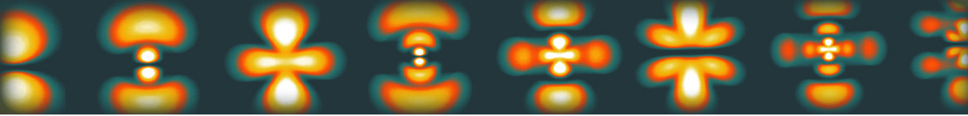

CONTEMPLATING ATOMS

What hydrogen teaches us about modified gravity

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Chameleon-like theories

Consider theories with the Einstein-frame action

$$S = \int \mathrm{d}^4x\sqrt{-g}(L_{\mathrm{grav}} + L_{\mathrm{em}}) + S_{\mathrm{matter}}[\Omega^2(\phi)g_{\mu\nu}],$$

$$L_{\mathrm{grav}} = \frac{M_{\mathrm{Pl}}^2}{2}(R - \partial_\mu\phi\,\partial^\mu\phi) - V(\phi), \quad L_{\mathrm{em}} = -\frac{1}{4}\varepsilon(\phi)F_{\mu\nu}F^{\mu\nu}.$$

Matter and photon couplings are, respectively,

$$\beta_m(\phi) = (\log\Omega)_{,\phi}, \quad \beta_\gamma(\phi) = (\log\varepsilon)_{,\phi}.$$

Imprints in the hydrogen spectrum

- Chameleons cause shifts in the energy spectrum of atoms.
  [Brax and Burrage, 2011]
- Contributions to gross and fine structure are known.
- Can put (weak) constraints on β_m and combination $\beta_m \beta_\gamma$.

Why study hydrogen . . . again?

Recent progress in our understanding of this simple system:

1. New fine-structure terms
2. Curved backgrounds / unscreened environments
3. Compare Einstein and Jordan frames

Not in this talk: Hyperfine structure, but can be done. [Wong and Davis, 2017]

1. Fine structure

Coupling the chameleon to the electron

Semiclassical approach: Quantize electron ψ coupled to classical fields $\{g_{\mu\nu}, \phi, A_\mu\}$ due to nucleus.

Electron ψ satisfies the *Jordan-frame* Dirac equation,

$$\left[i\gamma^a e_a^\mu(\partial_\mu + \omega_\mu + ieA_\mu) - m_e\right]\psi = 0.$$

Chameleon couples to the electron via:

1. Vierbein e_a^μ and spin connection ω_μ;
2. Inducing corrections to A_μ through $\varepsilon(\phi)$.

Corrections to the Coulomb field

In static case with $\mathbf{B} = 0$, Maxwell equations $\partial_\mu(\varepsilon F^{\mu\nu}) = 0$ become

$$\nabla \cdot \mathbf{D} := \nabla \cdot (\varepsilon \mathbf{E}) = 0, \quad \nabla \times \mathbf{E} = 0.$$

From spherical symmetry, also have $\nabla \times \mathbf{D} = \nabla\varepsilon \times \mathbf{E} = 0$.

Exact solution to $\mathbf{D}$ with boundary conditions (charge Ze at origin) is

$$|\mathbf{D}| = \frac{Ze}{4\pi r^2}.$$

Then easily get

$$|\mathbf{E}| = \frac{|\mathbf{D}|}{\varepsilon(\phi)} \approx \frac{Ze}{4\pi\epsilon(\phi_0)r^2}(1 - \beta_\gamma \delta\phi + \ldots).$$

Chameleon profile

Chameleon profile can be solved perturbatively. Let $\phi = \phi_0 + \delta\phi$ to get

$$\delta\phi = -\beta_m \frac{2Gm_N}{r} - \beta_\gamma \frac{GZ^2\alpha}{2r^2}.$$

- Linear regime valid because nucleus is light enough
- Assumed $m_\phi^{-1} \gg$ Bohr radius
- Have rescaled $\Omega^2(\phi_0)G \longrightarrow G$ and $\alpha/\varepsilon(\phi_0) \longrightarrow \alpha$
- Coupling constants $\beta \equiv \beta(\phi_0)$

In limit $\beta_m \gg 1$, can ignore Newtonian potential, so

$$g_{\mu\nu} \simeq (1 + 2\beta_m \delta\phi)\eta_{\mu\nu}.$$

Fine structure corrections

The perturbation Hamiltonian is

$$\delta H = \gamma^0 m_e \beta_m \delta\phi + e\delta A_0 + \underbrace{\frac{3}{2}\gamma^0\gamma^i(-i\partial_i)\beta_m\delta\phi}_{\text{Ignore for now}}.$$

First two terms give, explicitly,

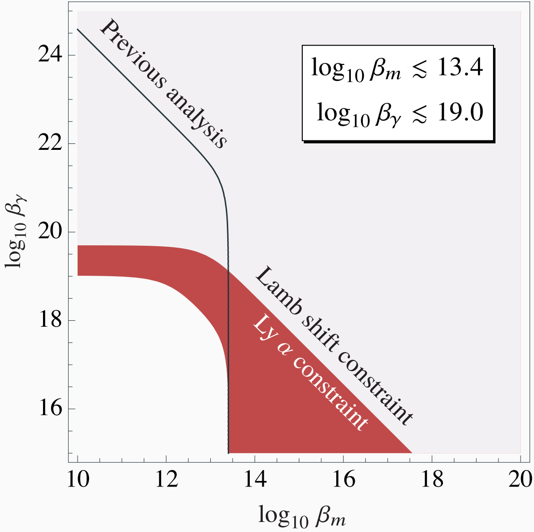
$$\delta H \supset -\gamma^0\beta_m^2\frac{2Gm_e m_N}{r} - \gamma^0\beta_m\beta_\gamma\frac{GZ^2\alpha m_e}{2r^2} \underbrace{-\beta_m\beta_\gamma\frac{GZ\alpha m_N}{r^2} - \beta_\gamma^2\frac{GZ^3\alpha^2}{6r^3}}_{\text{New; comes from }\delta A_0}.$$

Fine structure corrections (cont'd.)

$$\delta H \supset -\gamma^0 \beta_m^2 \frac{2Gm_e m_N}{r} - \gamma^0 \beta_m \beta_\gamma \frac{GZ^2 \alpha m_e}{2r^2} - \beta_m \beta_\gamma \frac{GZ\alpha m_N}{r^2} - \beta_\gamma^2 \frac{GZ^3 \alpha^2}{6r^3}$$

- 3rd term is larger than 2nd by factor of $\sim 2m_N/m_e$. Dominates corrections to the Lamb shift.
- Inclusion of δA_0 effect gives a β_γ^2 term, so now can constraint β_γ independently.

Fine structure corrections (cont'd.)



2. Curved backgrounds

Curved backgrounds

Pick Fermi coordinates x^μ with nucleus fixed at $x^i = 0$. Background is

$$\bar{g}_{\mu\nu}(x) = \eta_{\mu\nu} - \delta^0_\mu \delta^0_\nu \, 2a_i x^i + O(r^2),$$

$$\bar{\phi}(x) = \phi_0 + \phi_{,i} x^i + O(r^2).$$

Could have included quadratic terms (tidal forces).

[Done for GR by Parker, 1980; Parker and Pimentel, 1982]

Solve Maxwell equations perturbatively in this spacetime to get

$$A_\mu = A^{(0)}_\mu + \Delta A_\mu(a_i, \phi_{,i}).$$

Adiabatic approximation: Atom sees $\{a_i, \phi_{,i}\}$ as time-independent.

Curved backgrounds (cont'd.)

At first order, can ignore mixing between curved background terms and singular terms due to nucleus.

$$H = H_0 + \underbrace{\delta H}_{1/r^n \text{ terms from earlier}} + \underbrace{\Delta H.}_{\text{Curved bg. effects}}$$

Acceleration and electric correction

$$\Delta H \supset \gamma^0 m_e a_i x^i + \beta_\gamma Z\alpha \frac{\phi_{,i} x^i}{2r} + (\text{subleading terms } \propto a)$$

- Odd-parity terms, analagous to Stark effect.
- Washed out due to larger "QED corrections" (e.g. Lamb shift) that lift needed degeneracies between opposite-parity states.
- To see this effect, need

$$Z\beta_\gamma |\phi_{,i}| \gg 10^{19} g_\oplus, \quad \text{or} \quad a \gg 10^{15} g_\oplus.$$

(c.f. neutron stars have surface gravities $\sim 10^{11} g_\oplus$)

3. Einstein or Jordan frame?

A tale of two theories

We found a chameleon–spin interaction

$$\delta H_{\text{spin}} \supset \frac{3}{2}\gamma^0\gamma^i(-i\partial_i)\beta_m\delta\phi$$

in the Jordan frame.

$\left\langle \delta H_{\text{spin}} \right\rangle = 0$ for spherically symmetric $\delta\phi$ [Adkins, 2008], but in general nonvanishing, e.g. by including magnetic dipole contribution to $\delta\phi$.

Such a term does not emerge in the Einstein frame: Is electron theory frame-dependent?

To canonically normalize or not?

The Dirac action in the Jordan frame is

$$S = \int \mathrm{d}^4x\sqrt{-g}\bar{\psi}\left(i\not{e}^{\mu}D_{\mu} - m_e\right)\psi\,.$$

Write $g_{\mu\nu} = \Omega^2\tilde{g}_{\mu\nu}$ to get

$$S = \int \mathrm{d}^4x\sqrt{-\tilde{g}}\Omega^3\bar{\psi}\left(i\tilde{\not{e}}^{\mu}\tilde{D}_{\mu} - \Omega m_e + \tfrac{3}{2}i\tilde{\not{e}}^{\mu}\partial_{\mu}\log\Omega\right)\psi\,.$$

Extra term is exactly responsible for δH_{spin}. Canceled by canonical normalization: Let $\tilde{\psi} = \Omega^{-3/2}\psi$ to get

$$S = \int \mathrm{d}^4x\sqrt{-\tilde{g}}\bar{\tilde{\psi}}(i\tilde{\not{e}}^{\mu}\tilde{D}_{\mu} - \Omega m_e)\tilde{\psi}\,.$$

Matter of choice

- Electron observables are sensitive to field redefinitions, especially in a perturbative, semiclassical approach.
- A choice is required: either ψ is the electron and $\tilde{\psi} = \Omega^{-3/2}\psi$ is a composite electron–chameleon degree of freedom, or vice versa.
- ψ seems more natural, but $\tilde{\psi}$ certainly easier.

What does hydrogen teach us about modified gravity?

Conclusions

1. More complete picture of the hydrogen spectrum in chameleon-like gravity.
2. Uncompetitive constraints: $\log_{10} \beta_m \lesssim 13.4$ and $\log_{10} \beta_\gamma \lesssim 19.0$.
3. $\varepsilon(\phi)$ causes the vacuum to behave like a dielectric which induces corrections δA_μ. These must be taken into account as they can sometimes dominate.
4. Careful definition of particles needed.

Thank you for listening!

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