Vainshtein flows (?)

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Motivation

Theories for dark energy and inflation usually treated as effective theories tested against observations.

Rich structure: Non—trivial interactions and screening mechanisms

Study of theories with derivative interactions has a long history

Derivative expansions for the effective action
O(N) scalar field theories
….

k-essence
Galileons
Horndeski
…

- *What about their consistency and initial conditions from a more fundamental viewpoint?*
- *What can we say about their short-scale properties within and beyond EFT?*
- *This talk: A study within the Wilsonian framework for QFT's*

Scalar fields and derivative interactions

P(X) theories: Dark energy, primordial inflation $X \equiv \frac{1}{2}(\partial\phi)^2$

Vainshtein mechanism: Dominance of non-linear derivative interactions to ''switch off'' fifth-force effects

An old ''trick'': $\sim \frac{1}{\alpha} F^{a}_{\mu\nu} F^{\mu\nu}_{a}$

$\alpha \to 0$

Background configuration for Vainshtein screening is important

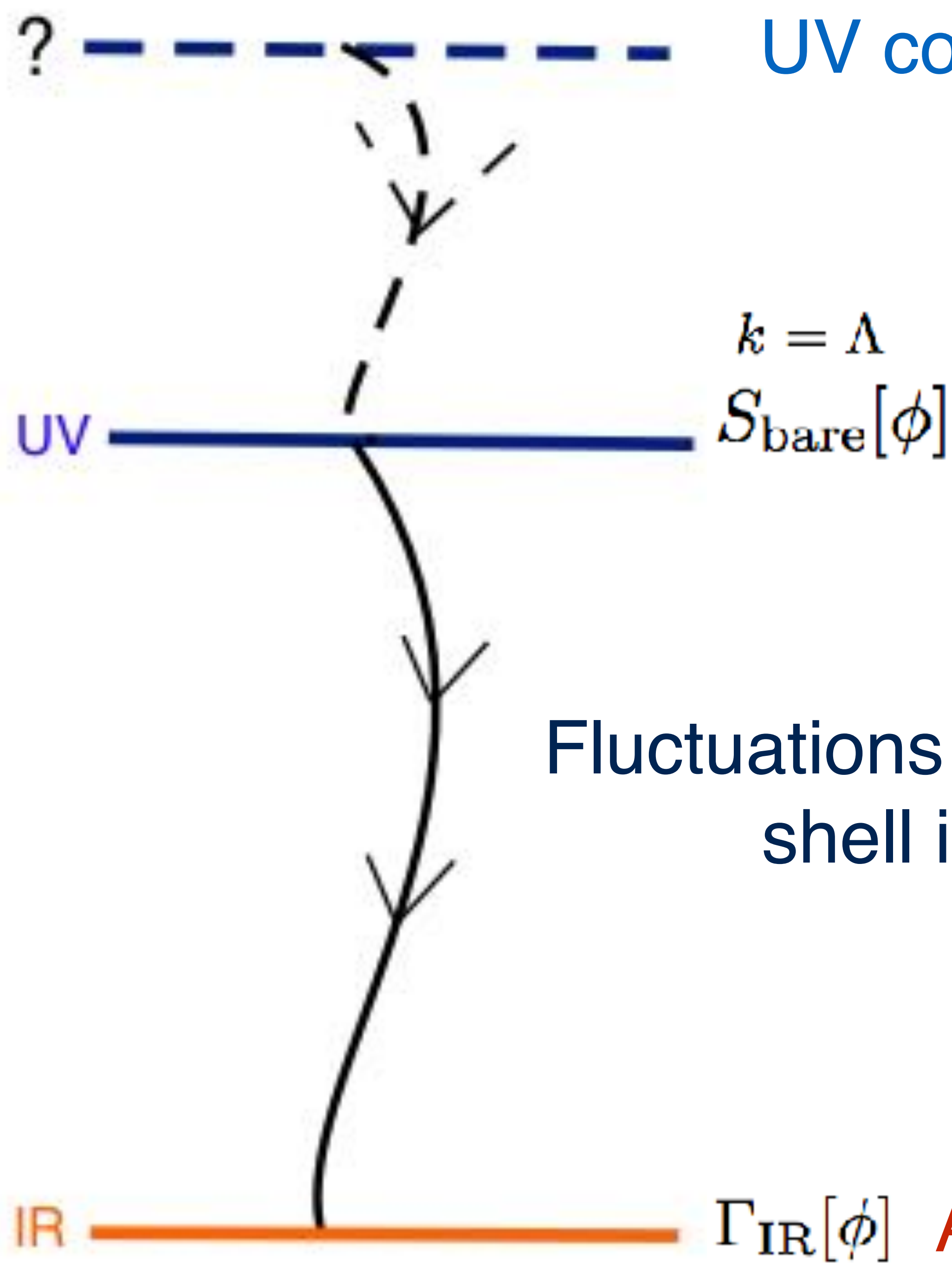
Stability of higher-order operators under quantum corrections?

- Higher-order operators remain under control for small- and large-derivative configurations *

* C. de Rham & R. H. Ribeiro (2014), arXiv: 1405.5213

Our tool: The Wilsonian framework for QFT

$$e^{W[J]} = \int \mathcal{D}\phi\, e^{-S[\phi]+J\cdot\phi-\boxed{\frac{1}{2}\phi\cdot R_k\cdot\phi}}$$



UV completion?

Theory is usually defined from some scale Λ down to the IR

Fluctuations are integrated out shell-by shell in momentum space

All fluctuations are integrated out

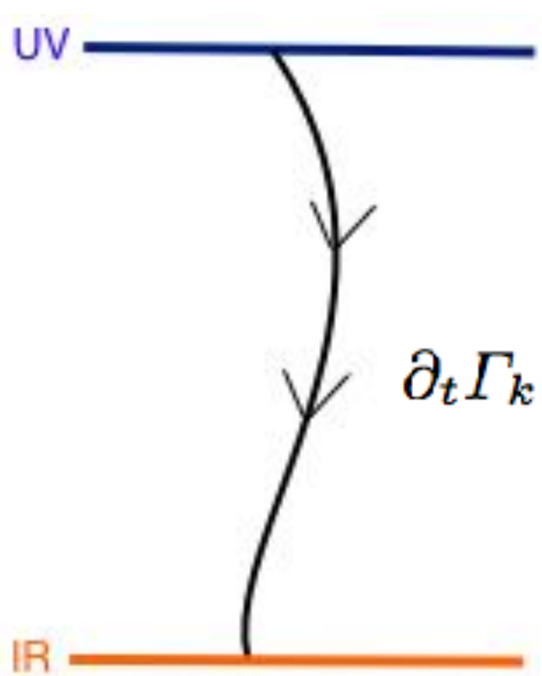
Our tool: The Wilsonian framework for QFT

Calculation of the effective action based on an exact integro-differential equation*

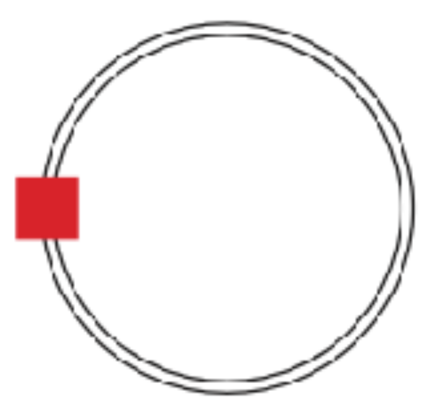
$$\frac{\partial}{\partial \ln k}\Gamma_k = \frac{1}{2}\mathrm{Trace}\left[\left(\Gamma_k^{(2)} + R_k\right)^{-1} \cdot k\partial_k R_k\right]$$

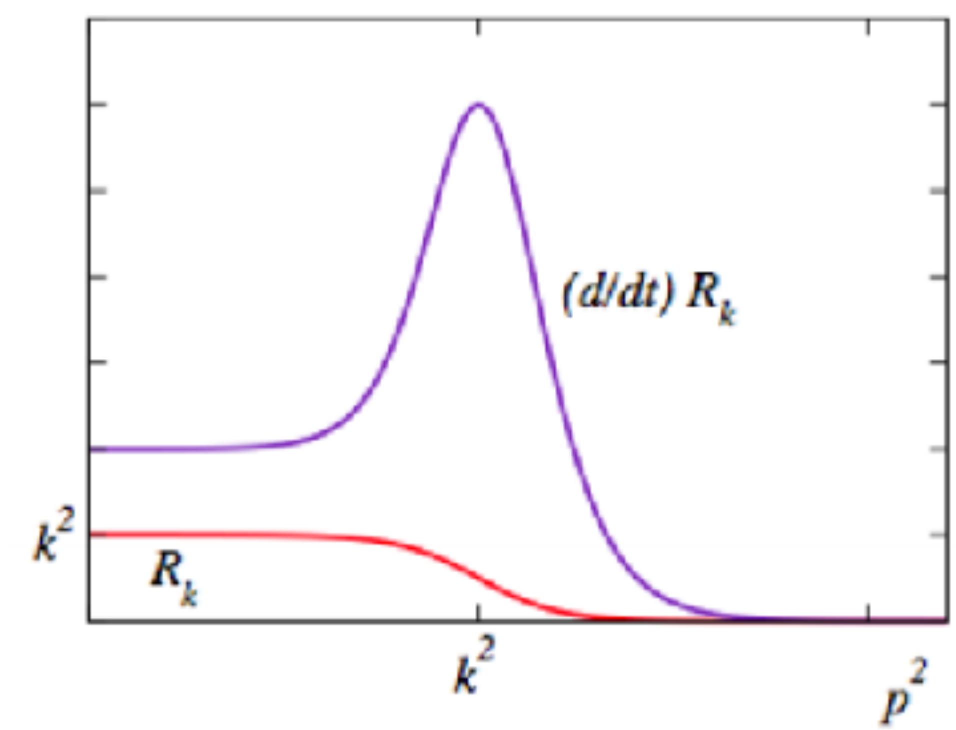
Family of eff. actions parametrised by the Wilsonian scale k

Trace over momenta is localised and "Wilsonian"



$$\partial_t \Gamma_k[\phi] = \frac{1}{2}\,\partial_t R_k$$



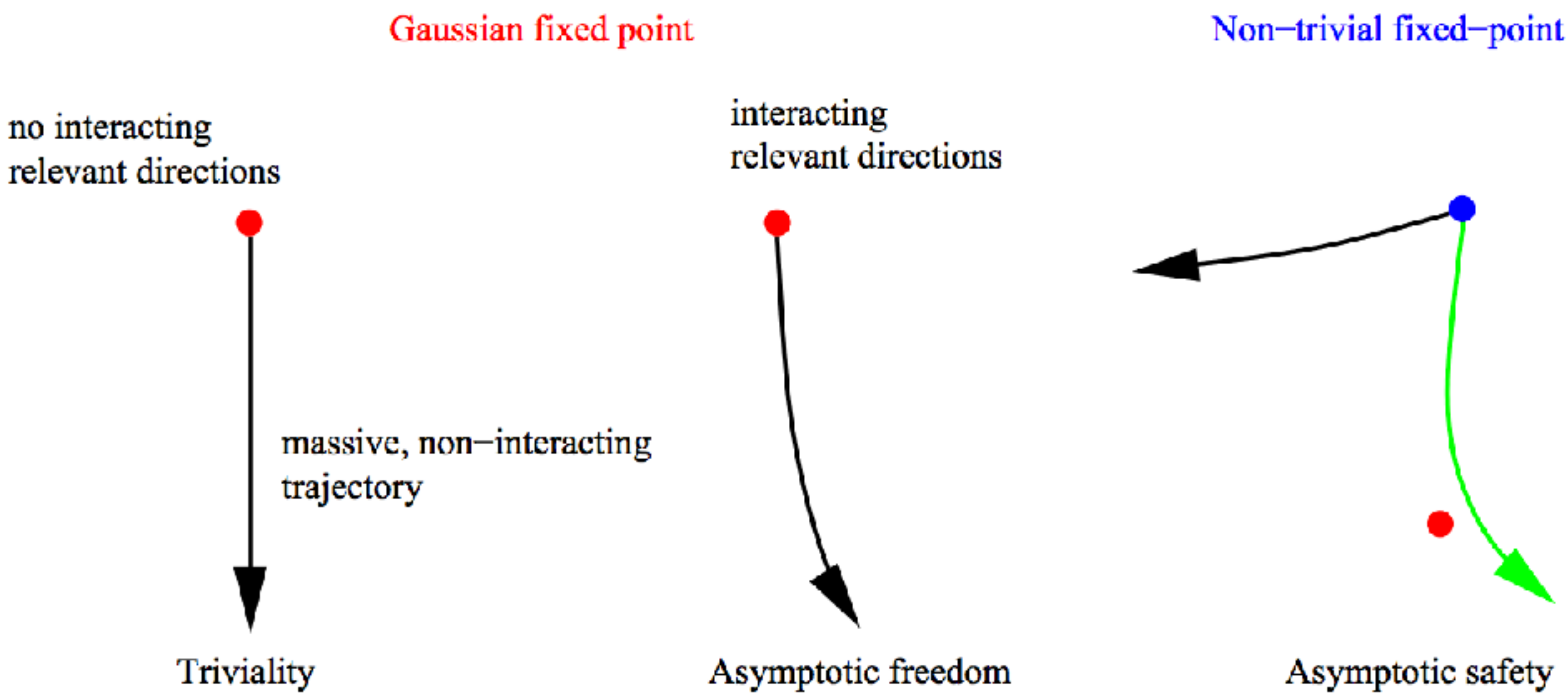


End result: Flow of effective couplings from some UV scale down to IR

* C. Wetterich (1993), T. R. Morris (1994) | Mid/Right picture from: H. Gies (2006) arXiv: 0611146

Our tool: The Wilsonian framework for QFT

Essentials of the Wilsonian RG: Running couplings and fixed points



Picture taken from: Oliver J. Rosten, ''Fundamentals of the exact RG'', Physics Reports 511 (2012)

Triviality and the local potential approximation for scalar QFT's

$$S = \int d^4x \left(\frac{1}{2}(\partial\phi)^2 + U(\phi) \right)$$

$$k\partial_k U(\phi) = \mathcal{I}\left[U, U', U''\right]$$

In d < 4 dimensions: Wilson-Fisher fixed point (interacting)

Higher dimensions: No interacting fixed point yet found —> theory is trivial

P(X) theory: A ''local potential'' approximation for X with ''potential'' P(X)

EFT expansion ⟶ $P(X) \sim X + \frac{X^2}{\Lambda^4} + \frac{X^3}{\Lambda^8} + \ldots$

Wilson & Fisher (1972) | *For reviews see*: O. J. Rosten, Phys. Reports 511 (2012), D. J.E. Callaway, Phys. Reports 167 (1988)

P(X) theory beyond the EFT framework?

- **UV completion:** A potential handle upon the model's initial conditions

P(X) defined at (arbitrary) scale k: $P(X) = \mathcal{Z}(k)X + c_2(k)X^2 + \ldots + c_n(k)X^n$

$$\phi = \bar{\phi} + \psi \quad \Rightarrow \quad \delta^{(2)}\Gamma = \int d^4x \; \psi \cdot Z_{\alpha\beta}\partial^\alpha \partial^\beta \cdot \psi$$

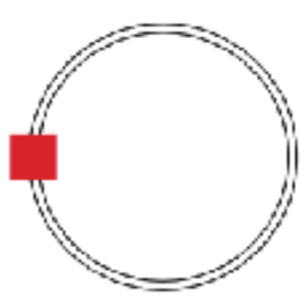
Fluctuating piece

Effective metric of fluctuations: $Z_{\alpha\beta} = P_X \delta_{\alpha\beta} - P_{XX}\phi_\alpha \phi_\beta$

"Wilsonian" trace over modes with p < k: $\mathcal{Z}(k)(-\Box) \to \mathcal{Z}(k)(-\Box) + R_k$

Existence of a small parameter: $\dfrac{X}{k^2} \ll 1$

- Small-derivative configuration
- Deep UV regime

$\frac{1}{2}\partial_t R_k$ 

$$= \frac{1}{2}\int_p \frac{\partial_t R_k(-p^2)}{\Gamma^{(2)}(-p^2) + R_k(-p^2)} = \mathcal{I}\left[k\partial_k \mathcal{Z}, P(X), P'(X)\right]$$

P(X) theory beyond the EFT framework?

$$k\partial_k \tilde{c}_i = \beta_i(\tilde{c}_j)$$

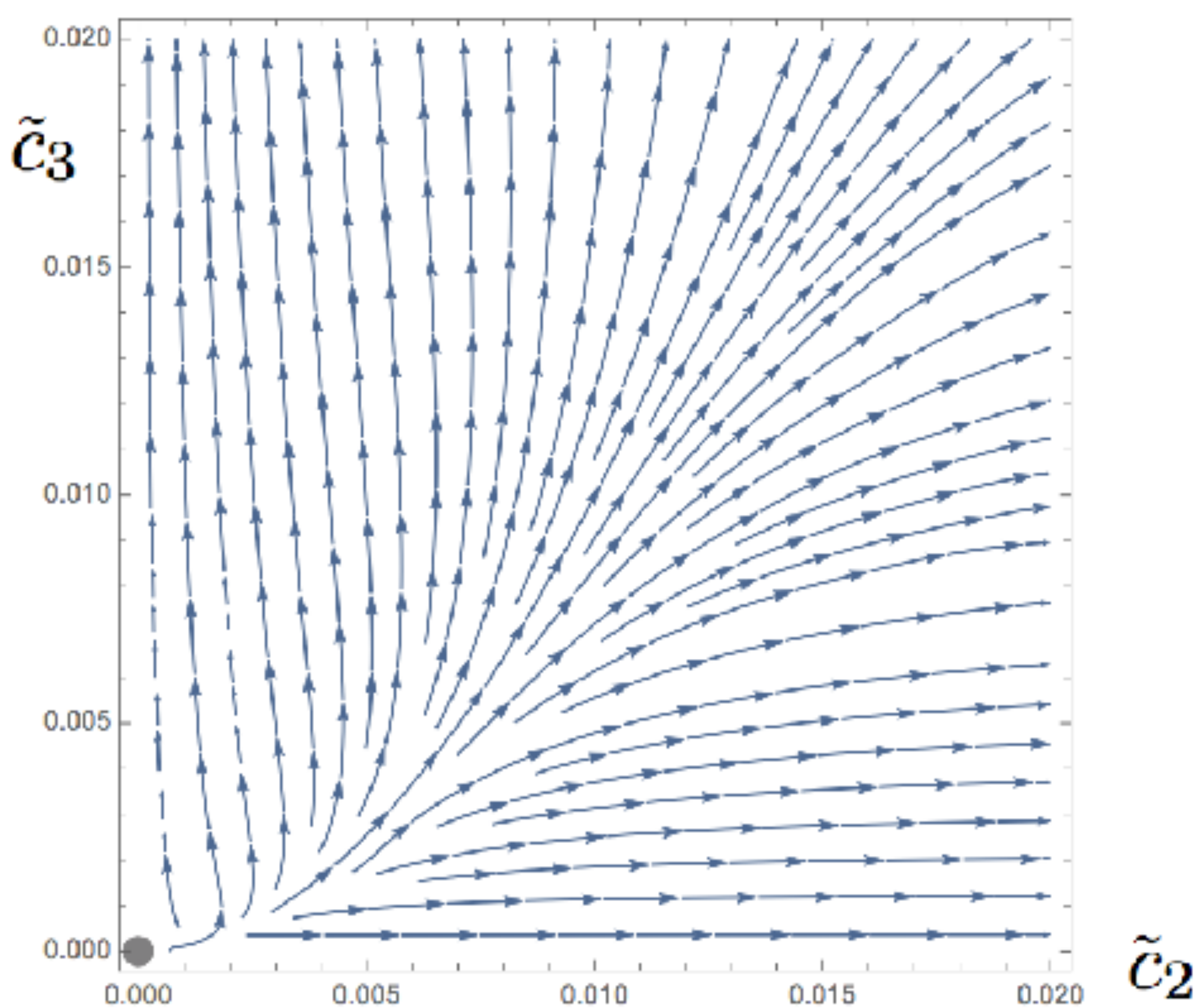
$$P(X) = \mathcal{Z}(k)X + c_2(k)X^2 + \ldots + c_n(k)X^n$$

Triviality $\beta_i(\tilde{c}_j) = 0$

$$\tilde{c}_i = 0$$

Stability
$$M_{ij} = \left.\frac{\partial \beta_i}{\partial \tilde{c}_j}\right|_{\tilde{c}_k=0} = \begin{bmatrix} 0 & 0 & 0 & \ldots & 0 \\ x_{21} & 4 & 0 & \ldots & 0 \\ x_{31} & x_{32} & 8 & \ldots & 0 \\ x_{n1} & x_{n2} & x_{n3} & \ldots & 4\cdot n \end{bmatrix}$$

$$\lambda_{n+1} = \lambda_n + 4 > 0$$



No UV-attractive directions

UV completion?

What is the moral?

(Non-) Running in the Vainshtein regime

No available small, background parameter

$$\Gamma = \int d^4x\, P(X) = \int d^4x \sum_i c_i (X - X_0)^i$$

Generic background configuration

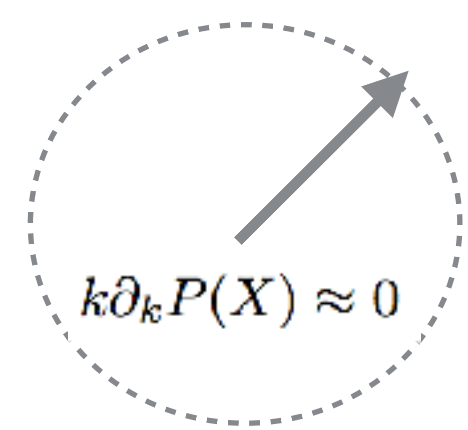
$$k\partial_k P(X) = \frac{1}{2}\mathrm{Tr}\frac{\partial_t R_k(-\Box)}{\Gamma^{(2)} + R_k(-\Box)} = \frac{2\pi^2}{(4\pi)^4}\sum_{i=0}^{n} I_i$$

Series of hypergeometric functions parametrised by n

- $$I_n \sim \frac{k^4}{(n+2)}\frac{1}{\Omega_0 \mathcal{Z}^{n-1}} \cdot F_1(1 + n/2, n, n/2 + 2, -\Omega_0/\mathcal{Z})$$

Background configuration $\Omega_0 \sim Z^{\alpha\beta}\hat{p}_\alpha \hat{p}_\beta$

- Large-derivative configurations: $\lim_{\Omega_0 \to \infty} \frac{I_n}{k^4} = 0$
- Leading order for $\Omega_0, n \gg 1$ $\quad \frac{I_n}{k^4} \sim \left(\frac{1}{n}\right)^{3/2} \cdot \Omega_0^{-(n/2+2)}$



Beyond the P(X) approximation

Quantum corrections generate higher-order derivative interactions. In principle, they should be included in the original derivative expansion

A more general, higher-derivative action: $L(X, B) \qquad B \equiv \Box\phi$

Results for the P(X) extend to the more general theory

No non—trivial UV fixed point:
Theory can be only viewed as an EFT

Running approaches zero
in the Vainshtein regime
irrespective of the form of L(X,B)

Dominance of B- or X- configurations

Implications

Lessons from the (non-perturbative) Wilsonian approach for L(X,B) theories:

- In the deep UV, expectations based on simple power counting hold true
- No apparent UV completion
- Can show analytically that within the Vainshtein radius, suppression of interactions switches off the RG flow irrespective of the form of L(X,B)

Should one worry about the absence of a UV completion?

Yes and no. Still, all of our realistic theories up to now are EFTs

"Freeze" of the RG flow for large-derivative configurations

A worrisome feature: Potentially strong sensitivity on initial conditions

Summary

Understanding the initial conditions and short-scale properties of effective dark energy theories is important

Wilsonian approach: a very important tool at hand

Screening plays an important role for the quantum dynamics of the theory

Theories with derivative interactions

Going beyond EFT: UV completion?

Initial conditions in Vainshtein regime?

Thank you