Vainshtein Screening

Galileon employs Vainshtein Screening and highly non-linear Lagrangian, but equations of motion only have second order time derivatives

$$\mathcal{L} = -\frac{c_2}{2}(\partial\phi)^2 - \frac{c_3}{\Lambda^3}\Box\phi(\partial\phi)^2 - \frac{c_4}{\Lambda^6}\mathcal{L}_4 - \frac{c_5}{\Lambda^9}\mathcal{L}_5 + \sum_i \frac{c_0^i\phi}{m_{\rm Pl}}T_i - \sum_i \frac{c_G^i}{\Lambda^4}\partial_\mu\phi\partial_\nu\phi T_i^{\mu\nu} \ ,$$

where c_0 and c_G are the conformal and disformal couplings

we take $\Lambda^3 = H_0^2 m_{\rm Pl}$ to be of cosmological interest

and $c_2 > 0$ to avoid ghosts in Minkowski space

The coupling to the metric is $\tilde{g}^i_{\mu\nu} = A^i(\phi)g_{\mu\nu} + \frac{2}{M_i^4}\partial_\mu\phi\partial_\nu\phi$. with $A^i(\phi) = 1 + \frac{c_0^i\phi}{m_{\rm Pl}}$

$$\mathcal{L}_4 = (\partial\phi)^2\left[2(\Box\phi)^2 - 2D_\mu D_\nu\phi D^\nu D^\mu\phi - R\frac{(\partial\phi)^2}{2}\right]$$

$$\mathcal{L}_5 = (\partial\phi)^2\left[(\Box\phi)^3 - 3(\Box\phi)D_\mu D_\nu\phi D^\nu D^\mu\phi + 2D_\mu D^\nu\phi D_\nu D^\rho\phi D_\rho D^\mu\phi - 6D_\mu\phi D^\mu D^\nu\phi D^\rho\phi G_{\nu\rho}\right]$$

these are the remaining terms and play a role cosmologically

For c3 non-zero the spherically symmetric solution is

$$\frac{d\phi}{dr} = -\frac{\Lambda^3 r}{4}\left(1 - \sqrt{1 + \left(\frac{R_*}{r}\right)^3}\right) ,$$

Non-linearities dominate inside the Vainshtein radius to screen the fifth force

$$R_* = \frac{1}{\Lambda}\left(\frac{c_0^b m}{2\pi c_3 m_{\rm Pl}}\right)^{1/3} \qquad \frac{F_\phi}{F_N} = (c_0^b)^2\left(\frac{r}{R_*}\right)^{3/2}$$

Both conformal and disformal couplings to matter are severely constrained.

c_0 leads to large variations in particle masses when coupled to baryons

c_G coupled to baryons is constrained by LHC and to photons by variation of the speed of light

giving the duality relation $d_L = \left(\frac{c_{\rm obs}}{c_{\rm emit}}\right)^2 (1+z)^2 d_A$

Shape Dependence

Bloomfield, Burrage and ACD 1408.4759

Vainshtein screening usually considered for spherical objects, except one case where planar symmetry was considered. We extend the analysis to cylindrical symmetry, appropiate for many astrophysical bodies.

Consider the equations of motion for the Galileon under the assumption of static configurations.

Assume matter configurations of energy density s.t.

$$T^{\mu}_{\mu} = -\rho$$

We now analyse different cosmological shapes.

Planar Symmetry

The metric is

$$ds^2 = -dt^2 + dx^2 + dy^2 + dz^2$$

Taking the field to depend on z we find that only the quadratic and coupling term survives, in agreement with Brax, Burrage and ACD, and

$$\frac{\beta}{M_P}\rho(z) = \partial_z^2 \phi$$

letting

$$\rho(z) = \rho_0, \pm z_0$$

we find

$$\phi = \begin{cases} \dfrac{\beta\rho_0}{2M_P} z^2 & |z| < z_0 \\ \dfrac{\beta\rho_0 z_0}{M_P}\left(z - \dfrac{z_0}{2}\right) & |z| \geq z_0 \end{cases}$$

to give

$$\frac{F_\phi}{F_G} = 2\beta^2 \,.$$

The fifth force is unscreened!

Cylindrical Symmetry

Using the metric

$$ds^2 = -dt^2 + dr^2 + r^2 d\theta^2 + dz^2$$

with $\phi = \phi(r), \rho = \rho(r)$ the quadratic, cubic and coupling terms contributes

giving

$$\frac{\beta}{M_P}\rho(r) = \phi'' + \frac{\phi'}{r} + \frac{2\phi'\phi''}{r\Lambda^3}$$

taking $\rho = \rho_0, r < r_0$ the Vainshtein radius is

$$r_v = \sqrt{\frac{2\beta\rho_0 r_0^2}{M_P\Lambda^3}} = \sqrt{\frac{2\beta\lambda}{\pi M_P\Lambda^3}} \qquad \Lambda^3 = H_0^2 M_{Pl}$$

where $\lambda = \pi r_0^2 \rho_0$ is the linear mass density

We then get

$$\phi = \frac{\Lambda^3}{4}\left(\sqrt{1+\frac{r_v^2}{r_0^2}}-1\right)r^2 \quad \text{for} \quad r < r_0$$

$$\phi = \frac{\Lambda^3}{4}\left[r^2\left(\sqrt{1+\frac{r_v^2}{r^2}}-1\right) + r_v^2 \ln\left(\frac{r+\sqrt{r^2+r_v^2}}{r_0+\sqrt{r_0^2+r_v^2}}\right)\right] \quad \text{for} \quad r > r_0$$

giving

$$\frac{F_\phi}{F_G} = 4\beta^2 \frac{r}{r_v}$$

Note less screened than spherical case

Spherical Symmetry

Using the metric

$$ds^2 = -dt^2 + dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2$$

Only the quintic term doesn't contribute, giving the equations of motion

$$\frac{\beta}{M_P}\rho(r) = \phi'' + \frac{2\phi'}{r} + \frac{2\phi'^2}{r^2\Lambda^3} + \frac{4\phi'\phi''}{r\Lambda^3} + \frac{6\lambda_4\phi'^2\phi''}{r^2\Lambda^6}$$

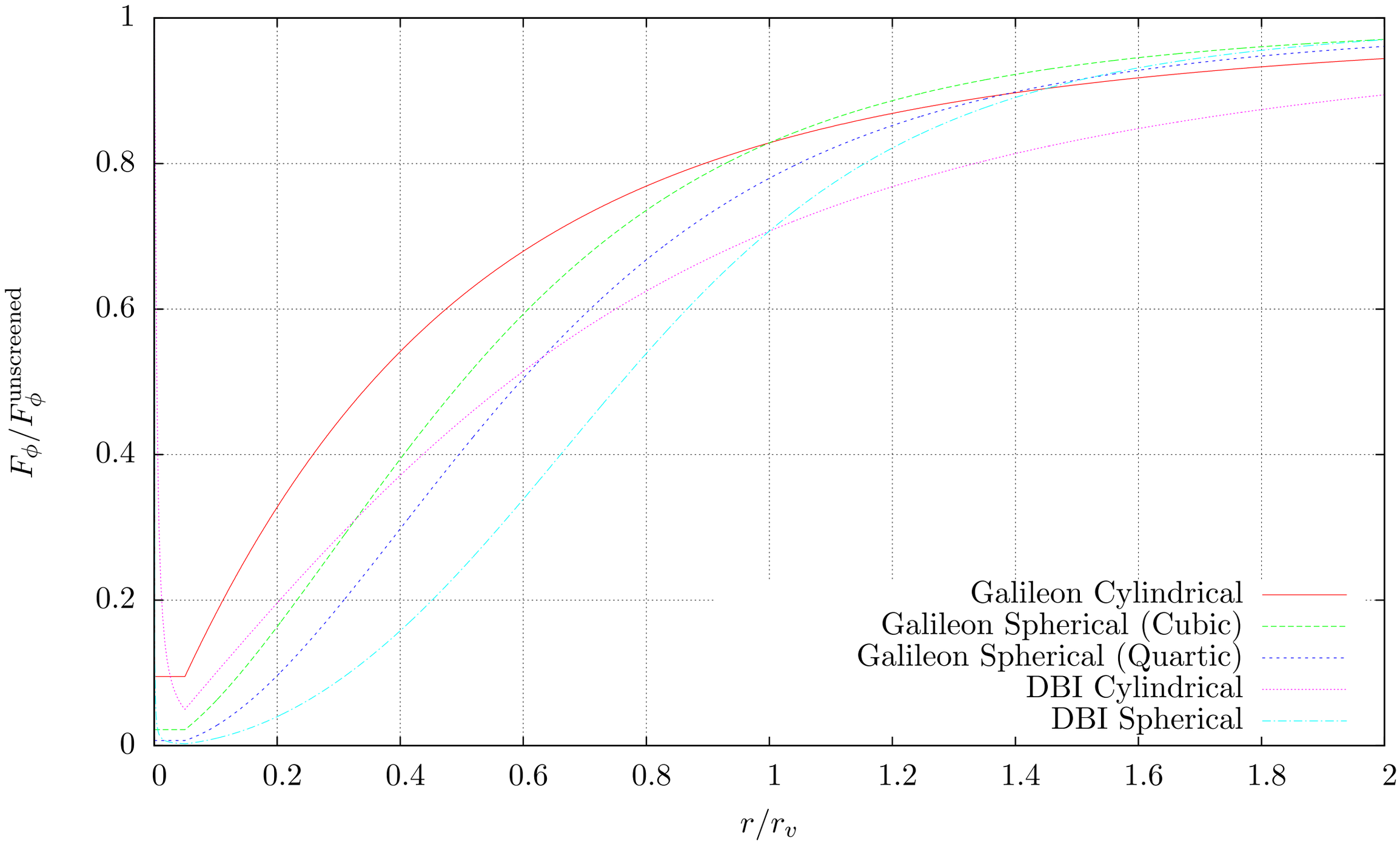
taking $\rho = \rho_0, r < r_0$ we can find solutions without and with the quartic term

In the general case the Vainshtein radius is

$$r_v = \left(\frac{3}{4}\right)^{1/3}\left(\frac{2\beta M}{\pi M_P \Lambda^3}\right)^{1/3}$$

and the fifth force is

$$\frac{F_\phi}{F_G} = 6\beta^2\frac{r^2}{r_v^2}$$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2
$F_\phi/F_\phi^{\rm unscreened}$
r/r_v
Galileon Cylindrical
Galileon Spherical (Cubic)
Galileon Spherical (Quartic)
DBI Cylindrical
DBI Spherical

Chameleon Screening

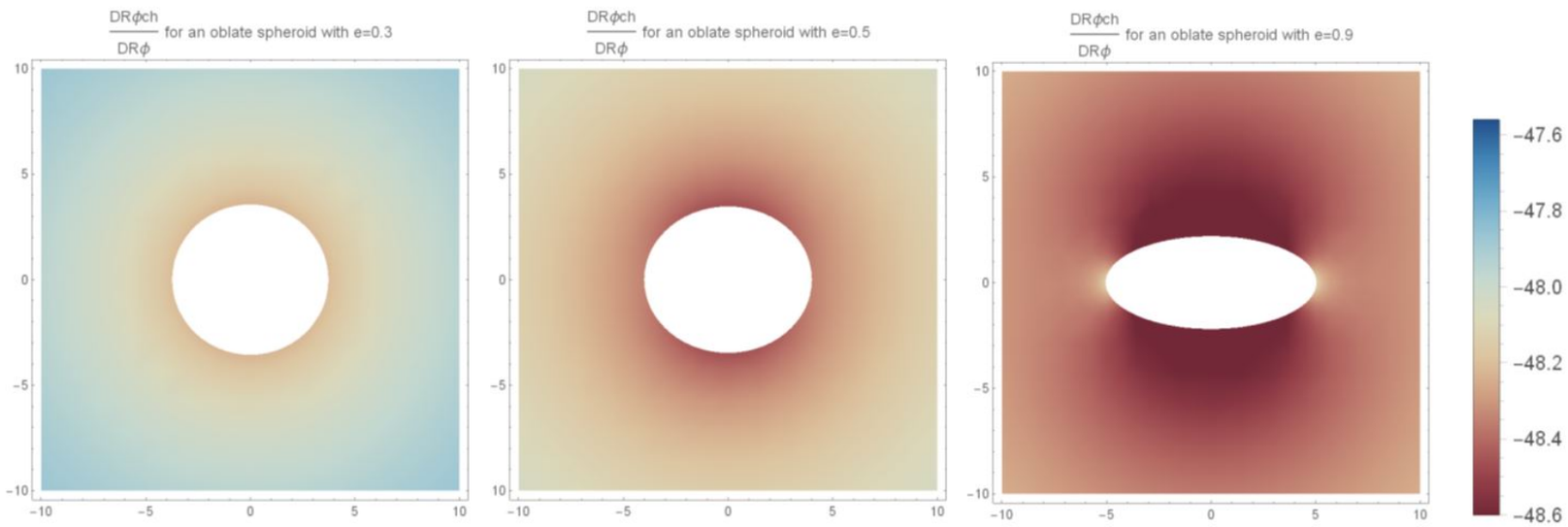

Figure 1: Radial screening for an oblate object

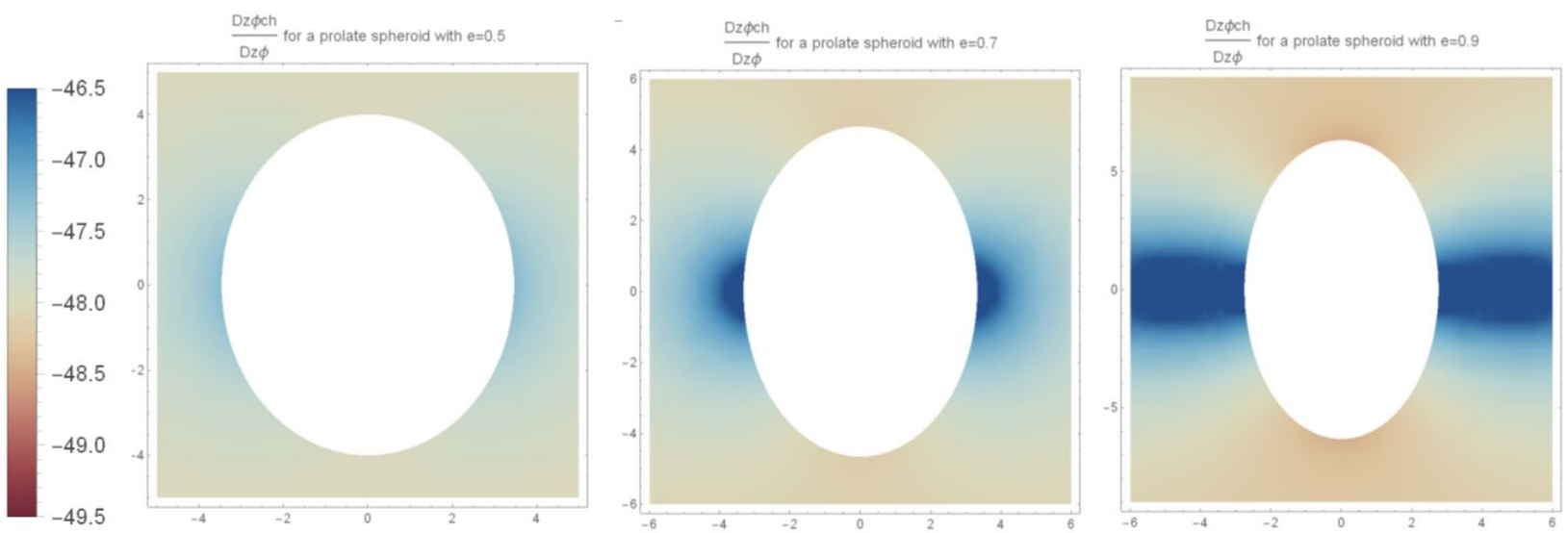

Figure 2: Vertical screening for a prolate object