

# Chiral extrapolation of the hadronic vacuum polarization contribution to the muon anomalous magnetic moment

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## Hadronic vacuum polarization contribution to muon anomalous magnetic moment:

expression:  $a_\mu^{\text{HVP}} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty \frac{dQ^2}{Q^2} w(Q^2) [\Pi(Q^2) - \Pi(0)]$  (Blum, '03)

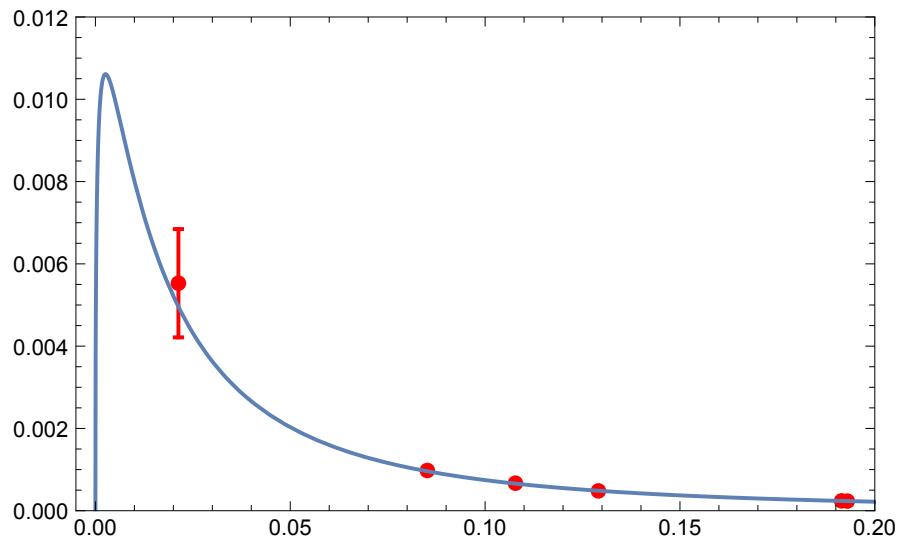
with  $w(Q^2)$  a known weight function, and  $\Pi(Q^2)$  the HVP obtained from

$$\Pi_{\mu\nu}(Q) = (\delta_{\mu\nu} Q^2 - Q_\mu Q_\nu) \Pi(Q^2)$$

integrand looks like

new statistics '15

AMA (Blum et al., '13)

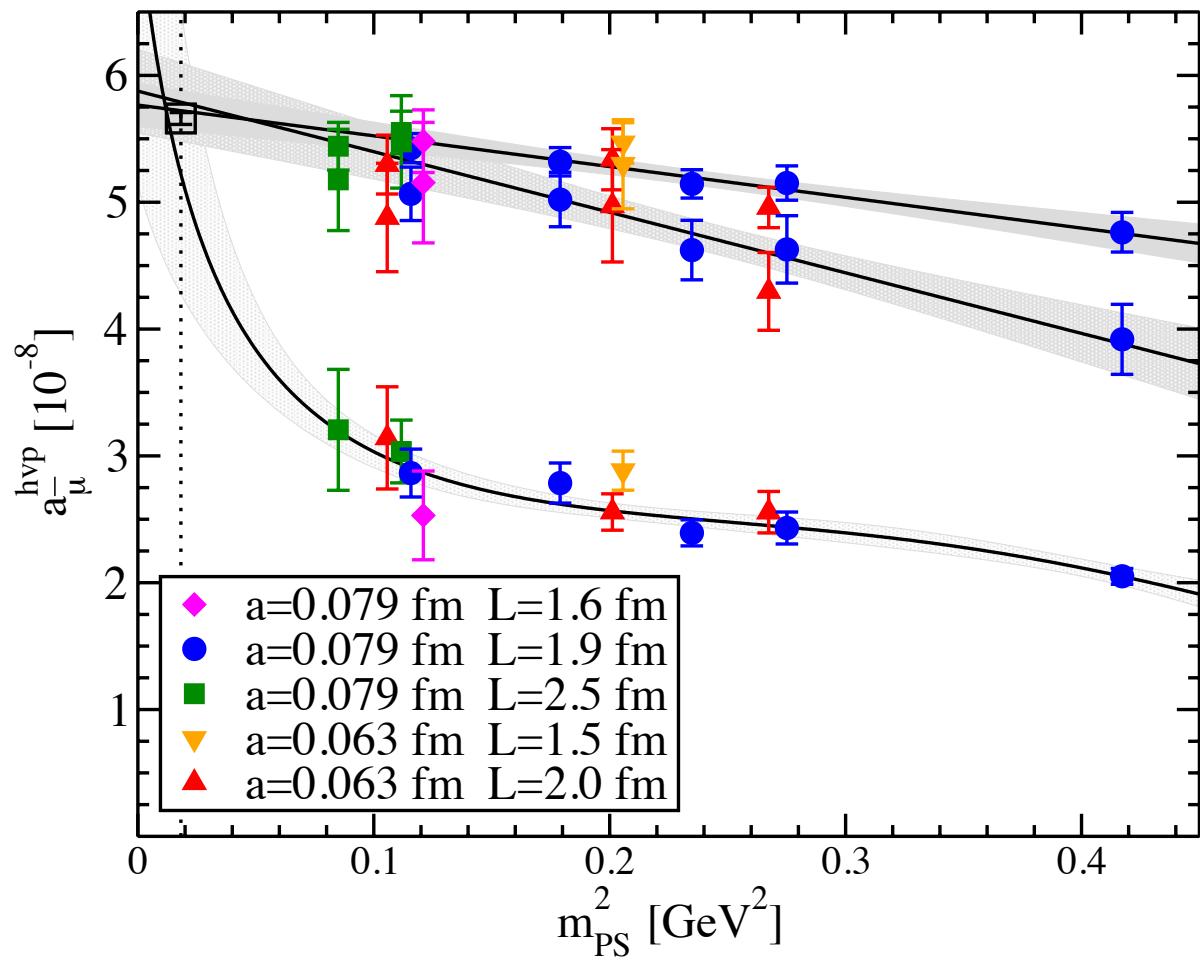


Aim: computation to better than 1%:

- Need very good, *model-independent* low-  $Q^2$  representation
- Finite-volume effects:  $\sim 5\%$  even at  $m_\pi L = 4$  ?
- Disconnected contribution and isospin breaking
- Strange and charm contributions, 4 (3) dynamical flavors
- Pion mass dependence of  $a_\mu^{\text{HVP}}$  -- **this talk**

## ETMC “trick”

Feng et al. 2011



Dependence of  $a_\mu^{\text{HVP}}$  on  $m_\pi^2$ . Lower curve: “uncorrected”  
Upper curve: “corrected”

## ETMC trick

Improve expression for  $a_\mu^{\text{HVP}}$ :

from

$$a_\mu^{\text{HVP}} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty \frac{dQ^2}{Q^2} w(Q^2) [\Pi(Q^2) - \Pi(0)]$$

to

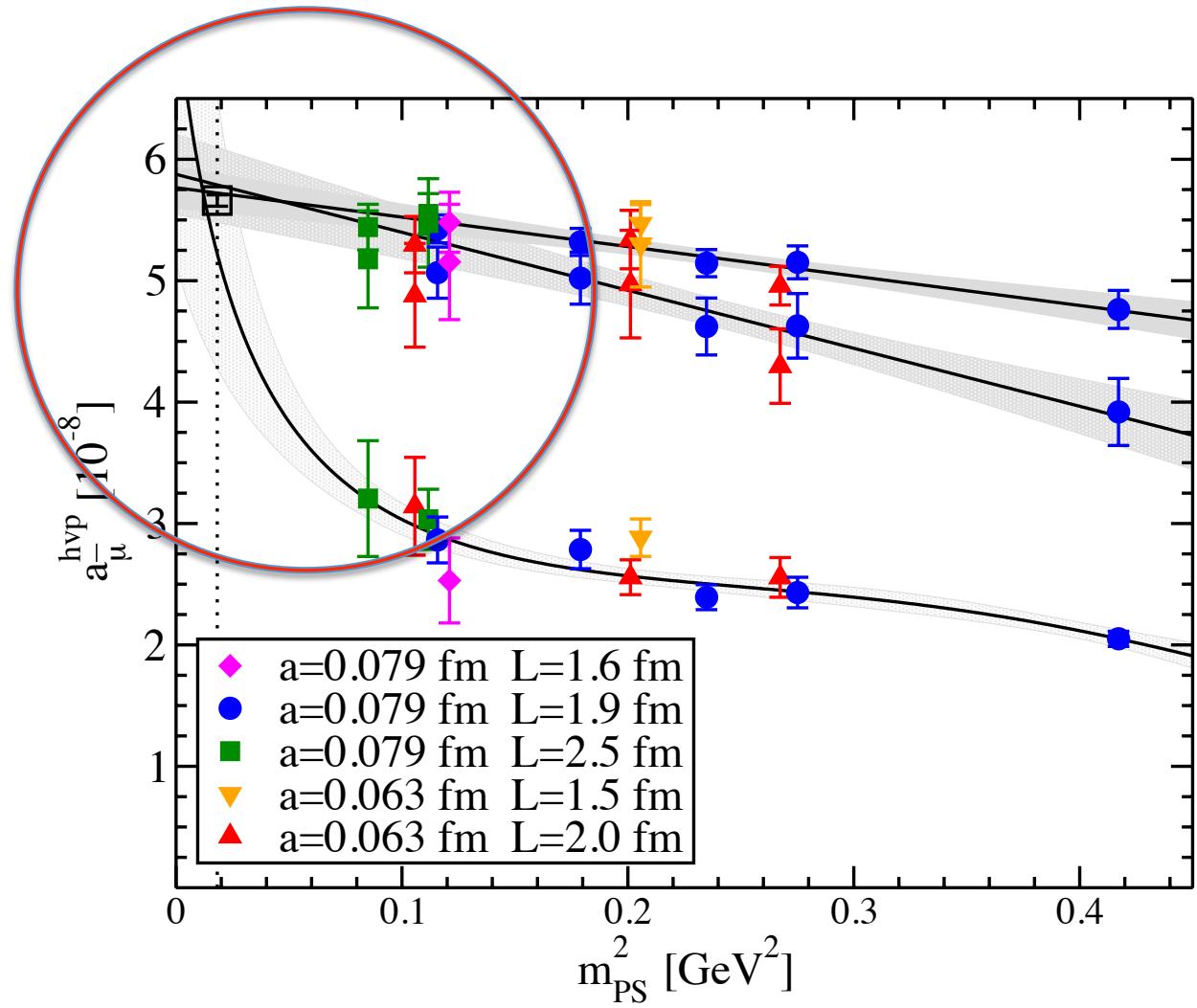
$$a_\mu^{\text{HVP}} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty \frac{dQ^2}{Q^2} w(Q^2) \left[ \Pi\left(\frac{m_{\rho,\text{latt}}^2}{m_\rho^2} Q^2\right) - \Pi(0) \right]$$

VMD:

$$\Pi(Q^2) = \frac{f_\rho^2 m_\rho^2}{Q^2 + m_\rho^2} + \Pi_{\text{PT}}(Q^2) \quad \Rightarrow$$

$$\Pi_{\text{latt}}\left(\frac{m_{\rho,\text{latt}}^2}{m_\rho^2} Q^2\right) = \frac{f_\rho^2 m_{\rho,\text{latt}}^2}{\frac{m_{\rho,\text{latt}}^2}{m_\rho^2} Q^2 + m_{\rho,\text{latt}}^2} + \dots = \frac{f_\rho^2 m_\rho^2}{Q^2 + m_\rho^2} + \dots$$

HPQCD variant: take out lowest-order pion loop first, and put it back at the physical pion mass at the end



Dependence of  $a_\mu^{\text{HVP}}$  on  $m_\pi^2$ . Lower curve: “uncorrected”  
 Upper curve: “corrected”

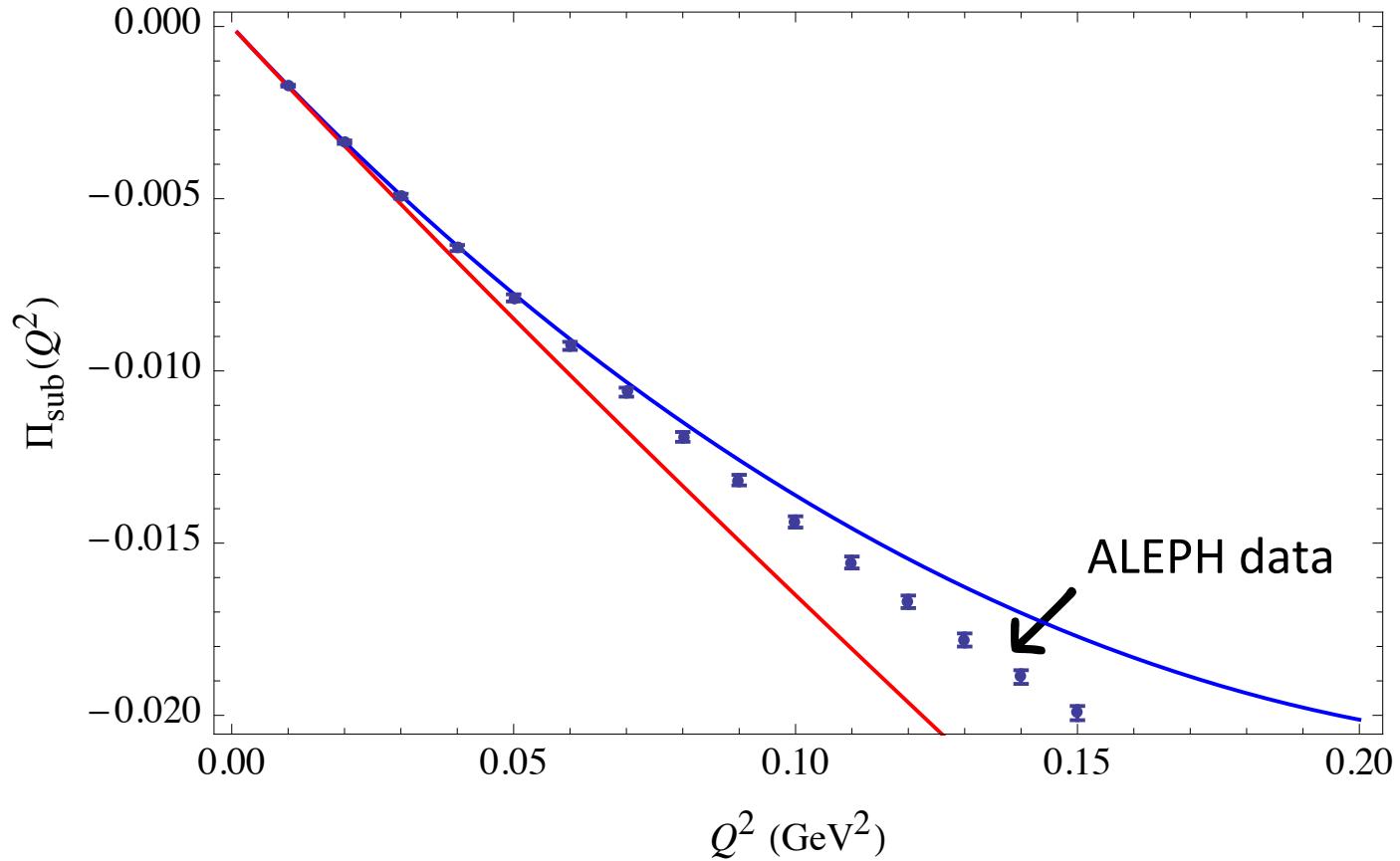
## Chiral perturbation theory

Amoros, Bijnens and Talavera (2000):

$$\begin{aligned}\Pi^{I=1}(Q^2) - \Pi^{I=1}(0) = & -4(F(Q^2) - F(0)) \\ & - \frac{4Q^2}{f_\pi^2} F^2(Q^2) + \frac{16Q^2}{f_\pi^2} L_9 F(Q^2) + 8C_{93} Q^2 \\ & + C'(Q^2)^2\end{aligned}$$

- $F(Q^2)$  known function (two-pion and two kaon cuts)
- $L_9 = 0.00593(43)$  NLO LEC (Bijnens and Talavera (2002))
- $C_{93} = -0.01536(44) \text{ GeV}^{-2}$  NNLO LEC (new, from ALEPH tau-decay data)
- $C' = 0.289 \text{ GeV}^{-4}$  analytic NNNLO term (from ALEPH data)  
(all at 0.77 GeV)

## Chiral perturbation theory



red ChPT without  $C'$   
blue ChPT with  $C'$  agreement with data to about 0.1 GeV $^2$

## Model pion mass dependence of $\tilde{a}_\mu \equiv a_\mu^{I=1}(Q^2 = 0.1 \text{ GeV}^2)$ :

- From MILC (HISQ):

$$m_\pi = 223, 262, 313, 382, 440 \text{ MeV}$$

$$m_K = 514, 523, 537, 558, 582 \text{ MeV}$$

$$f_\pi = 98, 101, 104, 109, 114 \text{ MeV}$$

$$m_\rho = 826, 836, 859, 894, 929 \text{ MeV}$$

- $C_{93} \sim 1/m_\rho^2$  hence replace  $C_{93} \rightarrow C_{93,\text{latt}}^{\text{eff}} = C_{93} \frac{m_\rho^2}{m_{\rho,\text{latt}}^2}$   
 $C' \sim 1/m_\rho^4$   
 $C' \rightarrow C'_{\text{latt}}^{\text{eff}} = C' \frac{m_\rho^4}{m_{\rho,\text{latt}}^4}$

(takes residual quark-mass dependence at higher orders into account;  
clearly visible in ALEPH data)

$$\tilde{a}_\mu \equiv a_\mu^{I=1}(Q_{\max}^2 = 0.1) = 9.73 \times 10^{-8}$$

(ChPT)

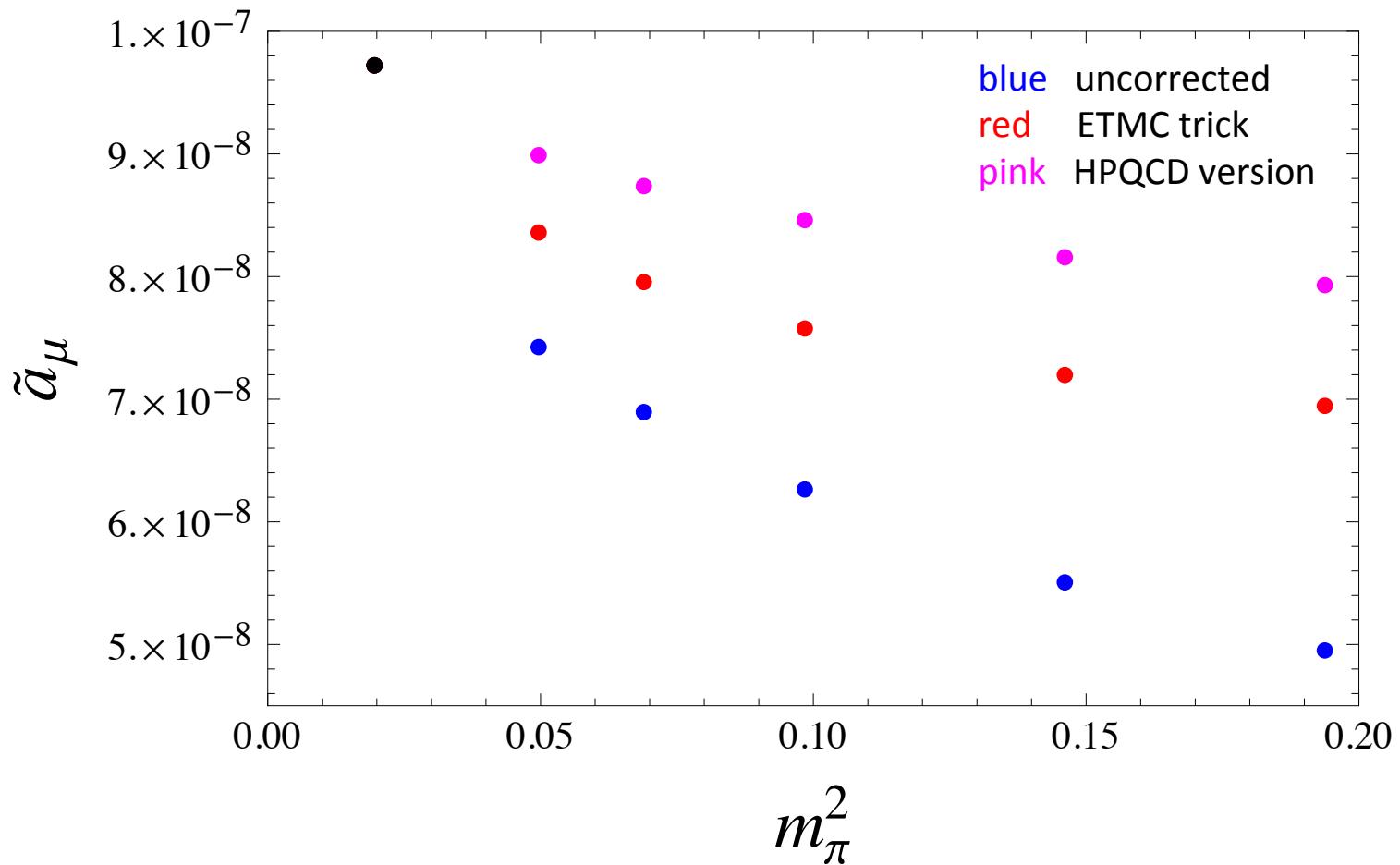
$$= 9.81 \times 10^{-8}$$

(ALEPH data)

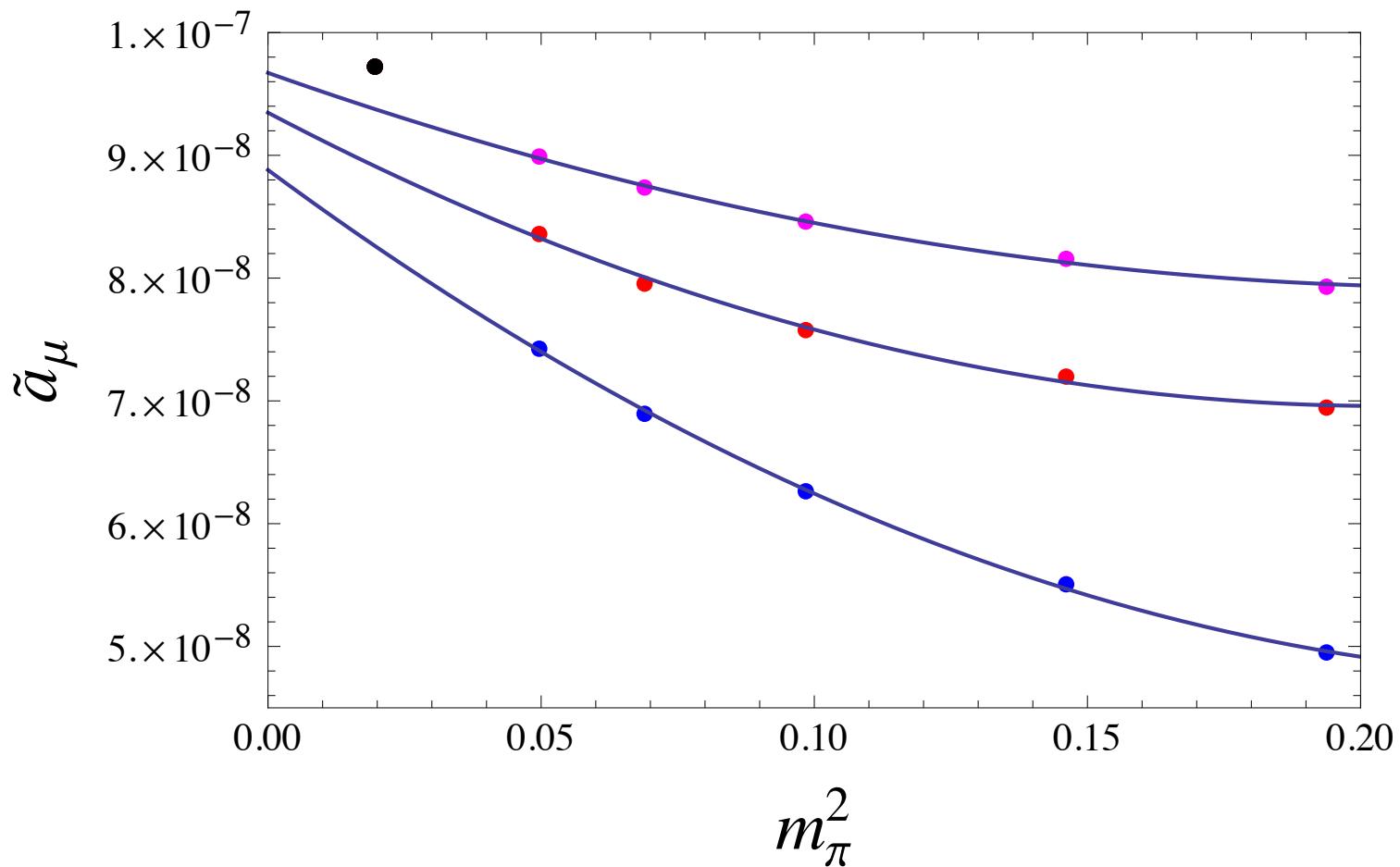
$$a_\mu^{I=1}(Q_{\max}^2 = \infty) = 11.95 \times 10^{-8}$$

(22% larger)

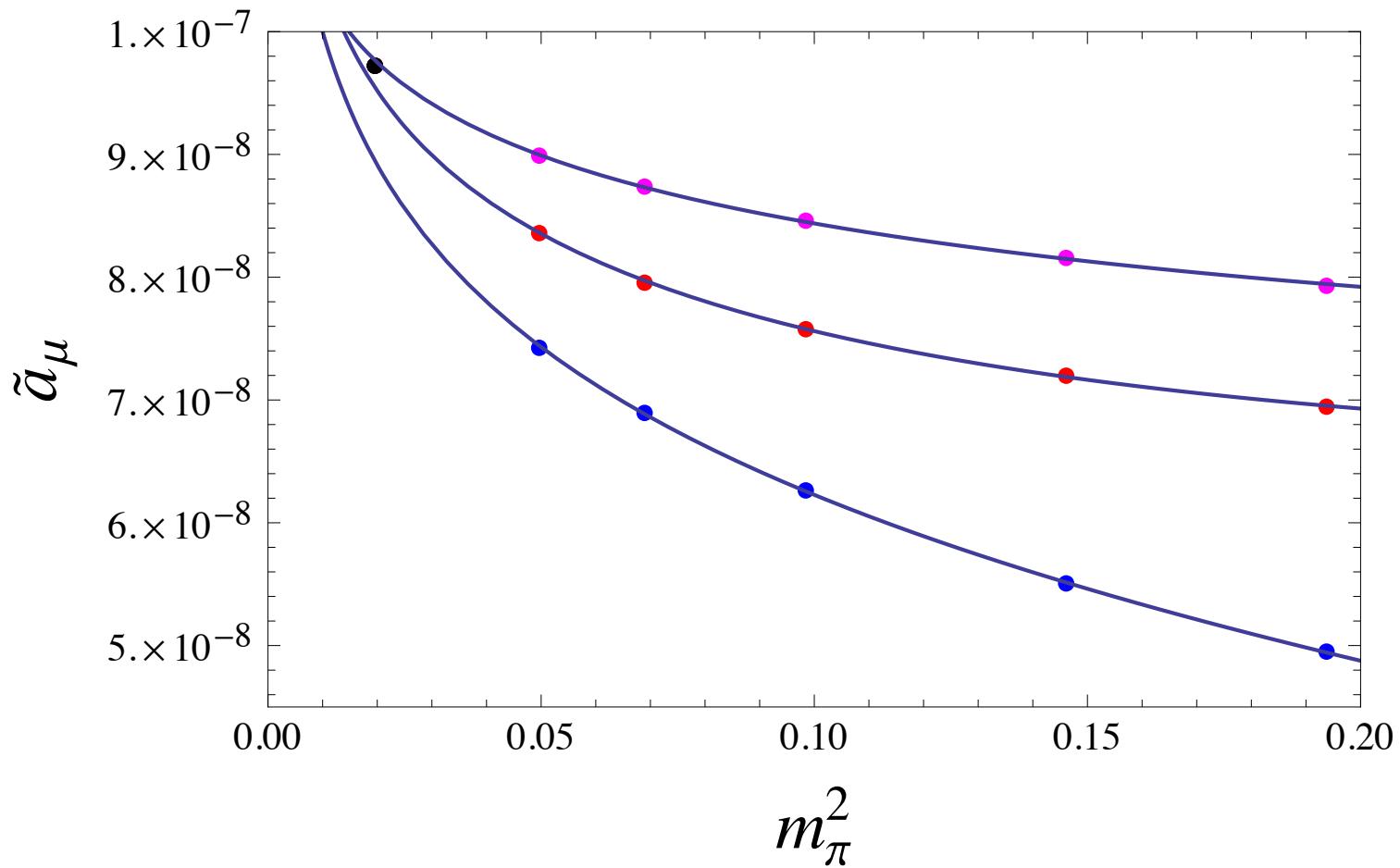
at physical  
pion mass



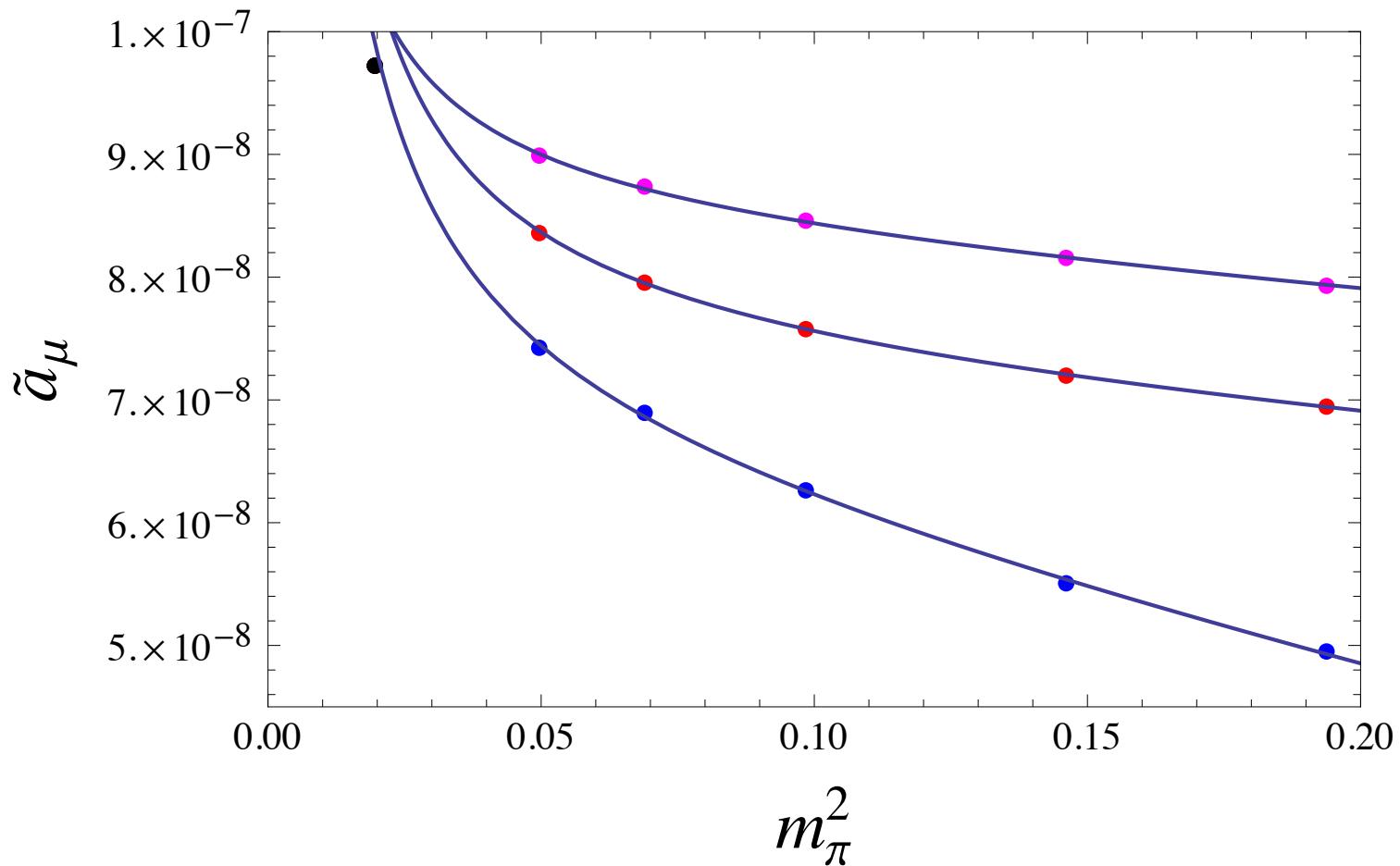
quadratic fit:  $am_{\pi}^4 + bm_{\pi}^2 + c$



logarithmic fit:  $a \log(m_\pi^2/m_{\pi,\text{phys}}^2) + b m_\pi^2 + c$



linear inv. fit:  $a/m_\pi^2 + bm_\pi^2 + c$



## Observations about fits:

- log fit much better than quadratic fit (same number of parameters)

	ETMC trick	HPQCD
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- relative errors -- quadratic: 15% 8% 4%  
logarithmic: 8% 2% 0.4%  
linear inv: 2% 8% 6%

- however, can predict coefficient of logarithm (for  $m_\pi \rightarrow 0$ ) :

theory:  $-\alpha^2/(12\pi^2) = -4.5 \times 10^{-7}$

fits:  $-1.5 \times 10^{-8}$  to  $-0.8 \times 10^{-8}$

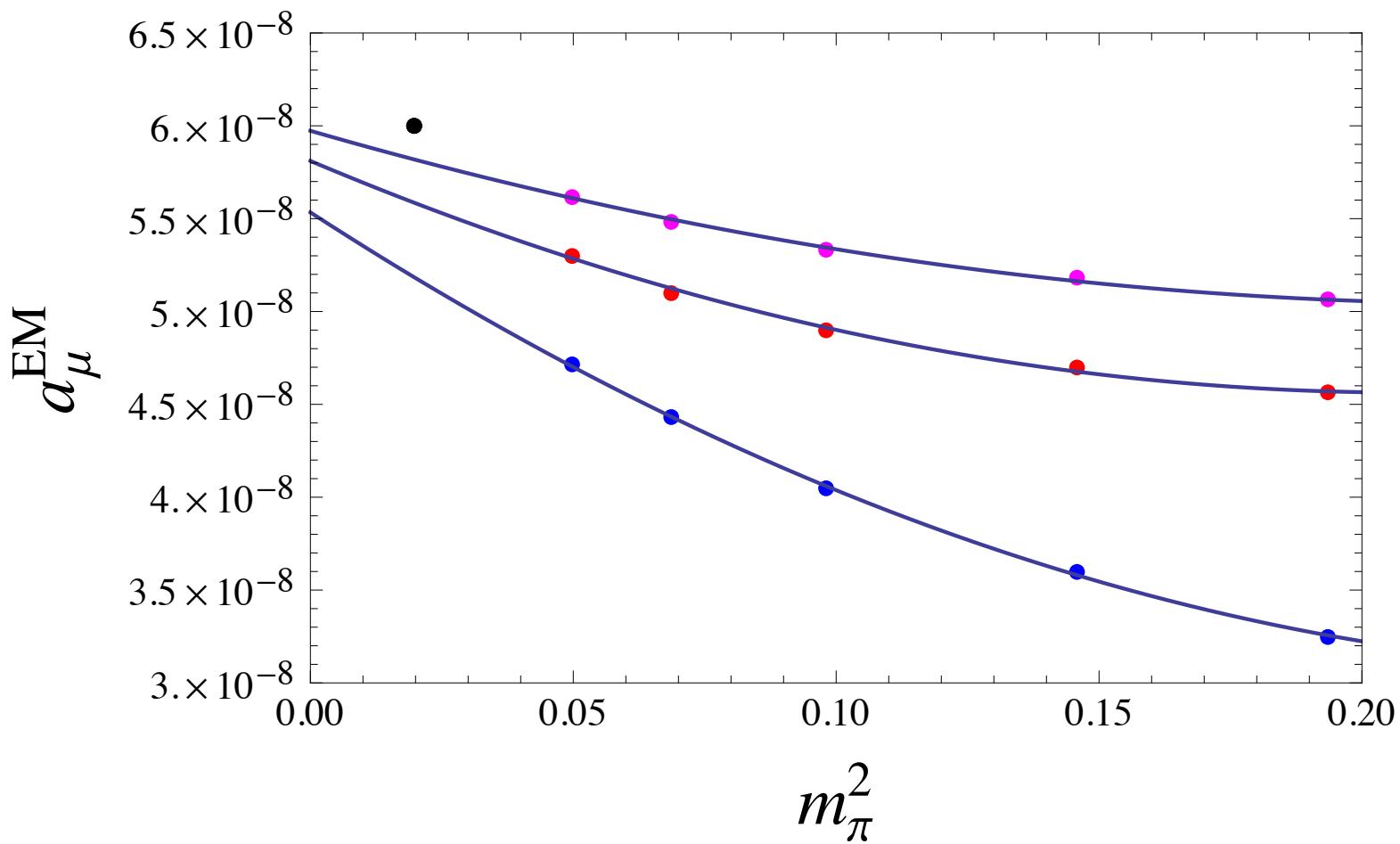
⇒ all fits just models (fit with log fixed at theory value is disaster)

⇒ need to estimate errors from comparing fits, e.g. quadratic and log

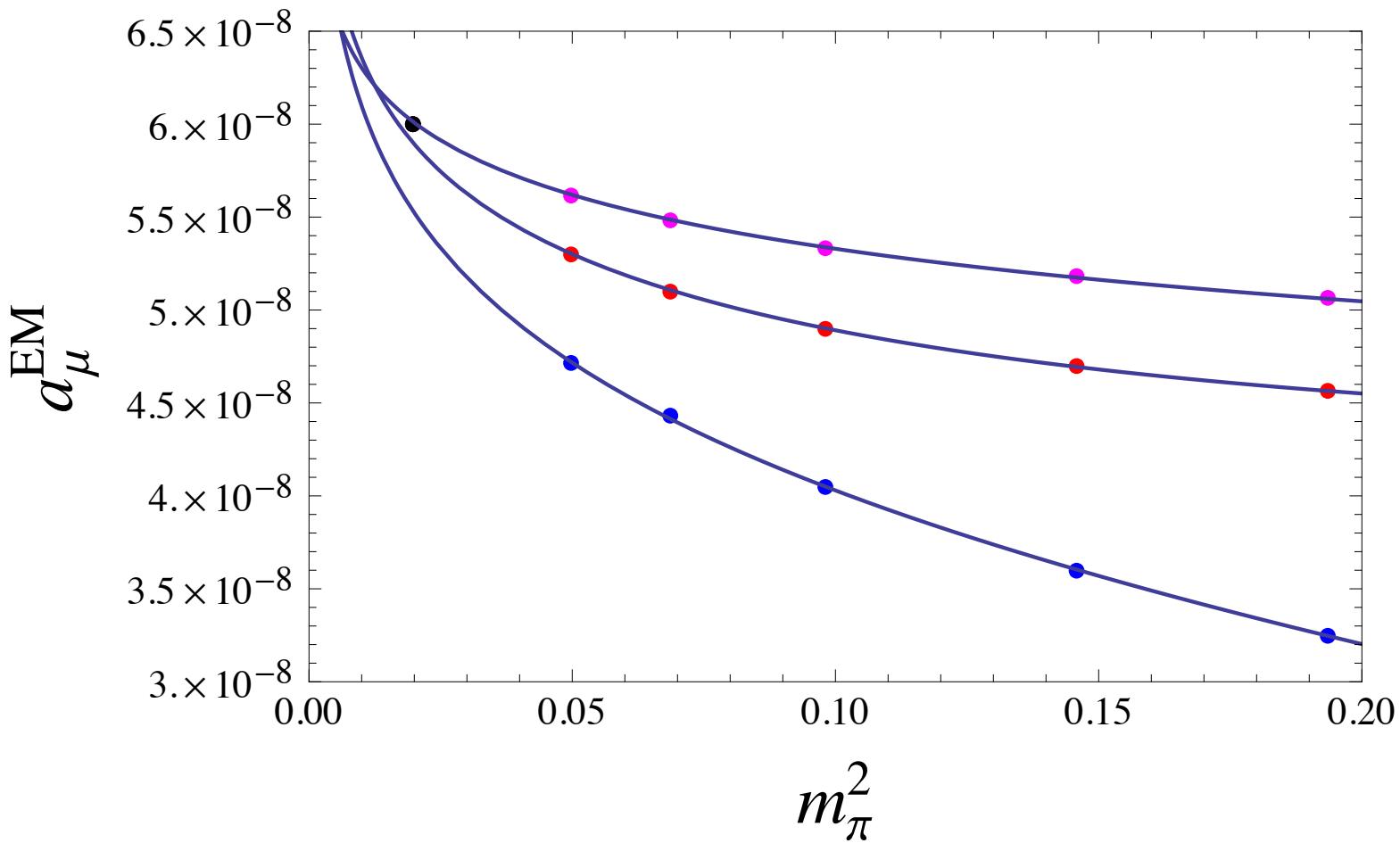
- need to add integral from  $0.1 \text{ GeV}^2$  to  $\infty$  ; unlikely to change lessons  
(no qualitative change if we integrate up to  $0.2 \text{ GeV}^2$ )

Same game with EM current:

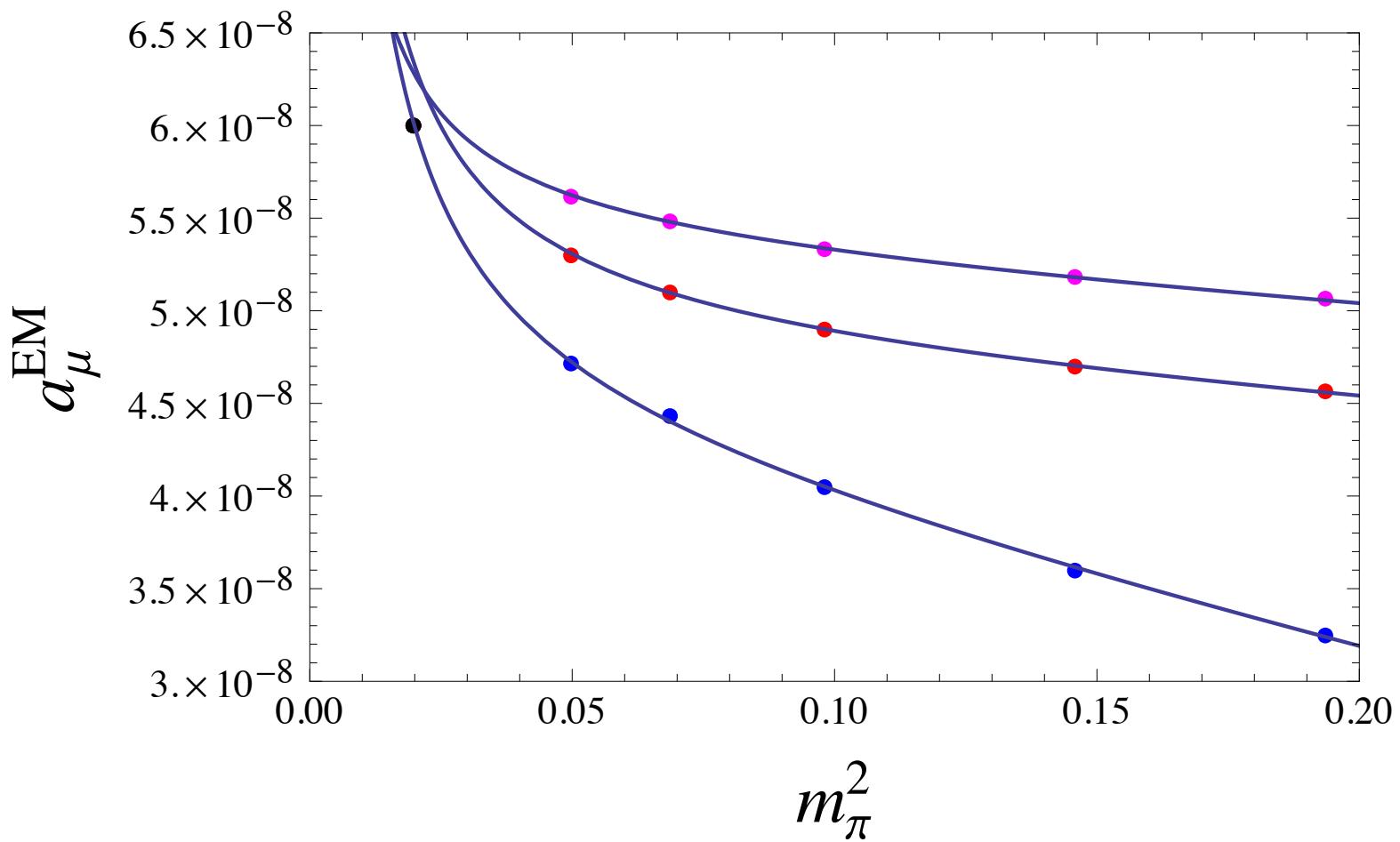
quadratic fit:  $am_{\pi}^4 + bm_{\pi}^2 + c$



logarithmic fit:  $a \log(m_\pi^2/m_{\pi,\text{phys}}^2) + b m_\pi^2 + c$



linear inv. fit:  $a/m_\pi^2 + bm_\pi^2 + c$



## Observations about fits (EM case):

- log fit much better than quadratic fit (same number of parameters)
- relative errors -- quadratic: 14%  
logarithmic: 8%  
linear inv: 0.7%

	ETMC trick	HPQCD
quadratic:	14%	7%
logarithmic:	8%	2%
linear inv:	0.7%	6%
- again, can predict coefficient of logarithm:  
theory:  $-\alpha^2/(24\pi^2) = -2.2 \times 10^{-8}$   
fits:  $-0.8 \times 10^{-8}$  to  $-0.4 \times 10^{-8}$   
 $\Rightarrow$  all fits just models (fit with log fixed at theory value is disaster)  
 $\Rightarrow$  need to estimate errors from comparing fits, e.g. quadratic and log
- need to add integral from  $0.1 \text{ GeV}^2$  to  $\infty$ ; unlikely to change lessons  
(no qualitative change if we integrate up to  $0.2 \text{ GeV}^2$ )

## Conclusion

- $a_\mu^{\text{HVP}}$  is very sensitive to the pion mass; long extrapolation from  $m_\pi = 220 \text{ MeV}$
- ETMC/HPQCD trick helps, but is not sufficient for <1% accuracy
- 2-pion cut important, in addition to pion-mass dependence of  $m_\rho$
- No “easy” chiral extrapolation to physical pion mass:  
need lattice computations at values well below 200 MeV  
with sufficient accuracy for sub-percent error in  $a_\mu^{\text{HVP}}$