# Pseudoscalar poles in HLbL in $g-2$

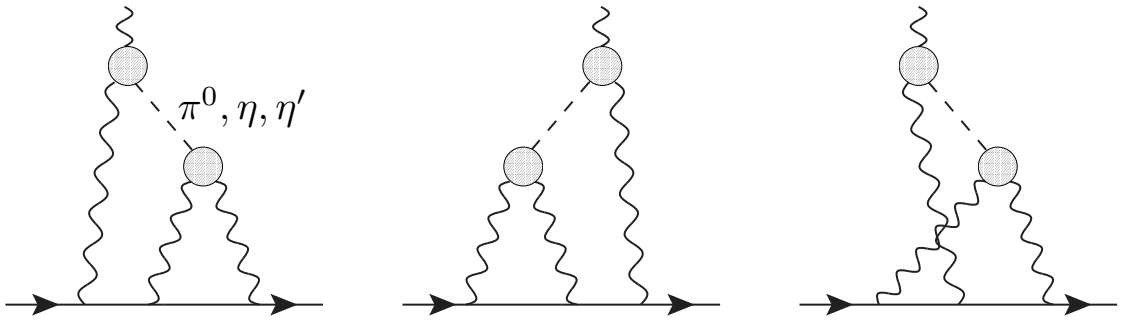
Andreas Nyffeler

PRISMA Cluster of Excellence, Institut für Kernphysik,
Helmholtz-Institut Mainz
Johannes Gutenberg Universität Mainz, Germany
nyffeler@kph.uni-mainz.de

First Workshop of the Muon $g-2$ Theory Initiative
Q Center, St. Charles (near Fermilab), USA, June 3-6, 2017

# Pion-pole contribution to $a_\mu^{\mathrm{HLbL};\pi^0}$ (analogously for $\eta, \eta'$)

[Jegerlehner + Nyffeler '09]




$$a_\mu^{\mathrm{HLbL};\pi^0} = \left(\frac{\alpha_e}{\pi}\right)^3 \left(a_\mu^{\mathrm{HLbL};\pi^0(1)} + a_\mu^{\mathrm{HLbL};\pi^0(2)}\right)$$

where

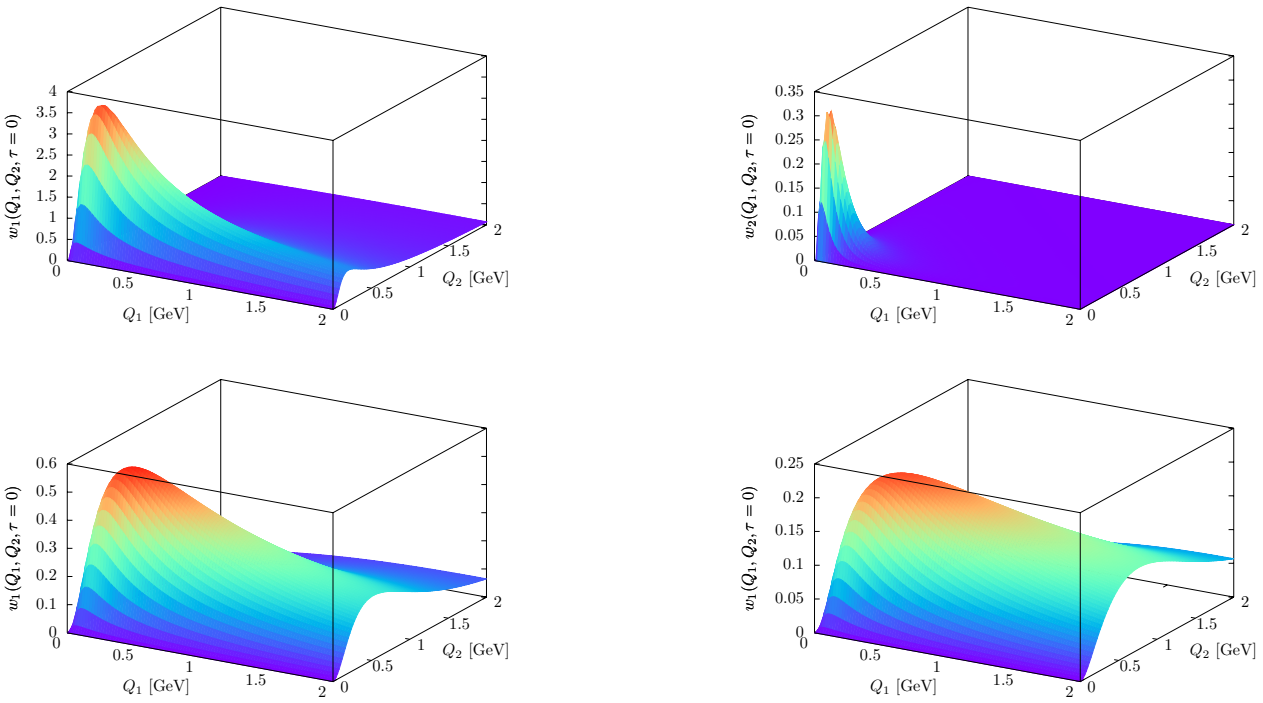
$$a_\mu^{\mathrm{HLbL};\pi^0(1)} = \int_0^\infty dQ_1 \int_0^\infty dQ_2 \int_{-1}^1 d\tau \; w_1(Q_1, Q_2, \tau) \; \mathcal{F}_{\pi^0\gamma^*\gamma^*}(-Q_1^2, -(Q_1+Q_2)^2) \; \mathcal{F}_{\pi^0\gamma^*\gamma^*}(-Q_2^2, 0)$$

$$a_\mu^{\mathrm{HLbL};\pi^0(2)} = \int_0^\infty dQ_1 \int_0^\infty dQ_2 \int_{-1}^1 d\tau \; w_2(Q_1, Q_2, \tau) \; \mathcal{F}_{\pi^0\gamma^*\gamma^*}(-Q_1^2, -Q_2^2) \; \mathcal{F}_{\pi^0\gamma^*\gamma^*}(-(Q_1+Q_2)^2, 0)$$

$w_{1,2}(Q_1, Q_2, \tau)$ are model-independent weight functions which are concentrated at small momenta below 1 $\mathrm{GeV}$ [Nyffeler '16].
3-dim. integration over lengths $Q_i = |(Q_i)_\mu|, i = 1, 2$ of the two Euclidean momenta and angle $\theta$ between them $Q_1 \cdot Q_2 = Q_1 Q_2 \cos\theta$ with $\tau = \cos\theta$.

# Relevant momentum regions and results in literature

Weight functions $w_i$ (Nyffeler '16):




Top: weight functions $w_{1,2}(Q_1, Q_2, \tau)$ for $\pi^0$ with $\theta = 90°(\tau = 0)$.
Bottom: weight functions $w_1(Q_1, Q_2, \tau)$ for $\eta$ (left) and $\eta'$ (right).

- Relevant momentum regions below 1 GeV for $\pi^0$, below 1.5 GeV for $\eta, \eta'$.
- Most calculations for neutral pion and all light pseudoscalars (poles or exchanges) agree at level of 15%, but full range of estimates (central values) much larger:

$$a_\mu^{\mathrm{HLbL};\pi^0} = (50-80)\times 10^{-11} = (65 \pm 15)\times 10^{-11} \quad (\pm 23\%)$$

$$a_\mu^{\mathrm{HLbL};\mathrm{P}} = (59-114)\times 10^{-11} = (87 \pm 27)\times 10^{-11} \quad (\pm 31\%)$$

# Pseudoscalar poles / exchanges in HLbL: old and new results

| Model for $\mathcal{F}_{P^{(*)}\gamma^*\gamma^*}$ | $a_\mu(\pi^0) \times 10^{11}$ | $a_\mu(\pi^0, \eta, \eta') \times 10^{11}$ |
|---|---|---|
| VMD / HLS (off-shell ?) [HKS, HK] | 57( 4 ) | 83( 6 ) |
| ENJL (modified) (off-shell ?) [BPP] | 59( 9 ) | 85( 13 ) |
| LMD+V ($h_2 = -10$ GeV$^2$) [KN] | 63( 10 ) | 88( 12 ) |
| LMD+V (constant FF at ext. vertex) [MV] | 77( 7 ) | 114( 10 ) |
| LMD+V (off-shell) [N] | 72( 12 ) | 99( 16 ) |
| Rational approximants [Talk Sanchez-Puertas] | 63.6(2.7) | 94.3(5.3) |
| Lattice QCD (fitted LMD+V) [Talk Gérardin] | 65.0(8.3) | – |
| Dyson-Schwinger Equations [Talk Roig] | 62.6(0.8) | – |

HKS, HK = Hayakawa, Kinoshita, Sanda '95, '96; Hayakawa, Kinoshita '98, '02
BPP = Bijnens, Pallante, Prades '95, '96, '02
KN = Knecht, Nyffeler '02
MV = Melnikov, Vainshtein '04
N = Nyffeler '09

Note: BPP, KN, MV, N use VMD for $\eta, \eta'$

Talk Gérardin: statistical error only

# HLbL scattering anno 2009: Glasgow consensus "versus" JN '09

Some selected results for the various contributions to $a_\mu^{\mathrm{HLbL}} \times 10^{11}$:

| Contribution | BPP | HKS, HK | KN | MV | BP, MdRR | PdRV | N, JN |
|---|---|---|---|---|---|---|---|
| $\pi^0$, $\eta$, $\eta'$ | 85±13 | 82.7±6.4 | 83±12 | 114±10 | – | 114±13 | 99 ± 16 |
| axial vectors | 2.5±1.0 | 1.7±1.7 | – | 22±5 | – | 15±10 | 22±5 |
| scalars | −6.8±2.0 | – | – | – | – | −7±7 | −7±2 |
| $\pi$, $K$ loops | −19±13 | −4.5±8.1 | – | – | – | −19±19 | −19±13 |
| $\pi$, $K$ loops +subl. $N_C$ | – | – | – | 0±10 | – | – | – |
| quark loops | 21±3 | 9.7±11.1 | – | – | – | 2.3 (c-quark) | 21±3 |
| Total | 83±32 | 89.6±15.4 | 80±40 | 136±25 | 110±40 | 105 ± 26 | 116 ± 39 |

BPP = Bijnens, Pallante, Prades '95, '96, '02; HKS = Hayakawa, Kinoshita, Sanda '95, '96; HK = Hayakawa, Kinoshita '98, '02; KN = Knecht, Nyffeler '02; MV = Melnikov, Vainshtein '04; BP = Bijnens, Prades '07; MdRR = Miller, de Rafael, Roberts '07; PdRV = Prades, de Rafael, Vainshtein '09; N = Nyffeler '09, JN = Jegerlehner, Nyffeler '09;

- Pseudoscalar-exchanges dominate numerically. Other contributions not negligible. Cancellation between $\pi$, $K$-loops and quark loops !
- PdRV: Do not consider dressed light quark loops as separate contribution ! Assume it is already taken into account by using short-distance constraint of MV '04 on pseudoscalar-pole contribution (no form factor at external vertex). Added all errors in quadrature !
- N, JN: New evaluation of pseudoscalars. Took over most values from BPP, except axial vectors from MV. Added all errors linearly. As done before in BPP.