Cosmology of the de Sitter Horndeski Models

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The Horndeski Lagrangian

$$\mathcal{L} = \frac{R}{2} + \sum_i \mathcal{L}_i + \mathcal{L}_m$$

It is the most general scalar field theory in 4D with second order equations of motion

$$\begin{aligned}
\mathcal{L}_2 &= K(\phi, X) \\
\mathcal{L}_3 &= -G_3(\phi, X)\Box\phi \\
\mathcal{L}_4 &= G_4(\phi, X)R + G_{4,X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)(\nabla^\mu\nabla^\nu\phi)\right] \\
\mathcal{L}_5 &= G_5(\phi, X)G_{\mu\nu}(\nabla^\mu\nabla^\nu\phi) - \\
&\frac{1}{6}G_{5,X}\left[(\Box\phi)^3 - 3(\Box\phi)(\nabla_\mu\nabla_\nu\phi)(\nabla^\mu\nabla^\nu\phi) + \right. \\
&\left. 2(\nabla^\mu\nabla_\alpha\phi)(\nabla^\alpha\nabla_\beta\phi)(\nabla^\beta\nabla_\mu\phi)\right]
\end{aligned}$$

Recipe for self-tuning Lagrangians, in concept

$$L(\phi, \dot{\phi}, a, \dot{a}) = a^3 \sum_{i=0}^{3} Z_i(\phi, \dot{\phi}, a) H^i$$

where H is Hubble rate

$$Z_i(\phi, \dot{\phi}, a) = X_i(\phi, \dot{\phi}) - \frac{k}{a^2} Y_i(\phi, \dot{\phi})$$

and X_i and Y_i are functions of the Horndeski or Deffayet free functions.

1. The theory must admit "the vacuum" for any value of the cosmological constant;
2. This should remain true before and after the phase transition where the cosmological constant jumps instantaneously by a finite amount;
3. The theory allows for a non-trivial cosmology.

Charmousis et al. 2011

Recipe for self-tuning Lagrangians, physically

We require that an abrupt change in the matter sector is absorbed by the scalar field leaving the vacuum unchanged.

1. The field equation must be trivially satisfied at the critical point to allow the field to self-adjust $(L_{\rm cp}(a,\phi,\dot\phi)=L_{\rm cp}(a))$;
2. At the critical point, the Hamiltonian must depend on $\dot\phi$ so that the continuous field can absorb discontinuities of the vacuum energy $(\mathcal{H}\propto\rho_{\rm vac}\Rightarrow\mathcal{H}_{\rm cp}\propto f(\dot\phi))$;
3. The scalar field equation of motion must depend on $\dot H$, such that the cosmological evolution is non-trivial before screening takes place $(\dot\phi\propto\dot H)$.

Charmousis et al. 2011

Recipe for self-tuning Lagrangians, in equations

1. $L_{\rm cp}(a,\phi,\dot\phi)=L_{\rm cp}(a)$;

$$L_{\rm cp}=c(a)+\frac{1}{a^3}\dot\zeta(a,\phi)$$

2. $\mathcal{H}\propto\rho_{\rm vac}\Rightarrow\mathcal{H}_{\rm cp}\propto f(\dot\phi)$;

$$\sum_{i=1}^{3} iZ_{i,\dot\phi}H^i\neq 0$$

3. $\dot\phi\propto\dot H$

$$Z_{i,\dot\phi}H^i\neq 0$$

at least for one i.

Charmousis et al. 2011

The Fab 4

The Fab Four potentials (Charmousis *et al.*) are indeed able to self-tune for $k = -1$

$$\mathcal{L}_{\rm John} = V_{\rm J}(\phi) G^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi,$$
$$\mathcal{L}_{\rm Paul} = V_{\rm P}(\phi) P^{\mu\nu\alpha\beta} \nabla_\mu \nabla_\alpha \phi \nabla_\nu \nabla_\beta \phi,$$
$$\mathcal{L}_{\rm George} = V_{\rm G}(\phi) R,$$
$$\mathcal{L}_{\rm Ringo} = V_{\rm R}(\phi) G,$$

- The cosmological models approach a patch of Minkowski with $k = -1$ when it is an attractor, and describe matter domination before that.
- $V_{\rm J}, V_{\rm P} \sim$"stiff fluid"; $V_{\rm G} \sim$"radiation"; and $V_{\rm R} \sim$"curvature".
- Unclear how to obtain a late time accelerated universe.

Extending the self tuning to a de Sitter vacuum

- Complete screening must not necessarily lead to Minkowski;
- If the field self-tunes to a de Sitter vacuum, the late time accelerated expansion can be naturally described by a dynamical approach to a stable critical point;
- Λ of the critical point is not determined by the $\rho_{\rm vac}$ of the matter field but is of purely gravitational origin.

Self tuning to a spatially flat de Sitter vacuum

At the critical point, $H_{\rm cp} = \sqrt{\Lambda}$.

The Lagrangian that at the critical point that satisfies all the constraints, i.e., $L_{\rm cp}(a, \phi, \dot{\phi}) = L_{\rm cp}(a)$ and $\mathcal{H}_{\rm cp} \propto f(\dot{\phi})$, is

$$\mathcal{L}_{\rm H}^{\rm cp} = \sum_{i=0}^{3} X_i(\phi,\, \dot{\phi})\Lambda^{i/2} = 3\sqrt{\Lambda}\, h(\phi) + \dot{\phi}\, h_{,\phi}(\phi)$$

Martin-Moruno, NJN, Lobo (2015)

Self tuning to a spatially flat de Sitter vacuum

$$\mathcal{L}_{\rm H}^{\rm cp} = \sum_{i=0}^{3} X_i(\phi,\, \dot{\phi})\Lambda^{i/2} = 3\sqrt{\Lambda}\, h(\phi) + \dot{\phi}\, h_{,\phi}(\phi)$$

What are the $X_i(\phi, \dot{\phi})$?

1. X_i are terms linear in $\dot{\phi}$

$$X_i = 3\sqrt{\Lambda} U_i(\phi) + \dot{\phi} W_i(\phi)$$

2. X_i are terms with a non-linear dependence on $\dot{\phi}$ which contribution has to vanish at the critical point, i.e., $\mathcal{L}_{\rm H}^{\rm cp} = 0$

I.
Linear models

The linear Lagrangian

Considering also matter, the linear Lagrangian and Hamiltonian are

$$L = L_{\rm EH} + L_{\rm linear} + L_{\rm m} \qquad \mathcal{H} = \mathcal{H}_{\rm EH} + \mathcal{H}_{\rm linear} + \mathcal{H}_{\rm m} = 0$$

where

$$L_{\rm linear} = a^3 \sum_i \left(3\sqrt{\Lambda} U_i(\phi) + \dot{\phi} W_i(\phi)\right) H^i$$

$i = 0, ..., 3$, subject to the constraint at the critical point,

$$\sum_i W_i(\phi)\Lambda^{i/2} = \sum_j U_{j,\phi}(\phi)\Lambda^{j/2},$$

8 functions - 1 constraint $=$ 7 free functions $\Rightarrow$ Mag 7!

W_i and U_i are related to the the κ_j functions of the Horndeski Lagrangian and G_j functions of the Deffayet et al. functions.

Equations of motion

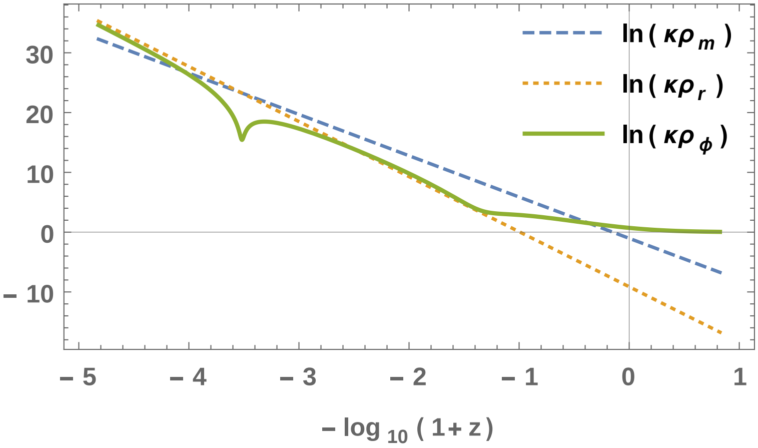
Together they give respectively the field equation for H' and the Friedmann equation

$$H' = 3\frac{\sum_i H^i \left(\sqrt{\Lambda}\, U_{i,\phi}(\phi) - H\, W_i(\phi)\right)}{\sum_i i\, H^i W_i(\phi)}$$

$$\phi' = \sqrt{\Lambda}\frac{(1-\Omega)\, H^2 - 3\sum_i (i-1)\, H^i\, U_i(\phi)}{\sum_i i\, H^{i+1} W_i(\phi)}$$

Tripod model: Energy densities

Example for: $U_2 = e^{\lambda\phi} + \frac{4}{3}e^{\beta\phi}$ and $W_2 = \lambda e^{\lambda\phi} + \beta e^{\beta\phi}$.



II.
Non-linear models

Non-linear Lagrangian

$$L_{\rm nl} = a^3 \sum_{i=0}^{3} X_i(\phi, \dot\phi) H^i$$

To ensure that any non-linear dependence of the Lagrangian on $\dot\phi$ to vanish at the critical point,

$$\sum_{i=0}^{3} X_i(\phi, \dot\phi) \Lambda^{i/2} = 0$$

Again, X_i are related to the the κ_j functions of the Horndeski Lagrangian and G_j functions of the Deffayet et al. functions.

Equations of motion

Proceed to shift-symmetric case and the redefinition $\psi = \dot{\phi}$

$$H' = \frac{3(1+w)Q_0 P_1 - Q_1 P_0}{Q_1 P_2 - Q_2 P_1}$$

$$\psi' = \frac{3(1+w)Q_0 P_2 - Q_2 P_0}{Q_2 P_1 - Q_1 P_2}$$

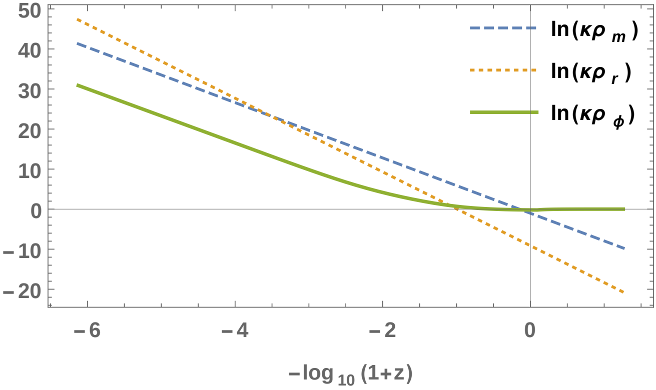
where Q_0, Q_1, Q_2, P_0, P_1, P_2, are complex functions of X_i and H, and the average equation of state parameter of matter fluids is

$$1 + w = \frac{\sum_s \Omega_s (1 + w_s)}{\sum_s \Omega_s}$$

With X_0, X_1 and X_2

Considering

$$X_2(\psi) = \alpha\psi^n, \qquad X_1(\psi) = -\alpha\psi^n + \frac{\beta}{\psi^m}, \qquad X_0(\psi) = -\frac{\beta}{\psi^m}$$

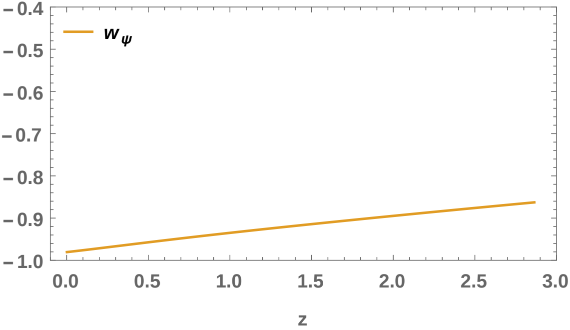


4. Extension with X_0, X_1 and X_2

Considering

$$X_2(\psi) = \alpha\psi^n, \qquad X_1(\psi) = -\alpha\psi^n + \frac{\beta}{\psi^m}, \qquad X_0(\psi) = -\frac{\beta}{\psi^m}$$

We can obtain a model with $w_\psi = w_0 + w_a(1-a)$ s.t. $w_0 = -0.98$ and $w_a = 0.04$



Slotheonic Galileon

Consider the coupling

$$\frac{1}{2M_*^2}G^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$$

Shown to have a tracking trajectory = "no matter what the initial conditions are, the field will asymptotically tend to that trajectory".
Germani and Martin-Moruno (2017)

More to do

- Need study of perturbations.
- Look for combination of linear and non-linear models?