Healthy degenerate theories with arbitrary higher-order derivatives

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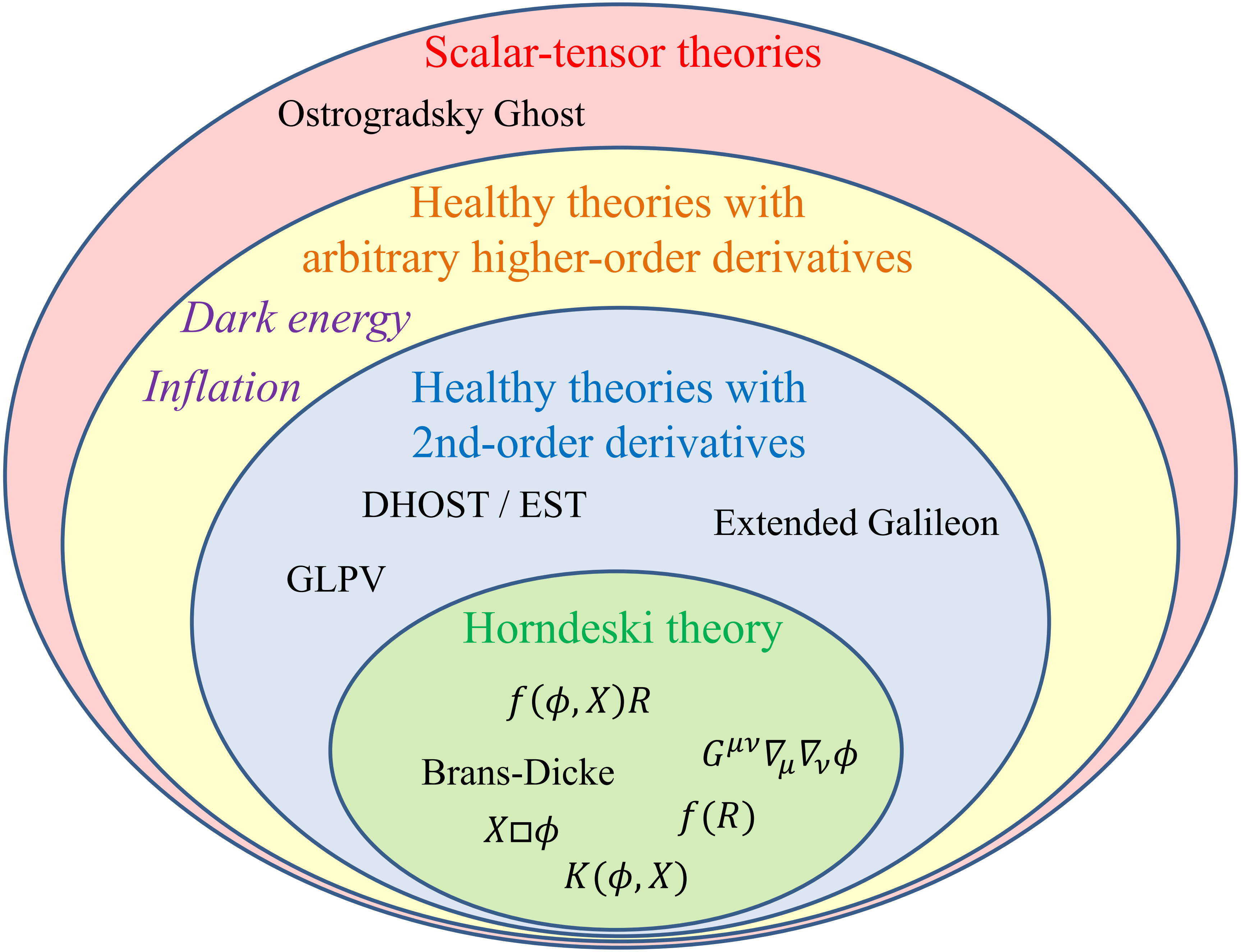

Scalar-tensor theories
Ostrogradsky Ghost
Healthy theories with
arbitrary higher-order derivatives
Dark energy
Inflation
Healthy theories with
2nd-order derivatives
DHOST / EST
Extended Galileon
GLPV
Horndeski theory
$f(\phi, X)R$
$G^{\mu\nu}\nabla_\mu\nabla_\nu\phi$
Brans-Dicke
$f(R)$
$X\Box\phi$
$K(\phi, X)$

Ostrogradsky ghost for $L = \frac{1}{2}\ddot{\phi}^2$ $\phi = \phi(t)$

- 4th order Euler-Lagrange eq

$$\ddddot{\phi} = 0$$

1 DOF = 2 ini conds

requires 4 initial conditions = 2 DOF.

$$\Phi \equiv \frac{\phi - \psi}{\sqrt{2}}, \Psi \equiv \frac{\phi + \psi}{\sqrt{2}}$$

- Hamiltonian is unbounded

$$L = \ddot{\phi}\psi - \frac{1}{2}\psi^2 = -\dot{\phi}\dot{\psi} - \frac{1}{2}\psi^2$$

$$= \frac{1}{2}\dot{\Phi}^2 - \frac{1}{2}\dot{\Psi}^2 - \frac{1}{4}(\Phi - \Psi)^2$$

$$H = \frac{1}{2}p_\Phi^2 - \frac{1}{2}p_\Psi^2 + \frac{1}{4}(\Phi - \Psi)^2$$

2 DOF = 1 healthy + 1 ghost

Problematic when coupled to a normal system

Ostrogradsky theorem for $L(\ddot{\phi}, \dot{\phi}, \phi)$

For Lagrangian $L(\ddot{\phi}, \dot{\phi}, \phi)$ with $\phi = \phi(t)$,
$K \equiv \partial^2 L / \partial \ddot{\phi}^2 \neq 0 \Longrightarrow H$ is unbounded.

Woodard, 1506.02210

'L is nondegenerate'.

- EL eq

$$K \ddddot{\phi} + \dot{K} \dddot{\phi} = (\text{terms up to } \ddot{\phi})$$

$K \neq 0 \Longrightarrow$ 4th order system = 2 DOF

- Hamiltonian is unbounded

Ostrogradsky theorem for $L(\ddot{\phi}, \dot{\phi}, \phi)$

For Lagrangian $L(\ddot{\phi}, \dot{\phi}, \phi)$ with $\phi = \phi(t)$,
$K \equiv \partial^2 L / \partial \ddot{\phi}^2 \neq 0 \Longrightarrow H$ is unbounded.

Let us impose $K = 0$ ← Degeneracy condition

✓ Remove $\ddddot{\phi}$ and $\dddot{\phi}$ from EL eq

✓ EL eq

$$K \ddddot{\phi} + \dot{K} \dddot{\phi} = (\text{terms up to } \ddot{\phi})$$

$K = 0 \Longrightarrow$ 2nd order system = 1 DOF

✓ Hamiltonian is bounded

✓ The most general ghost-free Lagrangian is

$$L = \ddot{\phi} f(\dot{\phi}, \phi) + g(\dot{\phi}, \phi) \sim G(\dot{\phi}, \phi)$$

$\Longrightarrow$ Detailed model-dependent analysis

Eliminating Ostrogradsky ghost

- ✓ $L(\ddot{\phi}, \dot{\phi}, \phi) \Longrightarrow L(\dot{\phi}, \phi)$

Eliminating Ostrogradsky ghost

✓ $L(\ddot{\phi}, \dot{\phi}, \phi) \Longrightarrow L(\dot{\phi}, \phi)$

- $L(\ddot{\phi}^a, \dot{\phi}^a, \phi^a)$ HM, Suyama, 1411.3721
- $L(\phi^{a(d)}, \phi^{a(d-1)}, \dots, \phi^a)$

Ostrogradsky theorem for $L(\ddot{\phi}^a, \dot{\phi}^a, \phi^a)$

For $L(\ddot{\phi}^a, \dot{\phi}^a, \phi^a)$ with $\phi^a = \phi^a(t)$ and $a = 1, \cdots n$,

$\det K \neq 0 \Longrightarrow H$ is unbounded,

where $K_{ab} \equiv \dfrac{\partial^2 L}{\partial \ddot{\phi}^a \partial \ddot{\phi}^b}$.

‘kinetic matrix’

Woodard, 1506.02210

$$M_{ab} \equiv \frac{\partial^2 L}{\partial \ddot{\phi}^a \partial \dot{\phi}^b} - \frac{\partial^2 L}{\partial \ddot{\phi}^b \partial \dot{\phi}^a}$$

- EL eq

$$K_{ab} \ddddot{\phi}^b + \left(\dot{K}_{ab} + M_{ab}\right) \dddot{\phi}^b = \left(\text{terms up to } \ddot{\phi}^a\right)$$

$\det K \neq 0 \Longrightarrow$ 4th order system.

- H is unbounded.

  n healthy + n ghost DOFs.

Eliminating Ostrogradsky ghost

For $L(\ddot{\phi}^a, \dot{\phi}^a, \phi^a)$ with $\phi^a = \phi^a(t)$ and $a = 1, \cdots n$,

$\det K \neq 0 \Longrightarrow H$ is unbounded,

where $K_{ab} \equiv \dfrac{\partial^2 L}{\partial \ddot{\phi}^a \partial \ddot{\phi}^b}$.

$\Rightarrow$ $K_{ab} = 0$ ⟶ ✓ Remove $\ddddot{\phi}^a$ (highest)

$$M_{ab} \equiv \frac{\partial^2 L}{\partial \ddot{\phi}^a \partial \dot{\phi}^b} - \frac{\partial^2 L}{\partial \ddot{\phi}^b \partial \dot{\phi}^a}$$

- EL eq

$$K_{ab} \ddddot{\phi}^b + \left(\dot{K}_{ab} + M_{ab}\right) \dddot{\phi}^b = \left(\text{terms up to } \ddot{\phi}^a\right)$$

- Still remain ghosts.

$K_{ab} = 0,\ \det M \neq 0\ (n\text{: even}) \Longrightarrow$ 3rd order system

- H is unbounded. $3n$ ini. conds. are required.
- Need to update the theorem.

HM, Suyama, 1411.3721

Eliminating Ostrogradsky ghost

For $L(\ddot{\phi}^a, \dot{\phi}^a, \phi^a)$ with $\phi^a = \phi^a(t)$ and $a = 1, \cdots n$,

$\det K \neq 0$ or $\det M \neq 0$ $\Leftrightarrow$ H is unbounded,

where $K_{ab} \equiv \frac{\partial^2 L}{\partial \ddot{\phi}^a \partial \ddot{\phi}^b}$ and $M_{ab} \equiv \frac{\partial^2 L}{\partial \ddot{\phi}^a \partial \dot{\phi}^b} - \frac{\partial^2 L}{\partial \ddot{\phi}^b \partial \dot{\phi}^a}$.

$\Rightarrow$ $K_{ab} = 0$ ⟶ ✓ Remove $\ddddot{\phi}^a$ (highest)

$M_{ab} = 0$ ⟶ ✓ Remove $\dddot{\phi}^a$ (next-highest)

✓ EL eq is 2nd order system.

✓ H is bounded. All ghost DOFs are removed.

✓ The most general ghost-free Lagrangian: $L \sim G\left(\dot{\phi}^a, \phi^a\right)$

HM, Suyama, 1411.3721

Eliminating Ostrogradsky ghost

For $L(\phi^{a(d)}, \phi^{a(d-1)}, \ldots, \phi^{a})$,

$\det K \neq 0$ or $\det M \neq 0$ $\Rightarrow H$ is unbounded,

where $K_{ab} \equiv \dfrac{\partial^2 L}{\partial \phi^{a(d)} \partial \phi^{b(d)}}$ and

$$M_{ab} \equiv \frac{\partial^2 L}{\partial \phi^{a(d)} \partial \phi^{b(d-1)}} - \frac{\partial^2 L}{\partial \phi^{b(d)} \partial \phi^{a(d-1)}} .$$

$\Rightarrow$ $K_{ab} = 0$ $\longrightarrow$ ✓ Remove $\phi^{a(2d)}$ (highest)

$M_{ab} = 0$ $\longrightarrow$ ✓ Remove $\phi^{a(2d-1)}$ (next-highest)

- Still remain ghosts from lower (> 2) derivatives.

HM, Suyama, 1411.3721

Eliminating Ostrogradsky ghost

- ✓ $L(\ddot{\phi}, \dot{\phi}, \phi) \Longrightarrow L(\dot{\phi}, \phi)$
- ✓ $L(\ddot{\phi}^a, \dot{\phi}^a, \phi^a) \Longrightarrow L(\dot{\phi}^a, \phi^a)$ HM, Suyama, 1411.3721
- $L(\phi^{a(d)}, \phi^{a(d-1)}, \dots, \phi^a)$

Eliminating Ostrogradsky ghost

✓ $L(\ddot{\phi}, \dot{\phi}, \phi) \Longrightarrow L(\dot{\phi}, \phi)$

✓ $L(\ddot{\phi}^a, \dot{\phi}^a, \phi^a) \Longrightarrow L(\dot{\phi}^a, \phi^a)$ HM, Suyama, 1411.3721

- $L(\phi^{a(d)}, \phi^{a(d-1)}, \dots, \phi^a)$

- Ostrogradsky theorem ↔ Highest deriv. in EL eq.
- To eliminate all ghost DOFs, we need to update the theorem for each L (Degeneracy condition).

- So far, higher-order derivatives in L and/or EL eq are not allowed.
- However, for L with mixed order of derivatives (⊃ ST theory), the situation is crucially different.

Horndeski theory

3 DOF = 2 for $g_{\mu\nu}$ + 1 for ϕ

Most general theory for 2nd order EL eq is Horndeski theory or generalized Galileon

Horndeski, 1974

Deffayet, Gao, Steer, Zahariade, 1103.3260

Kobayashi, Yamaguchi, Yokoyama, 1105.5723

$$L_2 = G_2(\phi, X) \qquad X \equiv \nabla_\mu\phi\nabla^\mu\phi$$

$$L_3 = G_3(\phi, X)\Box\phi$$

$$L_4 = G_4(\phi, X)R - 2G_{4X}(\phi, X)\left[(\Box\phi)^2 - \left(\nabla_\mu\nabla_\nu\phi\right)^2\right]$$

$$L_5 = G_5(\phi, X)G^{\mu\nu}\nabla_\mu\nabla_\nu\phi + \frac{1}{3}G_{5X}(\phi, X)\left[(\Box\phi)^3 - 3\Box\phi\left(\nabla_\mu\nabla_\nu\phi\right)^2 + \left(\nabla_\mu\nabla_\nu\phi\right)^3\right]$$

✓ 2nd order EL eq $\Longrightarrow$ Ostrogradsky ghost-free

Transformation

One can consider invertible disformal transformation

$$g_{\mu\nu} \to A(\phi, X) g_{\mu\nu} + B(\phi, X) \partial_\mu \phi \partial_\nu \phi$$

of Horndeski.

Zumalacárregui, García-Bellido, 1308.4685

While EL eq becomes a priori higher order, it is still healthy second order system. ⇒ beyond Horndeski

✓ Disformal invariance of ζ

HM, White, 1504.00846

✓ For general field theories, Lagrangians related through general invertible transformation are physically equivalent.

Takahashi, HM, Suyama, Kobayashi, 1702.01849

Example: $L = \frac{1}{2}(\dot{x} - \ddot{y})^2 + \frac{1}{2}\dot{y}^2$ ⟷ beyond Horndeski

- EL eqs are a priori 4th order

$$\frac{d}{dt}(\dot{x} - \ddot{y}) = 0, \qquad \frac{d^2}{dt^2}(\dot{x} - \ddot{y}) + \ddot{y} = 0$$

but can be rearranged to 2nd order system

$$\ddot{x} = 0, \qquad \ddot{y} = 0$$

- Invertible transformation

$$X = x - \dot{y}, \qquad Y = y$$
$$(x = X + \dot{Y}, \qquad y = Y)$$

⟷ Disformal transformation

leads Lagrangian to

$$L = \frac{1}{2}\dot{X}^2 + \frac{1}{2}\dot{Y}^2$$

⟷ Horndeski

Example: $L = \frac{1}{2}\frac{\dot{x}^2}{1+\ddot{y}} + \frac{1}{2}\dot{y}^2$

Gabadadze, Hinterbichler, Khoury, Pirtskhalava, Trodden, 1208.5773

- EL eqs are a priori 4th order

$$\frac{d}{dt}\left(\frac{\dot{x}}{1+\ddot{y}}\right) = 0, \qquad \frac{d^2}{dt^2}\left(\frac{\dot{x}^2}{(1+\ddot{y})^2}\right) + \ddot{y} = 0$$

but can be rearranged to 2nd order system

$$\ddot{x} = 0, \qquad \ddot{y} = 0$$

- Is it equivalent to $L = \frac{1}{2}\dot{X}^2 + \frac{1}{2}\dot{Y}^2$?

  At least, $X = x + a\dot{y},\ Y = y$ cannot transform it.

- Is there some other transformation?

  Not clear.

GLPV theory

$$L_2 = G_2(\phi, X) \qquad X \equiv \phi_{;\mu}\phi^{;\mu}$$

$$L_3 = G_3(\phi, X)\Box\phi$$

$$L_4 = G_4(\phi, X)R$$
$$-2G_{4X}(\phi, X)\left[(\Box\phi)^2 - \phi^{;\mu\nu}\phi_{;\mu\nu}\right]$$
$$+F_4(\phi, X)\epsilon^{\mu\nu\rho}{}_{\sigma}\epsilon^{\tilde{\mu}\tilde{\nu}\tilde{\rho}\sigma}\phi_{;\mu}\phi_{;\tilde{\mu}}\phi_{;\nu\tilde{\nu}}\phi_{;\rho\tilde{\rho}}$$

$$L_5 = G_5(\phi, X)G^{\mu\nu}\phi_{;\mu\nu}$$
$$+\frac{1}{3}G_{5X}(\phi, X)\left[(\Box\phi)^3 - 3\phi\phi^{;\mu\nu}\phi_{;\mu\nu} + \phi_{;\mu\nu}\phi^{;\mu\sigma}\phi^{;\nu}_{;\sigma}\right]$$
$$+F_5(\phi, X)\epsilon^{\mu\nu\rho\sigma}\epsilon^{\tilde{\mu}\tilde{\nu}\tilde{\rho}\tilde{\sigma}}\phi_{;\mu}\phi_{;\tilde{\mu}}\phi_{;\nu\tilde{\nu}}\phi_{;\rho\tilde{\rho}}\phi_{;\sigma\tilde{\sigma}}$$

Gleyzes, Langlois, Piazza, Vernizzi, 1404.6495, 1408.1952

Domenech et al, 1507.05390

Deffayet, Esposito-Farese, Steer, 1506.01974

- ✓ 3 DOF = 2 $g_{\mu\nu}$+ 1 ϕ / Ostrogradsky ghost-free
- ✓ Higher-order EL eqs are reducible to 2nd order system
- ✓ Includes subclass related to Horndeski through disformal transformation $g_{\mu\nu} \to A(\phi)g_{\mu\nu} + B(\phi, X)\partial_\mu\phi\partial_\nu\phi$

Healthy scalar-tensor theories

quadratic DHOST / EST

Langlois, Noui, 1510.06930, 1512.06820
Crisostomi, Koyama, Tasinato, 1602.03119
Achour, Langlois, Noui, 1602.08398

- Write down all possible derivative terms with coefficient functions at $(\nabla\nabla\phi)^2, (\partial\partial g)^2$ order

$$S = \int d^4x\sqrt{-g}\left[F_2(\phi, X)R + \sum_{i=1}^{5} A_i(\phi, X)L_i^{(2)}\right]$$

$$L_1^{(2)} = \left(\phi_{;\mu\nu}\right)^2, L_2^{(2)} = (\Box\phi)^2, L_3^{(2)} = (\Box\phi)\phi^{;\mu}\phi_{;\mu\nu}\phi^{;\nu},$$
$$L_4^{(2)} = \phi^{;\mu}\phi_{;\mu\nu}\phi^{;\nu\rho}\phi_{;\rho}, L_5^{(2)} = \left(\phi^{;\mu}\phi_{;\mu\nu}\phi^{;\nu}\right)^2.$$

- Impose degeneracy condition on F_2 and A_i.

Healthy scalar-tensor theories

quadratic DHOST / EST

- Needs to derive degeneracy condition for L with mixed orders of derivatives.

- Toy model was first studied Langlois, Noui, 1510.06930

$$L = \frac{1}{2} a \ddot{\phi}^2 + \frac{1}{2} k_0 \dot{\phi}^2 + \frac{1}{2} k_{ij} \dot{q}^i \dot{q}^j + b_i \ddot{\phi} \dot{q}^i + c_i \dot{\phi} \dot{q}^i - V(\phi, q)$$

which applies to only $(\nabla\nabla\phi)^2$ order theory.

- More general analysis is required beyond $(\nabla\nabla\phi)^2$.

HM, Noui, Suyama, Yamaguchi, Langlois, 1603.09355

See also Klein, Roest, 1604.01719; Crisostomi, Klein, Roest, 1703.01623

Eliminating Ostrogradsky ghost

- ✓ $L(\ddot{\phi}, \dot{\phi}, \phi) \Longrightarrow L(\dot{\phi}, \phi)$
- ✓ $L(\ddot{\phi}^a, \dot{\phi}^a, \phi^a) \Longrightarrow L(\dot{\phi}^a, \phi^a)$ HM, Suyama, 1411.3721
- $L(\phi^{a(d)}, \phi^{a(d-1)}, \dots, \phi^a)$

HM, Noui, Suyama, Yamaguchi, Langlois, 1603.09355

- $L(\ddot{\phi}, \dot{\phi}, \phi; \dot{q}, q)$
- $L(\ddot{\phi}, \dot{\phi}, \phi; \dot{q}^i, q^i) \sim \phi + g_{\mu\nu}$
- $L(\ddot{\phi}^a, \dot{\phi}^a, \phi^a; \dot{q}^i, q^i) \sim \phi^a + g_{\mu\nu}$

$$L(\ddot{\phi}, \dot{\phi}, \phi; \dot{q}, q)$$

$\phi(t), q(t)$ have different orders of derivatives.

Equivalent form

$$L(\dot{Q}, Q, \phi; \dot{q}, q) + \lambda(\dot{\phi} - Q).$$

Canonical momenta for ϕ and λ

$\Longrightarrow$ 2 primary constraints

8 ini. conds.

From $P = L_{\dot{Q}}$ and $p = L_{\dot{q}}$ $\quad (L_X \equiv \partial L / \partial X)$

$$\begin{pmatrix} \delta P \\ \delta p \end{pmatrix} = \begin{pmatrix} L_{\dot{Q}\dot{Q}} & L_{\dot{q}\dot{Q}} \\ L_{\dot{q}\dot{Q}} & L_{\dot{q}\dot{q}} \end{pmatrix} \begin{pmatrix} \delta \dot{Q} \\ \delta \dot{q} \end{pmatrix}.$$

kinetic matrix K

If $\det K \neq 0 \Longrightarrow$ No further primary constraints.

We thus impose $\det K = 0$.

$$L(\ddot{\phi}, \dot{\phi}, \phi; \dot{q}, q)$$

The degeneracy condition: $\det K = 0$

In particular, we consider the case

$L_{\dot{q}\dot{q}} \neq 0$, and $L_{\dot{Q}\dot{Q}} - L_{\dot{q}\dot{Q}}^2 / L_{\dot{q}\dot{q}} = 0$,

✓ q-sector is ``normal''

✓ Remove $\ddddot{\phi}$ & $\dddot{\phi}$

$$\begin{pmatrix} \delta P \\ \delta p \end{pmatrix} = \begin{pmatrix} L_{\dot{Q}\dot{Q}} & L_{\dot{q}\dot{Q}} \\ L_{\dot{q}\dot{Q}} & L_{\dot{q}\dot{q}} \end{pmatrix} \begin{pmatrix} \delta \dot{Q} \\ \delta \dot{q} \end{pmatrix}$$

$$\Longrightarrow \ \delta P = \frac{L_{\dot{q}\dot{Q}}}{L_{\dot{q}\dot{q}}} \delta p, \qquad \delta p = L_{\dot{q}\dot{Q}} \delta \dot{Q} + L_{\dot{q}\dot{q}} \delta \dot{q}$$

$$\Longrightarrow \ P = F(p, q, Q, \phi)$$

$L(\ddot{\phi}, \dot{\phi}, \phi; \dot{q}, q)$

The degeneracy condition: $\det K = 0$

In particular, we consider the case

$L_{\dot{q}\dot{q}} \neq 0$, and $L_{\dot{Q}\dot{Q}} - L_{\dot{q}\dot{Q}}^2 / L_{\dot{q}\dot{q}} = 0$,

↓ ↓

✓ q-sector is ``normal'' ✓ Remove $\ddddot{\phi}$ & $\dddot{\phi}$

↓

Additional 1 primary constraint $P = F(p, q, Q, \phi)$

$\Longrightarrow$ Additional 1 secondary constraint

$\Longrightarrow (8 - 2 - 2)/2 = 2$ healthy DOF.

✓ EL eqs can be reducible to 2nd order system.

$L(\ddot{\phi}, \dot{\phi}, \phi; \dot{q}^i, q^i)$ $\quad i = 1, \ldots, m$

The degeneracy condition: $\det K = 0$

In particular, we consider the case

$\det L_{\dot{q}^i \dot{q}^j} \neq 0,$ and $L_{\dot{Q}\dot{Q}} - L_{\dot{q}^i \dot{Q}} L^{-1}_{\dot{q}^i \dot{q}^j} L_{\dot{q}^j \dot{Q}} = 0,$

↓ ↓

✓ q^i-sector is ``normal'' ✓ Remove $\ddddot{\phi}$ & $\dddot{\phi}$

↓

Additional 1 primary constraint $P = F(p_i, q^i, Q, \phi)$

$\Longrightarrow$ Additional 1 secondary constraint

$\Longrightarrow$ $(2m + 6 - 2 - 2)/2 = m + 1$ healthy DOF.

✓ EL eqs can be reducible to 2nd order system.

$$L(\ddot{\phi}^a, \dot{\phi}^a, \phi^a; \dot{q}^i, q^i) \qquad a = 1, \ldots, n \qquad i = 1, \ldots, m$$

The degeneracy condition: $\det K = 0$

In particular, we consider the case

$\det L_{\dot{q}^i \dot{q}^j} \neq 0,$ and $L_{\dot{Q}^a \dot{Q}^b} - L_{\dot{q}^i \dot{Q}^a} L^{-1}_{\dot{q}^i \dot{q}^j} L_{\dot{q}^j \dot{Q}^b} = 0,$

✓ q^i-sector is ``normal''

✓ Remove $\ddddot{\phi}^a$ but not $\dddot{\phi}^a$

Additional n primary constraint

$$\Xi_a \equiv P_a - F_a(p_i, q^i, Q^b, \phi^b) = 0$$

$\Longrightarrow$ Impose $\{\Xi_a, \Xi_b\} = 0$ ⟶ ✓ Remove $\dddot{\phi}^a$

$\Longrightarrow$ Additional n secondary constraint

$\Longrightarrow$ $(2m + 6n - 2n - 2n)/2 = m + n$ healthy DOF.

✓ EL eqs can be reducible to 2nd order system.

Eliminating Ostrogradsky ghost

- ✓ $L(\ddot{\phi}, \dot{\phi}, \phi) \Longrightarrow L(\dot{\phi}, \phi)$
- ✓ $L(\ddot{\phi}^a, \dot{\phi}^a, \phi^a) \Longrightarrow L(\dot{\phi}^a, \phi^a)$ HM, Suyama, 1411.3721
- $L(\phi^{a(d)}, \phi^{a(d-1)}, \dots, \phi^a)$

HM, Noui, Suyama, Yamaguchi, Langlois, 1603.09355

- ✓ $L(\ddot{\phi}, \dot{\phi}, \phi; \dot{q}, q)$
- ✓ $L(\ddot{\phi}, \dot{\phi}, \phi; \dot{q}^i, q^i) \sim \phi + g_{\mu\nu}$
- ✓ $L(\ddot{\phi}^a, \dot{\phi}^a, \phi^a; \dot{q}^i, q^i) \sim \phi^a + g_{\mu\nu}$

Healthy scalar-tensor theories

cubic DHOST / EST

Achour, Crisostomi, Koyama, Langlois, Noui, Tasinato, 1608.08135

- At $(\nabla\nabla\phi)^3, (\partial\partial g)^2\nabla\nabla\phi$ order

$$S = \int d^4x\sqrt{-g}\left[F_3(\phi,X)R + \sum_{i=1}^{10} B_i(\phi,X)L_i^{(3)}\right]$$

$$L_1^{(3)} = (\Box\phi)^3, L_2^{(3)} = \Box\phi\left(\phi_{;\mu\nu}\right)^2, L_3^{(3)} = \phi_{;\mu\nu}\phi^{;\nu\rho}\phi_{;\rho}^{;\mu},$$

$$L_4^{(3)} = (\Box\phi)^2\phi^{;\mu}\phi_{;\mu\nu}\phi^{;\nu}, L_5^{(3)} = \cdots.$$

- Impose degeneracy condition on F_3 and B_i.

Healthy scalar-tensor theories

quadratic & cubic DHOST / EST

✓ Ghost-free Lagrangian for $(\nabla\nabla\phi)^2$ and $(\nabla\nabla\phi)^3$

Open questions

- Can we check whether ∃ redefinition of fields for given higher-order theory?
- Higher powers? Multi-field?

HM, Noui, Suyama, Yamaguchi, Langlois, 1603.09355

- Even higher order derivatives?

HM, Suyama, Yamaguchi, in prep.

Eliminating Ostrogradsky ghost

- ✓ $L(\ddot{\phi}, \dot{\phi}, \phi) \Longrightarrow L(\dot{\phi}, \phi)$
- ✓ $L(\ddot{\phi}^a, \dot{\phi}^a, \phi^a) \Longrightarrow L(\dot{\phi}^a, \phi^a)$ HM, Suyama, 1411.3721
- $L(\phi^{a(d)}, \phi^{a(d-1)}, \dots, \phi^a)$

HM, Noui, Suyama, Yamaguchi, Langlois, 1603.09355

- ✓ $L(\ddot{\phi}, \dot{\phi}, \phi; \dot{q}, q)$
- ✓ $L(\ddot{\phi}, \dot{\phi}, \phi; \dot{q}^i, q^i) \sim \phi + g_{\mu\nu}$
- ✓ $L(\ddot{\phi}^a, \dot{\phi}^a, \phi^a; \dot{q}^i, q^i) \sim \phi^a + g_{\mu\nu}$

HM, Suyama, Yamaguchi, in prep.

- $L(\dddot{\psi}, \ddot{\psi}, \dot{\psi}, \psi; \dot{q}^i, q^i)$
- $L(\dddot{\psi}^n, \ddot{\psi}^n, \dot{\psi}^n, \psi^n; \ddot{\phi}^a, \dot{\phi}^a, \phi^a; \dot{q}^i, q^i)$
- $L(\phi^{i_d(d+1)}, \dots; \phi^{i_{d-1}(d)}, \dots; \dots; \dot{\phi}^{i_0}, \phi^{i_0})$

$L(\dddot{\psi}, \ddot{\psi}, \dot{\psi}, \psi; \dot{q}^i, q^i)$ $\qquad i = 1, \dots, I$

Equivalent form

$$L(\dot{Q}, Q, R, \psi; \dot{q}^i, q^i) + \xi(\dot{\psi} - R) + \lambda(\dot{R} - Q).$$

Canonical momenta for $(Q, R, \psi, q^i, \xi, \lambda)$

$\Longrightarrow$ 4 primary constraints

$10 + 2I$ ini. conds.

Need more constraints to remove Ostrogradsky ghost.

Impose $\det K = 0$ or

$\det L_{\dot{q}^i \dot{q}^j} \neq 0$, and $L_{\dot{Q}\dot{Q}} - L_{\dot{q}^i \dot{Q}} L^{-1}_{\dot{q}^i \dot{q}^j} L_{\dot{q}^j \dot{Q}} = 0$.

$L(\dddot{\psi}, \ddot{\psi}, \dot{\psi}, \psi; \dot{q}^i, q^i)$ $\quad i = 1, \ldots, I$

The degeneracy condition: $\det K = 0$ or

$\det L_{\dot{q}^i \dot{q}^j} \neq 0$, and $L_{\dot{Q}\dot{Q}} - L_{\dot{q}^i \dot{Q}} L^{-1}_{\dot{q}^i \dot{q}^j} L_{\dot{q}^j \dot{Q}} = 0$,

→ ✓ Remove $\psi^{(6)}$ & $\psi^{(5)}$ ↓

Additional 1 primary constraint $\Psi \equiv P_Q - F(p_i, x) = 0$

$\Longrightarrow$ 1 secondary constraint $\Upsilon \equiv P_R - G(p_i, x) = 0$

Impose $\{\Upsilon, \Psi\} = 0$ → ✓ Remove $\psi^{(4)}$ & $\psi^{(3)}$

$\Longrightarrow$ 1 tertiary constraint $\Lambda \equiv \pi - I(p_i, x) = 0$

$\Longrightarrow$ 1 quaternary constraint $\Omega \equiv -\{\Lambda, H\} = 0$

Impose $Z \equiv \{\Omega, \Psi\} \neq 0$ → ✓ Stop Dirac algorithm

$\Longrightarrow (10 + 2I - 4 - 4)/2 = 1 + I$ healthy DOF.

✓ EL eqs can be reducible to 2nd order system.

Eliminating Ostrogradsky ghost

- ✓ $L(\ddot{\phi}, \dot{\phi}, \phi) \Longrightarrow L(\dot{\phi}, \phi)$
- ✓ $L(\ddot{\phi}^a, \dot{\phi}^a, \phi^a) \Longrightarrow L(\dot{\phi}^a, \phi^a)$ HM, Suyama, 1411.3721
- ✓ $L(\phi^{a(d)}, \phi^{a(d-1)}, \dots, \phi^a)$

HM, Noui, Suyama, Yamaguchi, Langlois, 1603.09355

- ✓ $L(\ddot{\phi}, \dot{\phi}, \phi; \dot{q}, q)$
- ✓ $L(\ddot{\phi}, \dot{\phi}, \phi; \dot{q}^i, q^i) \sim \phi + g_{\mu\nu}$
- ✓ $L(\ddot{\phi}^a, \dot{\phi}^a, \phi^a; \dot{q}^i, q^i) \sim \phi^a + g_{\mu\nu}$

HM, Suyama, Yamaguchi, in prep.

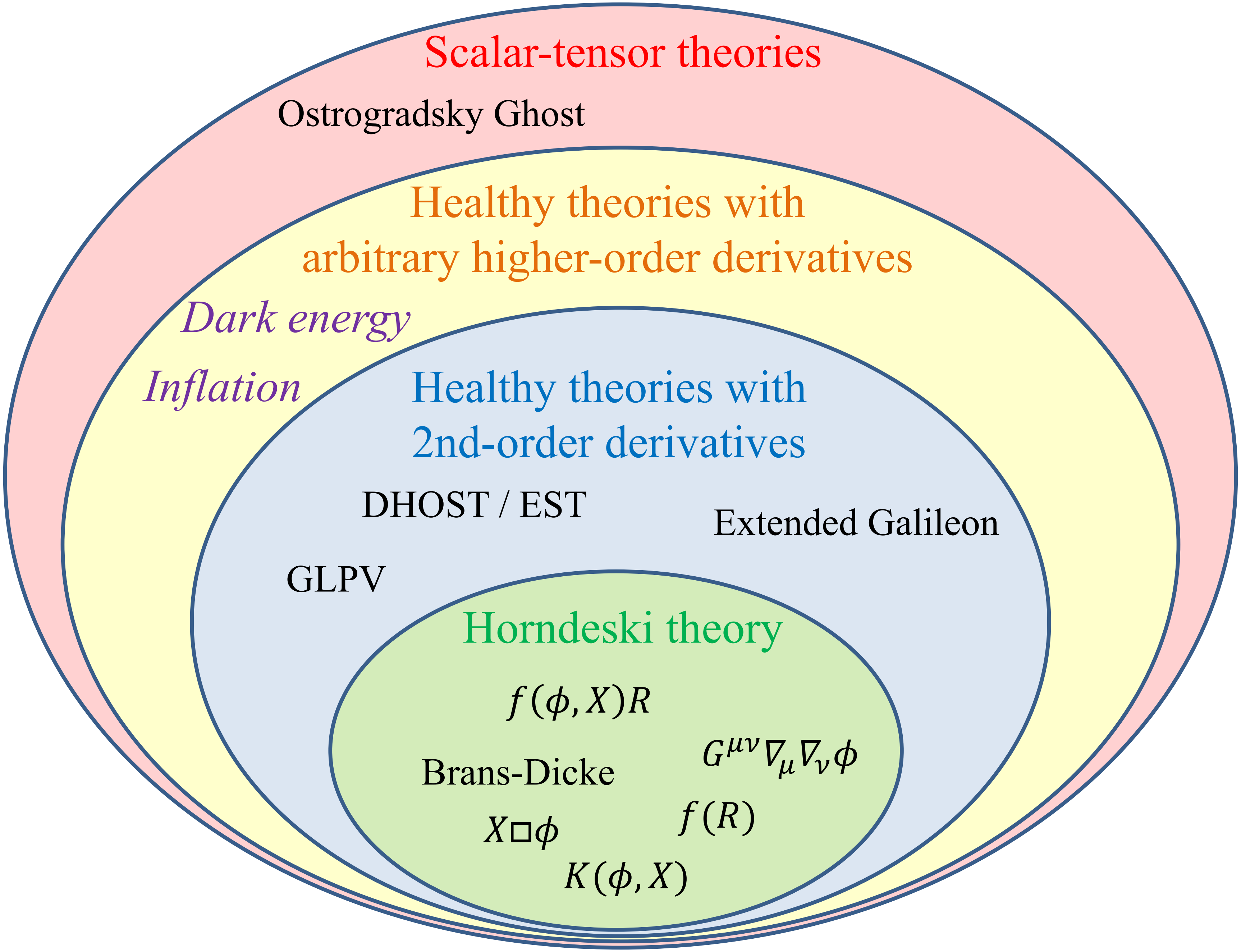
- ✓ $L(\dddot{\psi}, \ddot{\psi}, \dot{\psi}, \psi; \dot{q}^i, q^i)$
- ✓ $L(\dddot{\psi}^n, \ddot{\psi}^n, \dot{\psi}^n, \psi^n; \ddot{\phi}^a, \dot{\phi}^a, \phi^a; \dot{q}^i, q^i)$
- ✓ $L(\phi^{i_d(d+1)}, \dots; \phi^{i_{d-1}(d)}, \dots; \dots; \dot{\phi}^{i_0}, \phi^{i_0})$

Summary

- We derived the degeneracy condition for various types of Lagrangians to eliminate Ostrogradsky ghost, which allow us to construct healthy scalar-tensor theories, and proceed to detailed model-dependent analysis.

- We clarified that it is possible to construct healthy Lagrangian involving arbitrary higher-order derivatives.

- Application to scalar-tensor theories and their classifications are work in progress.


Scalar-tensor theories
Ostrogradsky Ghost
Healthy theories with
arbitrary higher-order derivatives
Dark energy
Inflation
Healthy theories with
2nd-order derivatives
DHOST / EST
Extended Galileon
GLPV
Horndeski theory
$f(\phi, X)R$
$G^{\mu\nu}\nabla_\mu\nabla_\nu\phi$
Brans-Dicke
$f(R)$
$X\Box\phi$
$K(\phi, X)$