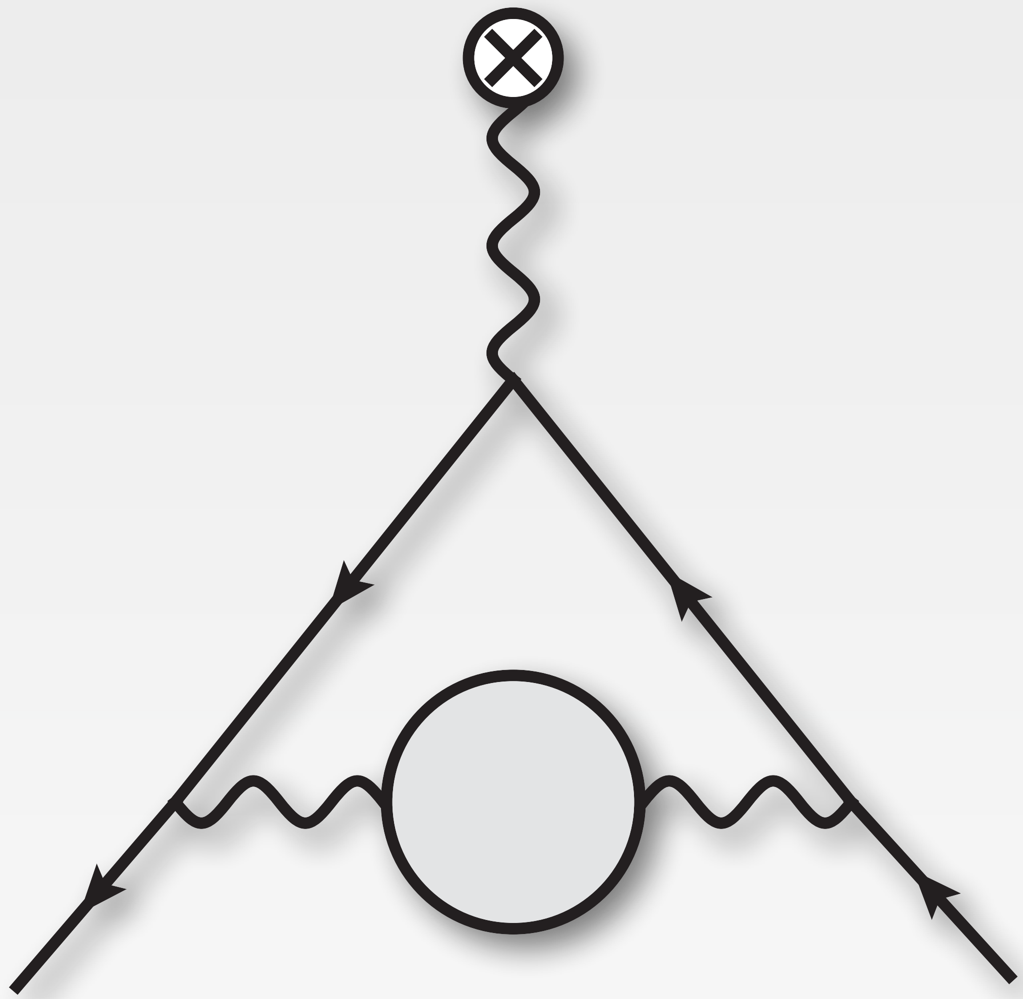

Isospin breaking corrections to the hadronic vacuum polarisation

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- Motivations & generalities
- Isospin corrections to the HVP from the lattice
- Preliminary results
- Summary & outlook

Motivations & generalities

Muon anomalous magnetic moment

[PDG 2016]

$$\boxed{\begin{aligned} a_{\mu,\text{exp.}} &= 116592089(54)(33)\cdot 10^{-11} \\ a_{\mu,\text{SM}} &= 116591802(2)(42)(26)\cdot 10^{-11} \end{aligned}}$$

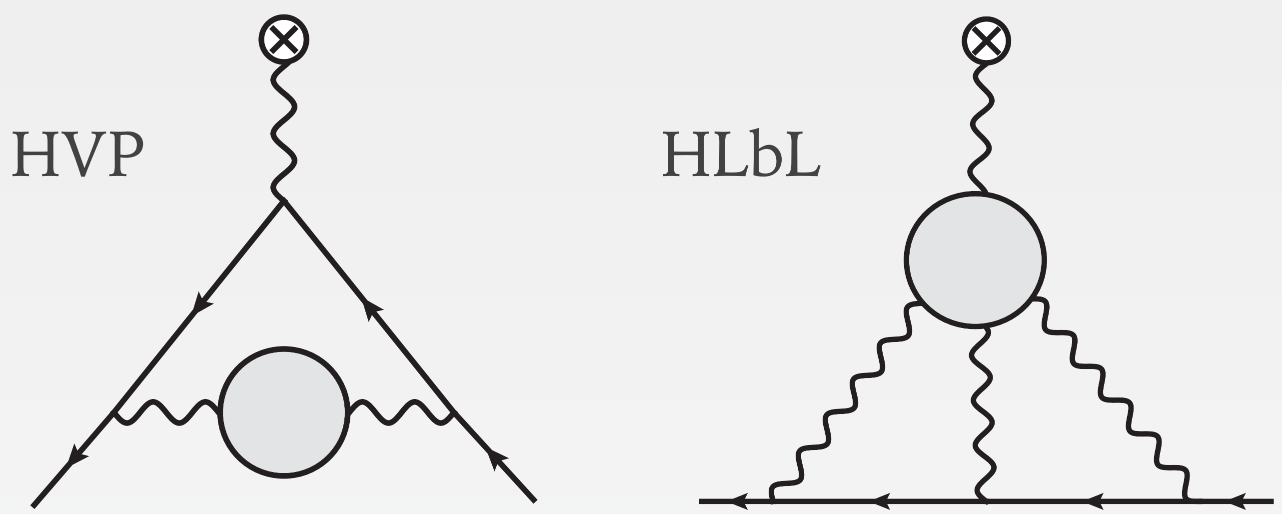
EW LO Had. NLO Had.

- $\sim 3.6\sigma$ discrepancy.

- The experimental error should be reduced by a factor 4 in the next years (FNAL & J-PARC experiments).

- Theoretical error completely dominated by hadronic uncertainties.

Hadronic contributions



- Order α^2: hadronic vacuum polarisation (HVP)
- Order α^3:
  - hadronic light-by-light (HLbL) scattering
  - QED corrections to the HVP

Hadronic contributions

HVP LO	$6932(42)(3) \times 10^{-11}$	[Davier *et al.* 2011]
HLbL	$105(26) \times 10^{-11}$	[Glasgow Consensus 2007]
HVP NLO	$-98.4(0.6) \times 10^{-11}$	[PDG 2015]

- Quantities dominated by non-perturbative QCD.
- **Lattice calculations can increase precision and reliability. Ideal target: per-mil precision.**
- It is very important to control isospin breaking effects ($O(1\%)$ effect).

Isospin corrections to the HVP from the lattice

Lattice QCD

- Lattice QCD: Monte-Carlo evaluation of the Euclidean QCD path integral on a finite and discrete space-time.
- Equivalent to an SU(3) gauged statistical system.
- Allows *ab-initio* non-perturbative calculations.
- It is now possible to perform realistic lattice simulations (physical quark masses, large volumes).

Non-compact lattice QED

- Naively discretised **Maxwell action:**

$$S[A_\mu] = \frac{1}{4}\sum_{\mu,\nu}(\partial_\mu A_\nu - \partial_\nu A_\mu)^2$$

- Pure gauge theory is free, it can be solved exactly.
- **Gauge invariance is preserved**.
- No mass gap: **large finite volume effects expected** (power law in the inverse spatial extent).

Zero-mode subtraction

Finite volume: **momentum quantisation**

$$\alpha\int\frac{\mathrm{d}^4k}{(2\pi)^4}\frac{1}{k^2}\cdots \quad\longmapsto\quad \frac{\alpha}{V}\sum_k\frac{1}{k^2}\cdots$$

Possibly IR divergent, but not for physical quantities

Contains a straight 1/0 !

Possible solution: **remove all the spatial zero-modes**, (*cf.* discussion in [Borsanyi *et al.*, Science, 2015]).

Electro-quenched approximation

- Electro-quenched approximation: **charged valence quarks**, but **neutral sea quarks.**

- **Non-unitary** theory (partially quenched)

- **Greatly reduce** the computational cost in both cases

- Missing contributions are large-N_c and SU(3) flavour suppressed: O(10%) of EM effects

- **Might be enough** for $g-2$! (to investigate)

Coupling to QCD

Two possible strategies to couple QED to QCD:

- **Stochastic method:**
  Simulate the EM field directly in the Monte-Carlo process, *cf.* for example [Borsanyi *et al.*, Science, 2015].

- **Perturbative method:**
  Expand the action and the observables at $\mathrm{O}(\alpha)$ and compute the corrections as pure QCD correlation functions, *cf.* for example [de Divitiis *et al.*, PRD, 2013].

So far **no direct comparison** of both approaches.

Stochastic method

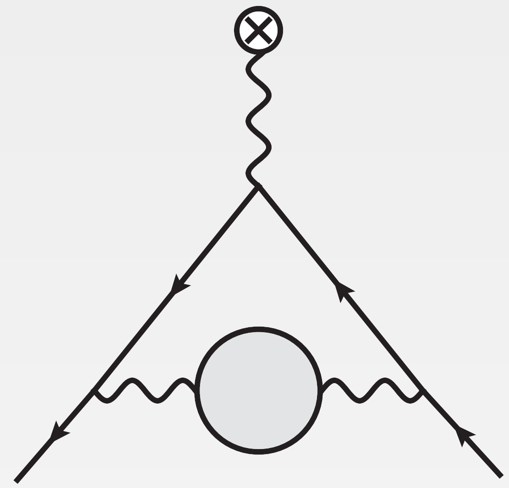
- Pros:
  - Ad hoc: Wick contractions to compute are the same, QED effects are automatically included.
  - Simpler contractions: less propagators to compute.
  - Sea EM effects included in the Monte-Carlo process, no additional disconnected diagrams.
- Cons:
  - Black box: no way to distinguish different corrections (order in α, isospin channel).
  - Photon propagation stochastic: more noisy?
  - EM coupling fixed once for all.

Perturbative method

$$\langle O_1(x_1) O_2(x_2)\rangle = \langle O_1(x_1) O_2(x_2)\rangle_{\alpha=0} \\ + \alpha \sum_{x,y,\mu,\nu} \langle O_1(x_1) J_\mu(x) J_\nu(y) O_2(x_2)\rangle_{\alpha=0} D_{\mu\nu}(x-y) + \mathrm{O}(\alpha^2)$$

- Pros:
  - Exact photon propagation.
  - Possibility to study different types of contributions.
  - Possibility to change the EM coupling.
  - Generally: more knobs to turn.
- Cons:
  - Much more complicated correlation functions (convolution with the photon propagator).
  - EM sea effects come through disconnected diagrams.

HVP vertex loop integral



- EM current 2-point function in Euclidean space-time:

$$\Pi_{\mu\nu}(q) = \int \mathrm{d}^4x \, \langle 0| \, \mathrm{T}[J_\mu(x) J_\nu(0)] \, |0\rangle \, e^{iq\cdot x} = (\delta_{\mu\nu} q^2 - q_\mu q_\nu)\Pi(q^2)$$

- Renormalisation: $\hat{\Pi}(q^2) = \Pi(q^2) - \Pi(0)$

- Vertex loop integral:

$$a_\mu^{(2)\mathrm{had.}} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^{+\infty} \mathrm{d}q^2 \, \hat{\Pi}(q^2) f(q^2)$$

where $f(q^2)$ is a know function of q^2.

Lattice HVP

- Conserved current $J_\mu^{\mathrm{C}} =$ not $\overline{\psi}\gamma_\mu\psi$.
  Comes from Noether's theorem and verifies $\partial_\mu J_\mu^{\mathrm{C}} = 0$.
  Depends on e: $J_\mu^{\mathrm{C}} = J_\mu^{\mathrm{C},0} - iqA_\mu T_\mu - q^2 A_\mu^2 J_\mu^{\mathrm{C},0} + \mathrm{O}(q^3)$.
- Local current $J_\mu^L = \overline{\psi}\gamma_\mu\psi$.
  Not conserved.
- $\langle J_\mu^{\mathrm{C}} J_\nu^{\mathrm{L}} \rangle$ is transverse!

Exploratory calculation setup

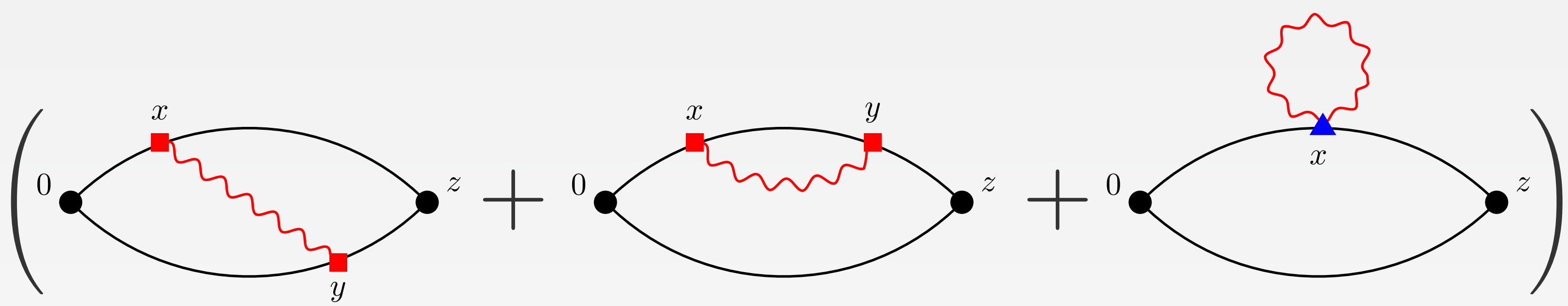
- $24^3 \times 64$ RBC-UKQCD ensemble
- 2+1 domain wall fermions and Iwasaki gauge action.
- $M_\pi \simeq 350\ \mathrm{MeV}$ and $a \simeq 0.12\ \mathrm{fm}$.
- Electro-quenched and no disconnected diagrams

EM corrections to the HVP

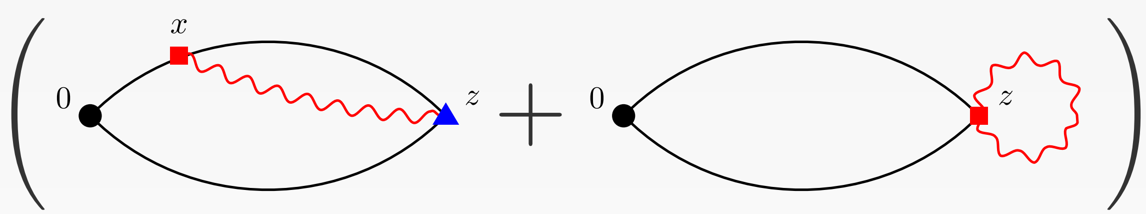
$$\langle J^{\mathrm{C}}_{\mu}(z) J^{\mathrm{L}}_{\nu}(0)\rangle = \langle J^{\mathrm{C},0}_{\mu}(z) J^{\mathrm{L}}_{\nu}(0)\rangle_{\alpha=0}$$

$J^{\mathrm{C},0}_{\mu}$ T_{μ}

Pure QCD HVP

$+\, q^2$ 

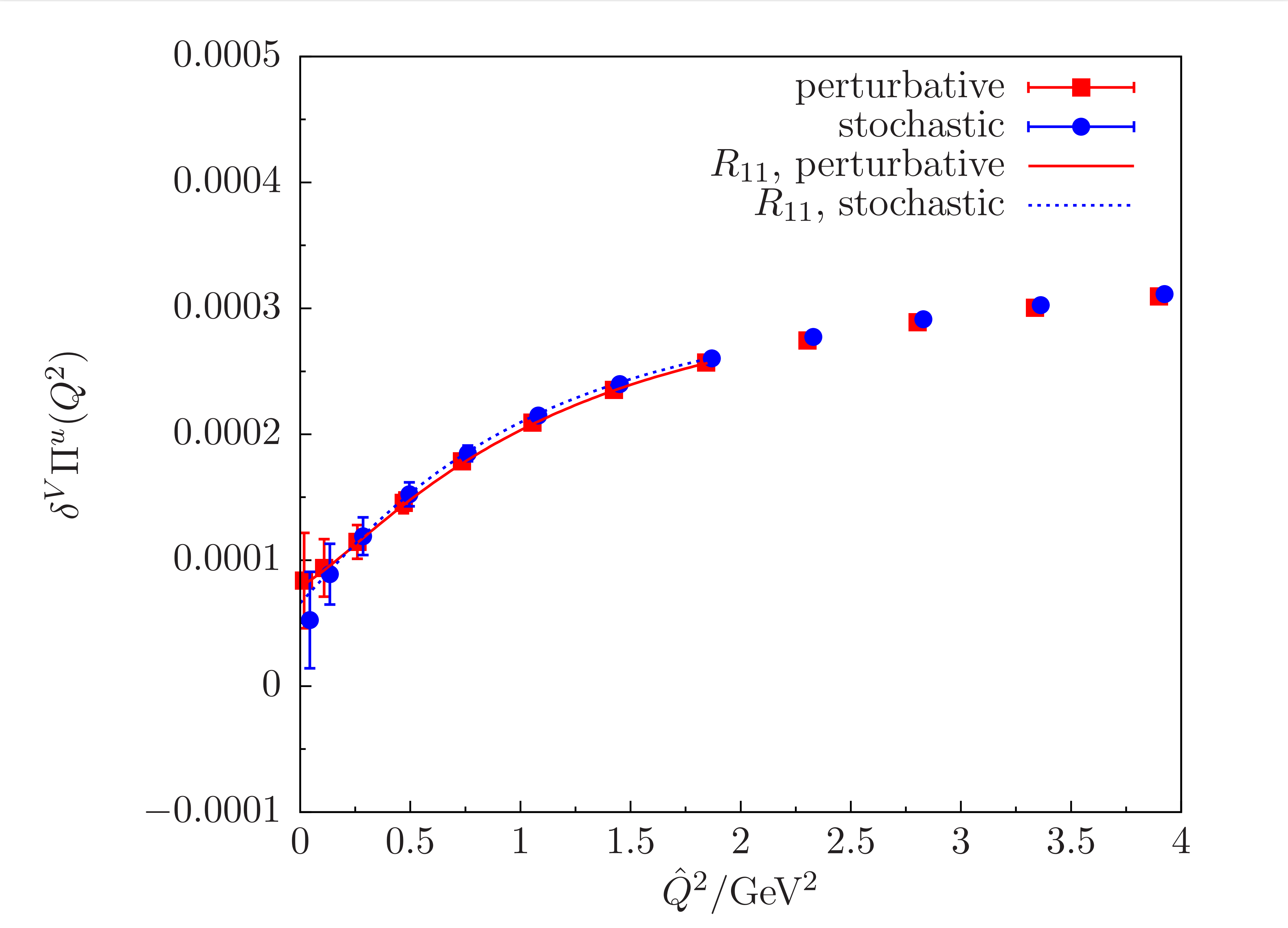
Expansion of the action (no disconnected, electro-quenched)

$+\, q^2$ 

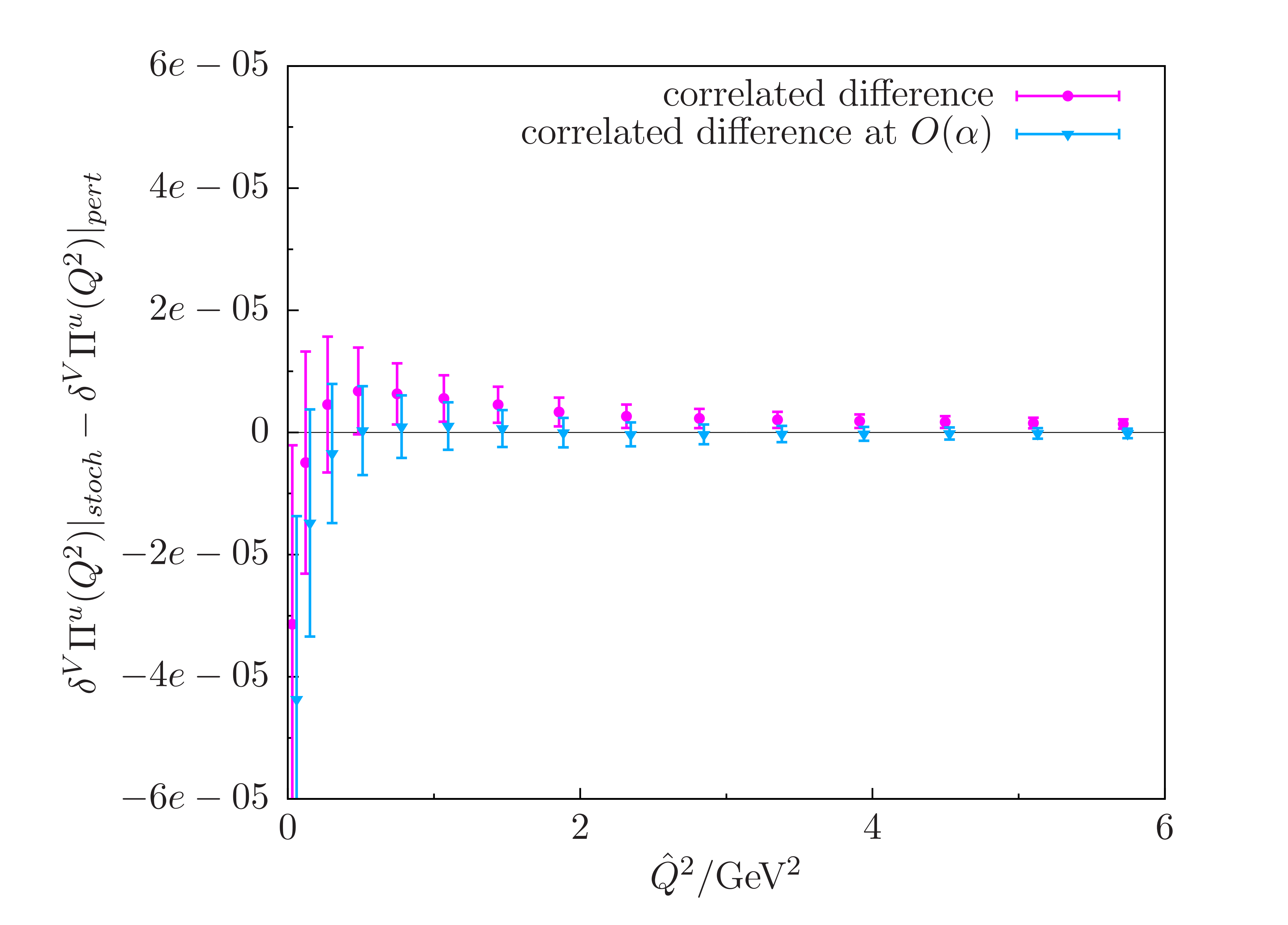
Expansion of the lattice conserved current

$$+\, \mathrm{O}(q^4)$$

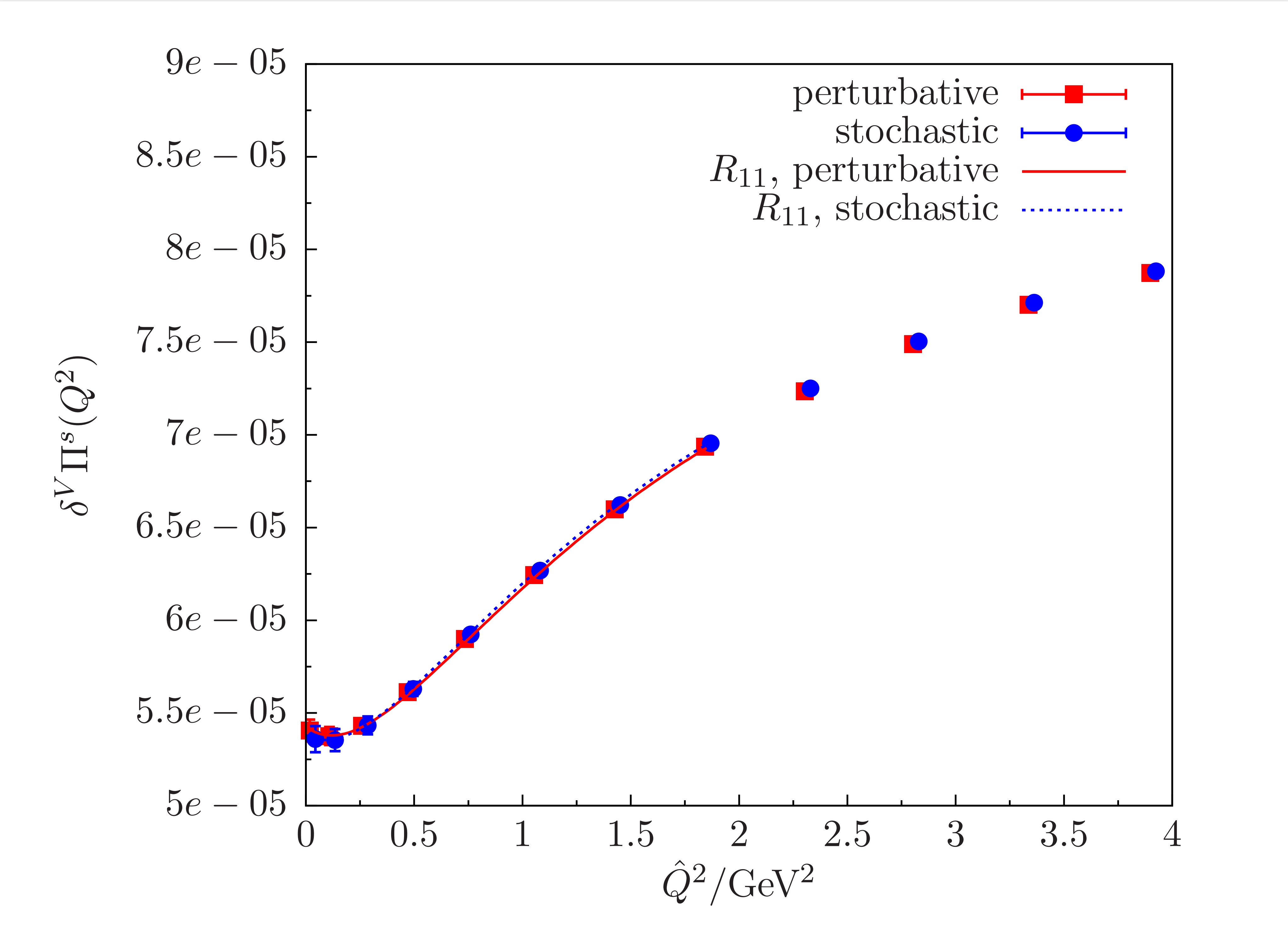
Preliminary results: HVP EM corrections



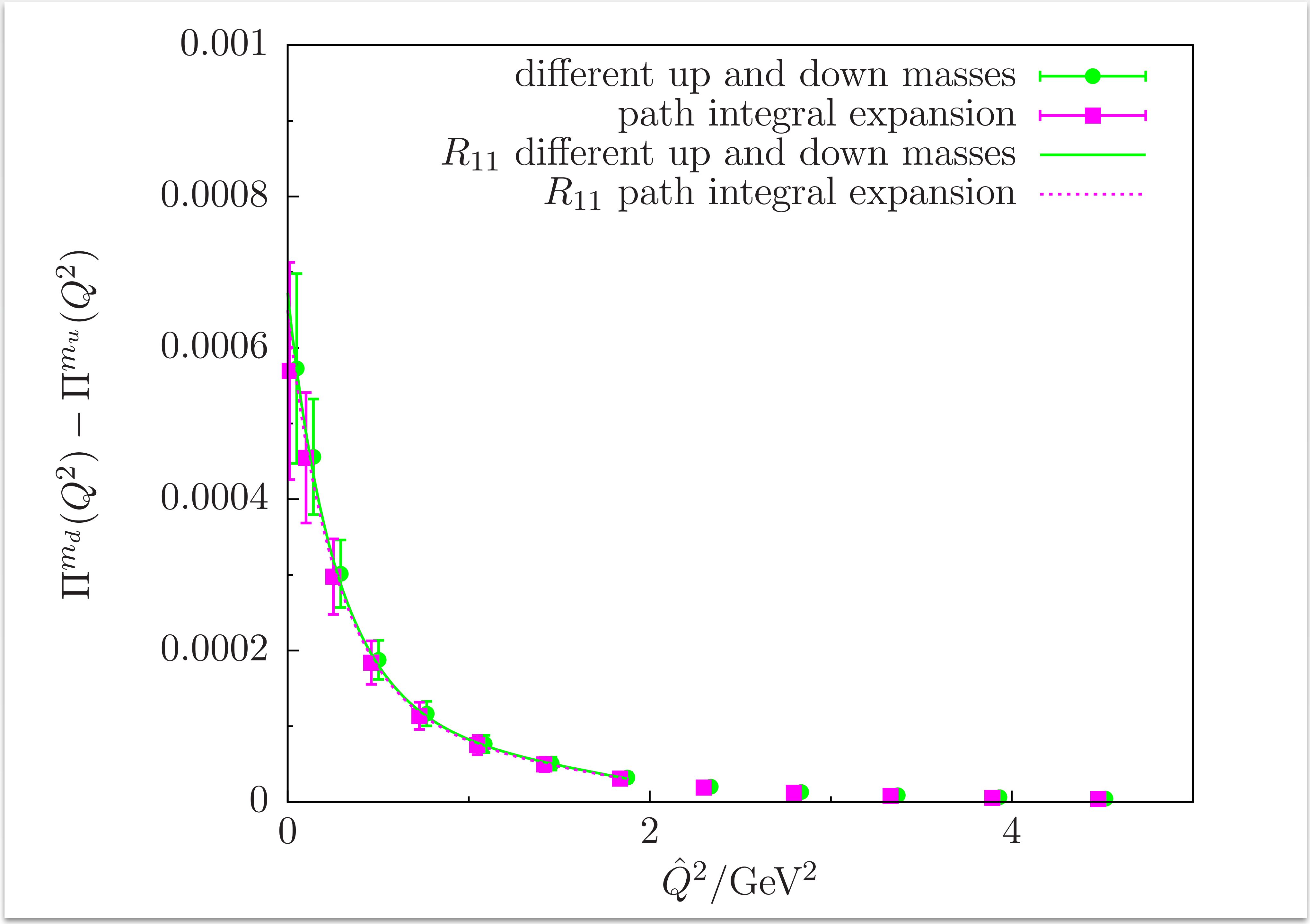
Preliminary results: HVP EM corrections



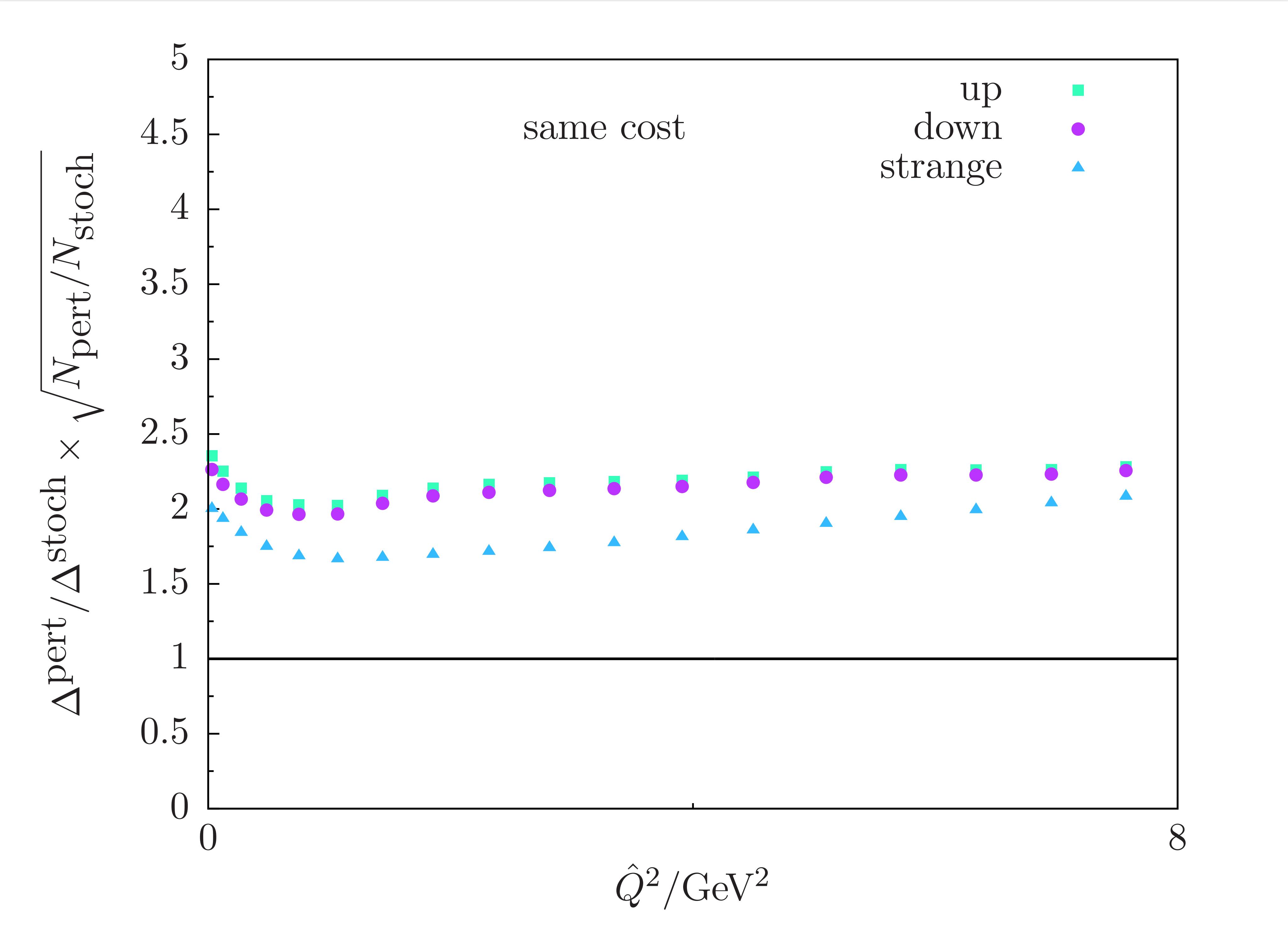
Preliminary results: HVP EM corrections



Preliminary results: HVP mass corrections



Preliminary results: efficiency



Preliminary results: g-2

EM corrections:

	$a_\mu^0 \times 10^{10}$	$\delta^V a_\mu^{\text{stoch}} \times 10^{10}$	$\delta^V a_\mu^{\text{pert}} \times 10^{10}$
u	318 ± 11	0.65 ± 0.31	0.37 ± 0.33
d	78.0 ± 2.3	0.040 ± 0.021	0.022 ± 0.16
s	47.98 ± 0.25	-0.0030 ± 0.0012	-0.0049 ± 0.0011

Strong corrections: about -5×10^{-10} .

Disclaimer: Unphysical pions, missing disconnected diagrams, electro-quenched.

We are working on it.

Summary & outlook

Summary

- This is the **first exploratory lattice computation** of the isospin breaking corrections to the HVP.
- **First direct comparison** of the stochastic and perturbative methods for including QED.
- For this quantity, the stochastic method has clearly the advantage on the perturbative one.
- **For more details, see V. Gülpers and J. Harrison talks at Lattice 2017.**

Outlook

- Can the perturbative method take the advantage by using a **better integration strategy**?
- Size of the **disconnected diagrams**?
- We need to go to **physical pion masses**.
- **Large finite volume effects essentially unknown**.
(work in progress with Southampton & Lund)

Thank you!