Dark Couplings

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Dark Couplings: What? Why? Who?

1. Scalar field models of dark energy or inflation
2. Multiple scalar field systems (Liddle 1998, Copeland 1999, Kim 2005, Tsujikawa 2006)
3. Scalar field must couple with the rest of the world
4. Couplings to baryons (Khoury); neutrinos (Wetterich)
5. Consider that dark matter particles propagate along geodesics defined with respect to a metric conformal to the gravitational metric $\Rightarrow$ Coupled quintessence (Wetterich 1995, Amendola 2000, Holden 2000)
6. Coupled quintessence with two matter components (Brookfield 2008, Baldi 2012)
7. Consider that the metrics are also related by derivatives of the scalar field $\Rightarrow$ Disformal couplings (Bekenstein 1993; Zumalacarregui 2010, 2013; Bruck 2015; Sakstein 2014, 2015)

Field coupled to dark matter(s)

Single field coupled quintessence:

- :) Scaling accelerating solutions;
- :) application to growing neutrino dark energy;
- :(dark matter never dominant for scaling solutions
- :(Instability problems in the matter density contrast.

Single field but 2 dark matter components (Brookfield, Baldi):

When couplings are symmetric one obtains dark matter domination followed by scalar field domination.

I.
Conformal Couplings

Conformal couplings

Let us consider the action

$$\mathcal{S} = \mathcal{S}_{\text{grav}}(g_{\mu\nu}) + S_{\text{field}}(g_{\mu\nu}, \phi) + \sum_i S_i(\psi_i, \tilde{g}^{(i)}_{\mu\nu})$$

where the fields ψ_i propagate on geodesics defined by the metrics

$$\tilde{g}^i_{\mu\nu} = C_i(\phi) g_{\mu\nu}$$

with $C_i(\phi)$, being the conformal function.

Equations of motion

From the action

$$\Box\phi = V_{,\phi} - \sum_i Q_i \ ,$$

where

$$Q_i = \frac{C_{i,\phi}}{2C_i} T_i$$

We find the following conservation equation for each i–component

$$\nabla^\mu T^i_{\mu\nu} = Q_i \nabla_\nu \phi \ .$$

Equations of motion for FLRW

The dynamical equations

$$\ddot{\phi}_i + 3H\dot{\phi}_i + V_{,\phi_i} = \kappa \sum_{\alpha} C_{i\alpha}\rho_{\alpha},$$

$$\dot{\rho}_{\alpha} + 3H\rho_{\alpha} = -\kappa \sum_{i} C_{i\alpha}\dot{\phi}_i\rho_{\alpha}.$$

$$\rho_{\alpha} = \rho_{\alpha 0}\exp\left(-3N - \kappa\sum_{i} C_{i\alpha}(\phi_i - \phi_{i0})\right)$$

$$\sum_{i}\rho_{\phi_i} = \sum_{i}\dot{\phi}_i^2/2 + V(\phi_1, ..., \phi_n)$$

The rate of change of the Hubble function and the Friedmann equation

$$\dot{H} = -\frac{\kappa^2}{2}\left(\sum_{\alpha}\rho_{\alpha} + \sum_{i}\dot{\phi}_i^2,\right), \qquad H^2 = \frac{\kappa^2}{3}\left(\sum_{\alpha}\rho_{\alpha} + \sum_{i}\rho_{\phi_i}\right).$$

Sum of exponential terms

$$V(\phi_1, ..., \phi_n) = M^4 \sum_i e^{-\kappa\lambda_i\phi_i}$$

$$x_i \equiv \frac{\kappa\dot{\phi}_i}{\sqrt{6}H}, \qquad y_i^2 \equiv \frac{\kappa^2 V_i}{3H^2}, \qquad z_\alpha^2 \equiv \frac{\kappa^2\rho_\alpha}{3H^2},$$

$$\begin{aligned}
x_i' &= -\left(3 + \frac{H'}{H}\right) x_i + \sqrt{\frac{3}{2}}\left(\lambda_i y_i^2 + \sum_\alpha C_{i\alpha} z_\alpha^2\right), \\
y_i' &= -\sqrt{\frac{3}{2}}\left(\lambda_i x_i + \sqrt{\frac{2}{3}}\frac{H'}{H}\right) y_i, \\
z_\alpha' &= -\sqrt{\frac{3}{2}}\left(\sum_i C_{i\alpha} x_i + \sqrt{\frac{3}{2}} + \sqrt{\frac{2}{3}}\frac{H'}{H}\right) z_\alpha, \\
\frac{H'}{H} &= -\frac{3}{2}\left(1 + \sum_i (x_i^2 - y_i^2)\right),
\end{aligned}$$

Critical points

1. Scalar field dominated solution

$$x_i = \frac{1}{\sqrt{6}} \frac{1}{\lambda_i \sum_j 1/\lambda_j^2} .$$

By using the Friedmann equation the effective equation of state is obtained as

$$w_{\rm eff} = \sum_i (x_i^2 - y_i^2) = -1 + \frac{1}{3}\lambda_{\rm eff}^2 ,$$

where the effective slope, $\lambda_{\rm eff}$, given by

$$\frac{1}{\lambda_{\rm eff}^2} = \sum_i \frac{1}{\lambda_i^2} ,$$

More fields $\Rightarrow$ inflation easier to achieve

Critical points

2. Scaling solution (example 2 fields x 2 dark-matter)

$$x_1 = \sqrt{\frac{3}{2}}\frac{1}{\lambda_1 - \gamma_1}, \qquad x_2 = \sqrt{\frac{3}{2}}\frac{1}{\lambda_2 - \gamma_2}.$$

where

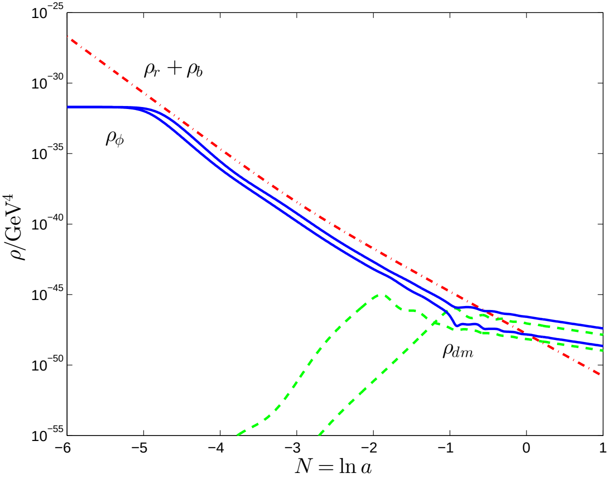
$$\gamma_1 = C_{11} + C_{21}\frac{\lambda_1}{\lambda_2}, \qquad \gamma_2 = C_{22} + C_{12}\frac{\lambda_2}{\lambda_1}.$$

$$w_{\rm eff} = \frac{C_{\rm eff}}{\lambda_{\rm eff} - C_{\rm eff}},$$

$$C_{\rm eff} \equiv \lambda_{\rm eff}\frac{\gamma_i}{\lambda_i}, \qquad \frac{1}{\lambda_{\rm eff}^2} = \frac{1}{\lambda_1^2} + \frac{1}{\lambda_2^2}.$$

Critical points

2. Scaling solution (example 2 fields x 2 dark-matter)



Exponential of a sum of terms

$$V(\phi_1, ..., \phi_n) = M^4 e^{-\sum_i \kappa\lambda_i\phi_i}$$

$$x_i \equiv \frac{\kappa\dot{\phi}_i}{\sqrt{6}H}, \qquad y^2 \equiv \frac{\kappa^2 V}{3H^2}, \qquad z_\alpha^2 \equiv \frac{\kappa^2\rho_\alpha}{3H^2}.$$

The evolution is now described by

$$\begin{aligned}
x_i' &= -\left(3+\frac{H'}{H}\right)x_i + \sqrt{\frac{3}{2}}\left(\lambda_i y^2 + \sum_\alpha C_{i\alpha} z_\alpha^2\right), \\
y' &= -\sqrt{\frac{3}{2}}\left(\sum_i \lambda_i x_i + \sqrt{\frac{2}{3}}\frac{H'}{H}\right) y, \\
z_\alpha' &= -\sqrt{\frac{3}{2}}\left(\sum_i C_{i\alpha} x_i + \sqrt{\frac{3}{2}} + \sqrt{\frac{2}{3}}\frac{H'}{H}\right) z_\alpha, \\
\frac{H'}{H} &= -\frac{3}{2}\left(1 + \sum_i x_i^2 - y^2\right),
\end{aligned}$$

Critical points

1. Scalar field dominated solution

$$x_i = \frac{\lambda_i}{\sqrt{6}},$$

$$w_{\rm eff} = -1 + \frac{1}{3}\lambda_{\rm eff},$$

where the effective slope, $\lambda_{\rm eff}$ is now

$$\lambda_{\rm eff}^2 = \sum_i \lambda_i^2.$$

More fields means inflation more difficult to achieve.

Critical points

2. Scaling solution (example 2 fields x 2 dark-matter)
It is useful to perform an orthogonal transformation Q, s.t.

$$\begin{aligned}
\hat{x}_i &= Q_{ij} x_j ,\\
\hat{\lambda}_i &= Q_{ij} \lambda_j ,\\
\hat{C}_{ij} &= Q_{il} C_{lj} ,
\end{aligned}$$

and then

$$\begin{aligned}
\hat{x}_1 &= \sqrt{\frac{3}{2}} \frac{1}{\lambda_{\rm eff} - C_{\rm eff}} ,\\
\hat{x}_2 &= 0.
\end{aligned}$$

Critical points

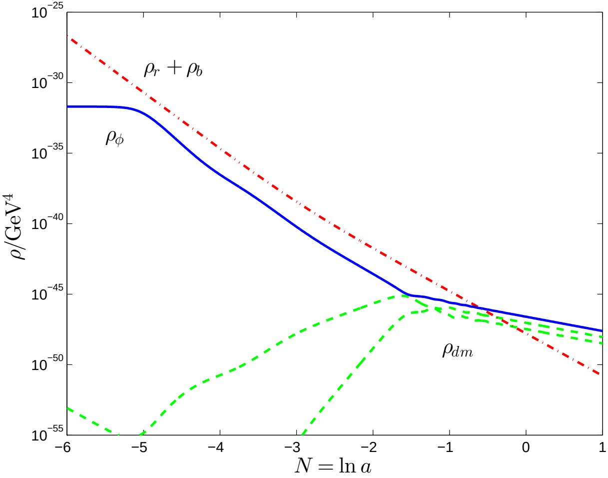
2. Scaling solution (example 2 fields x 2 dark-matter)

$$w_{\rm eff} = \frac{C_{\rm eff}}{\lambda_{\rm eff} - C_{\rm eff}}, \qquad \Omega_\phi = \frac{3 - \lambda_{\rm eff} C_{\rm eff} + C_{\rm eff}^2}{(\lambda_{\rm eff} - C_{\rm eff})^2}.$$

$$\begin{aligned} C_{\rm eff} = \hat{C}_{11} &= Q_{11} C_{11} + Q_{12} C_{21} \\ &= \frac{C_{22} C_{11} - C_{21} C_{12}}{\sqrt{(C_{11} - C_{12})^2 + (C_{22} - C_{21})^2}} \end{aligned}$$

$$\begin{aligned} \lambda_{\rm eff} = \hat{\lambda}_1 &= Q_{11} \lambda_1 + Q_{12} \lambda_2 \\ &= \frac{(C_{22} - C_{21})\lambda_1 + (C_{11} - C_{12})\lambda_2}{\sqrt{(C_{11} - C_{12})^2 + (C_{22} - C_{21})^2}} \end{aligned}$$

Critical points

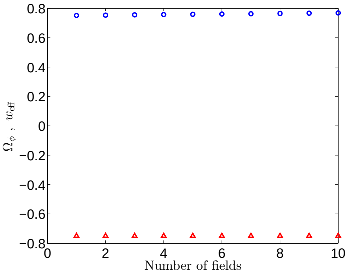


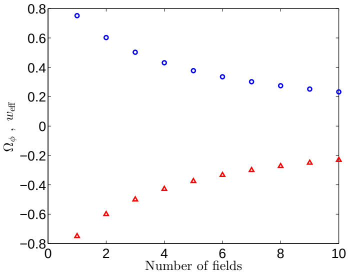
Critical points for scaling solution

When n fields are copy of field ϕ_1

$$V = M^4 \sum_i e^{-\kappa\lambda_i\phi_i}$$

$$V = M^4 e^{-\sum_i \kappa\lambda_i\phi_i}$$





Kinetic dominated solution

This solution is common to both potentials

$$w_{\rm eff} = \Omega_\phi = 1.$$

Conformal kinetic

This solution is common to both potentials

$$x_i = \sqrt{\frac{2}{3}} C_{i\alpha},$$

for any α.

$$w_{\rm eff} = \Omega_\phi = \sum_i x_i^2 = \frac{2}{3} \sum_i C_{i\alpha}^2.$$

Matter dominated solution

When this relation

$$\sum_{\alpha} C_{i\alpha} z_{\alpha}^2 = 0,$$

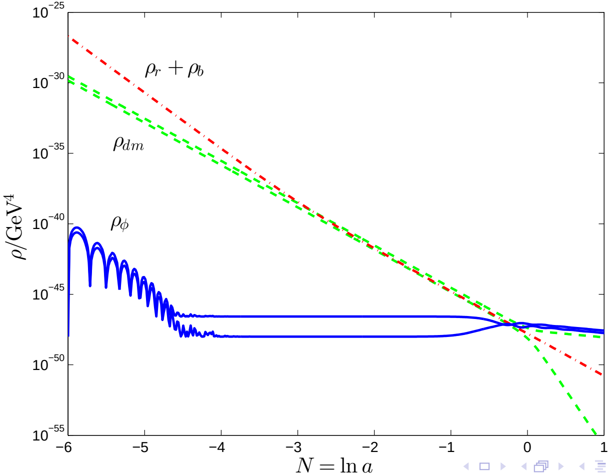
is satisfied and the couplings obey,

$$\frac{C_{11}}{C_{12}} = \frac{C_{21}}{C_{22}},$$

Fields settle at the bottom of the effective potential along a flat direction defined by,

$$\begin{aligned}(C_{11} - C_{12})(\phi_1 - \phi_{10}) + (C_{21} - C_{22})(\phi_2 - \phi_{20}) &= \frac{1}{\kappa} \ln\left(-\frac{C_{11}}{C_{12}}\frac{\rho_{10}}{\rho_{20}}\right) \\ &= \frac{1}{\kappa} \ln\left(-\frac{C_{21}}{C_{22}}\frac{\rho_{10}}{\rho_{20}}\right).\end{aligned}$$

Matter dominated solution



Matter density contrast

Density contrast for dark matter component α:

$$\delta_\alpha'' + \left(2 - \frac{3}{2}\sum_\beta \Omega_\beta - \sum_i (3x_i^2 + \sqrt{6}C_{i\alpha}x_i)\right)\delta_\alpha' - \frac{3}{2}\sum_\beta (1 + 2\sum_i C_{i\alpha}C_{i\beta})\Omega_\beta\delta_\beta = 0.$$

Beware of excessive growth or damping which may arise from the third term.

Comparing with observations

Typically observational results are presented as constraints on the combinations fg and $f\sigma_8$, since, for example, these quantities can be extracted directly from redshift space distortions.

Growth factor g

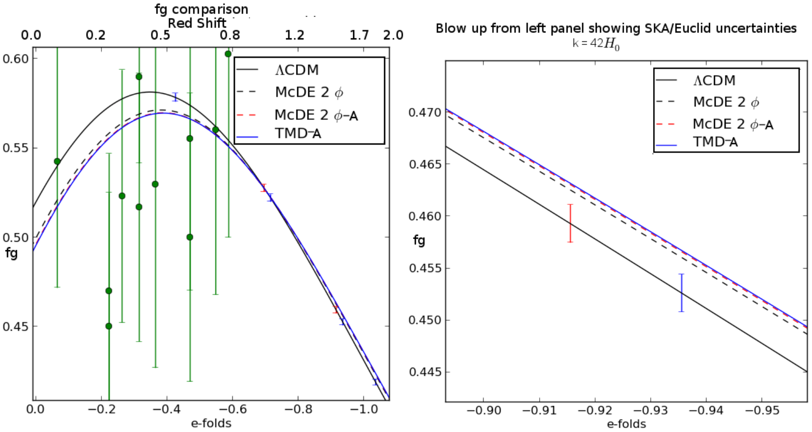
$$g = \frac{\delta}{\delta_0}\,,$$

Growth function, f

$$f = \frac{\delta'}{\delta}\,,$$

σ_8 is the amplitude of the matter power spectrum at a scale of $8h^{-1}$Mpc. We use $\sigma_8 = 0.81$.

Comparison with data



Summarising so far

1. The scalar field dominated solution: easier to obtain inflation for sum of exponentials;
2. Scaling solution: Effective coupling depends on Cs and ?s for sum of exponentials but only depends on Cs for the exponential of sum potential;
3. Matter dominated epoch: couplings must obey relation for early dust behavior. Fields settle at the bottom of the effective potential which is a flat direction;
4. Matter density contrast: Source term in the density contrast equation $\Rightarrow$ excessive growth for large Cs.
5. Possibility to discriminate between models with future surveys (Euclid, SKA).

II.
Disformal Couplings

Disformal couplings

Let us consider the action

$$\mathcal{S} = \mathcal{S}_{\rm grav}(g_{\mu\nu}) + S_{\rm field}(g_{\mu\nu}, \phi) + \sum_i S_i(\psi_i, \tilde{g}^{(i)}_{\mu\nu})$$

where the fields ψ_i propagate on geodesics defined by the metrics

$$\tilde{g}^i_{\mu\nu} = C_i(\phi) g_{\mu\nu} + D_i(\phi)\partial_\mu\phi\partial_\nu\phi \ ,$$

with $C_i(\phi), \ D_i(\phi)$ being the conformal and disformal coupling functions respectively.

Equations of motion

From the action

$$\Box\phi = V_{,\phi} - \sum_i Q_i \ ,$$

where

$$Q_i = \frac{C_{i,\phi}}{2C_i} T_i + \frac{D_{i,\phi}}{2C_i} T_i^{\mu\nu} \nabla_\mu\phi\nabla_\nu\phi - \nabla_\mu \left[\frac{D_i}{C_i} T_i^{\mu\nu} \nabla_\nu\phi\right]$$

We find the following conservation equation for each i–component

$$\nabla^\mu T^i_{\mu\nu} = Q_i \nabla_\nu\phi \ .$$

Equations of motion for FLRW

$$\ddot{\phi} + 3H\dot{\phi} + V_{,\phi} = \sum_i Q_i \, ,$$
$$\dot{\rho}_i + 3H\rho_i(1+w_i) = -Q_i\dot{\phi} \, ,$$

where

$$Q_1 = \frac{\mathcal{A}_2}{\mathcal{A}_1\mathcal{A}_2 - D_1 D_2 \rho_1\rho_2}\left(\mathcal{B}_1 - D_1\rho_1\frac{\mathcal{B}_2}{\mathcal{A}_2}\right) \, ,$$

$$Q_2 = \frac{\mathcal{A}_1}{\mathcal{A}_1\mathcal{A}_2 - D_1 D_2 \rho_1\rho_2}\left(\mathcal{B}_2 - D_2\rho_2\frac{\mathcal{B}_1}{\mathcal{A}_1}\right) \, ,$$

and $\mathcal{A}_i = C_i + D_i\left(\rho_i - \dot{\phi}^2\right)$, $\mathcal{B}_i =$
$\left[\frac{1}{2}C_{i,\phi}(-1+3w_i) - \frac{1}{2}D_{i,\phi}\dot{\phi}^2 + D_i\left(3(1+w_i)H\dot{\phi} + V_{,\phi} + \frac{C_{i,\phi}}{C_i}\dot{\phi}^2\right)\right]\rho_i$

Dynamical system analysis

Reduce the above system to a set of first order autonomous differential equations

$$
\begin{aligned}
x^2 &\equiv \frac{\kappa^2 \phi'^2}{6}, \quad y^2 \equiv \frac{\kappa^2 V}{3H^2}, \quad z_i^2 \equiv \frac{\kappa^2 \rho_i}{3H^2}, \quad \lambda_V \equiv -\frac{1}{\kappa}\frac{V_{,\phi}}{V}, \\
\lambda_C^i &\equiv -\frac{1}{\kappa}\frac{C_{i,\phi}}{C_i}, \quad \lambda_D^i \equiv -\frac{1}{\kappa}\frac{D_{i,\phi}}{D_i}, \quad \sigma_i \equiv \frac{D_i H^2}{\kappa^2 C_i},
\end{aligned}
$$

$$
\begin{aligned}
x' &= -\left(3 + \frac{H'}{H}\right) x + \sqrt{\frac{3}{2}} \left(\lambda_V y^2 + \frac{\kappa Q_1}{3H^2} + \frac{\kappa Q_2}{3H^2}\right) \\
y' &= -\sqrt{\frac{3}{2}} \left(\lambda_V x + \sqrt{\frac{2}{3}} \frac{H'}{H}\right) y \\
z_i' &= -\frac{3}{2} \left(1 + w_i + \frac{2}{3}\frac{H'}{H} + \frac{1}{3}\sqrt{\frac{2}{3}} \frac{\kappa Q_i}{H^2} \frac{x}{z_i^2}\right) z_i \\
\sigma_i' &= \left(\sqrt{6}(\lambda_C^i - \lambda_D^i) x + 2\frac{H'}{H}\right) \sigma_i
\end{aligned}
$$

Also important quantities

$$\frac{H'}{H} = -\frac{3}{2}\left(2x^2 + \sum_{i=1}^{2}(1+w_i)z_i^2\right) ,$$

$$x^2 + y^2 + \sum_{i=1}^{2} z_i^2 = 1 .$$

$$\Omega_\phi = x^2 + y^2 , \qquad w_\phi = \frac{x^2 - y^2}{x^2 + y^2} ,$$

$$Z_i \equiv \frac{1}{C_i^2}\sqrt{\frac{-\tilde{g}^i}{-g}} = \sqrt{1 - 6\sigma_i x^2} , \qquad w_i = \tilde{w}_i\left(1 - 6\sigma_i x^2\right)$$

$\tilde{w}_i$ is the equation of state parameter in the frame defined by $\tilde{g}^i_{\mu\nu}$. A potential problem for the theory is when $Z_i = 0$ due to a metric singularity (Sakstein 2014).

Single Fluid–Arbitrary EOS

Let us take the conformal, disformal functions and the potential to be

$$C(\phi) = e^{2\alpha\kappa\phi}, \qquad D(\phi) = \frac{e^{2(\alpha+\beta)\kappa\phi}}{M^4}, \qquad V(\phi) = V_0^4 e^{-\lambda\kappa\phi}$$

1, 2 Kination

$$x = \pm 1, \qquad y = 0, \qquad \sigma = 0, \qquad \Omega_\phi = 0$$

3, 4 Disformal

$$x = \frac{\sqrt{2}\beta \mp \sqrt{2\beta^2 - 3}}{\sqrt{3}}, \quad y = 0,$$

$$\sigma = \frac{1}{18}\left(2\beta\left(2\beta \pm \sqrt{4\beta^2 - 6}\right) - 3\right)$$

$$\Omega_\phi = \frac{1}{3}\left(\sqrt{2\beta^2 - 3} \mp \sqrt{2}\beta\right)^2, \quad w_\phi = 1, \quad Z = 0$$

5 Mixed

$$x = \frac{\sqrt{\frac{3}{2}}\gamma}{\alpha(4-3\gamma)+2\beta}, \quad y = 0, \quad \sigma \neq 0,$$

$$\Omega_\phi = \frac{3\gamma^2}{2(\alpha(4-3\gamma)+2\beta)^2}, \quad w_\phi = 1, \quad Z \in \mathbb{R}$$

Single Fluid–Arbitrary EOS

6 Conformal kinetic

$$x = \frac{\sqrt{\frac{2}{3}}\alpha(4-3\gamma)}{\gamma-2}, \qquad y = 0, \qquad \sigma = 0,$$
$$\Omega_\phi = \frac{2\alpha^2(4-3\gamma)^2}{3(\gamma-2)^2}, \qquad w_\phi = 1, \qquad Z = 1$$

7 Scalar field dominated

$$x = \frac{\lambda}{\sqrt{6}}, \qquad y \neq 0, \qquad \sigma = 0,$$
$$\Omega_\phi = 1, \qquad w_\phi = \frac{1}{3}\left(\lambda^2 - 3\right), \qquad Z = 1$$

8 Conformal scaling

$$x = \frac{\sqrt{\frac{3}{2}}\gamma}{(4-3\gamma)\alpha+\lambda}, \qquad y \neq 0, \qquad \sigma = 0$$
$$\Omega_\phi = \frac{\alpha^2(4-3\gamma)^2+\alpha(4-3\gamma)\lambda+3\gamma}{(\alpha(4-3\gamma)+\lambda)^2},$$
$$w_\phi = \frac{3\gamma^2}{\alpha^2(4-3\gamma)^2+\alpha(4-3\gamma)\lambda+3\gamma} - 1, \qquad Z = 1$$

Summary - single fluid

	Name	x	y	σ	Ω_ϕ	w_ϕ	Z
1	kination	-1	0	0	1	1	1
2	kination	1	0	0	1	1	1
3	Disformal	β	0	β	β	1	0
4	Disformal	β	0	β	β	1	0
5	Mixed	γ, α, β	0	γ, α, β	γ, α, β	1	γ, α, β
6	Conf. kinetic	γ, α	0	0	γ, α	1	1
7	ϕ dominated	λ	λ	0	1	λ	1
8	Conf. scaling	γ, α, λ	γ, α, λ	0	γ, α, λ	γ, α, λ	1

Two fluids - Dust and radiation

Take couplings and potential to be

$$C_i(\phi) = e^{2\alpha_i \kappa\phi}, \qquad D_i(\phi) = \frac{e^{2(\alpha_i+\beta_i)\kappa\phi}}{M_i^4}, \qquad V(\phi) = V_0 e^{-\lambda\kappa\phi}$$

The fixed points are all very similar to a single fluid. But there are two new ones.

a Conformal dust radiation

$$x = -\frac{1}{\sqrt{6}\alpha_1}, \quad y = 0, \quad z_1 = \frac{1}{\sqrt{3\alpha_1^2}}, \quad z_2 = \sqrt{1 - \frac{1}{2\alpha_1^2}},$$
$$\sigma_1 = 0, \quad \sigma_2 = 0,$$
$$\Omega_\phi = \frac{1}{6\alpha_1^2}, \quad w_\phi = 1, \quad Z_1 = 1, \quad Z_2 = 1, \quad w_{\text{eff}} = \frac{1}{3}$$

also found in (Amendola 2000).

Two fluids - Dust and radiation

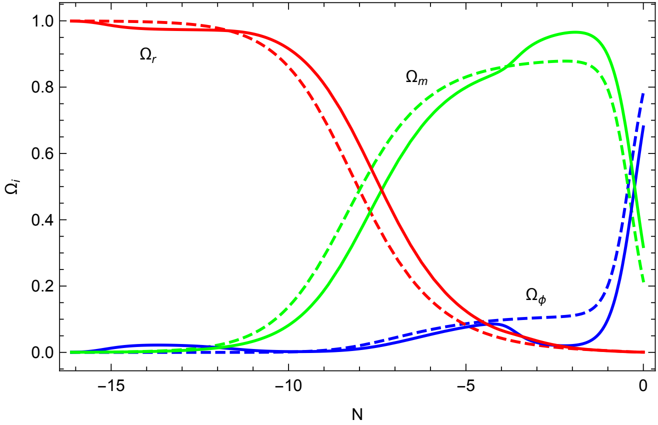
b Disformal dust radiation

$$x = \frac{\sqrt{\frac{2}{3}}}{\beta_1}, \quad y = 0, \quad z_1 = \frac{2}{\sqrt{3\beta_1^2}}, \; z_2 = \sqrt{1 - \frac{2}{\beta_1^2}}, \quad \sigma_1 = \frac{\beta_1^2}{4},$$

$$\sigma_2 = 0, \quad \Omega_\phi = \frac{2}{3\beta_1^2}, \quad w_\phi = 1, \; Z_1 = 0, \quad Z_2 = 0, \quad w_{\rm eff} = \frac{1}{3}$$

	Name	x	y	z_1	z_2	σ_1	σ_2	Ω_ϕ	w_ϕ	Z_1	Z_2	$w_{\rm eff}$
a	Conf. d-r	α_1	0	α_1	α_1	0	0	α_1	1	1	1	$1/3$
b	Disf. d-r	β_1	0	β_1	β_1	β_1	0	β_1	1	0	1	$1/3$

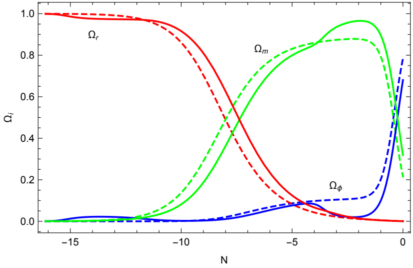
Example evolution



—— Conformal and disformal

- - - Only conformal

Things to pay attention to



1. Transient disformal fixed points (3r), (5r) and (b) during radiation domination
2. Radiation-matter equality changes
3. Transfer of matter to the scalar field = fixed point (6d)
4. Transfer of the scalar field to matter = fixed point (3d)
5. Final attractor is (7)

Two conformal-disformal dust fluids

There is one additional fixed point corresponding to a

c Conformal dust dominated

	Name	x	y	z_1	z_2	σ_1	σ_2	Z_1	Z_2	$w_{\rm eff}$
c	Conf. d.	0	0	α_1, α_2	α_1, α_2	0	0	1	1	0

$z_1 = \sqrt{\frac{\alpha_2}{\alpha_2 - \alpha_1}}$ and $z_2 = \sqrt{\frac{\alpha_1}{\alpha_1 - \alpha_2}}$

III. Disformal couplings and the variation of α

Disformal couplings and the variation of α

The metrics $\tilde{g}^{(m)}_{\mu\nu}$ and $\tilde{g}^{(r)}_{\mu\nu}$ are related to $g_{\mu\nu}$ via a disformal transformation:

$$\tilde{g}^{(m)}_{\mu\nu} = C_m(\phi) g_{\mu\nu} + D_m(\phi)\phi_{,\mu}\phi_{,\nu}$$

$$\tilde{g}^{(r)}_{\mu\nu} = C_r(\phi) g_{\mu\nu} + D_r(\phi)\phi_{,\mu}\phi_{,\nu} \ .$$

C_r and C_m are conformal factors
D_r and D_m are disformal factors
We can also write,

$$\tilde{g}^{(r)}_{\mu\nu} = \frac{C_r}{C_m}\tilde{g}^{(m)}_{\mu\nu} + \left(D_r - \frac{C_r D_m}{C_m}\right)\phi_{,\mu}\phi_{,\nu} \equiv A\tilde{g}^{(m)}_{\mu\nu} + B\phi_{,\mu}\phi_{,\nu}$$

Electromagnetic sector

The action

$$\mathcal{S}_{\rm EM} = -\frac{1}{4}\int d^4x\sqrt{-\tilde{g}^{(r)}}h(\phi)\tilde{g}^{\mu\nu}_{(r)}\tilde{g}^{\alpha\beta}_{(r)}F_{\mu\alpha}F_{\nu\beta} - \int d^4x\sqrt{-\tilde{g}^{(m)}}\tilde{g}^{\mu\nu}_{(m)}j_\mu A_\mu$$

- $F_{\mu\nu}$ is Faraday tensor; j^μ is the four–current;
- $h(\phi)$ is the coupling between the electromagnetism and ϕ.

In the frame in which matter is decoupled from the scalar field

$$\mathcal{S}_{\rm EM} = -\frac{1}{4}\int d^4x\sqrt{-\tilde{g}^{(m)}}hZ\left[\tilde{g}^{\mu\nu}_{(m)}\tilde{g}^{\alpha\beta}_{(m)} - 2\gamma^2\tilde{g}^{\mu\nu}_{(m)}\phi^{,\alpha}\phi^{,\beta}\right]F_{\mu\alpha}F_{\nu\beta} - \int d^4x\sqrt{-\tilde{g}^{(m)}}\tilde{g}^{\mu\nu}_{(m)}j_\mu A_\mu$$

where

$$Z = \left(1 + \frac{B}{A}\tilde{g}^{\mu\nu}_{(m)}\partial_\mu\phi\partial_\nu\phi\right)^{1/2}, \qquad \gamma^2 = \frac{B}{A + B\tilde{g}^{\mu\nu}_{(m)}\partial_\mu\phi\partial_\nu\phi}$$

The field equation for A_μ

Varying the action with respect to A_μ

$$\tilde{\nabla}_\epsilon \left(hZF^{\epsilon\rho}\right) - \tilde{\nabla}_\epsilon \left(hZ\gamma^2\phi^{,\beta}\left(\tilde{g}^{\epsilon\nu}_{(m)}\phi^{,\rho} - \tilde{g}^{\rho\nu}_{(m)}\phi^{,\epsilon}\right)F_{\nu\beta}\right) = j^\rho$$

With $\tilde{g}^{(m)}_{\mu\nu} = \eta_{\mu\nu}$, and $E^i = F^{i0}$

$$\nabla \cdot \mathbf{E} = \frac{Z\rho}{h}$$

where $\rho = j^0$. Integrating this equation over a volume $\mathcal{V}$ using, $\mathbf{E} = -\nabla V$, we get the electrostatic potential

$$V(r) = \frac{ZQ}{4\pi hr} \quad \Rightarrow \quad \boxed{\alpha \propto \frac{Z}{h}}$$

The fine structure constant depends on Z.

The evolution of α

For FLRW Universe,

$$Z = \left(\frac{1 - \frac{D_r}{C_r}\dot{\phi}^2}{1 - \frac{D_m}{C_m}\dot{\phi}^2} \right)^{1/2}$$

Time derivative of α,

$$\frac{\dot{\alpha}}{\alpha} = \frac{1}{Z}\left(\frac{\partial Z}{\partial \phi}\dot{\phi} + \frac{\partial Z}{\partial \dot{\phi}}\ddot{\phi} \right) - \frac{1}{h}\frac{dh}{d\phi}\dot{\phi}$$

Redshift evolution of α,

$$\frac{\Delta\alpha}{\alpha}(z) \equiv \frac{\alpha(z) - \alpha_0}{\alpha_0} = \frac{h_0 Z}{h Z_0} - 1$$

Gravity and matter field sector

Is the evolution of ϕ compatible with constraints on the evolution of α?

$$\mathcal{S} = \int d^4x\sqrt{-g}\left(\frac{1}{2}R - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right) + S_{\rm matter}(\tilde{g}^{(m)}_{\mu\nu})$$

with the equation of motion

$$\begin{aligned}
\ddot{\phi} + 3H\dot{\phi} + V' &= Q_m + Q_r, \\
\dot{\rho}_m + 3H(\rho_m + p_m) &= -Q_m\dot{\phi}, \\
\dot{\rho}_r + 3H(\rho_r + p_r) &= -Q_r\dot{\phi},
\end{aligned}$$

where Q_m and Q_r are complicated functions of ρ_m, ρ_r, $\dot{\phi}$, C_r, C_m, D_r, D_m and their field derivatives.

Couplings and parameters

We specify to exponential couplings and potential and to linear direct coupling $h(\phi)$:

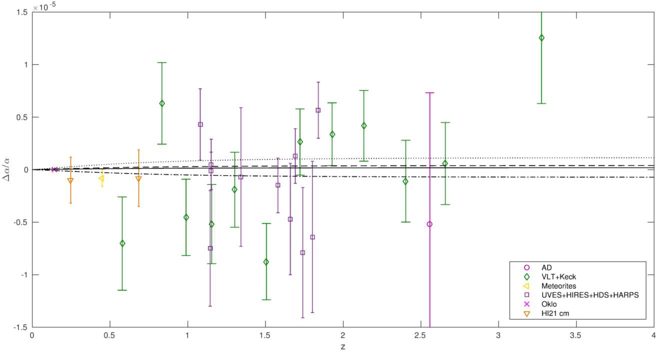
$$C_i(\phi) = c_i e^{2\alpha_i\phi}, \qquad D_i(\phi) = M_i^{-4} e^{2(\alpha_i+\beta_i)\phi},$$
$$h(\phi) = 1 - \zeta(\phi - \phi_0), \qquad V(\phi) = M_V^4 e^{-\lambda\phi}.$$

Parameters x_i, y_i, λ, β_i, M_i, M_V and ζ are tuned such that their are in agreement with constraints on α and on the cosmological parameters from Planck.

Parameter	Estimated value
$w_{0,\phi}$	-1.006 ± 0.045
H_0	(67.8 ± 0.9) km s^{-1}Mpc^{-1}
$\Omega_{0,m}$	0.308 ± 0.012

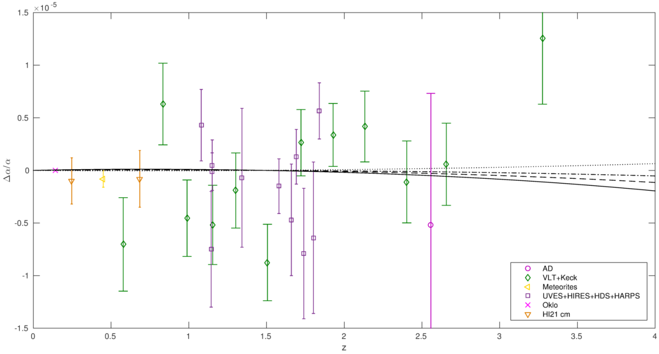
Disformal and electromagnetic couplings

M_r	M_m	M_V	U_m	$2\alpha_m$	$\lvert\zeta\rvert$	λ
$\sim$ 1 meV	$\sim$ 1 meV	2.69 meV	1	0	$< 5 \times 10^{-6}$	0.45



Disformal and conformal couplings

M_r	M_m	M_V	U_m	$2\alpha_m$	$\|\zeta\|$	λ
25-27 meV	15 meV	2.55 meV	8	0.14	0	0.45



Summary

1. A variation in the fine-structure constant can be induced by disformal couplings provided that the radiation and matter disformal coupling strengths are not identical.
2. Such a variation is enhanced in the presence of the usual electromagnetic coupling.
3. Laboratory measurements with molecular and nuclear clocks are expected to increase their sensitivity to as high as 10^{-21} yr^{-1}.
4. Better constrained data is expected from high-resolution ultra-stable spectrographs such as PEPSI at the LBT, ESPRESSO at the VLT and ELT-Hires at the E-ELT.