Dark Energy and Modified-Gravity cosmologies: DARKMOD

Cosmology of self-gravitating media

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based on:
arXiv:1704.00322
in collaboration with M. Celoria and D. Comelli
see also: arXiv: 605.05304 and arXiv:1603.02956
in collaboration with G. Ballesteros and D. Comelli

- What if Dark Energy is not a cosmological constant
- A number of probes are going to test its nature: EUCLID, DES ....
- Very large number of proposal:

quintessence, k-essence, f(R), massive gravity, brane-worlds, holographic DE, Einstein aether, vector models, phantoms, chameleons, Hordenski, ...

limited number of observables will available

difficult to tell the difference

Goal

Build a sufficient general effective description of DE based on symmetries

Starting point:

symmetries of the theory and the fields to realize them

Basic assumption: action principle and diff invariance

If the metric is the only field $\Rightarrow$ GR and 2 DoF

adding adding fields more $\Rightarrow$ DoF

Simplest option: scalar fields minimally coupled to gravity

Internal symmetries

Four scalars with shift symmetry $\varphi^A \quad \rightarrow \quad \varphi^A + c^A \qquad A = 0, 1, 2, 3$

With four scalars their eqs of motion $\Leftrightarrow$ $\nabla^\nu T_{\mu\nu} = 0$

rotational invariance $\varphi_a \quad \rightarrow \quad \mathcal{R}^a_b \varphi_b \qquad a = 1, 2, 3$

Sufficient to effectively describe the dynamics of large class of self-gravitating media based on an action principle:
fluids, superfluids, elastic solids, supersolids

leading order (derivative expansion): one derivative on each field

early references

Matarrese 1985

H. Leutwyler 1994, 1996;

R. Jackiw, V.P. Nair, S.Y Pi, A.P. Polychronakos 2004

S. Dubovsky, T. Gregoire, A. Nicolis, and R. Rattazzi 2006;

S. Dubovsky, L. Hui, A. Nicolis, and D.T. Son 2012

G. Ballesteros, B. Bellazzini, 2013

G. Ballesteros 2015

Action Principle

operators: rotational inv. in medium space and spacetime scalars

$$S = M_{Pl}^2 \int d^4x \sqrt{-g}\, R + \int d^4x \sqrt{-g}\, U(X,\, Y,\, \tau_n,\, y_n)$$

Operator	Definition
C^{AB}	$g^{\mu\nu}\, \partial_\mu \Phi^A\, \partial_\nu \Phi^B\,, \quad A, B = 0, 1, 2, 3$
B^{ab}	$g^{\mu\nu} \partial_\mu \Phi^a\, \partial_\mu \Phi^b\,, \quad a, b = 1, 2, 3$
Z^{ab}	$C^{a0}\, C^{b0}$
X	C^{00}
W^{ab}	$B^{ab} - Z^{ab}/X$
b	$\sqrt{\det \boldsymbol{B}}$
Y	$u^\mu \partial_\mu \Phi^0$
y_n	$\mathrm{Tr}\,(\boldsymbol{B}^n \cdot \boldsymbol{Z})\,, \quad n = 0, 1, 2, 3$
τ_n	$\mathrm{Tr}\,(\boldsymbol{B}^n)\,, \quad n = 1, 2, 3$
w_n	$\mathrm{Tr}\,(\boldsymbol{W}^n)\,, \quad n = 0, 1, 2, 3$
$\mathcal{O}_{\alpha\beta n}$	$(X/Y^2)^\alpha\, (y_n/Y^2)^\beta\,, \quad \alpha, \beta \in \mathbb{R}$
$\mathcal{O}_\alpha$	$(X/Y^2)^\alpha\,, \quad \alpha \in \mathbb{R}$

$$u^\mu = -\frac{\epsilon^{\mu\nu\alpha\beta}}{6\, b\sqrt{-g}} \epsilon_{abc}\, \partial_\nu \Phi^a\, \partial_\alpha \Phi^b\, \partial_\beta \Phi^c$$

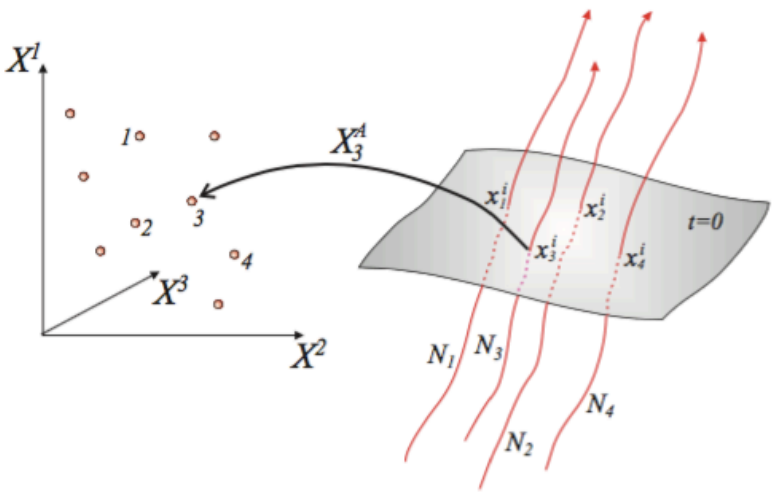
$$u^\mu \partial_\mu\, \Phi^a = 0 \qquad\qquad u^2 = -1$$

$$X = g^{\mu\nu} \partial_\mu \Phi^0\, \partial_\nu \Phi^0 = C^{00} < 0$$

$$\mathcal{V}^\mu = -\frac{\partial^\mu \Phi^0}{\sqrt{-X}}$$

Effective description of the low energy dynamics of a self-gravitating medium minimally coupled with gravity

Basic variables: four scalar fields



$\varphi^a = X^a$ $\quad a = 1, 2, 3$ spatial coordinates

φ^0 medium's clock

The presence of the medium breaks spontaneously Lorentz: scalars fluctuate around non-trivial configurations

$$\varphi^a = x^a + \pi^a$$

$$\varphi^0 = \phi(t) + \pi^0$$

fluctuations are the Goldstone bosons (phonons) of broken translations

Example

$$S_{fluid} = \int d^4x \sqrt{-g}\, U(b,\, Y,\,)$$

$$b = \left(\det\left(g^{\mu\nu}\partial_\mu\varphi^a\partial_\nu\varphi^b\right)\right)^{1/2} \qquad Y = u^\mu\partial_\mu\varphi^0$$

Energy Momentum tensor of a non-barotropic perfect fluid

$$T_{\mu\nu} = p\,\left(g_{\mu\nu} + u_\mu\nu\right) + \rho\, u_\mu\, u\nu$$

$$\rho = -U + Y\, U_Y \qquad p = U + b\, U_b$$

Thermodynamics can also be derived

Y fluid's temperature

b particle number density n

entropy per particle $\sigma = U_Y$

Symmetries

Four-dimensional media		
Symmetries of the action	**LO scalar operators**	**Type of medium**
$SO(3)_s$ & $\Phi^A \to \Phi^A + f^A$, $\partial_\mu f^A = 0$	X, Y, τ_n, y_n	supersolids
$\Phi^a \to \Phi^a + f^a(\Phi^0)$	X, w_n	
$\Phi^0 \to \Phi^0 + f(\Phi^a)$	Y, τ_n	
$\Phi^0 \to \Phi^0 + f(\Phi^0)$	$\tau_n, w_n, \mathcal{O}_{\alpha\beta n}$	
$\Phi^a \to \Phi^a + f^a(\Phi^0)$ & $\Phi^0 \to \Phi^0 + f(\Phi^0)$	w_n	
V_sDiff: $\Phi^a \to \Psi^a(\Phi^b)$, $\det\lvert\partial\Psi^a/\partial\Phi^b\rvert = 1$	b, Y, X	superfluids
$\Phi^0 \to \Phi^0 + f(\Phi^0)$ & V_sDiff	$b, \mathcal{O}_\alpha$	
$\Phi^0 \to \Phi^0 + f(\Phi^a)$ & V_sDiff	b, Y	perfect fluid
$\Phi^A \to \Psi^A(\Phi^B)$, $\det\lvert\partial\Psi^A/\partial\Phi^B\rvert = 1$	bY	perfect fluid with $\rho + p = 0$

Media with reduced internal dimensionality		
Symmetries of the action	**LO scalar operators**	**Type of medium**
$SO(3)_s$ & $\Phi^a \to \Phi^a + c^a$, $\partial_\mu c^a = 0$	τ_n	solid
V_sDiff	b	perfect fluid
$\Phi^0 \to \Phi^0 + c^0$, $\partial_\mu c^0 = 0$	X	

Dynamics of perturbations in terms of 5 mass parameters

$$M_0,\ M_1,\ M_2,\ M_3,\ M_4 \qquad M_\alpha \sim \left\{ \frac{\partial^2 U}{\partial z_N^2},\ \frac{\partial U}{\partial z_N} \right\} \qquad z = \{b,\ Y,\ X,\ \tau_m,\ y_n\}$$

Energy Momentum Tensor

$$T_{\mu\nu} = \rho\, u_\mu\, u_\nu + q_\mu\, q_\nu + q_\nu\, q_\mu + \Pi^{pf}_{\mu\nu} + \Pi_{\mu\nu}$$

EMT	Lagrangian	Medium	Masses
$q_\mu = 0,\ \Pi_{\mu\nu} = 0$	$U(b\,Y)$	Cosm. Constant	$M_{1,2} = 0$
$q_\mu = 0,\ \Pi_{\mu\nu} = 0$	$U(b,\ Y)$	Perfect Fluid	$M_{1,2} = 0$
$q_\mu = 0,\ \Pi_{\mu\nu} \neq 0$	$U(b,\ \tau_n,\ Y)$	Solid	$M_1 = 0$
$q_\mu \neq 0,\ \Pi_{\mu\nu} \propto\ q_\mu\, q_\nu$	$U(b,\ Y,\ X)$	SuperFluid	$M_2 = 0$
$q_\mu \neq 0,\ \Pi_{\mu\nu} \neq 0$	$U(b,\ Y,\ X,\ \tau_n,\ y_n)$	SuperSolid	$M_{1,2} \neq 0$

Mass Parameters

$$M_0 = \frac{\phi'^2}{2\,M_{pl}^2}\left[a^2\left(U_{Y^2} - 2\,U_X\right) - 4\,a\,\phi'\,U_{XY} + 4\,U_{X^2}\,\phi'^2\right]$$

$$M_1 = \frac{2\,\phi'^2}{M_{pl}^2}\left(\sum_{n=0}^{3} a^{-2\,n}\,U_{y_n} + a^2\,U_X\right)$$

$$M_2 = -\frac{2}{M_{pl}^2}\sum_{n=1}^{3} n^2\,a^{2(2-n)}\,U_{\tau_n}$$

$$M_3 = \frac{1}{M_{pl}^2}\left(2\sum_{m,n=1}^{3} m\,n\,a^{-2m-2n+4}\,U_{\tau_m\tau_n} + 2\sum_{n} n\,a^{1-2n}\,U_{b\tau_n} - \sum_{n,m=1}^{3} a^{4-2n}\,U_{\tau_n} + \frac{1}{2\,a^2}\,U_{b^2}\right)$$

$$M_4 = \frac{1}{M_{pl}^2}\left[\frac{\phi'}{2}\left(2\sum_{m,n=1}^{3} a^{3-2n}\,U_{Y\tau_n} - a^3\,U_Y + U_{bY}\right) + \phi'^2\left(-2\sum_{n=1}^{3} a^{2-2n}\,U_{X\tau_n} + a^2\,U_X - \frac{1}{a}\,U_{bX}\right)\right]$$

Scalar perturbations

$$ds^2 = a^2\,\eta_{\mu\nu}\,dx^\mu dx^\nu + 2\,a^2\left[\Psi(t,\,\vec{x})\,dt^2 + \Phi(t,\,\vec{x})\,d\vec{x}^2\right]$$

$$\varphi^0 = \phi^0(t) + \pi^0 \qquad \varphi^i = x^i + \partial_i \pi_l$$

Basic equations

$$c_b^2 = -\frac{M_4}{M_0}$$

$$\Psi - \Phi = M_2\left(A_0\,\Phi + A_1\,\Phi' + B\,\delta\sigma\right)$$

$$\Phi''\left(3\left(1+c_s^2\right)\mathcal{H} + \mathcal{F}_1\right)\Phi' + \frac{M_2\,\mathcal{H}\,\phi'}{6\,M_{pl}^2\,a^4\,k^2\,(w+1)\,\mathcal{H}^2}\,\delta\sigma' + \left(\frac{\phi'\,\left(c_b^2 - c_s^2\right)}{4\,M_{pl}^2\,a^2} + \mathcal{F}_2\right)\delta\sigma$$

$$+\left(3\,\mathcal{H}^2\,\left(c_s^2 - w\right) + k^2\,c_s^2 + \mathcal{F}_3\right)\Phi = 0$$

$$\left(\frac{\phi'^2\left(a^2\,\mathcal{H}^2\,(w+1) + M_1\right)}{a^2\,M_1\,M_{pl}^2\,\mathcal{H}^2\,(w+1)}\,\delta\sigma'\right)' + \left(\frac{4\,k^4\,M_2\,\phi'}{9\,\mathcal{H}^3\,(w+1)\,\left(a^2\,k^2\,(w+1) + 2\,M_2\right)}\right)\Phi'$$

$$-\left(\frac{2\,k^4\,\phi'\left(c_b^2 - c_s^2\right)}{3\,\mathcal{H}^2\,(w+1)} + \mathcal{F}_4\right)\Phi - \left(\frac{\phi'^2\,k^2\left(3\,a^2\,\mathcal{H}^2\,(w+1) + M_0^2\left(c_s^2 - 2\,c_b^2\right)\right)}{6\,a^2\,M_0\,M_{pl}^2\,\mathcal{H}^2\,(w+1)} + \mathcal{F}_5\right)\delta\sigma = 0$$

Results

Internal rotations: super solid

two independent scalar perturbations

non-adiabatic mode (entropy per particle perturbation)

anisotropic stress

Volume preserving diffs+internal time translation: fluids

one independent scalar perturbations

entropy per particle constant in time

Volume preserving diffs: superfluids

two independent scalar perturbations

entropy per particle is dynamical

Fluids

Lagrangian	M_0	M_1	M_2	M_3	M_4	M_1^{eff}	DoF	Features
$U(b)$	0	0	0	$\neq 0$	0	$\neq 0$	1	Barotropic, Isentropic
$U(Y)$	$\neq 0$	0	0	0	$\neq 0$	$\neq 0$	2	Barotropic, Adiabatic
$U(b,\ Y)$	$\neq 0$	0	0	$\neq 0$	$\neq 0$	$\neq 0$	2	Adiabatic
$U(X)$	$\neq 0$	$\neq 0$	0	0	0	0	1	Barotropic, Isent., Irrot.

Superfluids

Lagrangian	M_0	M_1	M_2	M_3	M_4	M_1^{eff}	DoF	Feature
$U(b,\ X)$	$\neq 0$	$\neq 0$	0	$\neq 0$	$\neq 0$	$\neq 0$	2	
$U(Y,\ X)$	$\neq 0$	$\neq 0$	0	0	$\neq 0$	$\neq 0$	2	
$U(b,\ Y,\ X)$	$\neq 0$	$\neq 0$	0	$\neq 0$	$\neq 0$	$\neq 0$	2	
$U(\mathcal{O}_\alpha)$	0	$\neq 0$	0	0	0	$\neq 0$	1	Isentropic
$U(b,\ \mathcal{O}_\alpha)$	0	$\neq 0$	0	$\neq 0$	0	$\neq 0$	1	Isentropic

Conclusions

$$\delta g_{00} = 2a^2\Psi$$

$$\delta g_{ij} = 2\delta_{ij}a^2\Phi$$

Symmetries determine:

- thermodynamics properties of the medium
- dynamics of linear cosmological perturbation

Supersolids

- two independent scalar perturbations: $\Phi,\ \delta\sigma$
- entropy per particle perturbation $\delta\sigma$
- anisotropic stress $\Psi \neq \Phi$
- massive dispersion relation for gravitons

Fluids and Superfluids

- one independent scalar perturbations (fluids):
- constant in time but still sources Φ
- $\delta\sigma$ dynamical for superfluids
- anisotropic stress is zero

Simple, self consistent. Observables easy to compute for DE surveys

Unitary Gauge

$$S = M_{pl} \int d^4x \sqrt{-g}\, R + \int d^4x \sqrt{-g}\, U(X, Y, \tau_n, y_n)$$

$$\Rightarrow \quad S = M_{pl}^2 \int d^4x \sqrt{-g}\, R + \int d^4x \sqrt{-g}\, U\left(N, \xi^i, \gamma^{ij}\right)$$

in the unitary gauge the medium action (fluids, solids ..) is interpreted as modification of gravity

in the unitary gauge diffs appear to be broken

in a generic gauge diffs are "restored" by the Stuckelberg fields Φ^A

what really matters is the number of DoF and underlying symmetries

GR has 2 DoF realized in a minimal way: just by the metric
other theories will typically have more DoF and more fields

Unitary Gauge

take a configuration such that $\det(\partial_\mu \Phi^A) \neq 0$

the 4 scalars can be taken as local coordinates and $\partial_\mu \Phi^A = \delta^A_\mu$

In the unitary gauge the scalars are gauged away,
all the dynamics is in the metric field
Take ADM form

$$g_{\mu\nu} = \begin{pmatrix} -N^2 + N^i N^j \gamma_{ij} & \gamma_{ij} N^j \\ \gamma_{ij} N^j & \gamma_{ij} \end{pmatrix}, \qquad g^{\mu\nu} = \begin{pmatrix} -1/N^2 & N^i/N^2 \\ N^j/N^2 & \gamma^{ij} - N^i N^j/N^2 \end{pmatrix}$$

Unitary Gauge

basic operators in the unitary gauge

$$X = C^{00} = -1/N^2\,, \quad C^i = C^{i0} = \xi^i/N\,, \quad B^{ij} = C^{ij} = \gamma^{ij} - \xi^i\,\xi^j \qquad \xi^i \equiv N^i/N$$

$$Y = \frac{1}{N\sqrt{1-\xi^2}}\,, \quad \tau_n = \mathrm{Tr}\left[(\gamma - \xi\otimes\xi)^n\right]\,, \quad y_n = \frac{\xi\cdot(\gamma-\xi\otimes\xi)^n\cdot\xi}{N^2}$$

$$b = \sqrt{(1-\xi^2)\det\gamma}\,, \quad w_n = \mathrm{Tr}\,(\gamma^n) \qquad u^\mu = \frac{\delta_0^\mu}{N\sqrt{1-\xi^2}}\,, \qquad \mathcal{V}^\mu = (-1/N,\,\xi^i)$$

$$\xi^2 \equiv \xi^i\,\gamma_{ij}\,\xi^j \text{ and } (\xi\otimes\xi)^{ij} \equiv \xi^i\xi^j.$$

Massive gravity reloaded

$$U = \mathcal{U}(\tau_n) + \sqrt{-X}\,\mathcal{E}(w_n)$$

special super solid

in the unitary gauge it appears as a set of massive gravity theories with 5 DoF, ghost free, no vDVZ discontinuity

expanding around flat space the graviton mass terms have only rotational invariance

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

$$\sqrt{-g}\,U = t^{\mu\nu}\,h_{\mu\nu} + \frac{M_{pl}^2}{4}\left(m_0^2\,h_{00}^2 + 2\,m_1^2\,h_{0i}h_{0i} - 2\,m_4^2\,h_{00}\,h_{ii} + m_3^2\,h_{ii}^2 - m_2^2\,h_{ij}\,h_{ij}\right)$$

the graviton mass scale is set by $U \equiv M_{Pl}^2\,m^2\,\tilde{U}$

is even possible to have mGR theories with 2 DoF

$$U = \sqrt{-X}\,\mathcal{E}(w_n) + \lambda$$

Scalars coupled with gravity

Four scalars with shift symmetry $\Phi^A \to \Phi^A + c^A\,, \qquad \partial_\mu c^A = 0$

$$S = \int d^4x \sqrt{-g}\left[M_{pl}^2\, R + U\left(C^{AB}\right) + L_m\right] \qquad C^{AB} = g^{\mu\nu}\partial_\mu\Phi^A\partial_\nu\Phi^B$$

EMT conservation <=> scalar field equations

$$T_{\mu\nu} = U\, g_{\mu\nu} - 2\,\frac{\partial U}{\partial C^{AB}}\,\partial_\mu\Phi^A\partial_\nu\Phi^B$$

Diff invariance of scalars' action encoded in the scalars equations of motion

Examples

Fluids $U(b,\, Y)$ $\qquad T_{\mu\nu} = (U - b\,U_b)g_{\mu\nu} + (Y\,U_Y - b\,U_b)u_\mu\,u_\nu$

$$p = w\,\rho$$

$$U(b) = b^{1+w}\,;$$
$$U(Y) = Y^{\frac{1+w}{w}}\,;$$
$$U(X) = X^{\frac{1+w}{2\,w}}\,;$$
$$U(b,\, Y) = b^{1+w}\,\mathcal{U}(Y\,b^{-w}) \quad \text{or} \quad b^{1+w} + \lambda\,Y^{\frac{1+w}{w}}$$

Super Fluids $U(b,\, Y,\, X)$

$$T_{\mu\nu} = (U - b\,U_b)\;g_{\mu\nu} + (Y\,U_Y - b\,U_b)\;u_\mu\,u_\nu + 2X\,U_X\,\mathcal{V}_\mu\,\mathcal{V}_\nu$$

Perfect Fluids Dynamics

$\nabla^\nu T_{\mu\nu} = 0$ encodes all

by using basic thermodynamics

$$n_\mu = n\, v_\mu, \qquad s_\mu = s\, v_\mu$$

$$\rho + p = T\, s + \mu\, n \qquad\qquad dp = s\, dT + n\, d\mu$$

$$v^\mu \nabla^\nu T_{\mu\nu} = 0 \quad \Rightarrow \quad T\, \nabla^\alpha s_\alpha + \mu\, \nabla^\alpha n_\alpha = 0 \qquad\qquad v^\mu = u^\mu,\ \mathcal{V}^\mu$$

$$\left(\delta^\alpha_\mu + v_\mu\, v^\alpha\right) \nabla^\nu T_{\alpha\nu} = 0 \quad \Rightarrow \quad (p+\rho)\, a_\mu + \left(\delta^\alpha_\mu + v_\mu\, v^\alpha\right) \nabla_\alpha\, p = 0$$