Large Scale Structure in mimetic Horndeski gravity

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Outline

- *A short introduction to mimetic gravity*
- *Mimetic Horndeski models: properties and motivations*
- *Cosmological perturbations*
- *Conclusions*

Based on

F. Arroja, N. B. , P. Karmakar, S. Matarrese, JCAP 1509, 051 (2015)

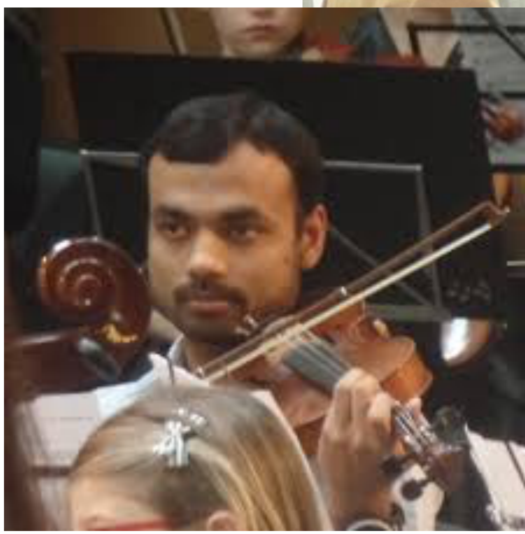
F. Arroja, N. B. , P. Karmakar, S. Matarrese, JCAP 1604, 042 (2016)

F. Arroja, T. Okamaura, N. B. , P. Karmakar, S. Matarrese, arXiv:1708.01850

Frederico Arroja

Sabino Matarrese

Purnendu Karmakar



Teppei Okumura

Mimetic gravity

- Original model: Mimetic Dark Matter
  proposed by *Chamseddine and Mukhanov, 1308.5410;*
  Chamseddine, Mukhanov and Vikman, 1403.3961

A modification of General Relativity where Dark Matter arises as a pure gravitational effect.

- $$S = \int d^4x \sqrt{-g}\, R + S_m$$

$$g_{\mu\nu} = -w\, \ell_{\mu\nu}$$

→ new auxiliary metric

$$w \equiv \ell^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi$$

Mimetic gravity

- Eqns. in mimetic Dark Matter varying S w.r.t to $\ell_{\mu\nu}$ and φ

$$G^{\mu\nu} - T^{\mu\nu} = (G - T)\, g^{\mu\alpha} g^{\nu\beta} \partial_\alpha \varphi \partial_\beta \varphi$$

$$\nabla_k ((G - T) \partial^k \varphi) = 0$$

N.B.1: the first equation is traceless

$(G - T)(1 - g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi) = 0$ but $\boxed{g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi = -1}$

Mimetic constraint

N.B.2: the first equation is equivalent to

$$G^{\mu\nu} = T^{\mu\nu} + \rho u^\mu u^\nu$$

→ The new degree of freedom (*of gravitational origin*) mimics an irrotational pressureless perfect fluid (Dark Matter).

$$\rho = (G - T) \qquad u^\mu = g^{\mu\alpha} \partial_\alpha \varphi$$

Mimetic gravity

- Why one gets new equations of motions?

The original theory is generically invariant under disformal transf.

$$g_{\mu\nu} = A(\varphi, w)\ell_{\mu\nu} + B(\varphi, w)\partial_\mu\varphi\partial_\nu\varphi$$

However there exists a particular subset for which eqn. of motion are no longer the original ones.

Shown by *Deruelle & Rua '14; Barvinsky '14* if you start from GR. We have generalized to any type of scalar-tensor theory.

⟷

Non-invertibility of the disformal transformation

$$B(\varphi, w) = -\frac{A(\varphi, w)}{w} + b(\varphi)$$

↓

Mimetic constraint

$$b(\varphi)g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - 1 = 0$$

Arroja, N. B. , Karmakar, Matarrese, JCAP 1509, 051 (2015)

N.B.: this is a very general statement, it is a kinematic constraint which i) does not depend on the original theory one starts from; ii) holds irrespecitve of whether the scalar field in the original action is the same or not as the one involved in the transf.

e.g.: mimetic dark matter corresponds to

$B(\varphi, X) = 0 \quad b(\varphi, w) = -1$ and $A(\varphi, w) = -w$

The mimetic constraint

- Suppose to have the disformal transformation

$$g_{\mu\nu} = A(\varphi, w)\ell_{\mu\nu} + B(\varphi, w)\partial_\mu\varphi\partial_\nu\varphi \quad \text{with} \quad B(\varphi, w) = -\frac{A(\varphi, w)}{w} + b(\varphi)$$

$$g^{\mu\nu} = \frac{\ell^{\mu\nu}}{A} + \frac{A - wb}{Abw^2}(\ell^{\mu\rho}\partial_\rho\varphi)(\ell^{\nu\sigma}\partial_\sigma\varphi)$$

$$\ell^{\mu\rho}\partial_\rho\varphi = b\, w\, g^{\mu\rho}\partial_\rho\varphi$$

- Since $w \equiv \ell^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi$ make the contraction

$$\ell^{\mu\rho}\partial_\rho\varphi\partial_\mu\varphi = w = b\, w\, g^{\mu\rho}\partial_\rho\varphi\partial_\mu\varphi \longrightarrow \boxed{b(\varphi)g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - 1 = 0}$$

Mimetic Horndeski

$$S = \int d^4x\sqrt{-g}\mathcal{L}[g_{\mu\nu}, \partial_{\lambda_1} g_{\mu\nu}, \dots, \partial_{\lambda_1}\dots\partial_{\lambda_p} g_{\mu\nu}, \varphi, \partial_{\lambda_1}\varphi, \dots, \partial_{\lambda_1}\dots\partial_{\lambda_q}\varphi] + S_m[g_{\mu\nu}, \phi_m]$$

$$+ \int d^4x\sqrt{-g}\lambda\left(b(\varphi)g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - 1\right) \quad **$$

Original starting theory ***(e.g. Horndeski)***.

Mimetic theory

- Some definitions of basic quantities

$$\Omega_\varphi = \frac{\delta\left(\sqrt{-g}\mathcal{L}\right)}{\delta\varphi} = \frac{\partial(\sqrt{-g}\mathcal{L})}{\partial\varphi} + \sum_{h=1}^{q}(-1)^h\frac{d}{dx^{\lambda_1}}\cdots\frac{d}{dx^{\lambda_h}}\frac{\partial(\sqrt{-g}\mathcal{L})}{\partial\left(\partial_{\lambda_1}\dots\partial_{\lambda_h}\varphi\right)},$$

$$E^{\mu\nu} = \frac{2}{\sqrt{-g}}\frac{\delta(\sqrt{-g}\mathcal{L})}{\delta g_{\mu\nu}} = \frac{2}{\sqrt{-g}}\left(\frac{\partial(\sqrt{-g}\mathcal{L})}{\partial g_{\mu\nu}} + \sum_{h=1}^{p}(-1)^h\frac{d}{dx^{\lambda_1}}\cdots\frac{d}{dx^{\lambda_h}}\frac{\partial(\sqrt{-g}\mathcal{L})}{\partial\left(\partial_{\lambda_1}\dots\partial_{\lambda_h} g_{\mu\nu}\right)}\right),$$

$$T^{\mu\nu} = \frac{2}{\sqrt{-g}}\frac{\delta(\sqrt{-g}\mathcal{L}_m)}{\delta g_{\mu\nu}}, \ \Omega_m = \frac{\delta(\sqrt{-g}\mathcal{L}_m)}{\delta\phi_m}, \ \text{where } S_m[g_{\mu\nu}, \phi_m] = \int d^4x\sqrt{-g}\mathcal{L}_m[g_{\mu\nu}, \phi_m]$$

** See *Golovnev '14; Barvinsky '14* for GR as the starting theory; *Chamseddine, Mukhanov and Vikman '14.*
Arroja, N. B. , Karmakar, Matarrese, JCAP 1509, 051 (2015)

Mimetic Horndeski

$$S = \int d^4x\sqrt{-g}\mathcal{L}[g_{\mu\nu}, \partial_{\lambda_1} g_{\mu\nu}, \dots, \partial_{\lambda_1}\dots\partial_{\lambda_p} g_{\mu\nu}, \varphi, \partial_{\lambda_1}\varphi, \dots, \partial_{\lambda_1}\dots\partial_{\lambda_q}\varphi] + S_m[g_{\mu\nu}, \phi_m]$$

$$+ \int d^4x\sqrt{-g}\lambda\left(b(\varphi)g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - 1\right)$$

Original starting theory ***(e.g. Horndeski)***.

Mimetic theory

- ***Mimetic Eqs. of motion:*** once you use $2\lambda = E + T$

$$b(\varphi)g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - 1 = 0$$

$$\nabla_\mu\left[(E+T)b(\varphi)\partial^\mu\varphi\right] - \frac{\Omega_\varphi}{\sqrt{-g}} = \frac{E+T}{2}\frac{1}{b(\varphi)}\frac{db(\varphi)}{d\varphi}$$

This equation is not independent from the others

$$E^{\mu\nu} + T^{\mu\nu} = (E+T)b(\varphi)\partial^\mu\varphi\partial^\nu\varphi$$

$$\Omega_m = 0$$

Using the mimetic constraint you can show that 0-0 component is not independent

Mimetic Horndeski

- Two simple examples of mimetic Horndeski cosmology.
- The original starting action is

$$S_H = \int d^4x\sqrt{-g}\mathcal{L}_H = \int d^4x\sqrt{-g}\sum_{n=0}^{3}\mathcal{L}_n$$

$$\begin{aligned}
\mathcal{L}_0 &= K\left(X,\varphi\right), \\
\mathcal{L}_1 &= -G_3\left(X,\varphi\right)\Box\varphi, \\
\mathcal{L}_2 &= G_{4,X}\left(X,\varphi\right)\left[\left(\Box\varphi\right)^2 - \left(\nabla_\mu\nabla_\nu\varphi\right)^2\right] + R\,G_4\left(X,\varphi\right), \\
\mathcal{L}_3 &= -\frac{1}{6}G_{5,X}\left(X,\varphi\right)\left[\left(\Box\varphi\right)^3 - 3\Box\varphi\left(\nabla_\mu\nabla_\nu\varphi\right)^2 + 2\left(\nabla_\mu\nabla_\nu\varphi\right)^3\right] + G_{\mu\nu}\nabla^\mu\nabla^\nu\varphi\,G_5\left(X,\varphi\right)
\end{aligned}$$

$$X = -1/2\nabla_\mu\varphi\nabla^\mu\varphi$$

Mimetic Horndeski

- **Example 1**: choose as a starting theory GR + a minimally coupled scalar field (with zero potential) and no matter (S_m=0)

$$K(X,\varphi) = c_2\, X \qquad G_3(X,\varphi) = 0 \qquad G_4(X,\varphi) = 1/2 \qquad G_5(X,\varphi) = 0$$

- This minimal scalar field mimetic model can mimic the background evolution of a perfect fluid universe with a constant equation of state ω.
  N.B.: of course not possible in the original theory where ω=1.

$$\left\{ \begin{array}{l} b(\varphi)\dot{\varphi}^2 + 1 = 0 \\ 6H^2 + 4\dot{H} + c_2\dot{\varphi}^2 = 0 \end{array} \right. \longrightarrow \left\{ \begin{array}{l} a(t) = t^{\frac{2}{3(1+\omega)}} \,, \varphi(t) = \pm\sqrt{\frac{-\alpha}{c_2}} \log\frac{t}{t_0} \\ b(\varphi) = -\frac{1}{\dot{\varphi}^2} = \frac{c_2}{\alpha} t^2 = \frac{c_2}{\alpha} t_0^2\, e^{\pm 2\sqrt{\frac{-c_2}{\varphi}}\varphi} \end{array} \right.$$

Here $\alpha = -\frac{8\omega}{3(1+\omega)^2}$

N.B.: see also *Lim, Sawicki, Vikman, 2010*, for a similar model.

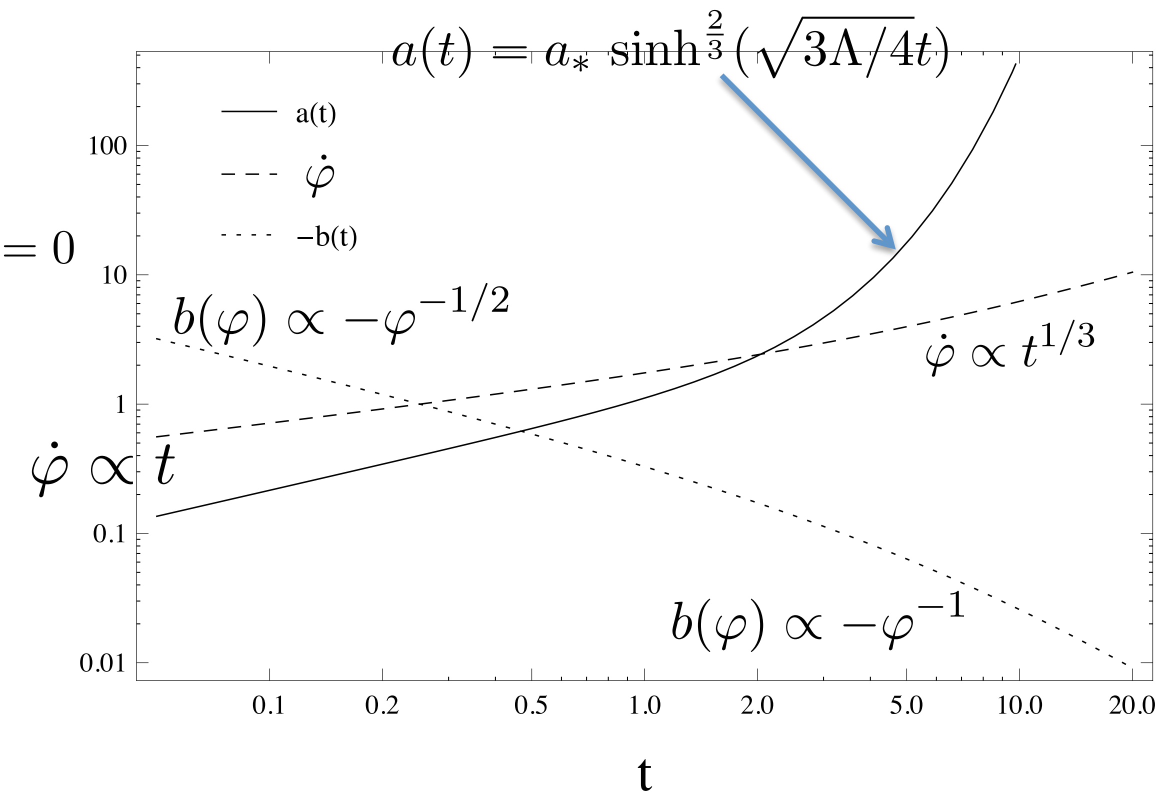
Mimetic Horndeski

- **Example 2**: no matter (S_m=0); mimetic cubic Galileon

$$K(X,\varphi) = c_2\, X \qquad G_3(X,\varphi) = 2c_3/\tilde{\Lambda}\, X \qquad G_4(X,\varphi) = 1/2 \qquad G_5(X,\varphi) = 0$$

- By suitably choosing an appropriate b(φ) one can mimic almost any background expansion history, including ΛCDM

$$\begin{cases} b(\varphi)\dot{\varphi}^2 + 1 = 0 \\ 6H^2 + 4\dot{H} + \dot{\varphi}^2(c_2 - 4c_3\ddot{\varphi}) = 0 \end{cases}$$



Mimetic Horndeski

- **Some interesting aspects/motivations are**

- it is a fairly general scalar-tensor theory.
  As such it can mimic any background expansion history

- it contains many mimetic gravity models studied in the literature.

- it can provide a model of (unified) dark matter and dark energy.
  It is a matter of fact that no dark matter *particles have been detected so far.*

- it can provide models only for DE, or only for DM (and possibly inflation models)

Mimetic Horndeski

- **Also some of the results of mimetic Horndeski models can be useful for**

 * extensions of mimetic DM models, where a term $(\Box\phi)^2$ is added : can address the small-scale ΛCDM problems if the sound speed is extremely small (Chamseddine, Mukhanov, Vikman '14; Capela & Ramazanov 2014).

 Notice that these extensions can find a UV justification: they are equivalent to the infrared limit of the projectable Horava-Lifshitz gravity (see Ramazanov, Arroja, Celoria, Matarrese, Pilo 2016)

 Extensions of mimetic dark matter models with higher-derivatives models can support vorticity (e.g., Mirzagholi & Vikman, 2015; Barvinsky 2014).

 * non-invertible disformal transformation applied to some (non-degenerate) actions that can lead to DHOST theories (see, e.g., Achour, Langlois, Noui, '16; Takahashi & Kobayashi '17).

Cosmological pert. in Mimetic Horndeski

- ***Linear perturbations:***

$$g_{00} = -a^2(\tau)\,(1+2\Phi)\,, \quad g_{0i} = 0, \quad g_{ij} = a^2(\tau)\,(1-2\Psi)\,\delta_{ij}$$

- ***Perturb Mimetic Eqs. of motion:***

$$b(\varphi)g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - 1 = 0 \quad E^{\mu i} + T^{\mu i} = (E+T)b(\varphi)\partial^\mu\varphi\partial^i\varphi \qquad \Omega_m = 0$$

For standard (non-interacting) "species" usual equations
(we consider just radiation and baryons: NO DM NOR DE PUT BY HAND)

$$\delta_{\mathfrak{f}}' + 3\mathcal{H}\left(c^2_{(\mathfrak{f})s} - w_{\mathfrak{f}}\right)\delta_{\mathfrak{f}} - 3(1+w_{\mathfrak{f}})\Psi' + (1+w_{\mathfrak{f}})\left[-k^2 - 9\mathcal{H}^2\left(c^2_{(\mathfrak{f})s} - c^2_{(\mathfrak{f})a}\right)\right]v_{\mathfrak{f}} = 0$$

$$v_{\mathfrak{f}}' + \mathcal{H}\left(1 - 3c^2_{(\mathfrak{f})s}\right)v_{\mathfrak{f}} + \frac{c^2_{(\mathfrak{f})s}}{1+w_{\mathfrak{f}}}\delta_{\mathfrak{f}} + \Phi - \frac{2k^2}{3\bar{\rho}_{\mathfrak{f}}\,(1+w_{\mathfrak{f}})}\Pi_{\mathfrak{f}} = 0.$$

Cosmological pert. In Mimetic Horndeski

- ***Perturb Mimetic Eqs. of motion:***

$$b(\varphi)g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - 1 = 0 \qquad E^{\mu i} + T^{\mu i} = (E+T)b(\varphi)\partial^\mu\varphi\partial^i\varphi \qquad \Omega_m = 0$$

mimetic constraint

Traceless i-j eqn. and 0-i

$$2\bar{b}\delta\varphi' + \bar{\varphi}'\bar{b}_{,\varphi}\delta\varphi - 2\bar{b}\bar{\varphi}'\Phi = 0$$

$$f_7\Psi + f_8\delta\varphi + f_9\Phi + a^2\Pi = 0$$

$$\Psi' + \mathcal{H}\Phi + \left(\frac{a^2(\bar{\rho}+\bar{p})}{f_{10}} + \mathcal{H}^2 - \mathcal{H}'\right)v_\varphi - \frac{a^2}{f_{10}}(\bar{\rho}+\bar{p})\,v = 0$$

* here f_i functions are a combination of the K, G_3,G_4, G_5 and their derivatives

Cosmological pert. in Mimetic Horndeski

- ***N.B.: from the mimetic constraint***

$$2\bar{b}\delta\varphi' + \bar{\varphi}'\bar{b}_{,\varphi}\delta\varphi - 2\bar{b}\bar{\varphi}'\Phi = 0$$

$$\Big\downarrow \quad v_\varphi \equiv -\frac{\delta\varphi}{\bar{\varphi}'}$$

$$v'_\varphi + \mathcal{H}v_\varphi + \Phi = 0$$

$$\Big\updownarrow \quad \text{vs}$$

$$v'_\mathfrak{f} + \mathcal{H}\left(1 - 3c^2_{(\mathfrak{f})s}\right)v_\mathfrak{f} + \frac{c^2_{(\mathfrak{f})s}}{1+w_\mathfrak{f}}\delta_\mathfrak{f} + \Phi - \frac{2k^2}{3\bar{\rho}_\mathfrak{f}\left(1+w_\mathfrak{f}\right)}\Pi_\mathfrak{f} = 0$$

the same as a perfect fluid with zero sound speed. This is a first indication that in mimetic models scalar perturbations have zero sound speed.

Mimetic cubic Horndenski

- Let us restrict to the models

$$G_4 = \frac{M_{Pl}^2}{2} \qquad G_5 = 0$$

while b(φ), K and G_3 are general.

- These models include the original mimetic dark matter models of Chamseddine and Mukhanov, 14+, and of Lim, Sawicki and Vikman '10. They include the cubic Galileon mimetic model explained before

- At the background level they can mimic any desired expansion history

- At the (linear) perturbation level

$$v'_\varphi + \mathcal{H} v_\varphi + \Phi = 0 \qquad \Phi = \Psi$$ (since for these models $f_8=0$, $f_7=M^2_{Pl}=-f_9=f_{10}/2$)

$$\Phi' + \mathcal{H}\Phi + \left(-\frac{a^2 \sum_{\mathfrak{f}=r,b} (\bar{\rho}_\mathfrak{f} + \bar{p}_\mathfrak{f})}{2M_{Pl}^2} + \mathcal{H}^2 - \mathcal{H}' \right) v_\varphi + \frac{a^2}{2M_{Pl}^2} \sum_{\mathfrak{f}=r,b} (\bar{\rho}_\mathfrak{f} + \bar{p}_\mathfrak{f}) v_\mathfrak{f} = 0$$

Equivalence with Pressureless Perfect Fluid DE

Suppose the model mimic the background expansion of a Perfect Fluid Dark energy model (PFDE)

$$2M_{Pl}^2(\mathcal{H}^2 - \mathcal{H}') = a^2 \sum_{\mathfrak{f}=r,m,DE} (\bar{\rho}_{\mathfrak{f}} + \bar{p}_{\mathfrak{f}})$$

$$\Phi' + \mathcal{H}\Phi + \frac{a^2}{2M_{Pl}^2}\left[(\bar{\rho}_{CDM} + \bar{p}_{CDM})v_{\varphi} + (\bar{\rho}_{DE} + \bar{p}_{DE})v_{\varphi} + \sum_{\mathfrak{f}=r,b}(\bar{\rho}_{\mathfrak{f}} + \bar{p}_{\mathfrak{f}})v_{\mathfrak{f}}\right] = 0$$

VS

$$\Phi' + 3\mathcal{H}\Phi + \frac{2}{a^2 M_{Pl}^2}\left[(\bar{\rho}_{CDM} + \bar{p}_{CDM})v_{CDM} + (\bar{\rho}_{DE} + \bar{p}_{DE})v_{DE} + \sum_{f=r,b}(\bar{\rho}_{f} + \bar{p}_{f})v_{f}\right] = 0$$

The two eqns. are the same if v_{DE} is a dust velocity, i.e., if c_s(DE)=0 and if Π_{DE} =0.

If the background expansion is that of PFDE (with any equation of state), these simple mimetic models predict the same solution for Φ as for pressurless PFDE.

Vanishing sound speed

- That mimetic Horndeski models can mimick ***pressureless*** PFDE might not come as a surprise:
  in *JCAP 1604, 042 (2016) and in arXiv:1708.01850* we have shown that (if the other fluids are dust) then the sound speed for scalar perturbations is zero.

- e.g.: for the mimetic cubic Horndeski models $G_4=1/2$ and $G_5=0$

$$\Phi'' + \Phi' \left(3\mathcal{H} + \tilde{\Gamma}\right) + \Phi\left(\mathcal{H}^2 + 2\mathcal{H}' + \tilde{\Gamma}\mathcal{H}\right) = 0$$

$$\tilde{\Gamma} \equiv \frac{-\mathcal{H}'' + \mathcal{H}\mathcal{H}' + \mathcal{H}^3}{\mathcal{H}' - \mathcal{H}^2} = 0 \quad \longleftrightarrow \quad \Lambda\text{CDM background}$$

So in particular if the background is ΛCDM then Φ is the same as in ΛCDM

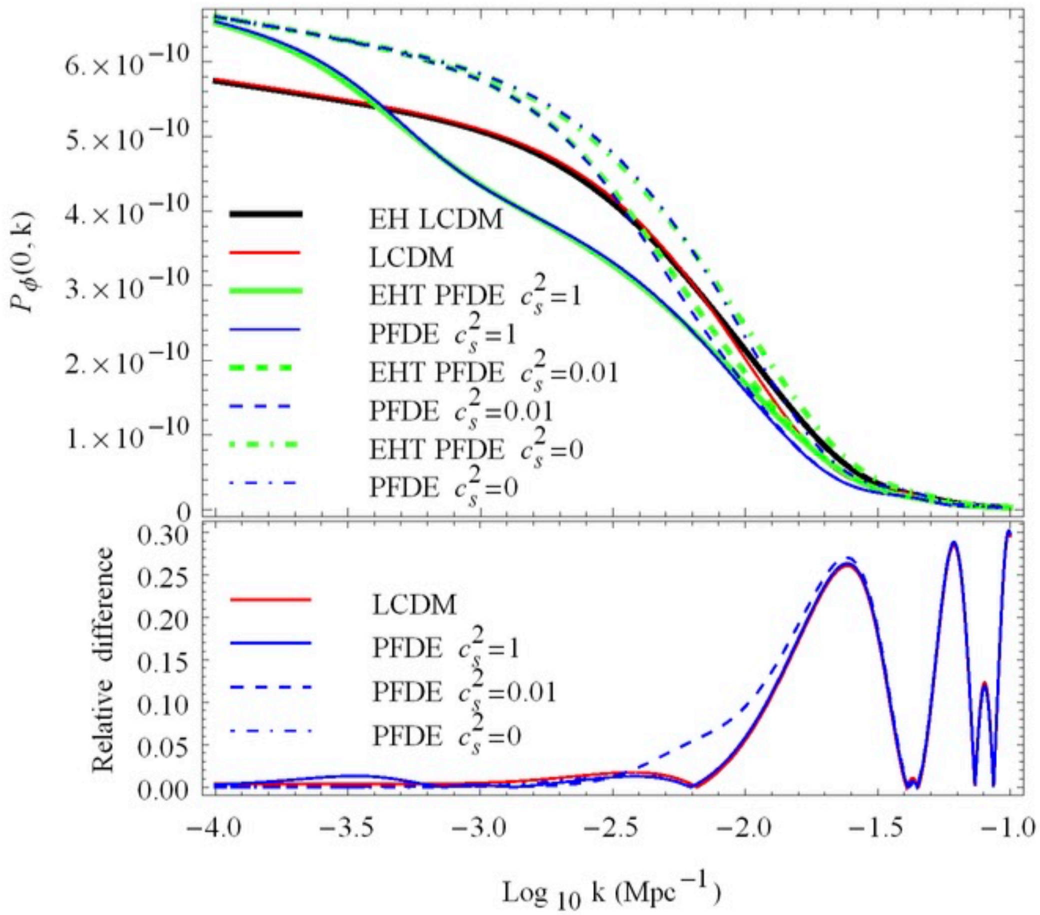
- N.B.: we have also shown that the sound speed is zero also in G^3 mimetic models.
  Also we have shown that the sound speed is exactly zero in any background in a fully non-perturbative way, see *JCAP 1604, 042 (2016).*

LSS in Mimetic Horndeski

- We have solved numerically the linear perturbations eqns. for mimetic models and for generic PFDE models under some simplifying approximations (e.g. hydrodynamical approx., instant recombination, and a toy model for decoupling). We compare our results with CAMB for ΛCDM and with fitting functions by Hu ('98), Eisenstein & Hu ('96) and Takada (06) → EHT for PFDE models

- Disclaimer: the main goal is not to constrain the parameters of the models with great accuracy (for the moment), but we have performed a first step to show that these mimetic models can give reasonable predictions for the linear power spectra

LSS in Mimetic Horndeski



Here for PFDE

$$w(a) = w_0 + w_a(1-a)$$

with

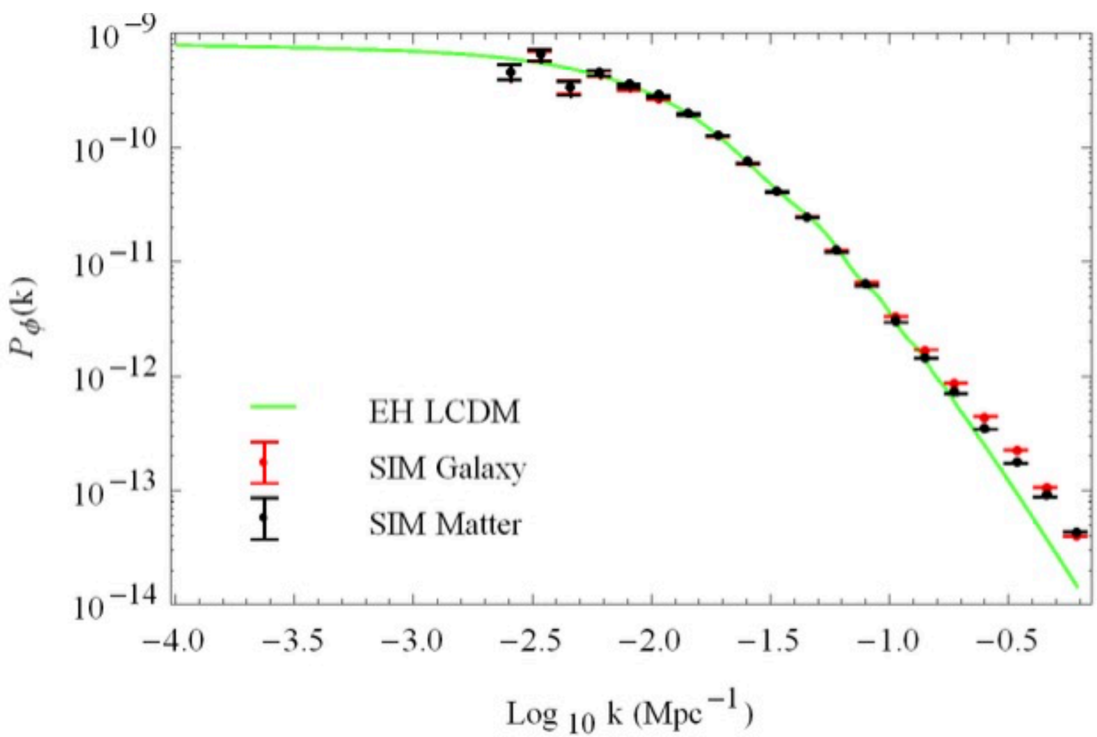
$$w_0 = -0.7,\ w_a = 0$$

Good agreement (to about 10%) of our code with EHT fitting functions for $k \lesssim 10^{-2}\mathrm{Mpc}^{-1}$

Distinguishing mimetic from PFDE models

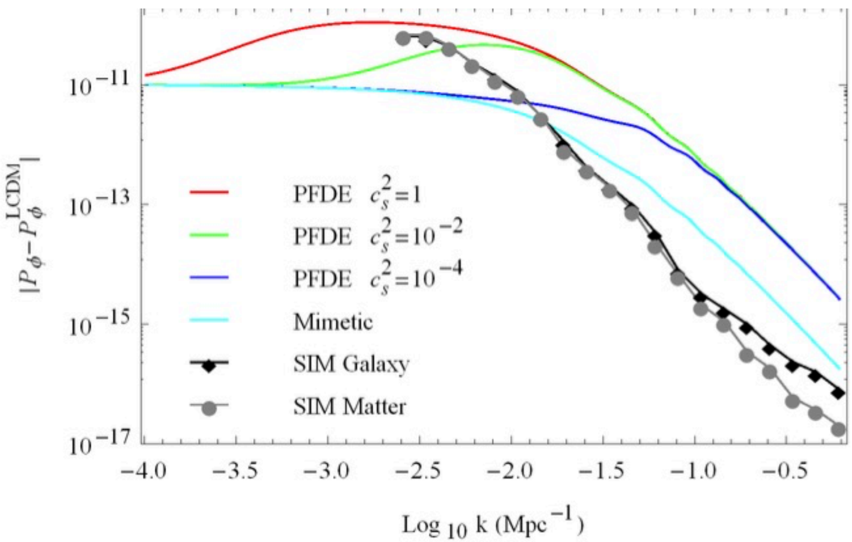
- We used matter and galaxy power spectra extracted by Okumura, Seljak, McDonald, Desjacques et al. '12+ from N-body simulations for ΛCDMproduced by Desjacques, Seljak, Iliev 09. From this get the power spectrum for ϕ.

- 12 independent realizations, and error bars computed by the dispersion (of the mean) among realizations (it does not include observational systematics). Use these error bars as proxy of the statistical error bars for future LSS surveys.

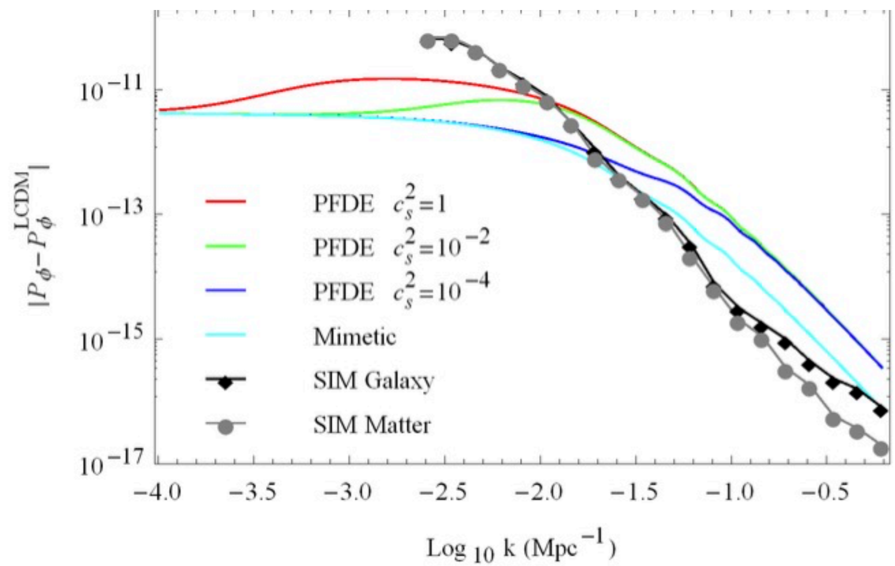


Distinguishing mimetic from PFDE models

Mimetic vs ΛCDM



$w_0 = -0.7,\ w_a = 0$

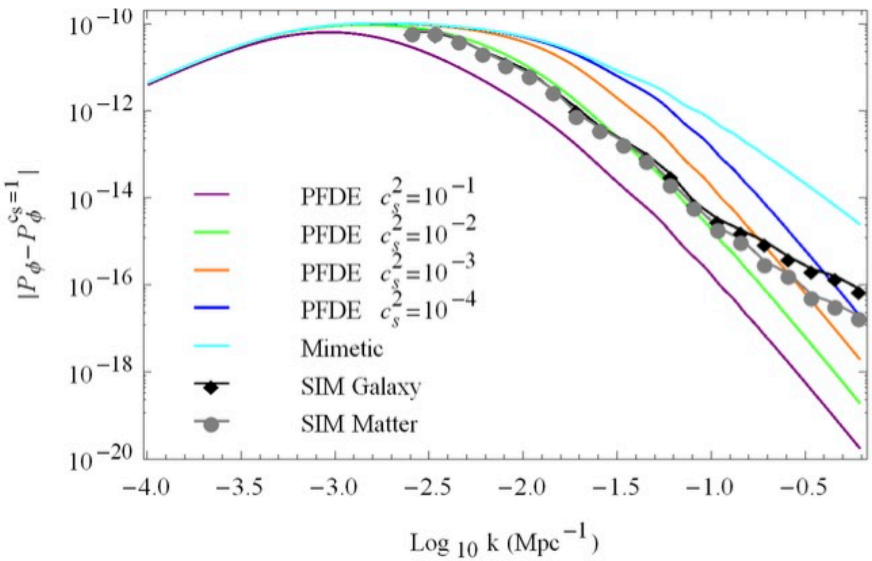


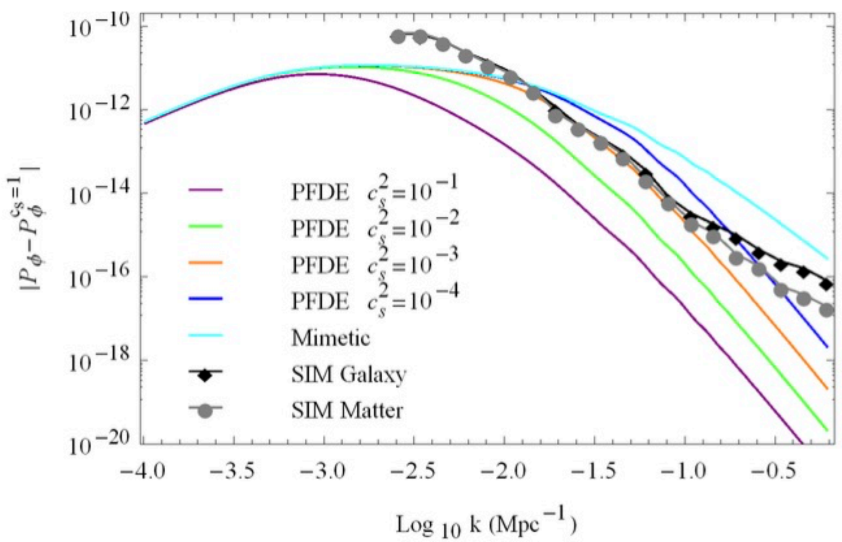
$w_0 = -0.95,\ \ w_a = 0$

N.B.: distinguishability of mimetic from other models is strongly dependent on the background: e.g., if the background is exactly fixed to the ΛCDM, then the mimetic model exactly predicts the same power spectrum of ϕ as in ΛCDM.

Distinguishing mimetic from PFDE models

Mimetic vs PFDE with c_s=1





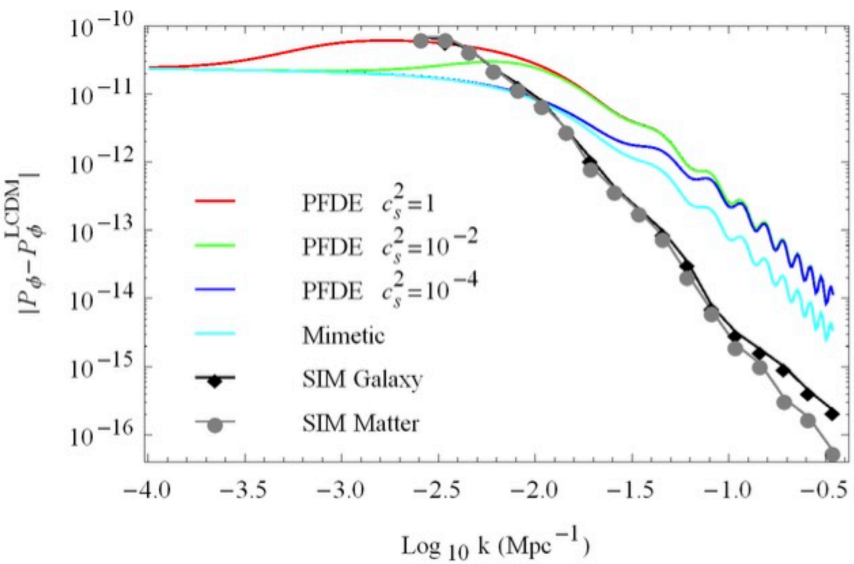
$w_0 = -0.7,\, w_a = 0$

$w_0 = -0.95,\;\; w_a = 0$

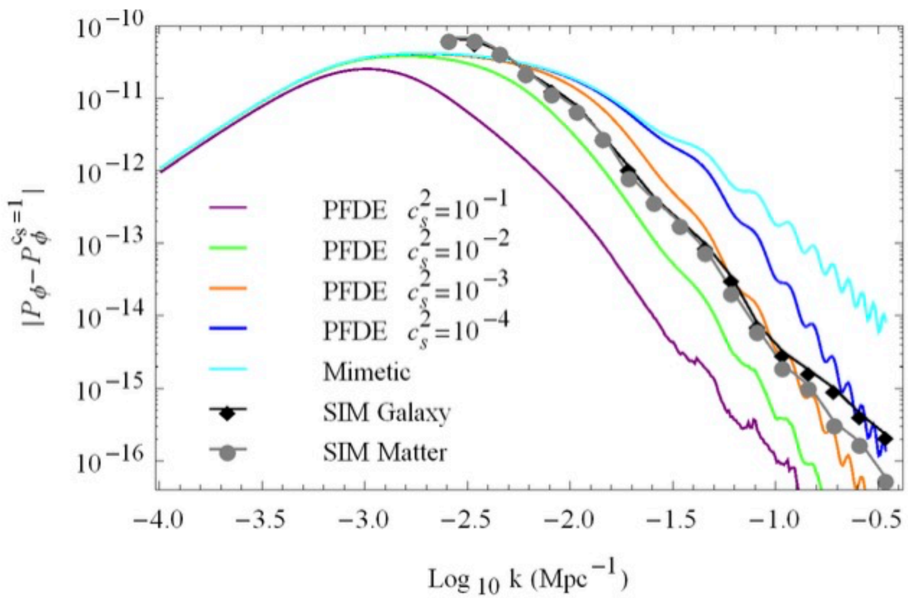
We can distinguish the mimetic model from the PFDE models with c_s=1

Distinguishing mimetic from PFDE models

Mimetic (with varying EOS) vs ΛCDM



Mimetic vs PFDE with c_s=1 (with varying EOS)



$$w_{DE} = -0.7 - 0.3(1-a)$$

We can distinguish the mimetic model from both ΛCDM and PFDE models with c_s=1

Some final comments

- If the predictions for Φ are the same as PFDE with zero sound speed, than why should we invoke the mimetic model?

  * in mimetic a single component, the mimetic field, accounts for both DM and DE
  * PFDE is more a phenomenological model. Mimetic can be regarded as a first step towards possible underlying theories for PFDE

- Also: there are in fact potential ways to distinguish mimetic models from PFDE with zero sound speed (to be explored)

  * the equivalence of mimetic models with PFDE holds for *adiabatic initial conditions*
  * such an equivalence holds *for linear scales, so non-linear evolution might be different*
  * *direct detection of DM particles would rule out this scenario (the mimetic field is assumed to be couplde only gravitationally with the standard model particles).*

Conclusions

- Mimetic gravity provides a scenario where both DM and DE can be mimicked by a modification of gravity (both at the background and linear perturbation level).
- Mimetic Horndeski is a fairly general model which encompasses many of the mimetic models proposed so far.
- We have studied in details its linear cosmological perturbations. In the simplest mimetic cubic Horndeski models the mimetic field can describe both DM and DE mimicking perfect fluid dark energy models with zero sound speed for the gravitational potentials (providing un underlying theory for PFDE).
- Possibility to distinguish these models from other popular DE models: e.g. they can be distinguished from PFDE with unity sound speed if the e.o.s. is -0.95 on the basis of matter (galaxy) power spectrum measurements.
- ***There are ways to rule out the models: e.g., non-adiabatic initial conditions, direct/ indirect detection of DM particles***
- **Various next steps are possible: extract other cosmological observables, like the usual Ξ and γ parameters; implementation within standard numerical codes; what about non-linear evolution; what about bias? ....**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In the following some technical details
(see relevant papers)

Mimetic gravity

- Why one gets new equations of motions?

The original theory is generically invariant under disformal transf. However there exists a particular subset such that the resulting equations of motion are no longer the original ones.

Non-invertibility of the disformal transformation

$$B(\varphi, w) = -\frac{A(\varphi, w)}{w} + b(\varphi)$$

Mimetic constraint

$$b(\varphi) g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi - 1 = 0$$

How to show this correspondence?

- Start from a given theory

$$S = \int d^4x\sqrt{-g}\mathcal{L}[g_{\mu\nu}, \partial_{\lambda_1} g_{\mu\nu}, \ldots, \partial_{\lambda_1} \ldots \partial_{\lambda_p} g_{\mu\nu}, \Psi, \partial_{\lambda_1}\Psi, \ldots, \partial_{\lambda_1} \ldots \partial_{\lambda_q}\Psi] + S_m[g_{\mu\nu}, \phi_m]$$

and compute the eqns. of motion

$$\delta S = \frac{1}{2}\int d^4x\sqrt{-g}(E^{\mu\nu} + T^{\mu\nu})\delta g_{\mu\nu} + \int d^4x\ \Omega_\Psi \delta\Psi + \int d^4x\ \Omega_m \delta\phi_m$$

- **Now, apply the disformal transf.** $g_{\mu\nu} = A(\Psi, w)\ell_{\mu\nu} + B(\Psi, w)\partial_\mu\Psi\partial_\nu\Psi$

and hence write eqns. of motion w.r.t. auxiliary metric $\ell_{\mu\nu}$ and Ψ

$$A(E^{\mu\nu} + T^{\mu\nu}) = \left(\alpha_1\frac{\partial A}{\partial w} + \alpha_2\frac{\partial B}{\partial w}\right)(\ell^{\mu\rho}\partial_\rho\Psi)(\ell^{\nu\sigma}\partial_\sigma\Psi)$$

$$\frac{1}{\sqrt{-g}}\partial_\rho\left\{\sqrt{-g}\,\partial_\sigma\Psi\left[B(E^{\rho\sigma} + T^{\rho\sigma}) + \left(\alpha_1\frac{\partial A}{\partial w} + \alpha_2\frac{\partial B}{\partial w}\right)\ell^{\rho\sigma}\right]\right\} - \frac{\Omega_\Psi}{\sqrt{-g}} = \frac{1}{2}\left(\alpha_1\frac{\partial A}{\partial \Psi} + \alpha_2\frac{\partial B}{\partial \Psi}\right)$$

$$\alpha_1 \equiv (E^{\rho\sigma} + T^{\rho\sigma})\ell_{\rho\sigma} \qquad \alpha_2 \equiv (E^{\rho\sigma} + T^{\rho\sigma})\partial_\rho\Psi\,\partial_\sigma\Psi$$

- Contract the eqns. with $\ell_{\mu\nu}$ and $\partial_\mu\Psi\partial_\nu\Psi$

$$\alpha_1\left(A - w\frac{\partial A}{\partial w}\right) - \alpha_2 w\frac{\partial B}{\partial w} = 0, \qquad \alpha_1\, w^2\frac{\partial A}{\partial w} - \alpha_2\left(A - w^2\frac{\partial B}{\partial w}\right) = 0$$

Now the point is that the solutions of this algebaric system for α_1 and α_2 is different according to its deteminant being zero or not.

GENERIC CASE: Det(M) ≠ 0 ⇒ $\alpha_1 = \alpha_2 = 0$ ⇒

$$E^{\mu\nu} + T^{\mu\nu} = 0$$
$$\Omega_\Psi = 0.$$

This shows that generically the starting theory is invariant under disformal tranformnations

MIMETIC CASE: Det(M) = 0 ⇒

$$B(\Psi, w) = -\frac{A(\Psi, w)}{w} + b(\Psi)$$

and in this case one obtains different eqns. of motion (the mimetic eqns. showed before)

- finally: it is not difficult to show that this is *exactly* the condition for the disformal transformation to be *non-invertible*