Chameleon-induced quantum dynamics

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Overview

What is a chameleon?

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- Scalar field from modified gravity

$$\mathcal{L} = -\frac{1}{2}(\partial\varphi)^2 - \frac{\lambda}{4!}\varphi^4 + \frac{\varphi}{\mathcal{M}} T_\mu{}^\mu$$

- General chameleon potential $V(\varphi) = \Lambda^4 \left(\frac{\Lambda}{\varphi}\right)^n$

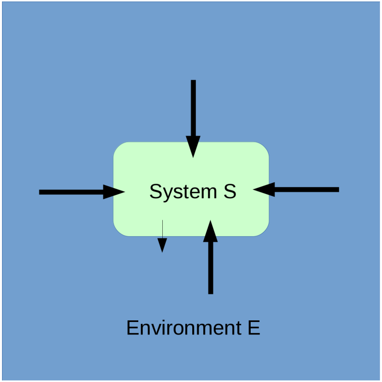
- Fifth force screening in environments of high mass density

- Unscreened force stronger than gravity for $\mathcal{M} < M_{\text{Planck}}$

Coherence and decoherence

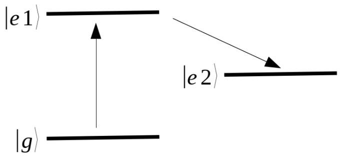
- Coherent state has non-vanishing off-diagonal density matrix elements
- Example: $|\Psi\rangle = \alpha|0\rangle + \beta|1\rangle \to \rho = \begin{pmatrix} |\alpha|^2 & \alpha^*\beta \\ \alpha\beta^* & |\beta|^2 \end{pmatrix}$
- Decoherence:
    - decay of off-diagonal elements $\to \rho = \begin{pmatrix} |\alpha|^2 & 0 \\ 0 & |\beta|^2 \end{pmatrix}$
    - Caused by environmental influences, e.g. gravitational decoherence [Blencowe (2012); Anastopoulos, Hu (2013);...]

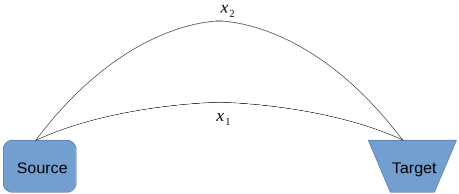
Open quantum systems

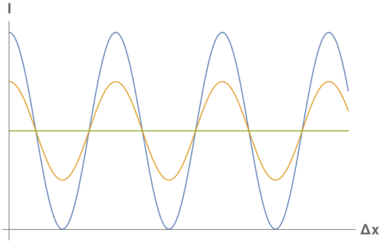


Born approximation: Environment barely affected by system $\rightarrow \rho(t) \approx \rho_S(t) \otimes \rho_E$

Atom interferometry

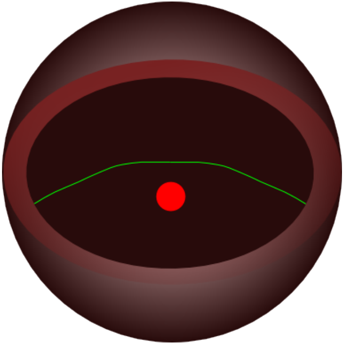






[Jaffe et al. (2016)]

Chameleon in a spherical vacuum chamber



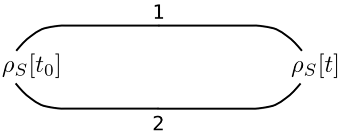
Constant chameleon mass $m_\varphi \approx \frac{-u}{\sqrt{2}L}$ with $u = -1.287$ and L : chamber radius

[Burrage, Copeland, Hinds (2014)]

Density matrix

$$\rho_S[\phi_f^1,\phi_f^2,t] = \int d\phi_0^{1,2} \int_{\phi_0^{1,2}(\vec{x})}^{\phi_f^{1,2}(\vec{x})} D\phi^{1,2} \exp\left[\frac{i}{\hbar}\left(S[\phi^1] - S[\phi^2] + S_{\mathrm{IF}}[\phi^1,\phi^2]\right)\right] \rho_S[\phi_0^1,\phi_0^2,t_0]$$

$$\exp\left\{\frac{i}{\hbar} S_{\mathrm{IF}}[\phi^1,\phi^2]\right\} = \int d\varphi_f^1(\vec{x}) d\varphi_0^1(\vec{x}) d\varphi_0^2(\vec{x}) \rho_S[\varphi_0^1,\varphi_0^2,t_0] \int_{\varphi_0^1(\vec{x})}^{\varphi_f^1(\vec{x})} D\varphi^1 \int_{\varphi_0^2(\vec{x})}^{\varphi_f^1(\vec{x})} D\varphi^2$$
$$\times \exp\left\{\frac{i}{\hbar}\left(S[\varphi^1] + S_{\mathrm{Int}}[\phi^1,\varphi^1] - S[\varphi^2] - S_{\mathrm{Int}}[\phi^2,\varphi^2]\right)\right\}$$



[Calzetta, Hu, *Nonequilibrium Quantum Field Theory* (2008)]

Evolution of the density matrix

- Time derivative of $\rho_S[\phi^1_f, \phi^2_f, t]$

$$\rightarrow \partial_t \rho_S[\phi^1_f, \phi^2_f, t] = -\frac{i}{\hbar}\{H[\phi^1_f] - H[\phi^2_f]\}\rho_S[\phi^1_f, \phi^2_f, t] + \frac{i}{\hbar}\frac{\partial S_{\mathrm{IF}}[\phi^1_f, \phi^2_f]}{\partial t}\rho_S[\phi^1_f, \phi^2_f, t]$$

- Perturbative calculation of influence action S_{IF} up to second order in λ and $\frac{\langle \varphi \rangle}{\mathcal{M}}$

- Canonical quantisation: $\phi^1(x)\phi^2(y)\rho \rightarrow \hat{\phi}(x)\hat{\rho}\hat{\phi}(y)$

- Projection on momentum basis of single-particle subspace

$$\begin{aligned} \hat{\rho}_S &= \int \frac{d^3Q d^3Q'}{(2\pi)^6} |Q\rangle\langle Q|\hat{\rho}_S|Q'\rangle\langle Q'| \\ &=: \int \frac{d^3Q d^3Q'}{(2\pi)^6} |Q\rangle\langle Q'|\rho_S(Q, Q') \end{aligned}$$

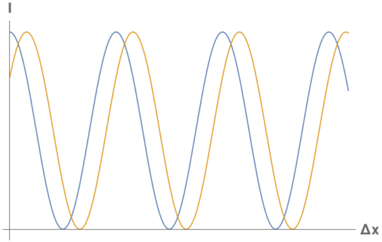
Master equation

$$\partial_t \rho_S[K, K'] = -\frac{i}{\hbar}[H, \rho_S] + \frac{i}{\hbar}\aleph[\rho_S] + \mathcal{D}[\rho_S] + i\mathcal{F}[\rho_S]$$

- Coherence shift $\aleph[\rho_S] \sim \langle\varphi\rangle/\mathcal{M}$
- Decoherence kernel $\mathcal{D}[\rho_S] \sim (\langle\varphi\rangle/\mathcal{M})^2$
- Diffusion kernel $\mathcal{F}[\rho_S] \sim (\langle\varphi\rangle/\mathcal{M})^2$

Coherence shift

$$\aleph[\rho_S] = -\frac{\langle\varphi\rangle m_{\mathrm{at}}^2}{\mathcal{M}}\left(\frac{1}{\omega_{\vec{K}}} - \frac{1}{\omega_{\vec{K}'}}\right)\rho_S(\vec{K},\vec{K}')$$



Decoherence kernel - Markov limit

$$\mathcal{D}[\rho_S] = -\frac{1}{\hbar\mathcal{M}^2}\Re\Bigg\{\int_0^t d\tau\int\frac{d^3k}{(2\pi)^3}\Bigg[\Bigg(f(K,k)e^{-i\tau(\omega^c_{\vec{K}-\vec{k}}+\omega_{\vec{K}}-\omega_{\vec{k}})}+\dots\Bigg)\rho_S(\vec{K},\vec{K'})$$

$$-\tilde{f}_1(K,K',k)e^{-i\tau(\omega^c_{\vec{k}-\vec{K}}+\omega_{\vec{k}}-\omega_{\vec{K}})}\rho_S(\vec{k},\vec{K'}+\vec{k}-\vec{K})$$

$$-\tilde{f}_2(K,K',k)e^{-i\tau(\omega_{\vec{K'}}-\omega^c_{\vec{k}-\vec{K}}-\omega_{\vec{k}})}\rho_S(\vec{K'}+\vec{k}-\vec{K},\vec{k})\Bigg]\Bigg\}$$

- Markov limit: $\int_0^t \rightarrow \int_0^\infty$
- $\int\limits_0^\infty d\tau e^{\pm ix\tau} = \pi\delta(x) \pm \frac{i}{x} \quad\rightarrow\quad \mathcal{D} = 0$ for $m_{\text{ch}} \leq 2m_{\text{at}}$

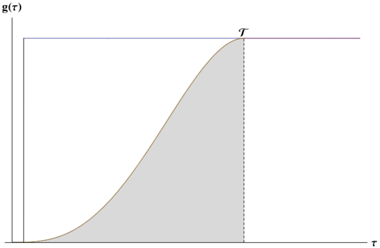
Decoherence kernel - Non-Markovian case

$$\mathcal{D}[\rho_S] = -\frac{1}{\hbar}\Re\Bigg\{\int_0^t d\tau \frac{g(\tau)}{\mathcal{M}^2}\int\frac{d^3k}{(2\pi)^3}\Bigg[\Bigg(f(K,k)e^{-i\tau(\omega^c_{\vec{K}-\vec{k}}+\omega_{\vec{K}}-\omega_{\vec{k}})}+\dots\Bigg)\rho_S(\vec{K},\vec{K}')$$
$$-\tilde{f}_1(K,K',k)e^{-i\tau(\omega^c_{\vec{k}-\vec{K}}+\omega_{\vec{k}}-\omega_{\vec{K}})}\rho_S(\vec{k},\vec{K}'+\vec{k}-\vec{K})$$
$$-\tilde{f}_2(K,K',k)e^{-i\tau(\omega_{\vec{K}'}-\omega^c_{\vec{k}-\vec{K}}-\omega_{\vec{k}})}\rho_S(\vec{K}'+\vec{k}-\vec{K},\vec{k})\Bigg]\Bigg\}$$

- Smooth coupling function $g(\tau)$ avoids initial time singularity

Decoherence kernel - Non-Markovian case

$$\text{Example:} \quad g(\tau) \quad := \quad \begin{cases} 0 & \tau \leq 0 \\ -3\left(\frac{\tau}{\mathcal{T}}\right)^4 + 4\left(\frac{\tau}{\mathcal{T}}\right)^3 & 0 \leq \tau \leq \mathcal{T} \\ 1 & \mathcal{T} \leq \tau \end{cases}$$



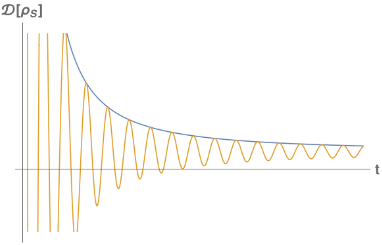
Decoherence kernel - Non-Markovian case

Stationary phase approximation →

$$\mathcal{D}[\rho_S] \sim \Re\Bigg\{\left(\frac{F_1(\vec{K},\vec{K}')}{\sqrt{t}^3}e^{-it\omega_1}+h_1(\mathcal{T},\vec{K},\vec{K}')\right)\rho_S(\vec{K},\vec{K}')$$
$$-\left(\frac{F_2(\vec{K},\vec{K}')}{\sqrt{t}^3}e^{-it\omega_2}+h_2(\mathcal{T},\vec{K},\vec{K}')\right)\rho_S(\frac{m_{\mathrm{at}}}{m_{\mathrm{at}}+m_{\mathrm{ch}}}\vec{K},\vec{K}'-\frac{m_{\mathrm{ch}}}{m_{\mathrm{at}}+m_{\mathrm{ch}}}\vec{K})$$
$$-\left(\frac{F_3(\vec{K},\vec{K}')}{\sqrt{t}^3}e^{-it\omega_3}+h_3(\mathcal{T},\vec{K},\vec{K}')\right)\rho_S(\vec{K}-\frac{m_{\mathrm{ch}}}{m_{\mathrm{at}}+m_{\mathrm{ch}}}\vec{K}',\frac{m_{\mathrm{at}}}{m_{\mathrm{at}}+m_{\mathrm{ch}}}\vec{K}')\Bigg\}$$

- $t^{-3/2}$ decay
- t-constant pieces depending on time scale $\mathcal{T}$
- Small population changes

Experimental implications



Assuming:

- Rb atom ($m_{\mathrm{at}} = 87\mathrm{u}$)
- Vacuum chamber radius $L = 1\mathrm{m}$
- Momenta $K = 1m\frac{\mathrm{m}}{\mathrm{s}}$ and $K' = 10m\frac{\mathrm{m}}{\mathrm{s}}$
- Coupling $\mathcal{M} = M_{\mathrm{Planck}}/10$

$$\rightarrow \quad \mathcal{D}[\rho_S] \quad \approx \quad \sqrt{\frac{\mathrm{s}^3}{t^3}} \times 10^{-37} \cos\left[c_0 \frac{t}{\mathrm{s}}\right] \frac{1}{\mathrm{s}}$$

$$\aleph[\rho_S] \quad \approx \quad 10^{-16}\frac{1}{\mathrm{s}}$$

Summary

- Chameleon present in quantum experiments
- Small coherence shift
- Small decoherence
- Formalism has other applications: other screened fields, axions, dilatons,...