

HVP strong IB effect

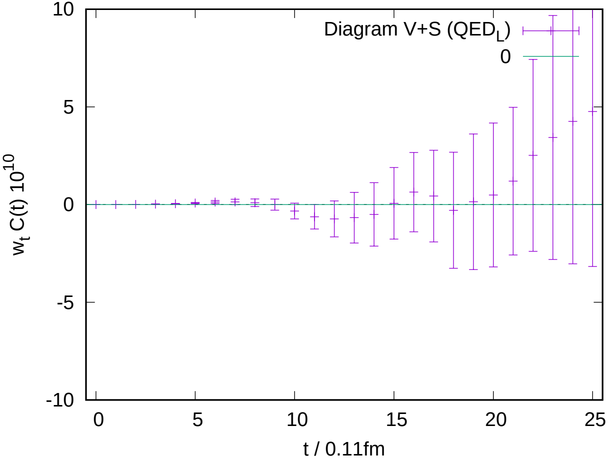


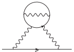


HVP QED+strong IB contributions

RBC 2017

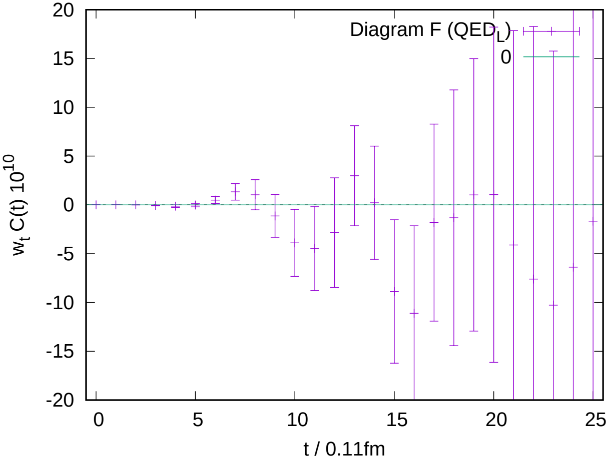
HVP QED diagram V+S





HVP QED+strong IB contributions

HVP QED diagram F



Straightforward improvements of statistics available

Current status of the RBC/UKQCD project

- Improved statistical estimators significantly reduced our statistical uncertainty and new Multi-Grid Lanczos further improves its efficiency. Currently $\delta a_\mu^{\rm stat} = 15 \times 10^{-10}$ for a pure lattice calculation and $\delta a_\mu^{\rm stat} \leq 5 \times 10^{-10}$ for a combined lattice+R-ratio calculation.

- At current statistical precision finite-volume behavior seems well described by scalar QED/leading-order ChPT. We are in the process of settling this uncertainty at significantly higher precision using dedicated runs at $L = 4.8$fm, $L = 6.4$fm, and $L = 9.6$fm.

- For the disconnected contributions the new method allowed for a precise calculation at physical pion mass (Phys.Rev.Lett. 116 (2016) 232002) with $\delta a_\mu = 4 \times 10^{-10}$.

- For the QED contributions at this point we only have a preliminary analysis at the physical point which suggests that the errors are controllable at the Fermilab E989 precision.

Plan to reach the Fermilab E989 precision

- Our current methods and anticipated computing time would suffice to control statistical noise of a combined lattice+R-ratio calculation at Fermilab E989 precision within 1-2 years. Reaching a comparable statistical precision for a pure lattice result may be achievable in the next 5 years.

- Our dedicated finite-volume study should be completed in the next 1-2 years and should allow for precise control of the largest systematic uncertainty.

- We will gather additional statistics to reduce the error of the quark-disconnected contribution computed in Phys.Rev.Lett. 116 (2016) 232002 to the Fermilab E989 precision. This only requires about 1 month of running and will be done at the appropriate time.

- We are planning to complete our QED correction project in a similar timeline; our preliminary studies suggest that this is realistic.

Thank you

The setup:

$$C(t) = \frac{1}{3V} \sum_{j=0,1,2} \sum_{t'} \langle \mathcal{V}_j(t+t') \mathcal{V}_j(t') \rangle_{\mathrm{SU(3)}} \tag{1}$$

where V stands for the four-dimensional lattice volume,
$\mathcal{V}_\mu = (1/3)(\mathcal{V}_\mu^{u/d} - \mathcal{V}_\mu^{s})$, and

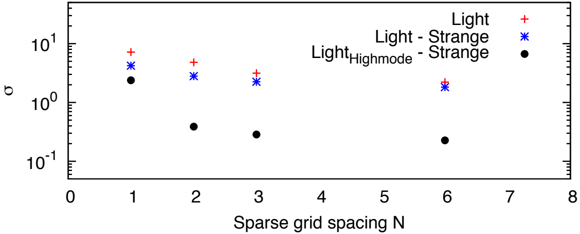
$$\mathcal{V}_\mu^f(t) = \sum_{\vec{x}} \mathrm{Im}\,\mathrm{Tr}[D^{-1}_{\vec{x},t;\vec{x},t}(m_f)\gamma_\mu]\,. \tag{2}$$

We separate 2000 low modes (up to around m_s) from light quark propagator as $D^{-1} = \sum_n v^n (w^n)^\dagger + D^{-1}_{\mathrm{high}}$ and estimate the high mode stochastically and the low modes as a full volume average Foley 2005.

We use a sparse grid for the high modes similar to Li 2010 which has support only for points x_μ with $(x_\mu - x_\mu^{(0)})$ mod $N = 0$; here we additionally use a random grid offset $x_\mu^{(0)}$ per sample allowing us to stochastically project to momenta.

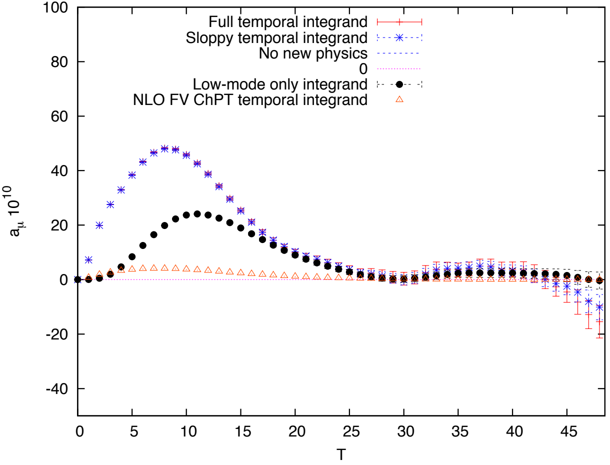
Combination of both ideas is crucial for noise reduction at physical pion mass!

Fluctuation of $\mathcal{V}_\mu$ (σ):

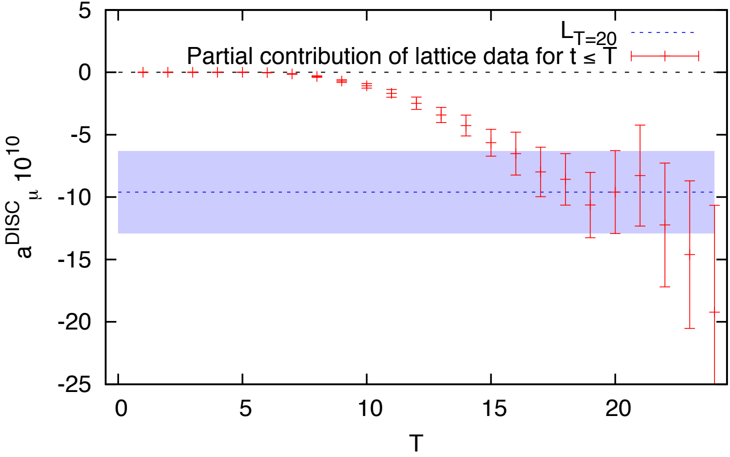


Since $C(t)$ is the autocorrelator of $\mathcal{V}_\mu$, we can create a stochastic estimator whose noise is potentially reduced linearly in the number of random samples, hence the normalization in the lower panel

Low-mode saturation for physical pion mass (here 2000 modes):

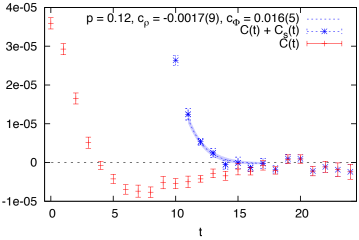


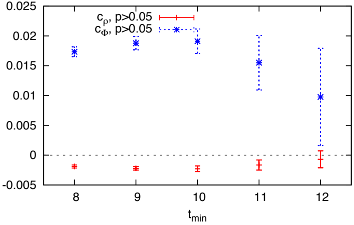
Result for partial sum $L_T = \sum_{t=0}^{T} w_t C(t)$:



For $t \geq 15$ $C(t)$ is consistent with zero but the stochastic noise is t-independent and $w_t \propto t^4$ such that it is difficult to identify a plateau region based only on this plot

Resulting correlators and fit of $C(t) + C_s(t)$ to $c_\rho e^{-E_\rho t} + c_\phi e^{-E_\phi t}$ in the region $t \in [t_{\min}, \ldots, 17]$ with fixed energies $E_\rho = 770$ MeV and $E_\phi = 1020$. $C_s(t)$ is the strange connected correlator.

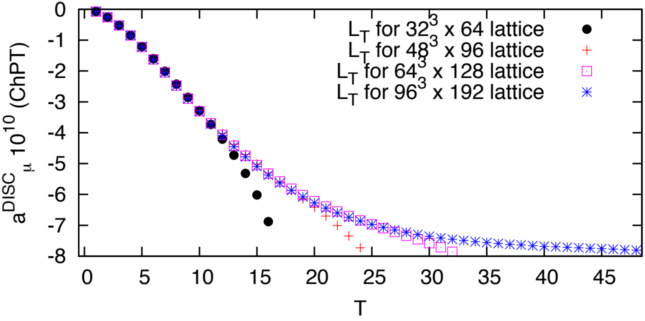




We fit to $C(t) + C_s(t)$ instead of $C(t)$ since the former has a spectral representation.

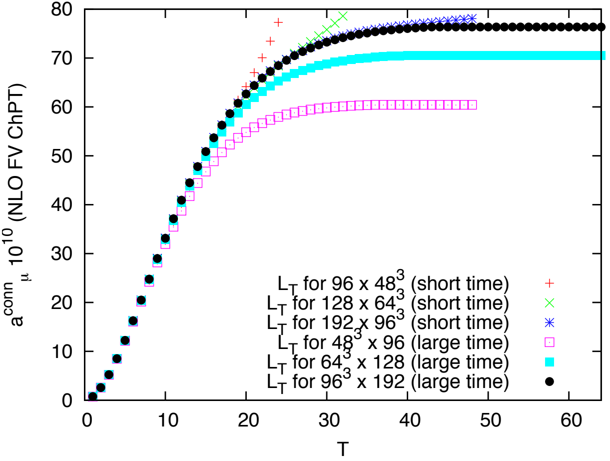
We could use this model alone for the long-distance tail to help identify a plateau but it would miss the two-pion tail

We therefore additionally calculate the two-pion tail for the disconnected diagram in ChPT:



A closer look at the NLO FV ChPT prediction (1-loop sQED):

We show the partial sum $\sum_{t=0}^{T} w_t C(t)$ for different geometries and volumes:



The dispersive approach to HVP LO

The dispersion relation

$$\begin{aligned}\Pi_{\mu\nu}(q) &= i\left(q_\mu q_\nu - g_{\mu\nu}q^2\right)\Pi(q^2)\\ \Pi(q^2) &= -\frac{q^2}{\pi}\int_{4m_\pi^2}^{\infty}\frac{ds}{s}\frac{\mathrm{Im}\Pi(s)}{q^2-s}\,.\end{aligned}$$

allows for the determination of $a_\mu^{\rm HVP}$ from experimental data via

$$a_\mu^{\rm HVP\ LO} = \left(\frac{\alpha m_\mu}{3\pi}\right)^2\left[\int_{4m_\pi^2}^{E_0^2} ds\,\frac{R_\gamma^{\rm exp}(s)\hat{K}(s)}{s^2} + \int_{E_0^2}^{\infty} ds\,\frac{R_\gamma^{\rm pQCD}(s)\hat{K}(s)}{s^2}\right],$$

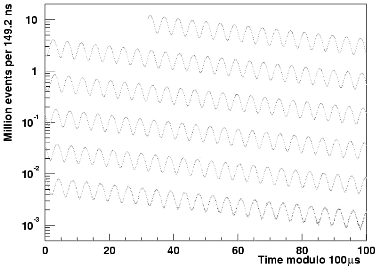
$$R_\gamma(s) = \sigma^{(0)}(e^+e^- \to \gamma^* \to \mathrm{hadrons})/\frac{4\pi\alpha^2}{3s}$$

Experimentally with or without additional hard photon (ISR: $e^+e^- \to \gamma^*(\to \mathrm{hadrons})\gamma$)

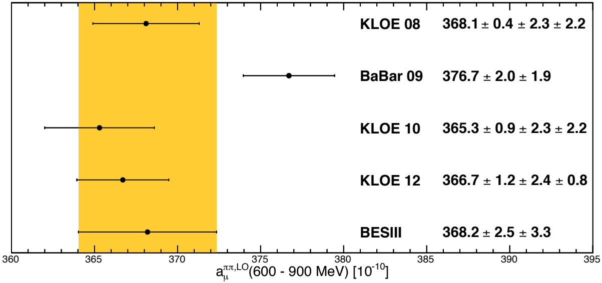
Experimental setup: muon storage ring with tuned momentum of muons to cancel leading coupling to electric field

$$\vec{\omega}_a = -\frac{q}{m}\left[a_\mu \vec{B} - \left(a_\mu - \frac{1}{\gamma^2 - 1}\right)\frac{\vec{\beta}\times\vec{E}}{c}\right]$$

Because of parity violation in weak decay of muon, a correlation between muon spin and decay electron direction exists, which can be used to measure the anomalous precession frequency ω_a:



BESIII 2015 update:

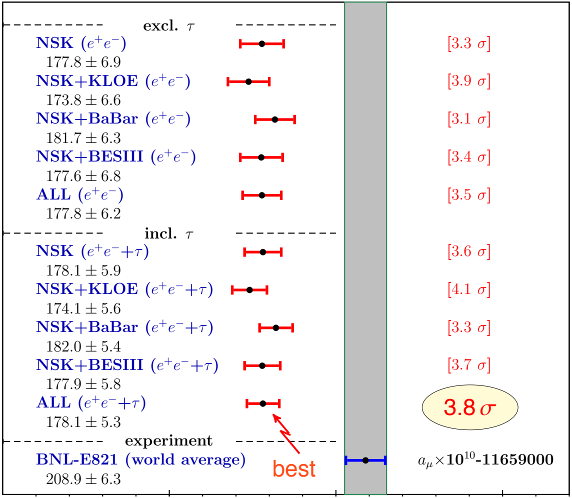


Jegerlehner FCCP2015 summary:

final state	range (GeV)	$a_\mu^{\mathrm{had}(1)} \times 10^{10}$ (stat) (syst) [tot]	rel	abs
ρ	(0.28, 1.05)	507.55 (0.39) (2.68)[2.71]	0.5%	39.9%
ω	(0.42, 0.81)	35.23 (0.42) (0.95)[1.04]	3.0%	5.9%
ϕ	(1.00, 1.04)	34.31 (0.48) (0.79)[0.92]	2.7%	4.7%
J/ψ		8.94 (0.42) (0.41)[0.59]	6.6%	1.9%
Υ		0.11 (0.00) (0.01)[0.01]	6.8%	0.0%
had	(1.05, 2.00)	60.45 (0.21) (2.80)[2.80]	4.6%	42.9%
had	(2.00, 3.10)	21.63 (0.12) (0.92)[0.93]	4.3%	4.7%
had	(3.10, 3.60)	3.77 (0.03) (0.10)[0.10]	2.8%	0.1%
had	(3.60, 9.46)	13.77 (0.04) (0.01)[0.04]	0.3%	0.0%
had	(9.46,13.00)	1.28 (0.01) (0.07)[0.07]	5.4%	0.0%
pQCD	(13.0,∞)	1.53 (0.00) (0.00)[0.00]	0.0%	0.0%
data	(0.28,13.00)	687.06 (0.89) (4.19)[4.28]	0.6%	0.0%
total		688.59 (0.89) (4.19)[4.28]	0.6%	100.0%

Results for $a_\mu^{\mathrm{had}(1)} \times 10^{10}$. Update August 2015, incl SCAN[NSK]+ISR[KLOE10,KLOE12,BaBar,BESIII]

Jegerlehner FCCP2015 summary ($\tau \leftrightarrow e^+e^-$):



Our setup:

$$C(t) = \frac{1}{3V} \sum_{j=0,1,2} \sum_{t'} \langle \mathcal{V}_j(t+t') \mathcal{V}_j(t') \rangle_{\mathrm{SU(3)}} \tag{3}$$

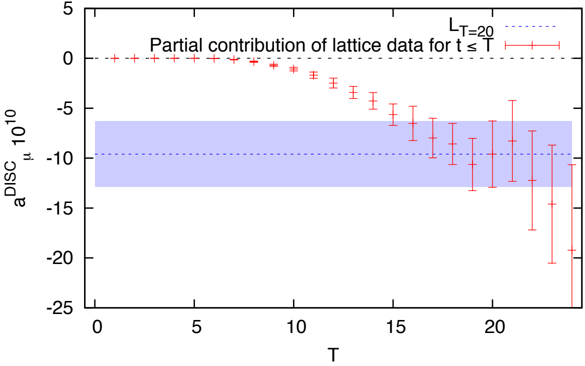
where V stands for the four-dimensional lattice volume, $\mathcal{V}_\mu = (1/3)(\mathcal{V}_\mu^{u/d} - \mathcal{V}_\mu^s)$, and

$$\mathcal{V}_\mu^f(t) = \sum_{\vec{x}} \mathrm{Im}\,\mathrm{Tr}[D^{-1}_{\vec{x},t;\vec{x},t}(m_f)\gamma_\mu]\,. \tag{4}$$

We separate 2000 low modes (up to around m_s) from light quark propagator as $D^{-1} = \sum_n v^n (w^n)^\dagger + D^{-1}_{\mathrm{high}}$ and estimate the high mode stochastically and the low modes as a full volume average Foley 2005.

We use a sparse grid for the high modes similar to Li 2010 which has support only for points x_μ with $(x_\mu - x_\mu^{(0)}) \bmod N = 0$; here we additionally use a random grid offset $x_\mu^{(0)}$ per sample allowing us to stochastically project to momenta.

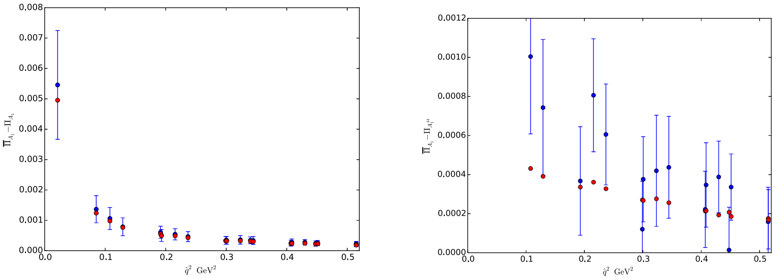
Study $L_T = \sum_{t=T+1}^{\infty} w_t C(t)$ and use value of T in plateau region (here $T = 20$) as central value. Use a combined estimate of a resonance model and the two-pion tail to estimate systematic uncertainty.



Combined with an estimate of discretization errors, we find

$$a_\mu^{\mathrm{HVP\ (LO)\ DISC}} = -9.6(3.3)_{\mathrm{stat}}(2.3)_{\mathrm{sys}} \times 10^{-10} \,. \qquad (5)$$

From Aubin et al. 2015 (arXiv:1512.07555v2)

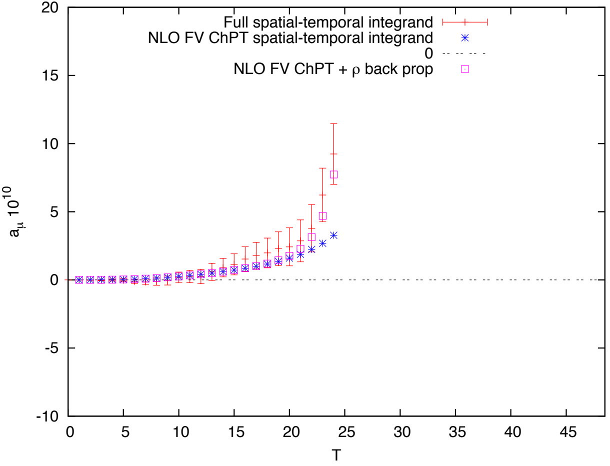


MILC lattice data with $m_\pi L = 4.2$, $m_\pi \approx 220$ MeV; Plot difference of $\Pi(q^2)$ from different irreps of 90-degree rotation symmetry of spatial components versus NLO FV ChPT prediction (red dots)

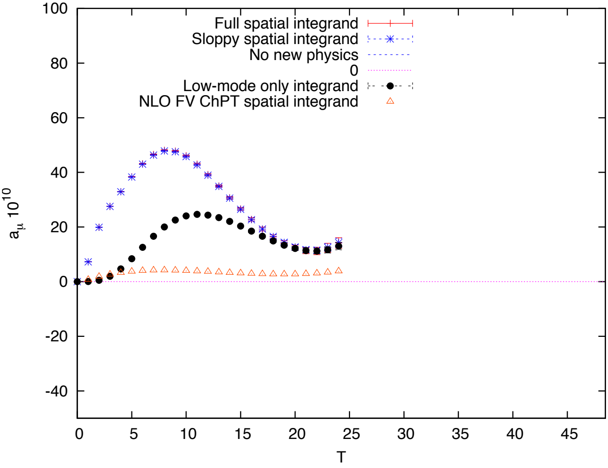
While the absolute value of a_μ is poorly described by the two-pion contribution, the volume dependence may be described sufficiently well to use ChPT to control FV errors at the 1% level; this needs further scrutiny

Aubin et al. find an $O(10\%)$ finite-volume error for $m_\pi L = 4.2$ based on the $A_1 - A_1^{44}$ difference (right-hand plot)

Compare difference of integrand of $48 \times 48 \times 96 \times 48$ (spatial) and $48 \times 48 \times 48 \times 96$ (temporal) geometries with NLO FV ChPT $(A_1 - A_1^{44})$:



$m_\pi = 140$ MeV, $p^2 = m_\pi^2/(4\pi f_\pi)^2 \approx 0.7\%$


Full spatial integrand
Sloppy spatial integrand
No new physics
0
Low-mode only integrand
NLO FV ChPT spatial integrand
$a_\mu\ 10^{10}$
T
100
80
60
40
20
0
-20
-40
0
5
10
15
20
25
30
35
40
45