Towards a dispersive determination of the η and η' transition form factors

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Dispersion relations for meson transition form factors

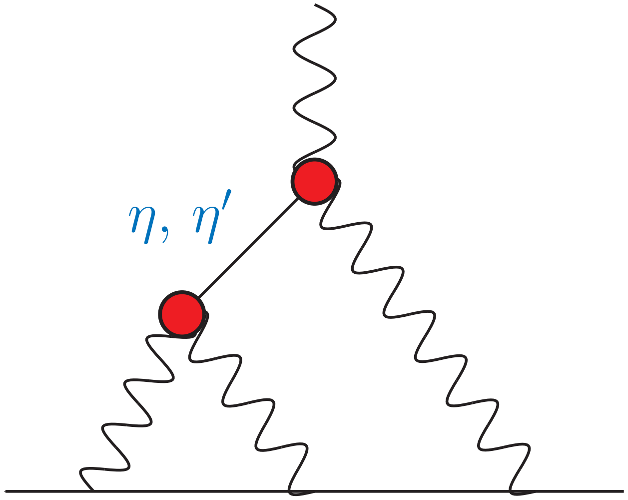
Ingredients for a data-driven analysis of $\boldsymbol{\eta, \eta' \to \gamma^*\gamma^{(*)}}$

$\longrightarrow$ goal: high precision at low energies

- pion vector form factor
- radiative decays: $\eta \to \pi^+\pi^-\gamma$
- crossed-channel dynamics: $\gamma\pi^- \to \pi^-\eta$
- $\eta' \to \pi^+\pi^-\gamma \longrightarrow \gamma^*\gamma$
- towards the *doubly*-virtual form factor:

$$e^+e^- \to \eta\pi^+\pi^-, \quad \eta' \to \pi^+\pi^-\pi^+\pi^-$$

Summary / Outlook



Dispersive analysis of $\pi^0/\eta \to \gamma^*\gamma^*$

- isospin decomposition: cf. previous talk by M. Hoferichter

$$F_{\pi^0\gamma^*\gamma^*}(q_1^2, q_2^2) = F_{vs}(q_1^2, q_2^2) + F_{vs}(q_2^2, q_1^2)$$

$$F_{\eta\gamma^*\gamma^*}(q_1^2, q_2^2) = F_{vv}(q_1^2, q_2^2) + F_{ss}(q_2^2, q_1^2)$$

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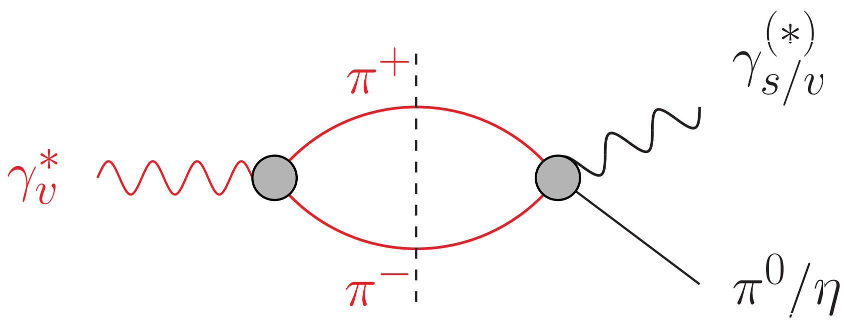
- analyse the leading hadronic intermediate states:

Hanhart et al. 2013, Hoferichter et al. 2014



▷ isovector photon: 2 pions

$\propto$ pion vector form factor $\times$ $\gamma\pi \to \pi\pi$ / $\eta \to \pi\pi\gamma$

all determined in terms of pion–pion P-wave phase shift

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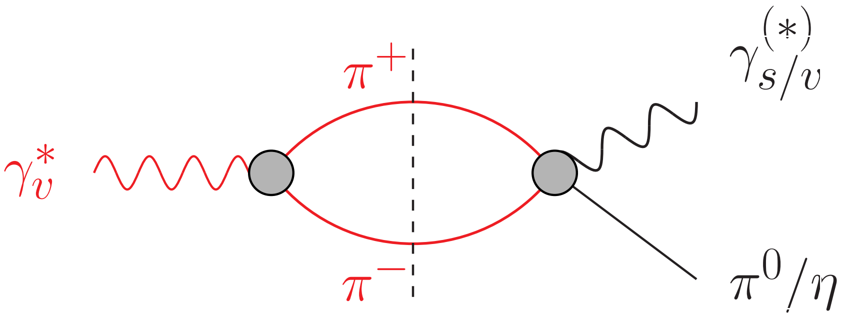

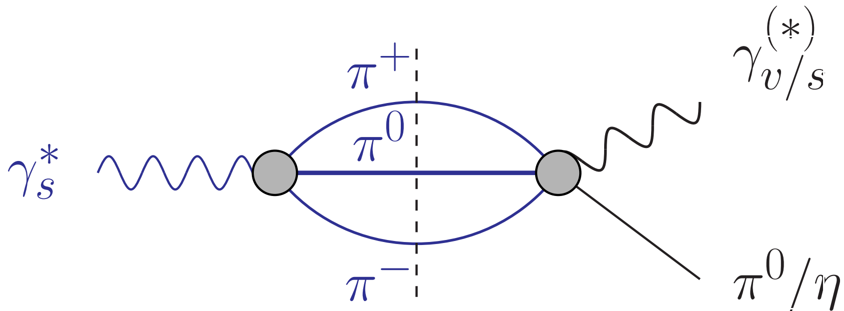

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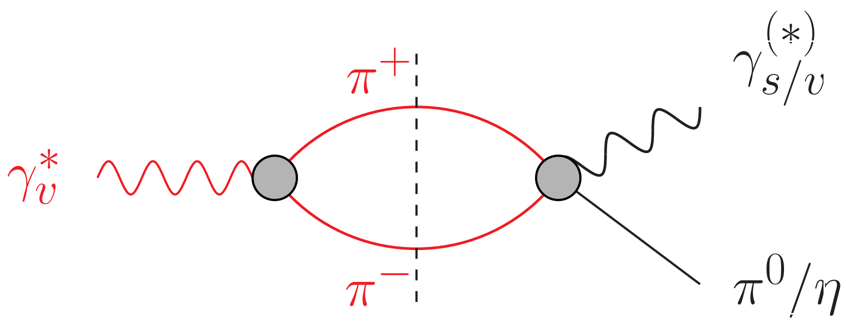
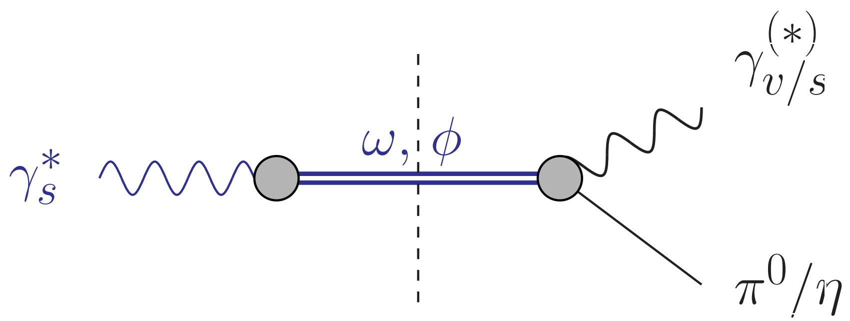




▷ isovector photon: 2 pions

$\propto$ pion vector form factor $\times$ $\gamma\pi \to \pi\pi$ / $\eta \to \pi\pi\gamma$

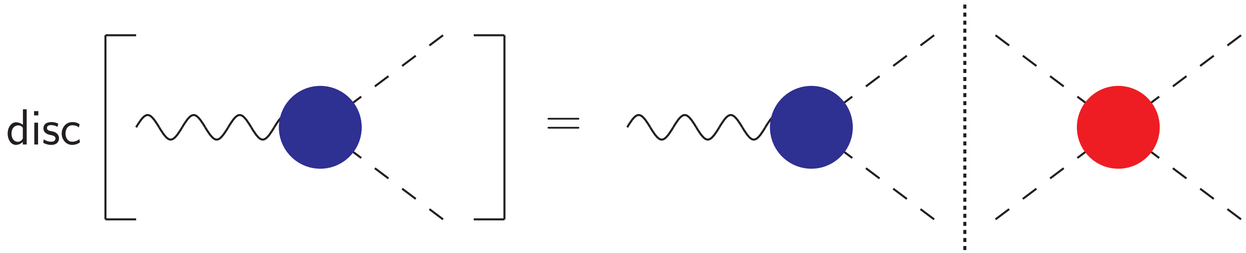
all determined in terms of pion–pion P-wave phase shift

▷ isoscalar photon: 3 pions $\longrightarrow$ dominated by narrow ω, ϕ

$\leftrightarrow \omega/\phi$ transition form factors; very small for the η

Warm-up: pion form factor from dispersion relations

- measured in $e^+e^- \to \pi^+\pi^-$, $\tau^- \to \pi^-\pi^0\nu_\tau$:



$$\text{Im}\, F(s) \;\propto\; F(s) \times \text{phase space} \times T^*_{\pi\pi}(s)$$

$\longrightarrow$ final-state theorem: phase of $F(s)$ is scattering phase $\delta(s)$

Watson 1954

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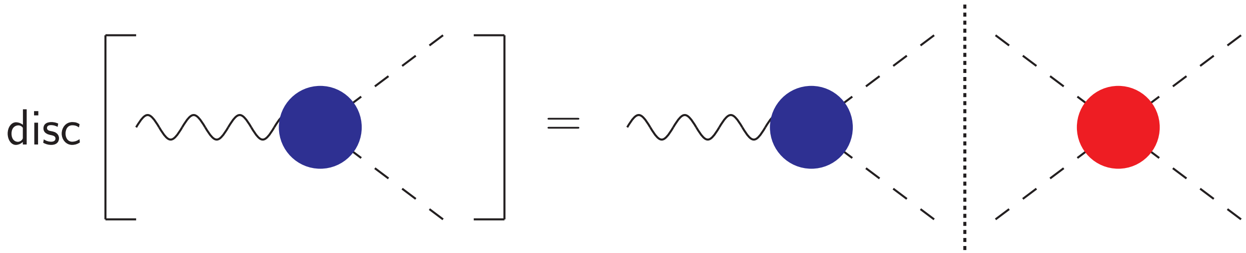

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Watson 1954

- dispersion relations allow to reconstruct form factor from imaginary part $\longrightarrow$ elastic scattering phase $\delta(s)$:

$$F(s) = P(s)\Omega(s) \,, \quad \Omega(s) = \exp\left\{ \frac{s}{\pi} \int_{4M_\pi^2}^{\infty} ds' \frac{\delta(s')}{s'(s'-s)} \right\}$$

$P(s)$ polynomial, $\Omega(s)$ Omnès function

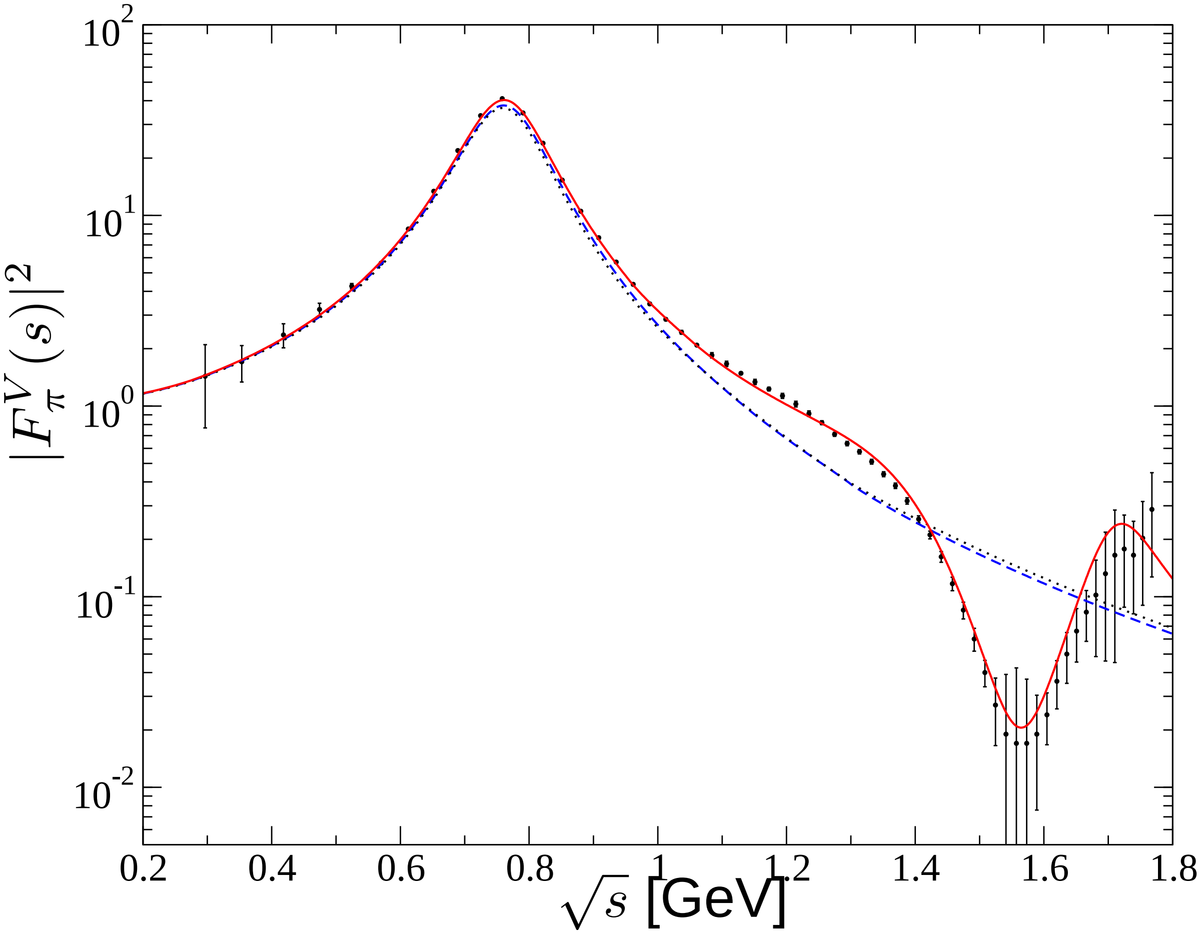
Omnès 1958

- today: high-accuracy $\pi\pi$ phase shifts available

Ananthanarayan et al. 2001, García-Martín et al. 2011

Pion vector form factor vs. Omnès representation

Data on pion form factor in $\tau^- \to \pi^-\pi^0\nu_\tau$ Belle 2008

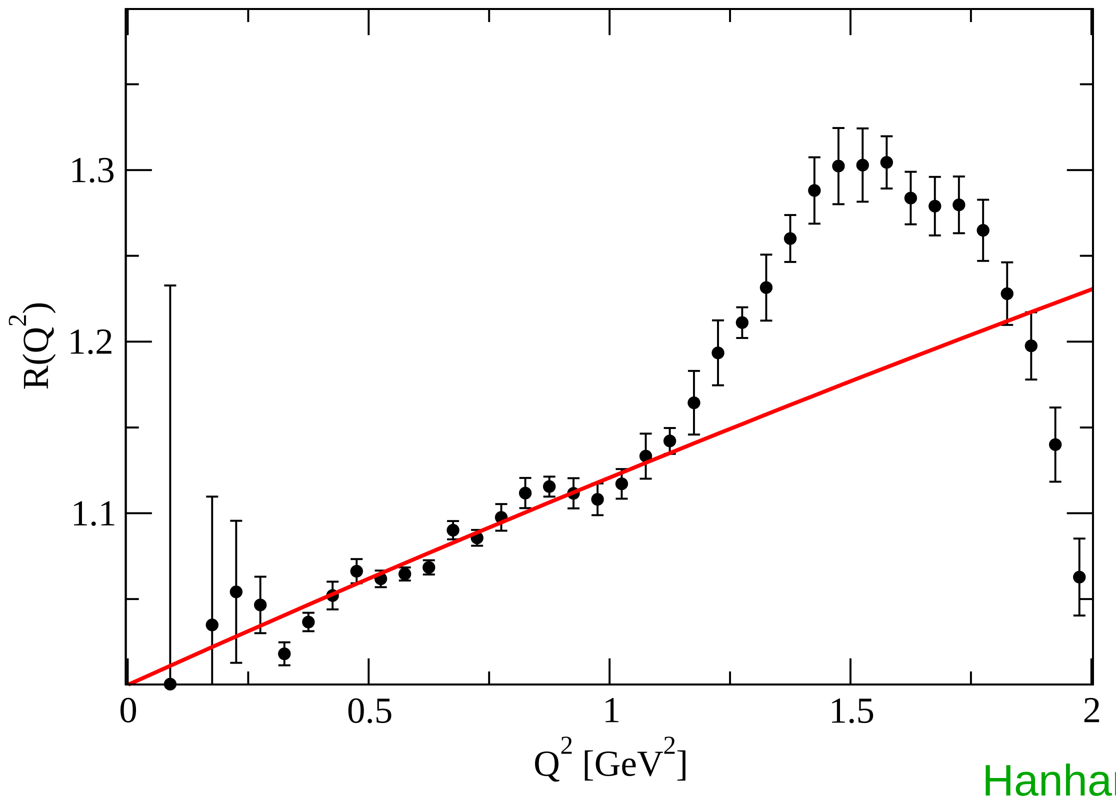


Schneider et al. 2012

Pion vector form factor vs. Omnès representation

Data on pion form factor in $\tau^- \to \pi^-\pi^0\nu_\tau$ Belle 2008

- divide $\tau^- \to \pi^-\pi^0\nu_\tau$ form factor by Omnès function:



Hanhart et al. 2013

$\longrightarrow$ linear below 1 GeV: $F_\pi^V(s) \approx (1 + 0.1\,\mathrm{GeV}^{-2} s)\Omega(s)$

$\longrightarrow$ above: inelastic resonances ρ', ρ''...

Final-state universality: η, $\eta' \to \pi^+\pi^-\gamma$

- $\eta^{(\prime)} \to \pi^+\pi^-\gamma$ driven by the chiral anomaly, $\pi^+\pi^-$ in P-wave
  $\longrightarrow$ final-state interactions the same as for vector form factor
- ansatz: $\mathcal{F}_{\pi\pi\gamma}^{\eta^{(\prime)}} = A \times P(t) \times \Omega(t)$, $P(t) = 1 + \alpha^{(\prime)} t$, $t = M_{\pi\pi}^2$

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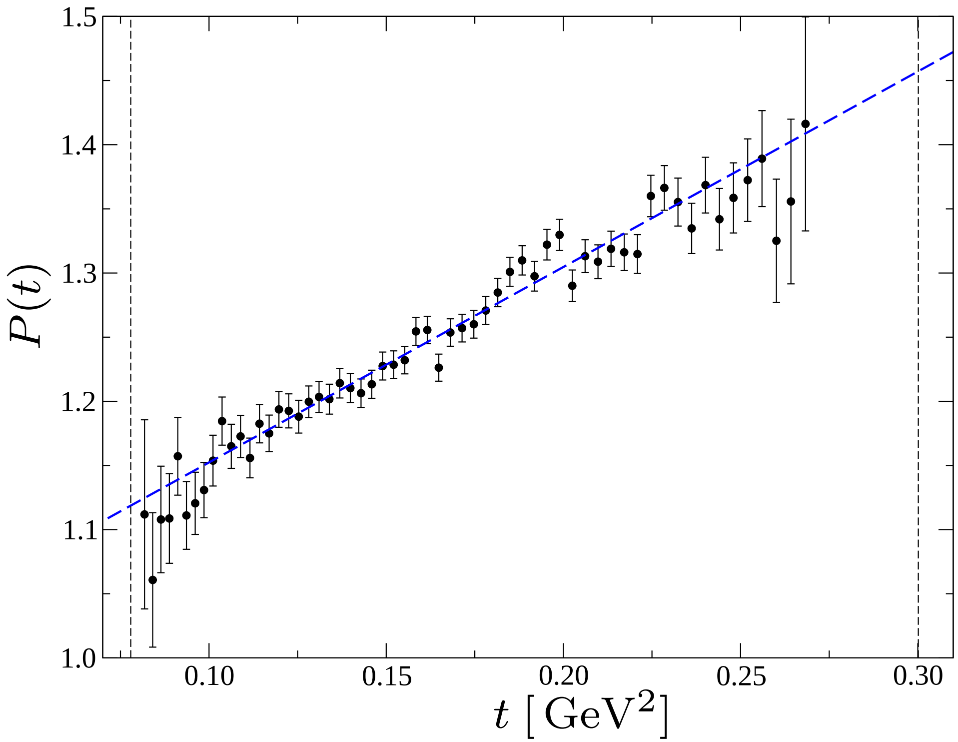
- ansatz: $\mathcal{F}^{\eta^{(\prime)}}_{\pi\pi\gamma} = A \times P(t) \times \Omega(t)$, $P(t) = 1 + \alpha^{(\prime)} t$, $t = M^2_{\pi\pi}$
- divide data by pion form factor $\longrightarrow P(t)$ Stollenwerk et al. 2012

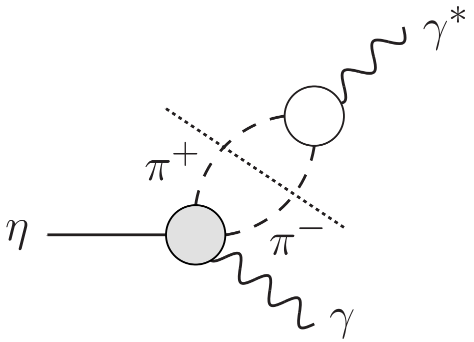


$\longrightarrow$ exp.: $\alpha_{\text{KLOE}} = (1.52 \pm 0.06)$ GeV^{-2} cf. KLOE 2013

Transition form factor $\eta \to \gamma^*\gamma$

Hanhart et al. 2013

$$\bar{F}_{\eta\gamma^*\gamma}(q^2,0) = 1 + \frac{\kappa_\eta q^2}{96\pi^2 F_\pi^2} \int_{4M_\pi^2}^{\infty} ds \sigma(s)^3 P(s) \frac{|F_\pi^V(s)|^2}{s-q^2}$$
$$+ \Delta F^{I=0}_{\eta\gamma^*\gamma}(q^2,0) \; [\longrightarrow \text{VMD}]$$



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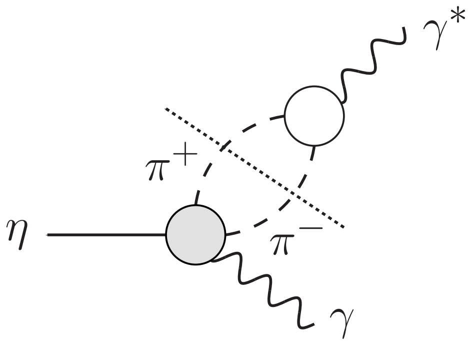
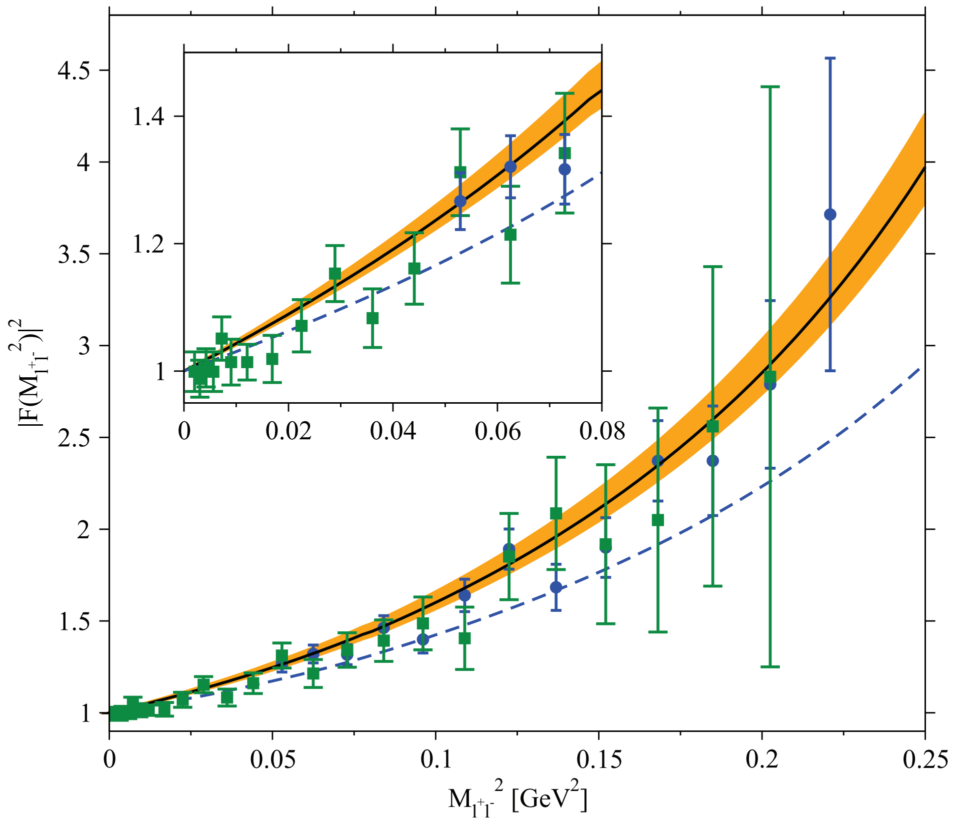




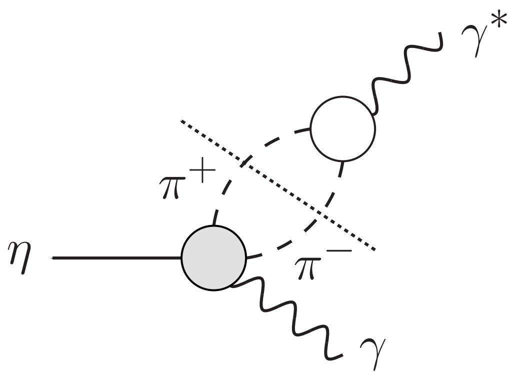
$\longrightarrow$ huge statistical advantage of using hadronic input for $\eta \to \pi^+\pi^-\gamma$ over direct measurement of $\eta \to e^+e^-\gamma$ (rate suppressed by α_{QED}^2)

figure courtesy of C. Hanhart

data: NA60 2011, A2 2014

Anomalous decay $\eta \to \pi^+\pi^-\gamma$

- $\alpha_{\text{KLOE}} = (1.52 \pm 0.06)\,\text{GeV}^{-2}$ large
  $\longrightarrow$ implausible to explain through ρ', ρ''...
- for large t, expect $P(t) \to$ const. rather
- $\eta \to \gamma^*\gamma$ transition form factor:
  $\longrightarrow$ dispersion integral covers
  larger energy range



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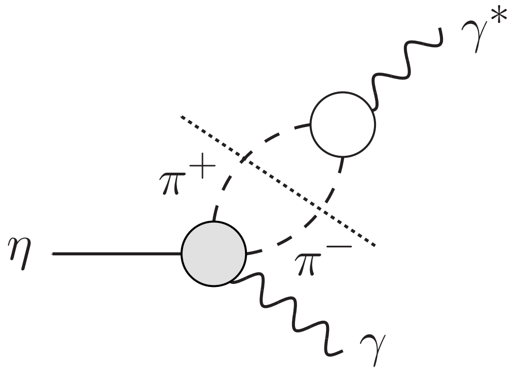


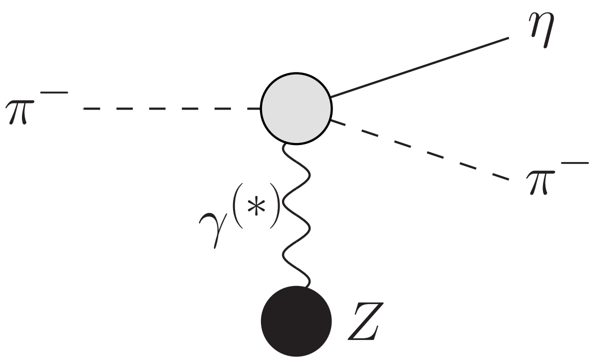
Intriguing observation:

- naive continuation of $\mathcal{F}^{\eta}_{\pi\pi\gamma} = A(1+\alpha t)\Omega(t)$ has zero at $t = -1/\alpha \approx -0.66\,\text{GeV}^2$
  $\longrightarrow$ test this in crossed process $\gamma\pi^- \to \pi^-\eta$
  $\longrightarrow$ "left-hand cuts" in $\pi\eta$ system?

BK, Plenter 2015

Primakoff reaction $\gamma\pi \to \pi\eta$

- can be measured in
  Primakoff reaction COMPASS
- $\pi\eta$ S-wave forbidden
  P-wave exotic: $J^{PC} = 1^{-+}$
  D-wave $a_2(1320)$ first resonance



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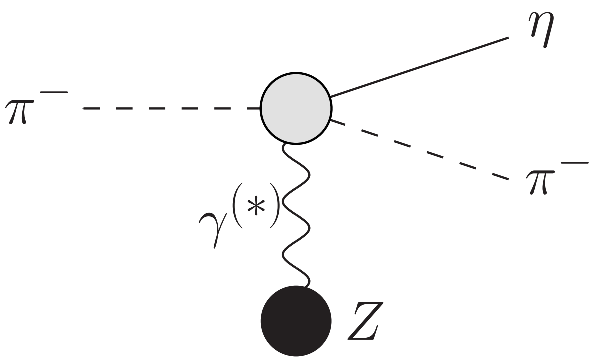

- $\pi\eta$ S-wave forbidden
  P-wave exotic: $J^{PC} = 1^{-+}$
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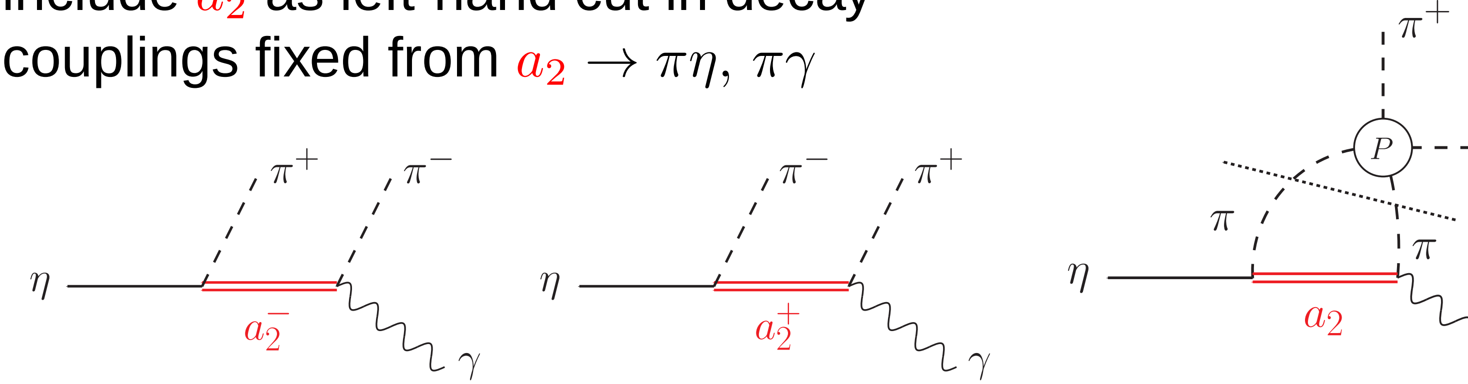
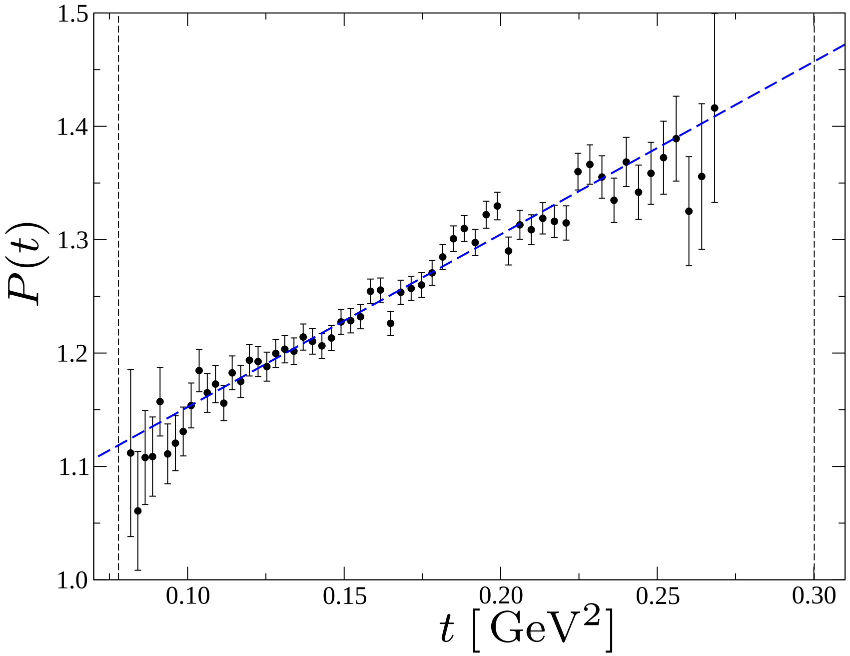
- include a_2 as left-hand cut in decay
  couplings fixed from $a_2 \to \pi\eta,\ \pi\gamma$



  ▷ compatible with decay data?
  ▷ predictions for $\gamma\pi \to \pi\eta$ cross sections and asymmetries
  [$\longrightarrow$ spares] BK, Plenter 2015

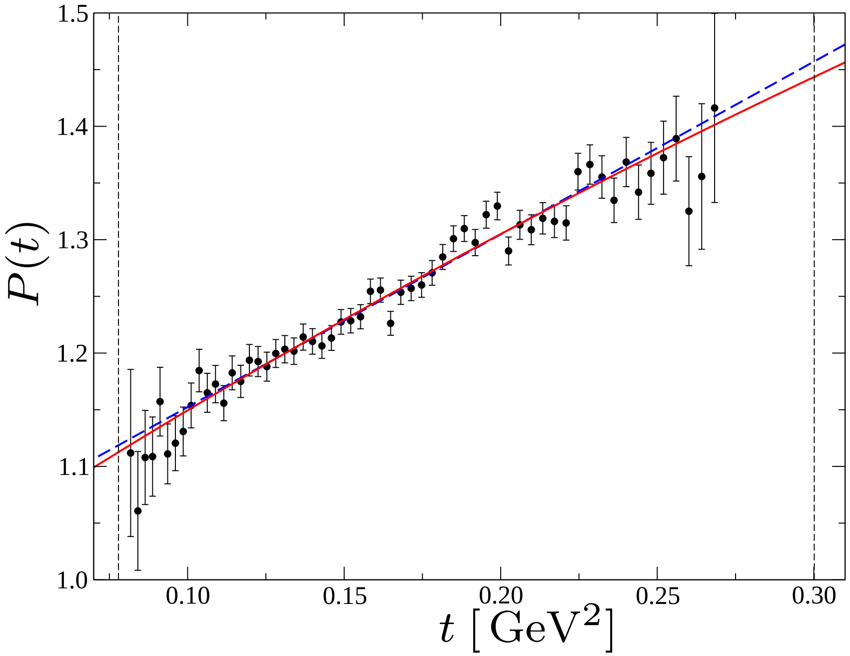
$\eta,\ \eta' \to \pi^+\pi^-\gamma$ with a_2



KLOE 2013

$\alpha = 1.52 \pm 0.06$, $\chi^2/\text{ndof} = 0.94$

$\eta,\ \eta' \to \pi^+\pi^-\gamma$ with a_2



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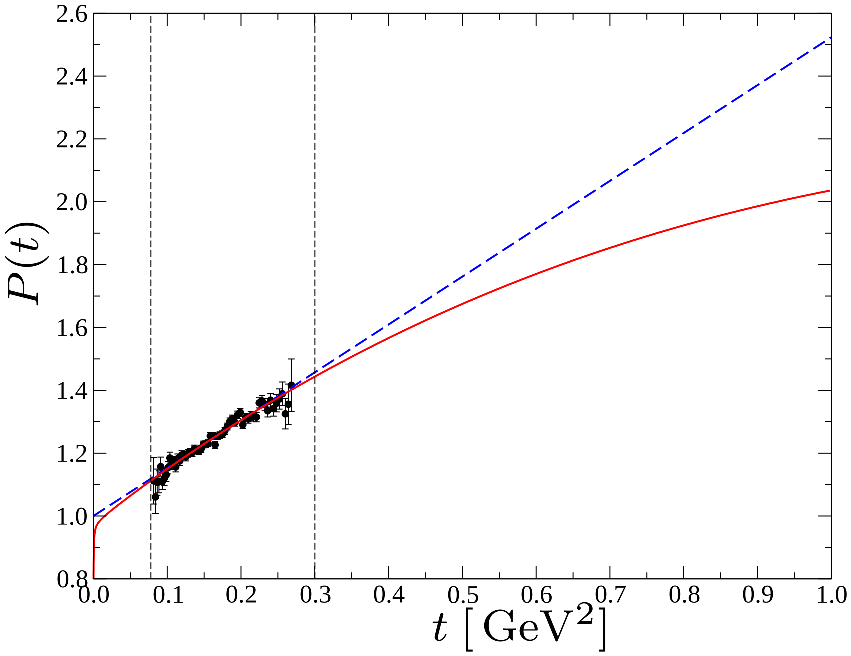
$\alpha = 1.52 \pm 0.06$, χ^2/ndof $= 0.94$

$\longrightarrow \alpha = 1.42 \pm 0.06$, χ^2/ndof $= 0.90$

$\eta,\ \eta' \to \pi^+\pi^-\gamma$ with a_2



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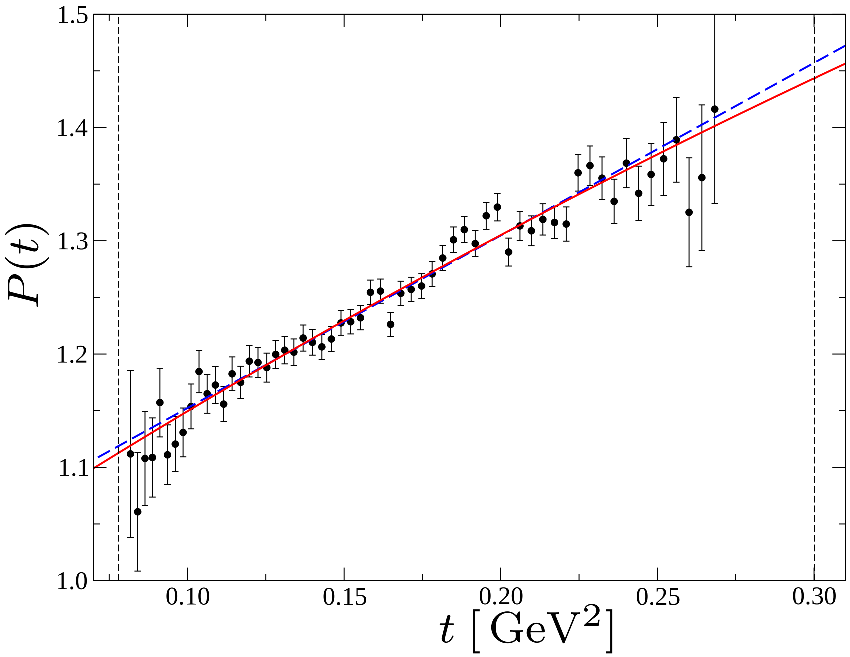
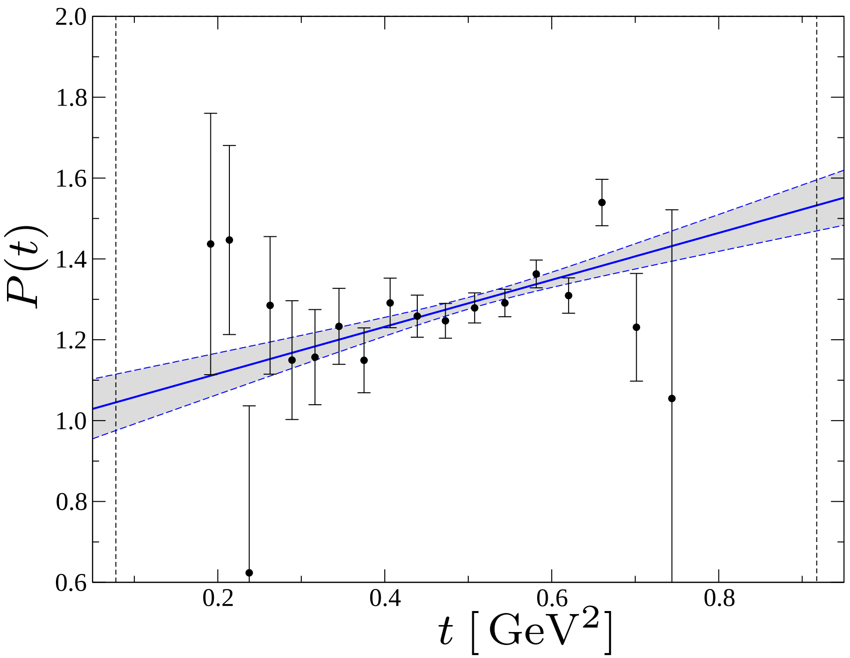
- equally good—why care? sum rule for $\eta \to \gamma^*\gamma$ transition form factor slope reduced by $7 - 8\%$ cf. Hanhart et al. 2013

$\eta,\ \eta' \to \pi^+\pi^-\gamma$ with a_2





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$\longrightarrow \alpha = 1.42 \pm 0.06$, $\chi^2/\text{ndof} = 0.90$

Crystal Barrel 1997

$\alpha' = 0.6 \pm 0.2$, $\chi^2/\text{ndof} = 1.2$

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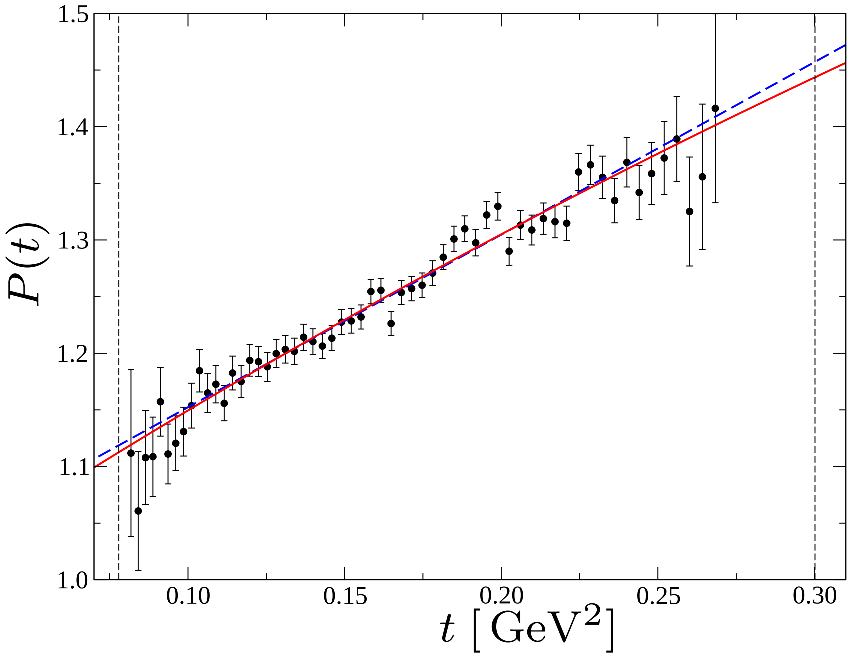
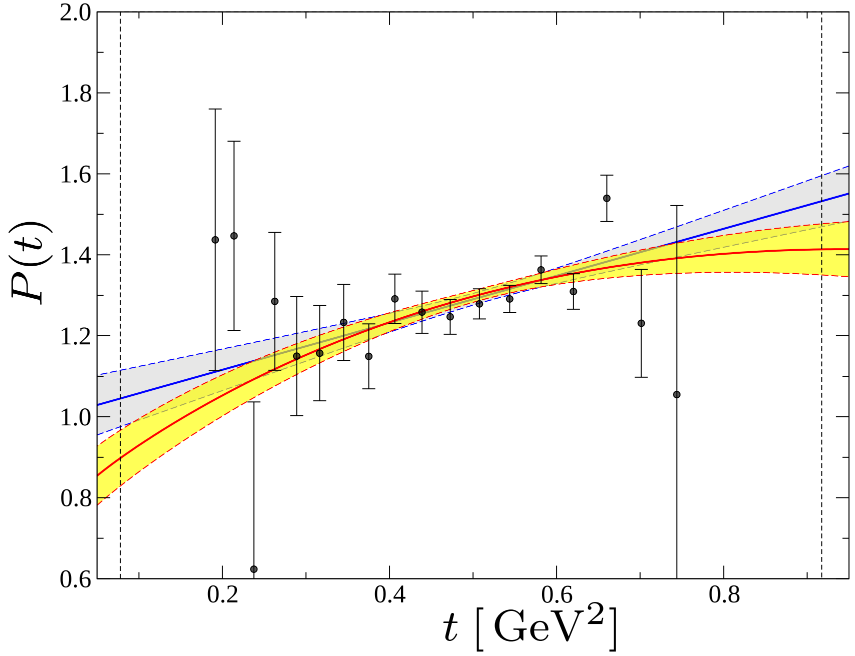
$\eta,\ \eta' \to \pi^+\pi^-\gamma$ with a_2





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$\longrightarrow \alpha = 1.42 \pm 0.06$, χ^2/ndof $= 0.90$

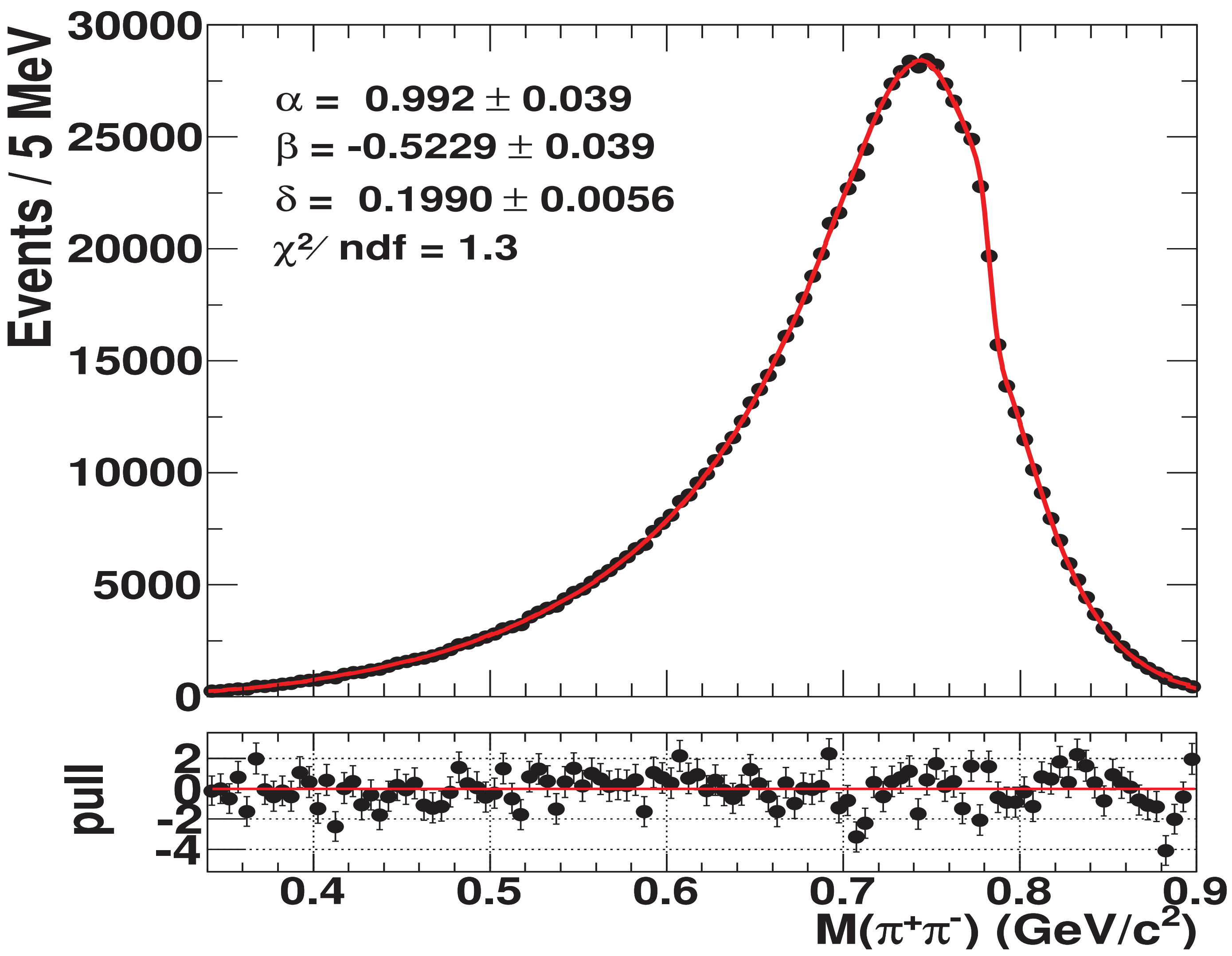
Crystal Barrel 1997

$\alpha' = 0.6 \pm 0.2$, χ^2/ndof $= 1.2$

$\longrightarrow \alpha' = 1.4 \pm 0.4$, χ^2/ndof $= 1.4$

- equally good—why care? sum rule for $\eta \to \gamma^*\gamma$ transition form factor slope reduced by $7-8\%$ cf. Hanhart et al. 2013
- $\alpha \approx \alpha'$ (large-N_c) better fulfilled including a_2 BK, Plenter 2015

New data on $\eta' \to \pi^+\pi^-\gamma$



BESIII preliminary, Fang 2015

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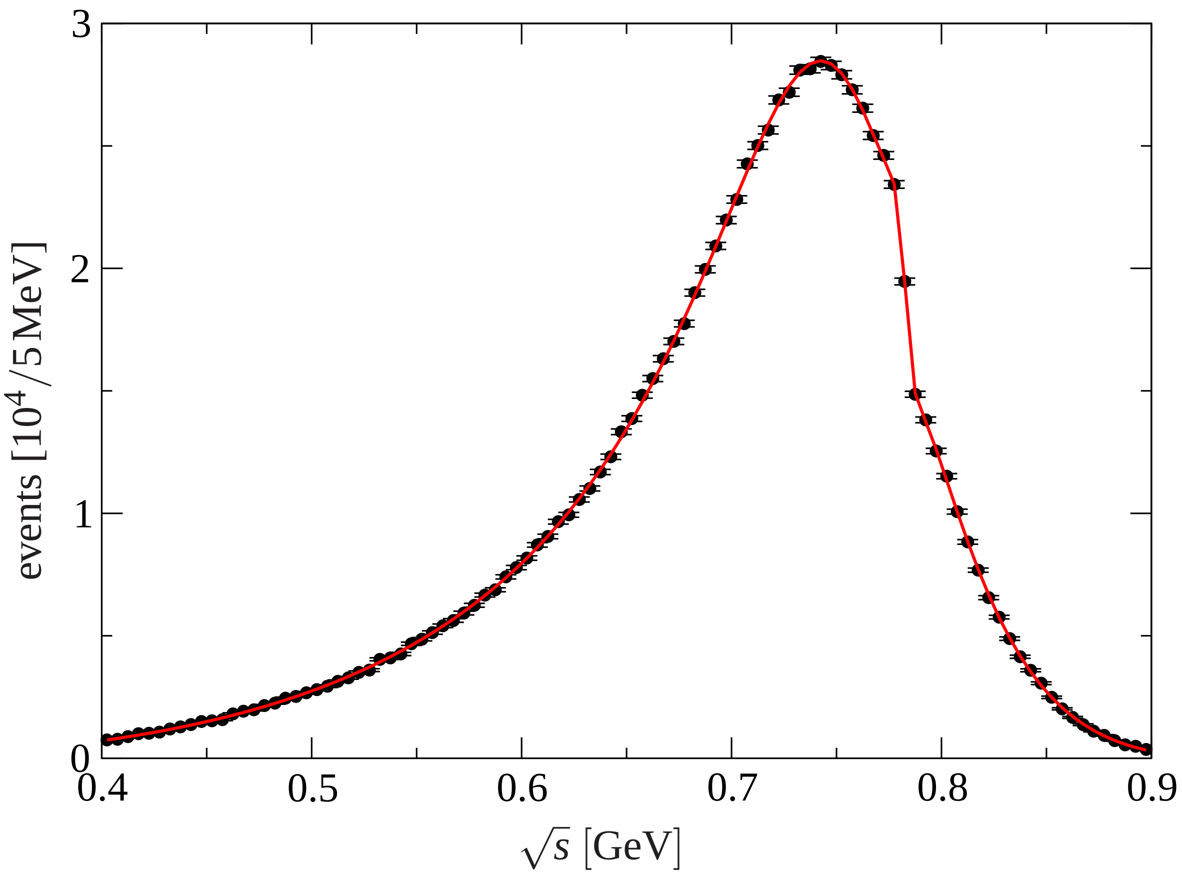


fit to pseudodata after BESIII preliminary

- fit form $$\left[A(1+\alpha t+\beta t^2)+\frac{\kappa}{m_\omega^2-t-im_\omega\Gamma_\omega}\right]\times\Omega(t)$$

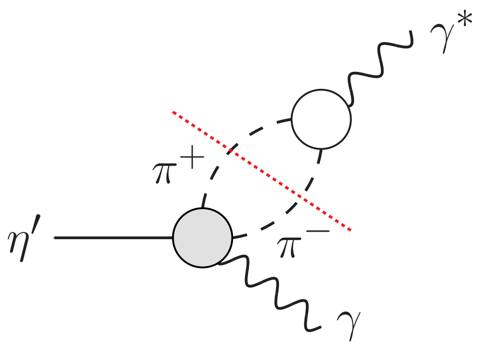
$\longrightarrow$ curvature $\propto \beta t^2$ essential (smaller than a_2 prediction)

$\longrightarrow$ even ρ–ω mixing clearly visible

Hanhart et al. 2017

Prediction for η' transition form factor

- isovector: combine high-precision data on $\eta' \to \pi^+\pi^-\gamma$ and $e^+e^- \to \pi^+\pi^-$
- isoscalar: VMD, couplings fixed from $\eta' \to \omega\gamma$ and $\phi \to \eta'\gamma$



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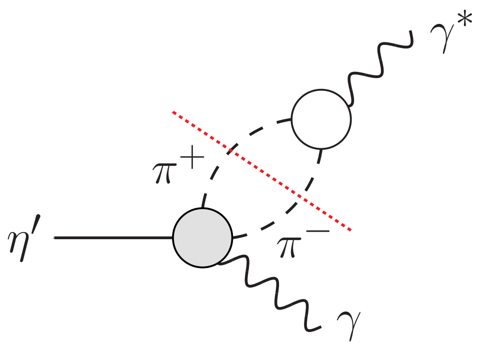
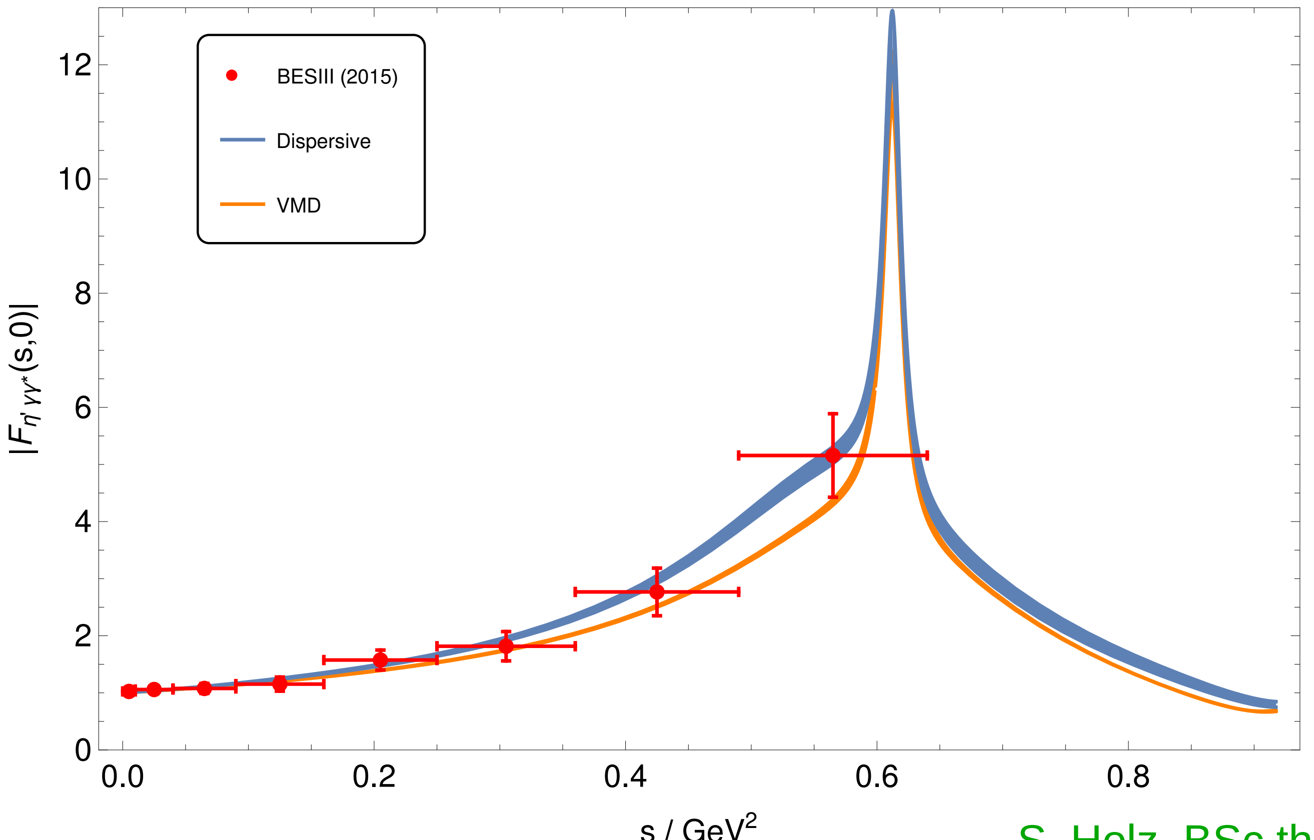
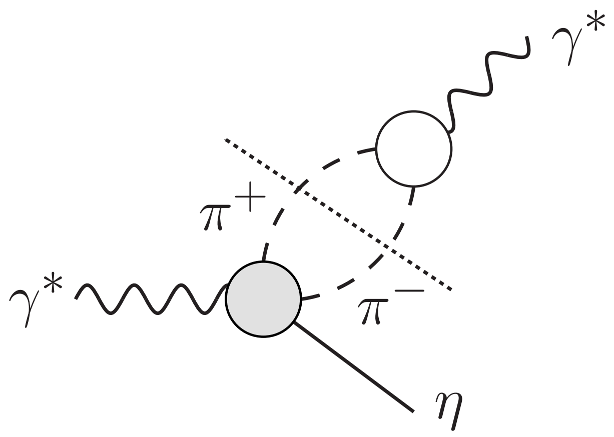




S. Holz, BSc thesis 2016

How to go *doubly* virtual? — $e^+e^- \to \eta\pi^+\pi^-$

- idea (again): beat α_{QED}^2 suppression of $e^+e^- \to \eta e^+e^-$ by measuring $e^+e^- \to \eta\pi^+\pi^-$ instead



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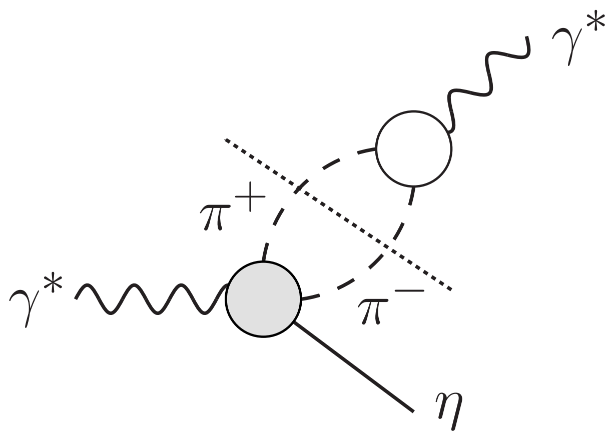


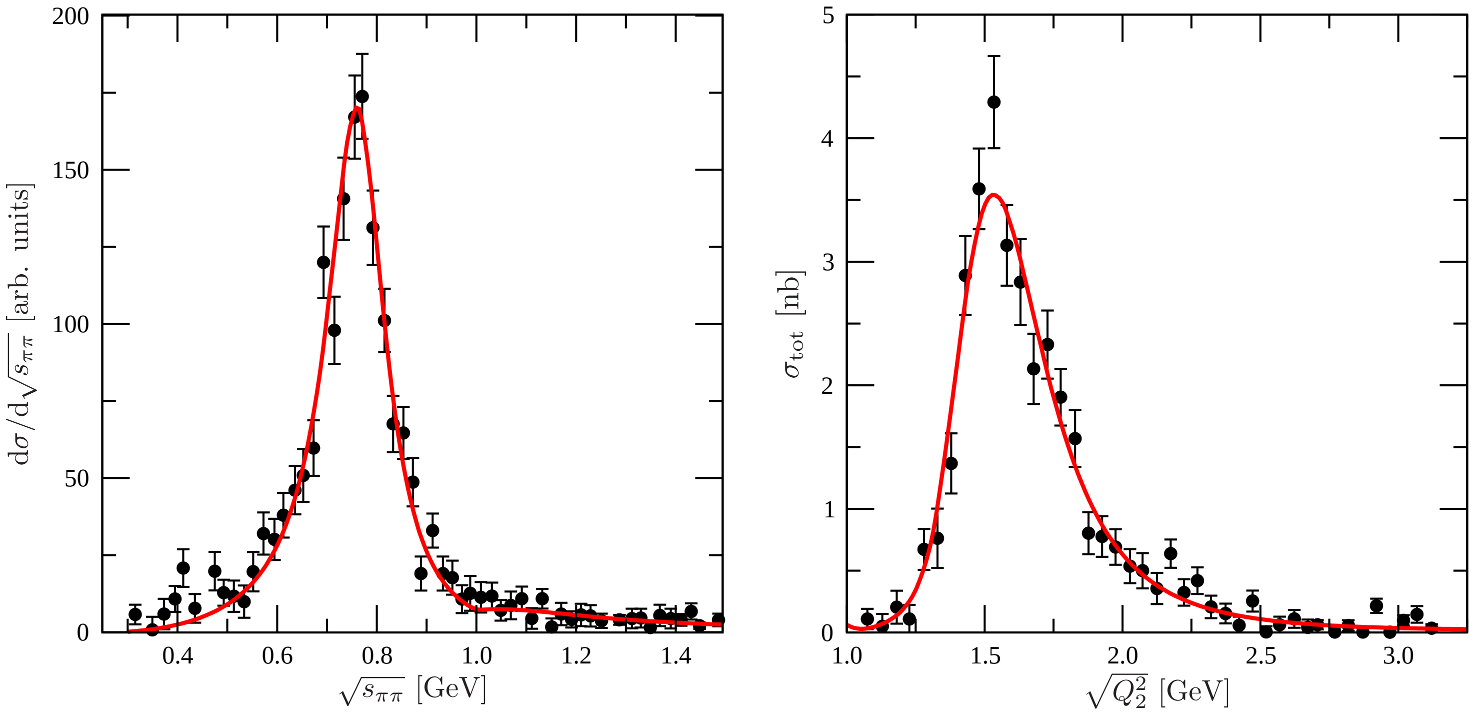
- test factorisation hypothesis in $e^+e^- \to \eta\pi^+\pi^-$:

$$F_{\eta\pi\pi\gamma^*}(s_{\pi\pi}, Q_2^2) \stackrel{!?}{=} F_{\eta\pi\pi\gamma}(s_{\pi\pi}) \times F_{\eta\gamma\gamma^*}(Q_2^2)$$

  - ▷ allow same form for $F_{\eta\pi\pi\gamma}(s_{\pi\pi})$ as in $\eta \to \pi^+\pi^-\gamma$
  - ▷ fit subtractions to $\pi^+\pi^-$ distribution in $e^+e^- \to \eta\pi^+\pi^-$
    $\longrightarrow$ are they compatible to the ones in $\eta \to \pi^+\pi^-\gamma$?
  - ▷ parametrise $F_{\eta\gamma\gamma^*}(Q_2^2)$ by sum of Breit–Wigners (ρ, ρ')

Xiao et al. (preliminary)

How to go *doubly* virtual? — $e^+e^- \to \eta\pi^+\pi^-$



$$\frac{d\sigma}{d\sqrt{s_{\pi\pi}}}$$

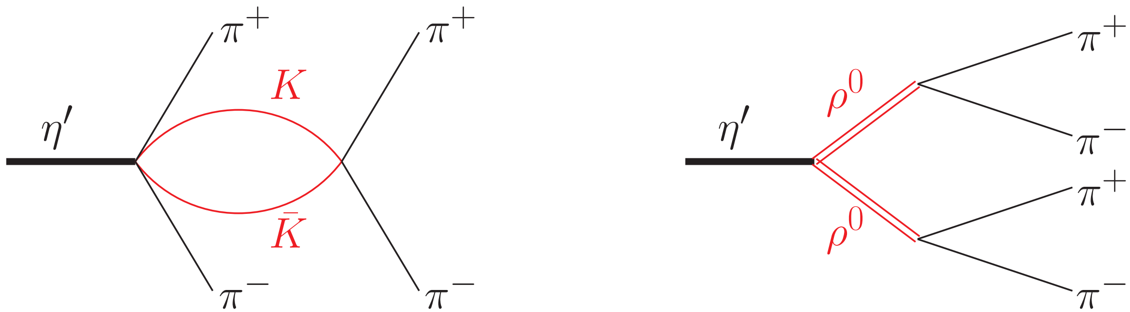
$$\sigma_{\text{tot}}(Q_2^2)$$

Xiao et al. (preliminary); data: BaBar 2007

- $d\sigma/d\sqrt{s_{\pi\pi}}$ integrated over $1\,\text{GeV} \leq \sqrt{Q_2^2} \leq 4.5\,\text{GeV}$
- factorisation seems to work only if a_2 contribution retained
- more differential/binned data highly desirable!

How to go *doubly* virtual? — $\eta' \to \pi^+\pi^-\pi^+\pi^-$

- prediction of $\eta' \to 4\pi$ branching ratios based on ChPT + VMD:



$\longrightarrow$ $\mathcal{B}(\eta' \to \pi^+\pi^-\pi^+\pi^-) = (10 \pm 3) \times 10^{-5}$ Guo, BK, Wirzba 2012

exp: $\mathcal{B}(\eta' \to \pi^+\pi^-\pi^+\pi^-) = (8.5 \pm 0.7 \pm 0.6) \times 10^{-5}$ BESIII 2014

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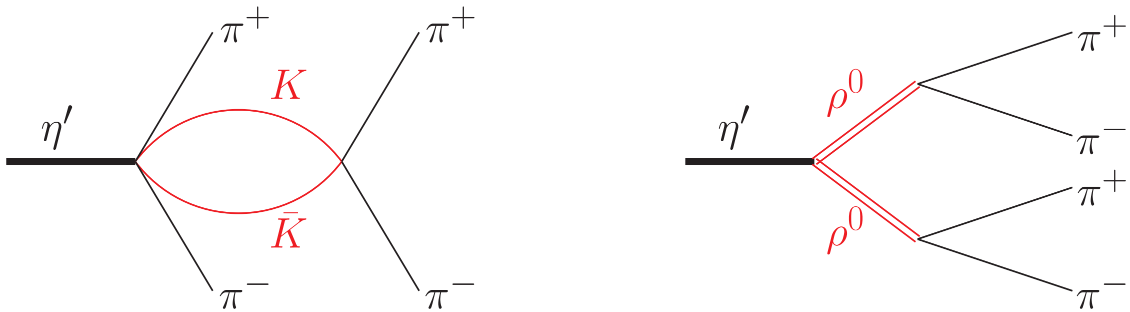

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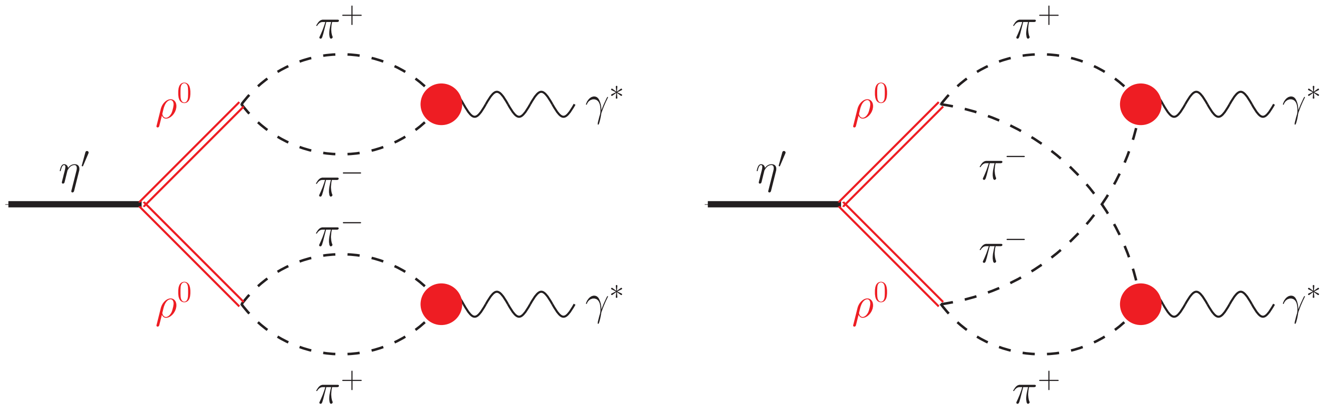
- start analysis of *doubly virtual* η' transition form factor from here?



factorising | non-factorising

$\longrightarrow$ more differential info on $\eta' \to \pi^+\pi^-\pi^+\pi^-$ highly desirable!

Summary / Outlook

Dispersive analyses of $\eta^{(\prime)}$ transition form factors:

- high-precision data on $\eta \to \pi^+\pi^-\gamma$ KLOE and $\eta' \to \pi^+\pi^-\gamma$ BESIII allow for high-precision dispersive predictions of $\eta^{(\prime)} \to \gamma\gamma^*$
- not discussed here: dispersive continuation of transition form factors to spacelike virtualities see M. Hoferichter for π^0

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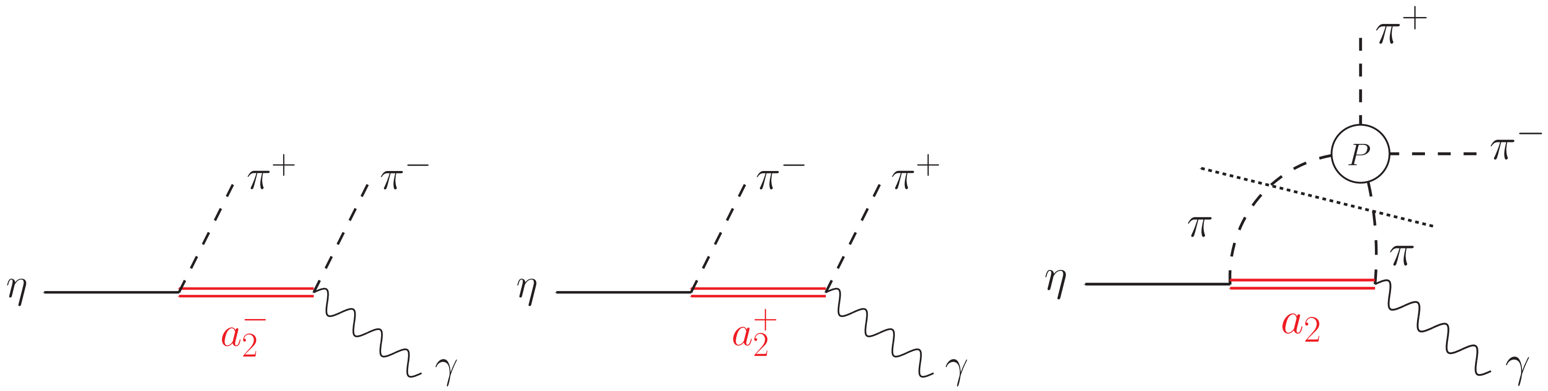
Further useful experimental input (mainly for doubly virtual):

- Primakoff reaction $\gamma\pi \to \pi\eta$ — COMPASS
- $e^+e^- \to \eta\pi^+\pi^-$ differential data — Xiao et al., in progress
- given $\eta' \to \pi^+\pi^-\gamma$ — can you do $\eta' \to \pi^+\pi^-e^+e^-$ with precision?
- more detailed data on $\eta' \to \pi^+\pi^-\pi^+\pi^-$? — Plenter et al., in progress

$\longrightarrow$ determine η, η' pole contributions to HLbL
with controlled uncertainty

Spares

Formalism including left-hand cuts



- a_2 + rescattering essential to preserve Watson's theorem
- formally:

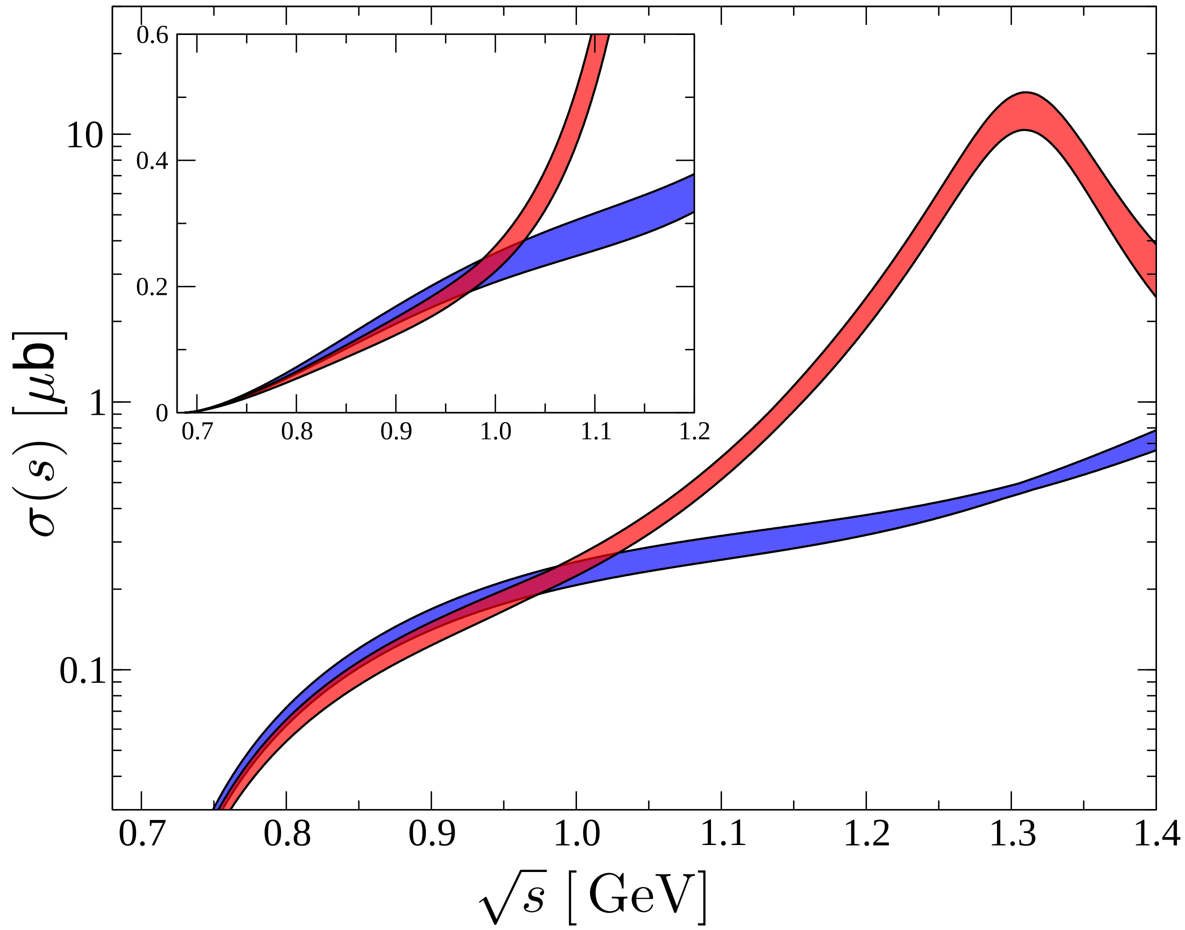
$$\mathcal{F}^{\eta}_{\pi\pi\gamma}(s,t,u) = \mathcal{F}(t) + \mathcal{G}_{a_2}(s,t,u) + \mathcal{G}_{a_2}(u,t,s)$$

$$\mathcal{F}(t) = \Omega(t)\left\{ A(1+\alpha t) + \frac{t^2}{\pi}\int_{4M_\pi^2}^{\infty}\frac{dx}{x^2}\frac{\sin\delta(x)\hat{\mathcal{G}}(x)}{|\Omega(x)|(x-t)}\right\}$$

$\hat{\mathcal{G}}$: t-channel P-wave projection of a_2 exchange graphs

- re-fit subtraction constants A, α

Total cross section $\gamma\pi \to \pi\eta$

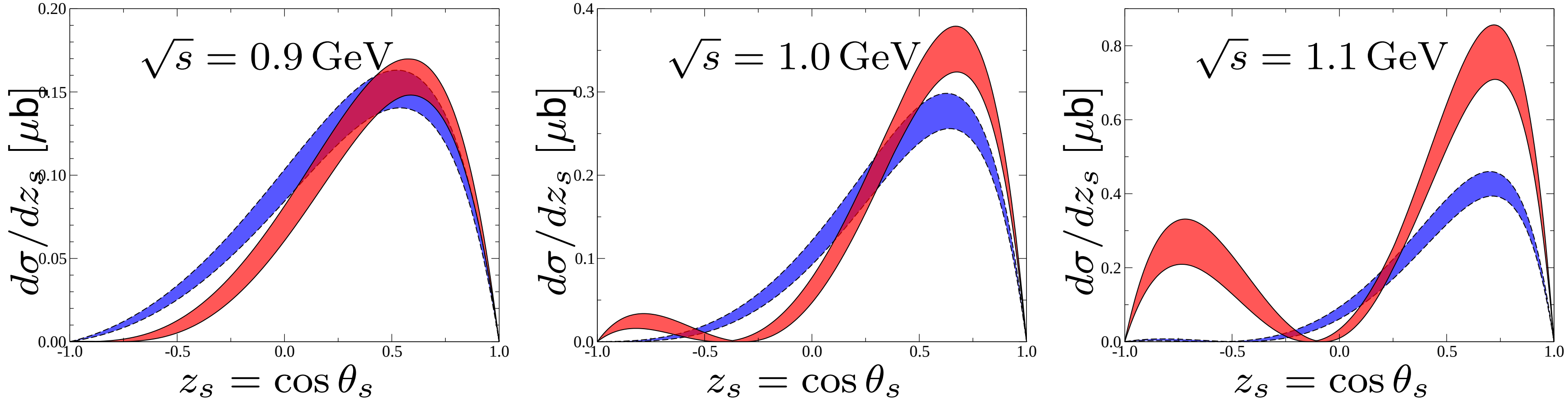


blue: t-channel dynamics / "ρ" only   red: full amplitude

- t-channel dynamics dominate below $\sqrt{s} \approx 1\,\mathrm{GeV}$
- uncertainty bands: $\Gamma(\eta \to \pi^+\pi^-\gamma)$, α, a_2 couplings BK, Plenter 2015

Differential cross sections $\gamma\pi \to \pi\eta$

- amplitude zero visible in differential cross sections:



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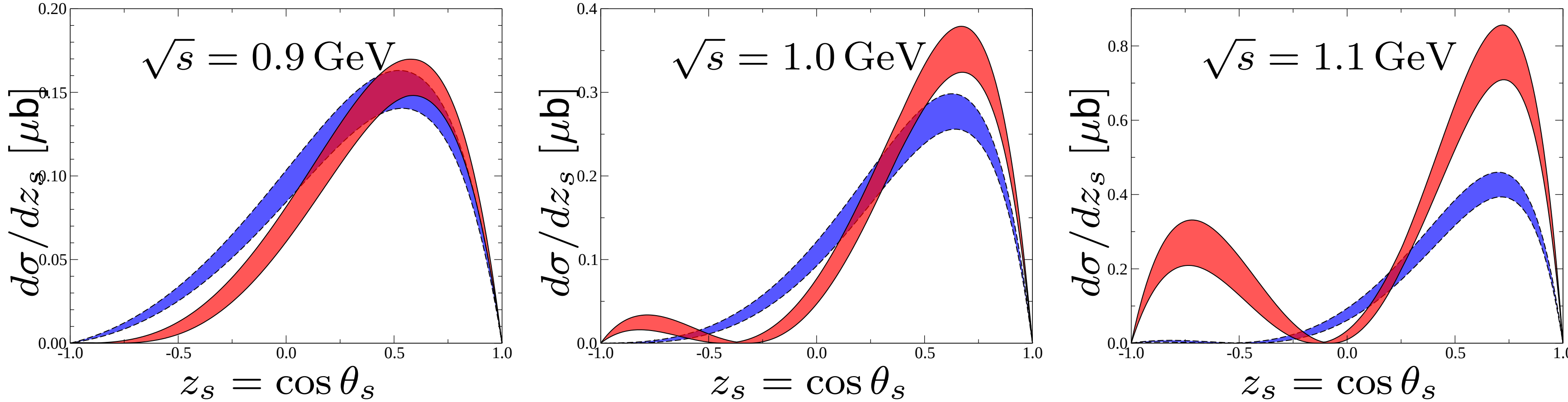

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- strong P-D-wave interference
- can be expressed as forward-backward asymmetry

$$A_{\text{FB}} = \frac{\sigma(\cos\theta > 0) - \sigma(\cos\theta < 0)}{\sigma_{\text{total}}}$$