Higher Derivative Theories

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Portsmouth

Based on [arXiv:1601.04658], [arXiv:1602.03119], [arXiv:1608.08135], [arXiv:1703.01623] and [arXiv:1710.soon]

in collaboration with:

Matthew Hull, Kazuya Koyama, Gianmassimo Tasinato, Jibril Ben Achour, David Langlois, Karim Noui, Remko Klein, Diederik Roest, Christos Charmousis

Historical Overview

- 1971 – **Lovelock**

  the most general metric theory of gravity yielding conserved second order equations of motion in arbitrary number of dimensions

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Second order field equations

What's wrong with higher order field equations?

Ostrogradsky theorem – 1850

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- Newton, i.e. $L = L(q,\, \dot{q})$

$$\frac{\partial L}{\partial q} - \frac{d}{dt}\frac{\partial L}{\partial \dot{q}} = 0 \qquad \overset{\text{nd}}{\Longrightarrow} \qquad \ddot{q} = F(q,\, \dot{q})$$

$$p \equiv \frac{\partial L}{\partial \dot{q}} \qquad \text{nd} \Longrightarrow \dot{q} = f(q,\, p) \qquad\qquad H(q,\, p) = p\, f - L(q,\, f)$$

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- Higher derivative $L = L(q,\, \dot q,\, \ddot q)$

$$\frac{\partial L}{\partial q} - \frac{d}{dt}\frac{\partial L}{\partial \dot q} + \frac{d^2}{dt^2}\frac{\partial L}{\partial \ddot q} = 0 \qquad \overset{\text{nd}}{\Longrightarrow} \qquad \ddddot q = F(q,\, \dot q,\, \ddot q,\, \dddot q)$$

$$Q \equiv \dot q\,, \quad p \equiv \frac{\partial L}{\partial \dot q} - \frac{d}{dt}\frac{\partial L}{\partial \ddot q}\,, \quad P \equiv \frac{\partial L}{\partial \ddot q}\,, \qquad \text{nd} \Longrightarrow \ddot q = f(q,\, Q,\, P)$$

$$H(q,\, Q,\, p,\, P) = p\, Q + P f - L(q,\, Q,\, f)$$

H linear in p $\Rightarrow$ unbounded energy

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WAY OUT: break the assumptions

Evading the Ostrogradsky instability

Assumptions: 1) two variables 2) degenerate Langlois & Noui [arXiv:1510.06930]

- $L = L(q_1,\, \dot q_1,\, \ddot q_1,\, q_2,\, \dot q_2) \qquad \longrightarrow \qquad L(q_1,\, Q,\, \dot Q,\, q_2,\, \dot q_2) + \lambda(\dot q_1 - Q)$

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Primary constraint $\psi(P,\, p_2) \approx 0 \iff \boxed{\det \mathbb{H} = 0}$ $\quad \mathbb{H} = \begin{pmatrix} \frac{\partial^2 L}{\partial \dot{Q}^2} & \frac{\partial^2 L}{\partial \dot{Q} \partial \dot{q}_2} \\ \frac{\partial^2 L}{\partial \dot{q}_2 \partial \dot{Q}} & \frac{\partial^2 L}{\partial \dot{q}_2^2} \end{pmatrix}$

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Generalization: Motohashi et al. [arXiv:1603.09355]; Klein & Roest [arXiv:1604.01719]

- $L = L(\ddot\phi_m, \dot\phi_m, \phi_m, \dot q_\alpha, q_\alpha) \qquad v^A_m = (\delta^n_m, V^\alpha_m) \qquad V^\alpha_m \equiv -L_{\ddot\phi_m \dot q_\beta} L^{-1}_{\dot q_\beta \dot q_\alpha}$

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Primary constraints $\iff 0 = P_{(mn)} \equiv v_m^A L_{\dot{\psi}_A \dot{\psi}_B} v_n^B$

Secondary constraints $\iff 0 = S_{[mn]} \equiv 2\, v_m^A L_{\dot{\psi}_{[A} \psi_{B]}} v_n^B$

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Field theories: [arXiv:1703.01623]

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1 primary constraint $\implies$ $\exists$ **1** secondary constraint

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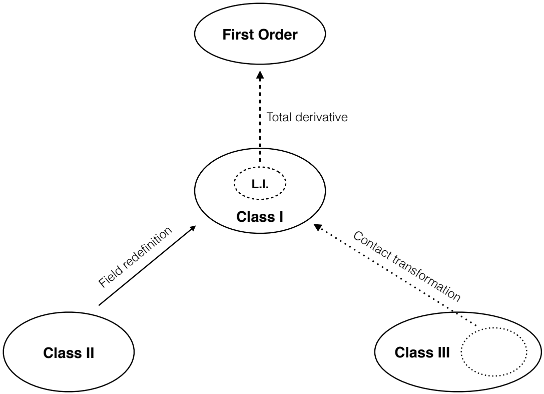

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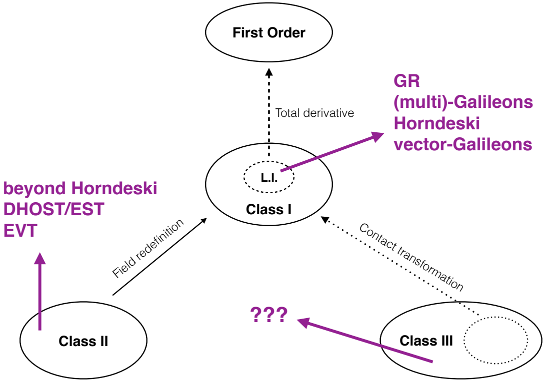

Scalar-Tensor Theories

two fields $(g_{\mu\nu},\, \phi)$

Second derivative of ϕ: $\quad L = L(\nabla_\mu \partial_\nu \phi) \quad \longrightarrow \quad L(\nabla_\mu A_\nu) + \lambda^\mu(\partial_\mu \phi - A_\mu)$

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Covariant 3+1 decomposition: $t^\mu = \partial/\partial t = N n^\mu + N^\mu \quad \begin{cases} g_{\mu\nu} = h_{\mu\nu} - n_\mu n_\nu \\ A_\mu = \hat{A}_\nu h^\nu_\mu - A_* n_\mu \end{cases}$

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det $\mathbb{H} = 0 \quad \Longrightarrow$ ghost free

Degenerate Scalar-Tensor Theories

$$S = \int d^4x \sqrt{-g} \left(f_2 \, R + C_{(2)}^{\mu\nu\rho\sigma} \, \phi_{\mu\nu} \, \phi_{\rho\sigma} + f_3 \, G_{\mu\nu} \phi^{\mu\nu} + C_{(3)}^{\mu\nu\rho\sigma\alpha\beta} \, \phi_{\mu\nu} \, \phi_{\rho\sigma} \, \phi_{\alpha\beta} \right)$$

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Phenomenological applications:

- ET of DE
  Langlois et al. [arXiv:1703.03797]
- Stable bouncing cosmology
  Creminelli et al. [arXiv:1610.04207]
- Breaking of Vainshtein mechanism
  Kobayashi et al. [arXiv:1411.4130] + many others

Pure Metric Theories

$$S[g_{\mu\nu}] = \int d^4x\,\sqrt{-g}\,L\left(g_{\mu\nu},\,\partial_\rho g_{\mu\nu},\,\partial_\rho\partial_\sigma g_{\mu\nu}\right) \qquad \supseteq \qquad \dot{K}_{ij} \qquad \supseteq \qquad \ddot{\gamma}_{ij}$$

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FULLY DEGENERATE THEORIES

6 primary constraints $\quad \Longleftrightarrow \quad \mathcal{A}^{ij,\lambda m}(x,y) \equiv \dfrac{\partial^2 L}{\partial\ddot{\gamma}_{ij}(x)\partial\ddot{\gamma}_{\lambda m}(y)} = 0$

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6 primary constraints $\iff$ $\mathcal{A}^{ij,\lambda m}(x,y) \equiv \dfrac{\partial^2 L}{\partial \ddot{\gamma}_{ij}(x) \partial \ddot{\gamma}_{\lambda m}(y)} = 0$

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C has NO secondary constraints $\longrightarrow$ 5 dof (3 Ostrogradsky modes)

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4 secondary constraints missing :(

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In the Unitary gauge is OK ! :)

Chiral Scalar-Tensor Theories

One new ingredient: Levi-Civita tensor $\varepsilon^{\mu\nu\rho\sigma}$

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First derivatives of the scalar field only:

$$L_1 \equiv \varepsilon^{\mu\nu\alpha\beta} R_{\alpha\beta\rho\sigma} R_{\mu\nu}{}^{\rho}{}_{\lambda} \phi^{\sigma} \phi^{\lambda} \,, \qquad L_2 \equiv \varepsilon^{\mu\nu\alpha\beta} R_{\alpha\beta\rho\sigma} R_{\mu\lambda}{}^{\rho\sigma} \phi_{\nu} \phi^{\lambda} \,,$$

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Including second derivatives of the scalar field:

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6 (+ 1) primary constraints $\Longrightarrow$ ✓ tuning the free functions

6 (+ 1) secondary constraints $\Longrightarrow$ X no way

Chiral Scalar-Tensor Theories

Unitary gauge:

First derivatives of the scalar field only

$$
\begin{aligned}
S_{UG} &= \frac{2\,\dot{\phi}^2\,\epsilon^{ij\lambda}}{N}\Big[2\,(2a_1+a_2+4a_4)\left(K K_{mi} D_\lambda K^m_j + {}^{(3)}R_{mi} D_\lambda K^m_j - K_{mi} K^{mn} D_\lambda K_{jn}\right) \\
&- (a_2+4a_4)\left(2\,K_{mi} K^n_j D_n K^m_\lambda + {}^{(3)}R_{j\lambda m}{}^n D_n K^m_i\right)\Big]
\end{aligned}
$$

Including second derivatives of the scalar field

$$
\begin{aligned}
S_{UG} &= \frac{\dot{\phi}^3}{N^4}\,\epsilon^{ij\lambda}\Big\{2\,N\left[b_1 N K_{mi} D_\lambda K^m_j + (b_4+b_5-b_3)\,\dot{\phi} K_{mi} K^n_j D_n K^m_\lambda\right] \\
&+ \dot{\phi}\left[b_3\;{}^{(3)}R_{j\lambda m}{}^n K^m_i D_n N - 2\,(b_4+b_5)\;{}^{(3)}R_{m\lambda} K^m_j D_i N\right]\Big\}
\end{aligned}
$$

Summary

- It is possible to avoid the Ostrogradsky instability in higher order theories in a non-trivial way
- Brand new theories
- New phenomenology – GW in parity breaking scalar-tensor theories

Thank you