Neutron stars in beyond Horndeski theory

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Beyond Horndeski theory

Horndeski theory

Vainshtein mechanism (weak gravity)
=>
Recover of General Relativity
=>
theory passes observational tests

beyond Horndeski theory

Breaking of the Vainshtein mechanism inside matter

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Beyond Horndeski theory

Breaking of the Vainshtein mechanism inside matter

↓

Neutron stars are different compared to those in General Relativity

We can use this to resolve some problems (see later)

Newtonian order (Linearised gravity)

$$ds^2 = (-1 + 2\Phi)\,dt^2 + (1 + 2\Psi)\,\delta_{ij}\,dx^i\,dx^j$$

- Φ and Ψ are Newtonian potentials ($\Phi = \Psi$ for GR)
- Corrections for Horndeski theory:

$$\frac{d\Phi}{dr} = \frac{G_N M}{r}\left[1 + 2\alpha^2\left(\frac{r}{r_v}\right)^n\right]$$

- α is the coupling between the scalar and matter, n is model dependent parameter
- For solar mass object $r_v \sim \mathcal{O}(0.1\ \mathrm{kpc})$ $\Rightarrow$ for $r \ll r_V$ GR is restored (solar system tests are passed)

Newtonian order (Linearised gravity)

$$ds^2 = (-1 + 2\Phi)\,\mathrm{d}t^2 + (1 + 2\Psi)\,\delta_{ij}\,\mathrm{d}x^i\,\mathrm{d}x^j$$

- For Horndeski theory GR is restored also inside matter
- For beyond Horndeski theory:

$$\frac{\mathrm{d}\Phi}{\mathrm{d}r} = \frac{G_{\mathrm{N}}M(r)}{r^2} + \frac{\Upsilon_1 G_{\mathrm{N}}}{4}\frac{\mathrm{d}^2M(r)}{\mathrm{d}r^2}$$

$$\frac{\mathrm{d}\Psi}{\mathrm{d}r} = \frac{G_{\mathrm{N}}M(r)}{r^2} - \frac{5\Upsilon_2 G_{\mathrm{N}}}{4r}\frac{\mathrm{d}M(r)}{\mathrm{d}r}$$

(Breaking of GR)

[Kobayashi et al'15]

$$M(r) \equiv 4\pi \int_0^r s^2 \rho(s) ds$$

- Υ_1 and Υ_2 are only non-zero for beyond Horndeski theory

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Current constraints on beyond Horndeski parameters

- $\Upsilon_1 > -2/3$ for a sensible stellar profile
- $-0.22 < \Upsilon_1 < 0.027$

Chandrasekhar mass of dwarf stars

Consistency of the minimum mass for hydrogen burning

Non-relativistic systems

What happens for relativistic stars?

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Simplest beyond Horndeski theory

$$\boxed{S = \int d^4x \sqrt{-g} \left[M_{\rm pl}^2 \left(\frac{R}{2} - \Lambda \right) - k_2 \mathcal{L}_2 + f_4 \mathcal{L}_{4,\rm bH} \right]}$$

$$\mathcal{L}_2 = \phi_\mu \phi^\mu \equiv X$$

$$\mathcal{L}_{4,\rm bH} = -X \left[(\Box\phi)^2 - (\phi_{\mu\nu})^2 \right] + 2\phi^\mu \phi^\nu \left[\phi_{\mu\nu} \Box\phi - \phi_{\mu\sigma} \phi^\sigma{}_\nu \right]$$

Λ is a (positive) cosmological constant

k_2 and f_4 are constant coefficients

$$M_{\rm pl}^2 = (8\pi G)^{-1}$$

STEP 1: cosmology

Cosmological solution sets boundary condition for local solution

$$\mathrm{d}s^2 = -\,\mathrm{d}\tau^2 + e^{2H\tau}\left(\mathrm{d}r'^2 + r'^2\,\mathrm{d}\Omega_2^2\right)$$

FLRW ansatz

$$\phi = q\tau$$

Scalar field ansatz

$$k_2 = -2\frac{M_{\rm pl}^2 H^2}{q^2}\left(1-\sigma^2\right)$$

$$f_4 = \frac{M_{\rm pl}^2}{6q^4}\left(1-\sigma^2\right)$$

$$\sigma^2 \equiv \Lambda/(3M_{\rm pl}^2 H^2)$$

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STEP 1: cosmology

Cosmological coordinates to
Schwarzschild coordinates:

$$\tau = t + \frac{1}{2H}\ln\left[1 - H^2 r^2\right] \quad \text{and} \quad r' = \frac{e^{-Ht}}{\sqrt{1 - H^2 r^2}}\, r$$

$$\mathrm{d}s^2 = -(1 - H^2 r^2)\,\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{1 - H^2 r^2}\,\mathrm{d}r^2 + r^2\,\mathrm{d}\Omega_2^2$$

$$\phi(r,t) = qt + \frac{q}{2H}\ln\left(1 - H^2 r^2\right)$$

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STEP 2: Solution outside the star

Exact vacuum solution:

$$\mathrm{d}s^2 = -e^{\nu(r)}\,\mathrm{d}t^2 + e^{\lambda(r)}\,\mathrm{d}r^2 + r^2\,\mathrm{d}\Omega_2^2$$

metric

$$\nu(r) = -\lambda(r) = \ln\left(1 - \frac{\mathcal{M}}{r} - H^2 r^2\right)$$

$$\phi(r) = qt - q\int \frac{\sqrt{\frac{\mathcal{M}}{r} + H^2 r^2}}{1 - \frac{\mathcal{M}}{r} - H^2 r^2}\,\mathrm{d}r$$

Schwarzschild-de-Sitter metric and non-trivial scalar field configuration as in the case of “self-tuned” black holes

$$\mathcal{M} = 2G_\mathrm{N} M$$

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STEP 3: Solution inside the star

$$\boxed{\mathrm{d}s^2 = -e^{\nu(r)}\,\mathrm{d}t^2 + e^{\lambda(r)}\,\mathrm{d}r^2 + r^2\,\mathrm{d}\Omega_2^2}$$

metric

$$T^{\mu}_{\nu} = \mathrm{diag}\left(-\varepsilon, P, P, P\right)$$

Solve modified Einstein equations inside matter (Numerics)

Example:

$$\varepsilon = \left(\frac{P}{K}\right)^{\frac{1}{2}} + P$$

More realistic equations of state: SLy4, BSK20

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Observations of neutron stars

Solving numerically modified Tolman-Oppenheimer-Volkoff equations, we find mass-radius relations and we can compare the results with observations

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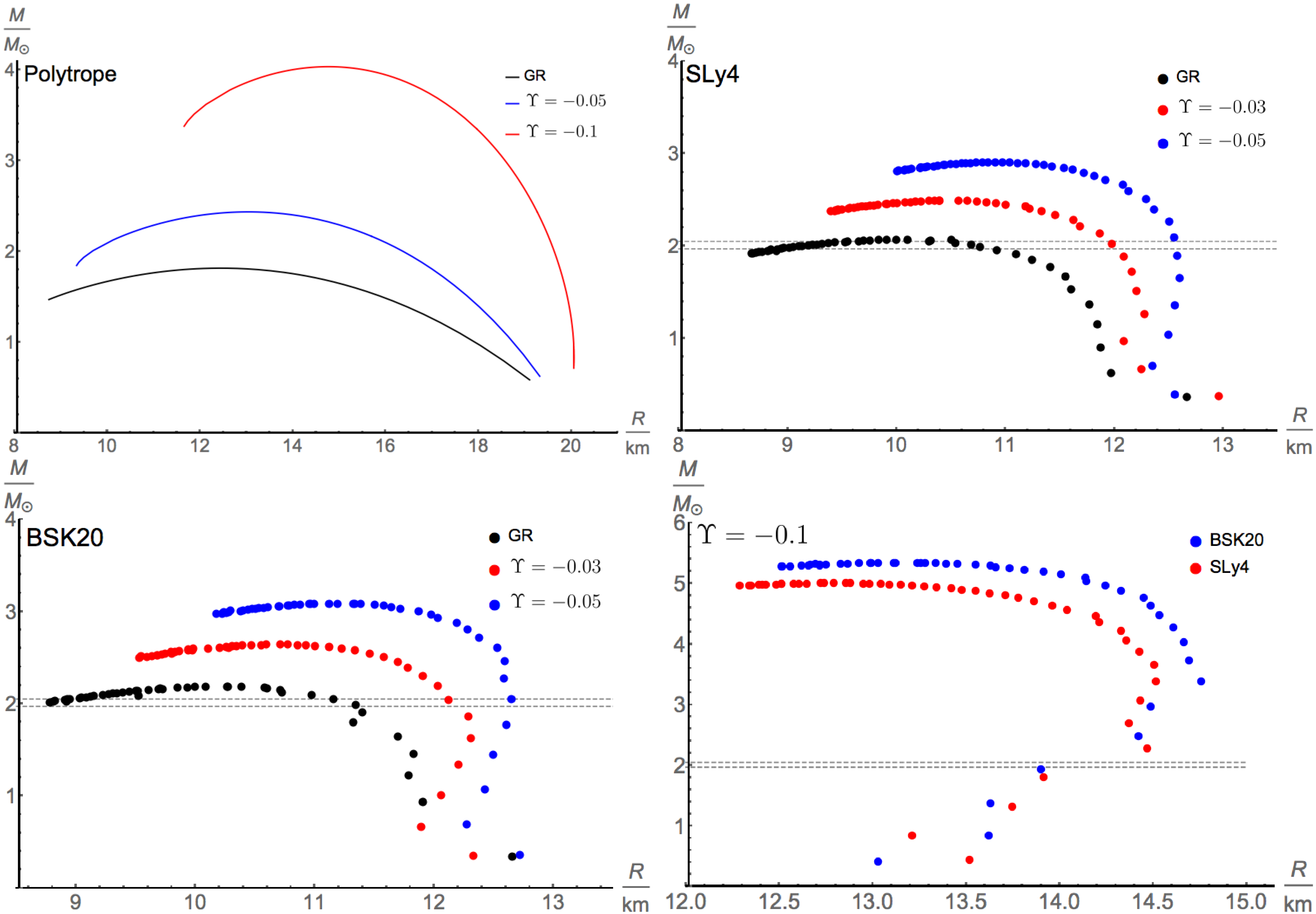
mass-radius relation



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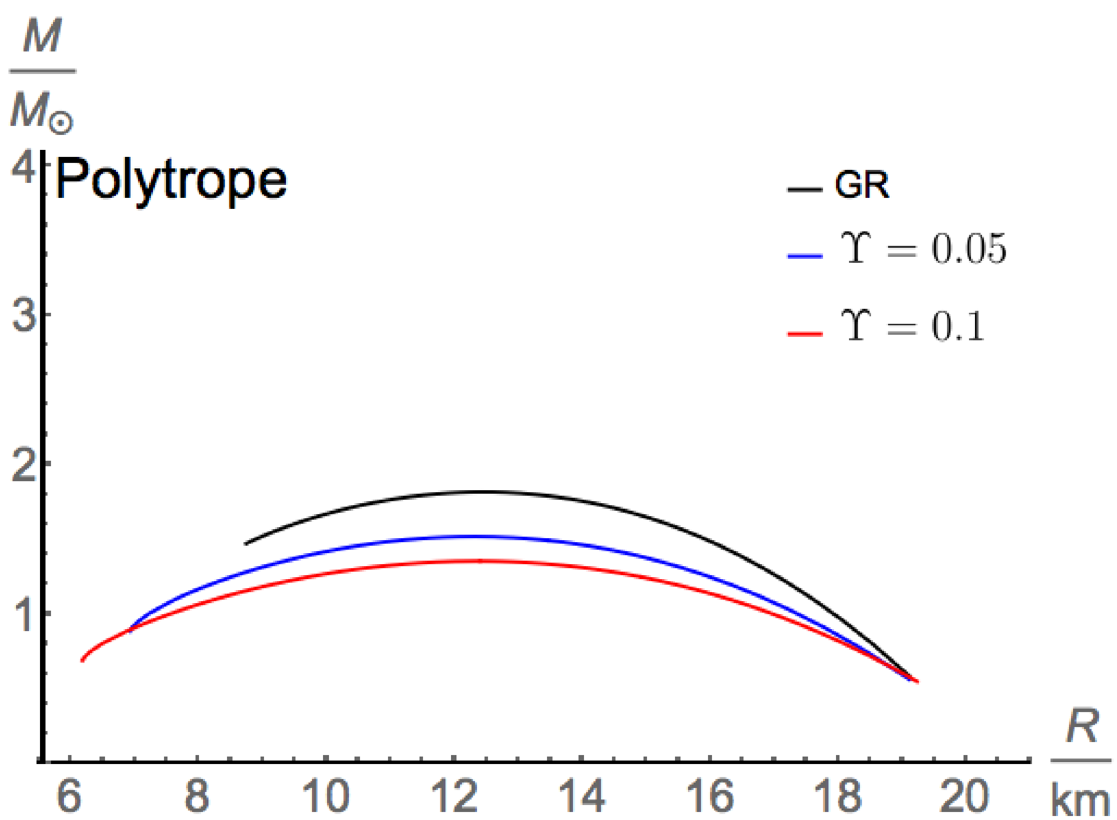
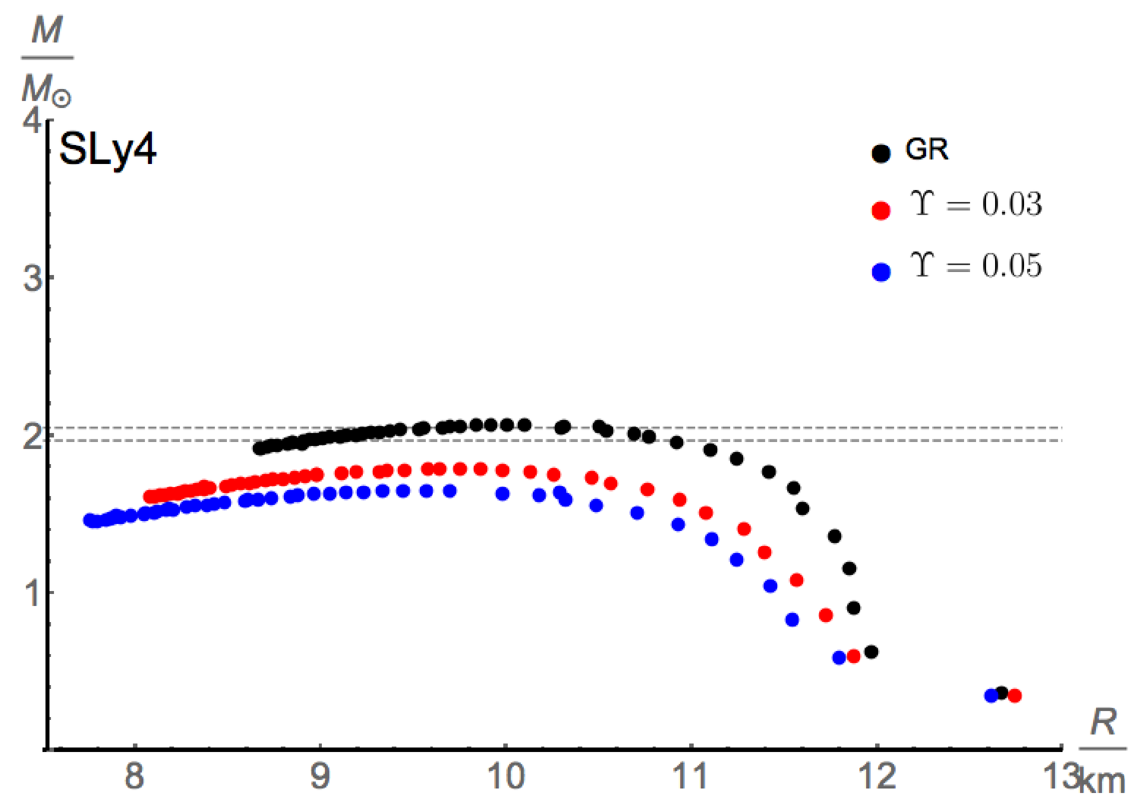
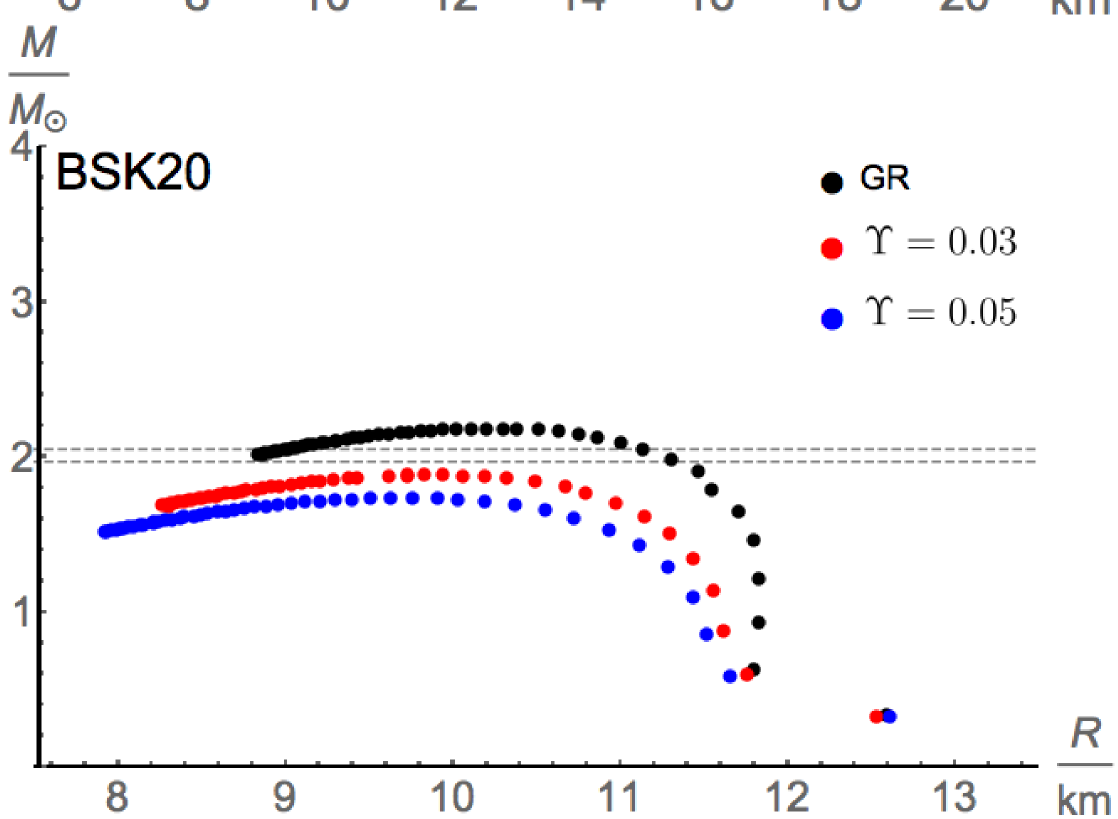
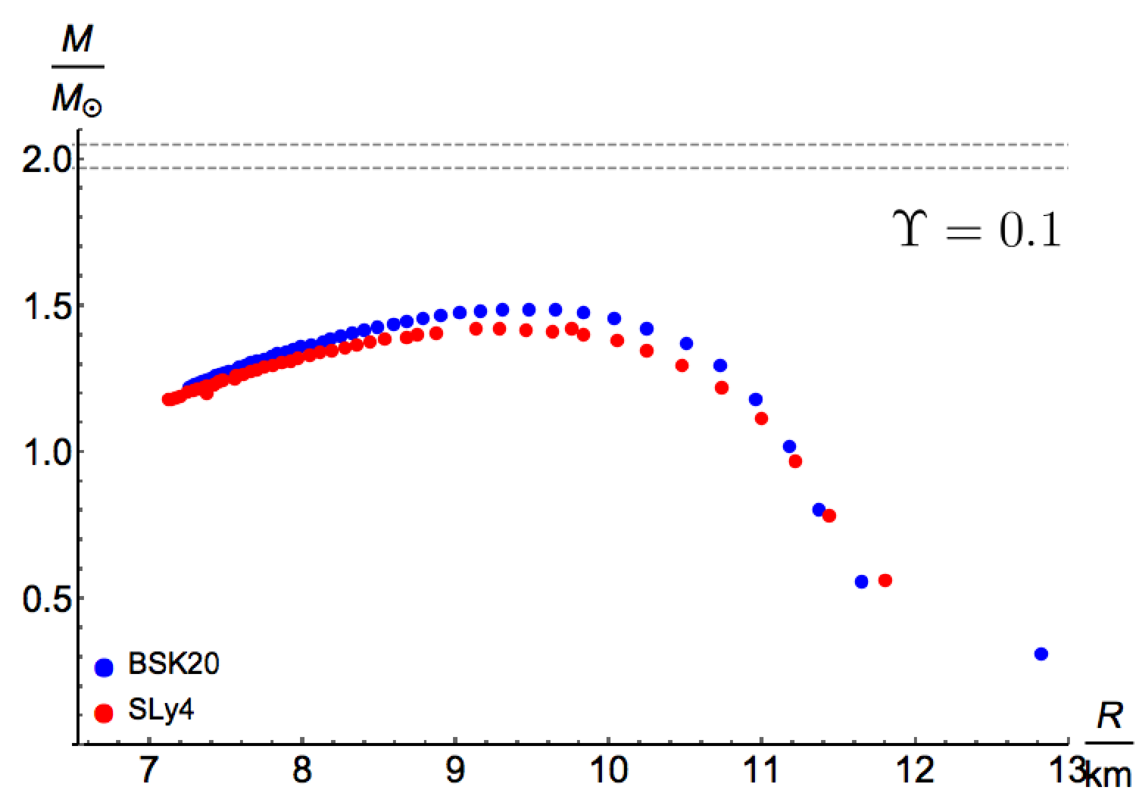
mass-radius relation

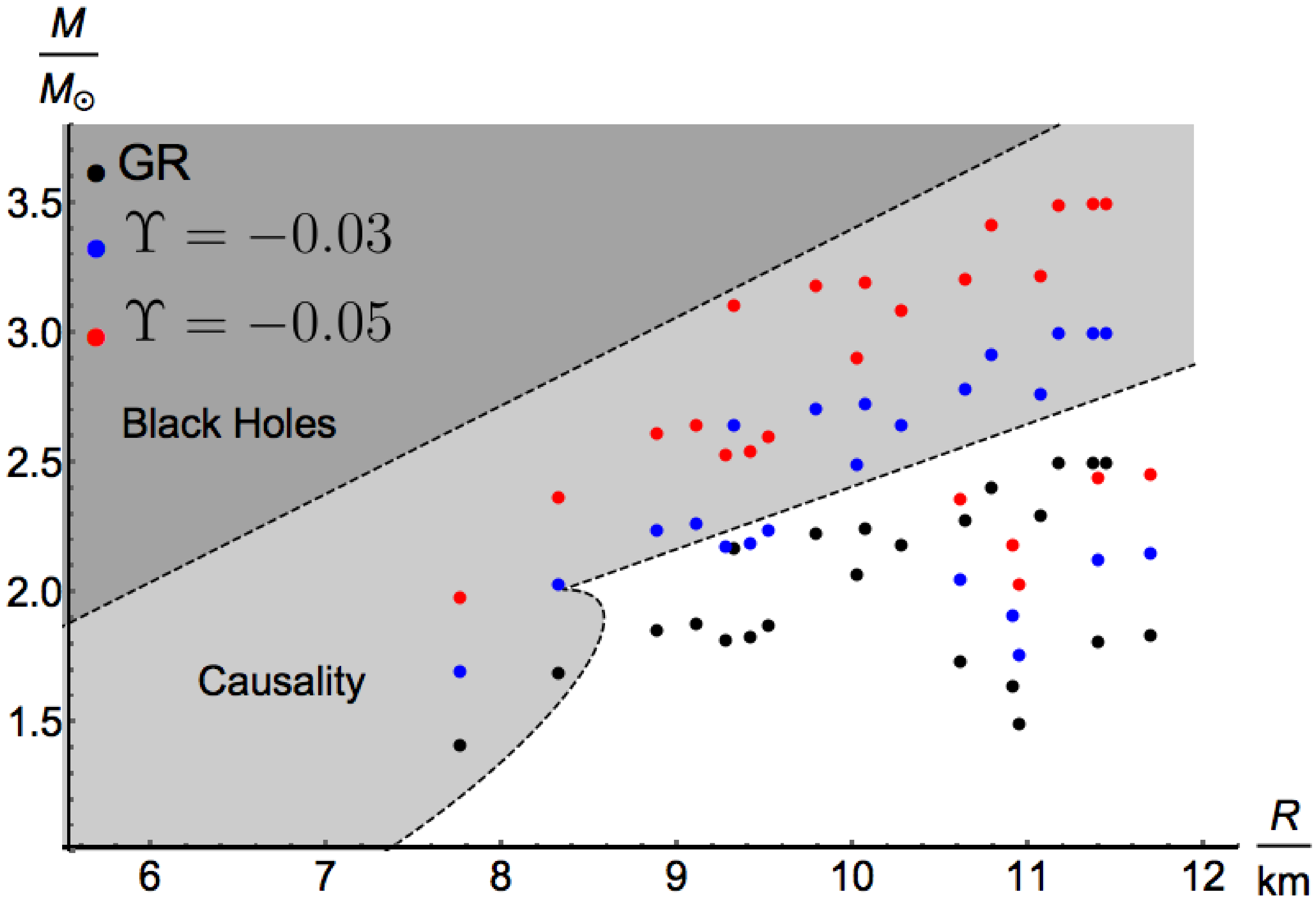








maximum mass and radius



32 equations of state

Rotating neutron stars

$$\mathrm{d}s^2 = -e^{\nu(r)}\,\mathrm{d}t^2 + e^{\lambda(r)}\,\mathrm{d}r^2 + r^2\,\mathrm{d}\Omega_2^2$$

$$\mathrm{d}s^2 = -e^{\nu(r)}\,\mathrm{d}t^2 + e^{\lambda(r)}\,\mathrm{d}r^2 + r^2\left(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2\right) - 2\varepsilon(\Omega - \omega(r))r^2\sin^2\theta\,\mathrm{d}t\,\mathrm{d}\phi.$$

geometry of space-time, containing a slowly rotating star with angular velocity Ω

+ one differential equation $\omega'' = \ldots$

Rotating neutron stars

$$
\begin{aligned}
\mathrm{d}s^2 = {} & -e^{\nu(r)}\,\mathrm{d}t^2 + e^{\lambda(r)}\,\mathrm{d}r^2 + r^2\left(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2\right) \\
& - 2\varepsilon(\Omega - \omega(r))r^2\sin^2\theta\,\mathrm{d}t\,\mathrm{d}\phi.
\end{aligned}
$$

Important star properties:

$$\bar{I} = \frac{Ic^2}{G^2M^3}$$ - dimensionless moment of inertia

$$\mathcal{C} = \frac{GM}{Rc^2}$$ - compactness

Universal relation in General Relativity

$$\bar{I} = a_1\mathcal{C}^{-1} + a_2\mathcal{C}^{-2} + a_3\mathcal{C}^{-3} + a_4\mathcal{C}^{-4}$$

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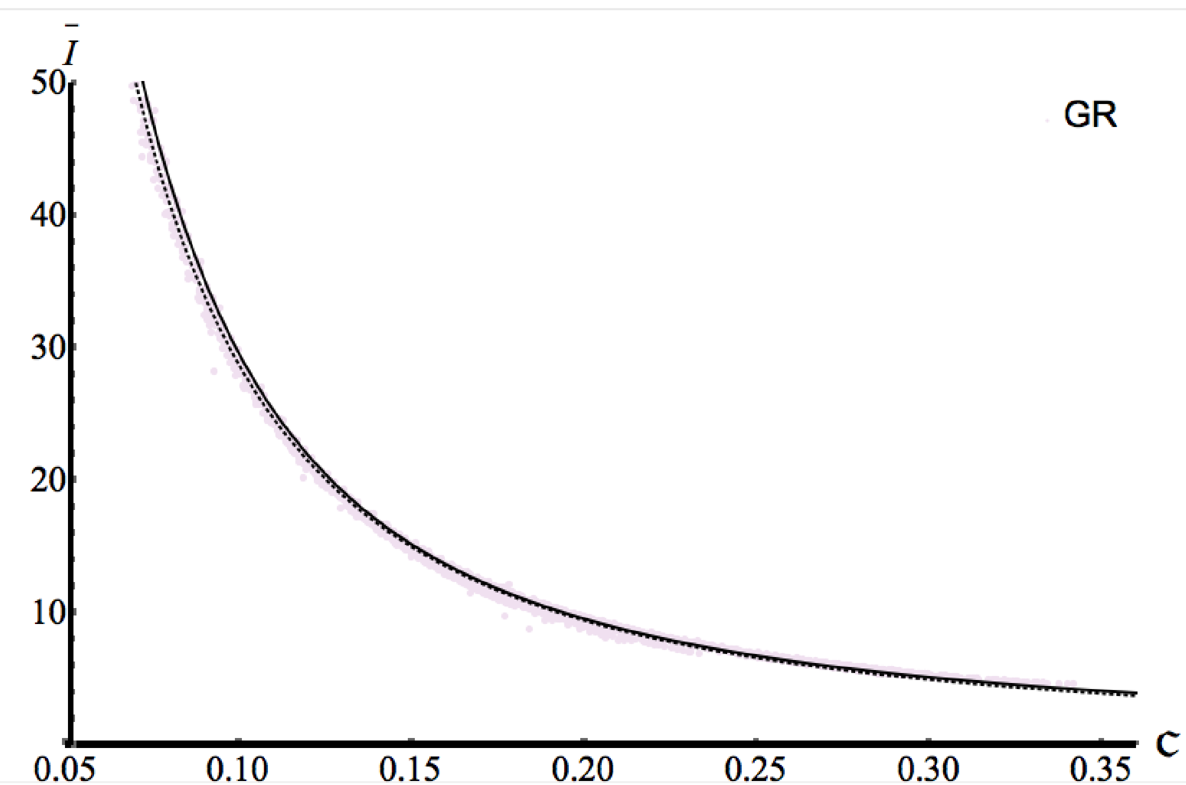
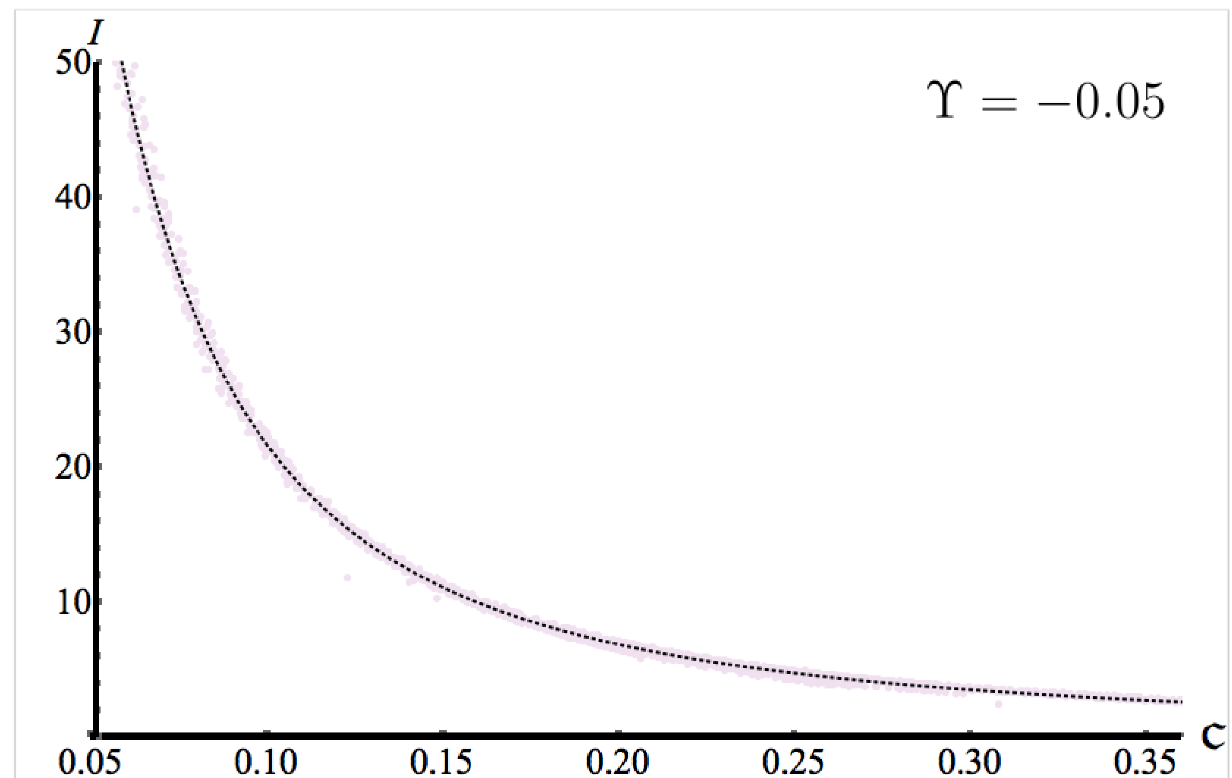
Rotating neutron stars

$\bar{I} - \mathcal{C}$ relation





Universal relation also holds in beyond Horndeski theory

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Hyperon and quark stars

Hyperons (baryons with non-zero strangeness) should appear at high densities

Hyperons contribute to the structure of neutron stars (in the core)

Equations of state which include hyperons are much *softer* than pure nucleonic ones

The resulting maximum mass of NSs is significantly lower than nucleonic NSs

“Hyperon puzzle ”

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Hyperon and quark stars

- + Quark stars
- Hyperon stars are unstable above a threshold mass
- A first-order phase transition occurs to deconfined quark matter
- EoS becomes stiffer => stars with similar mass but smaller radii

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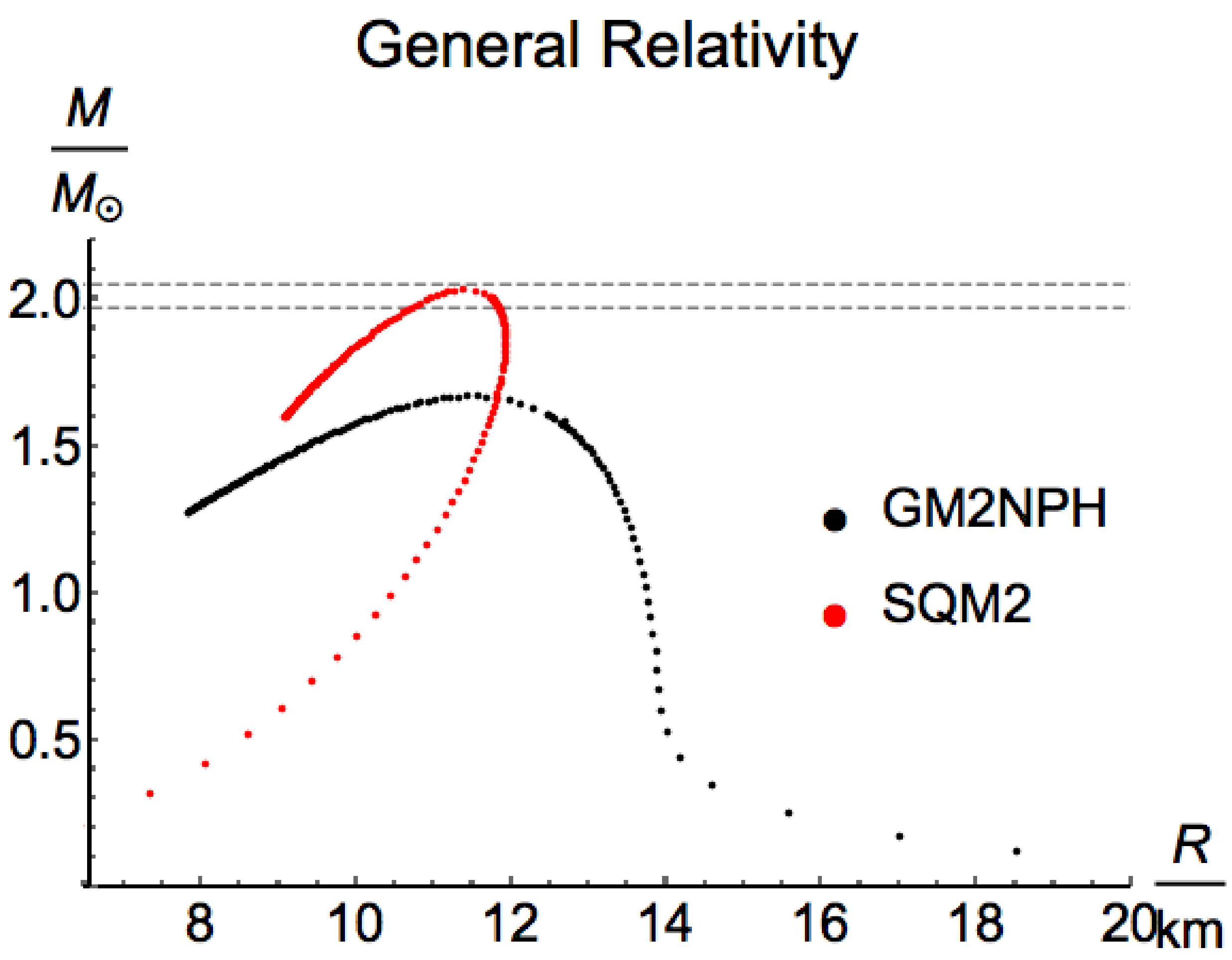
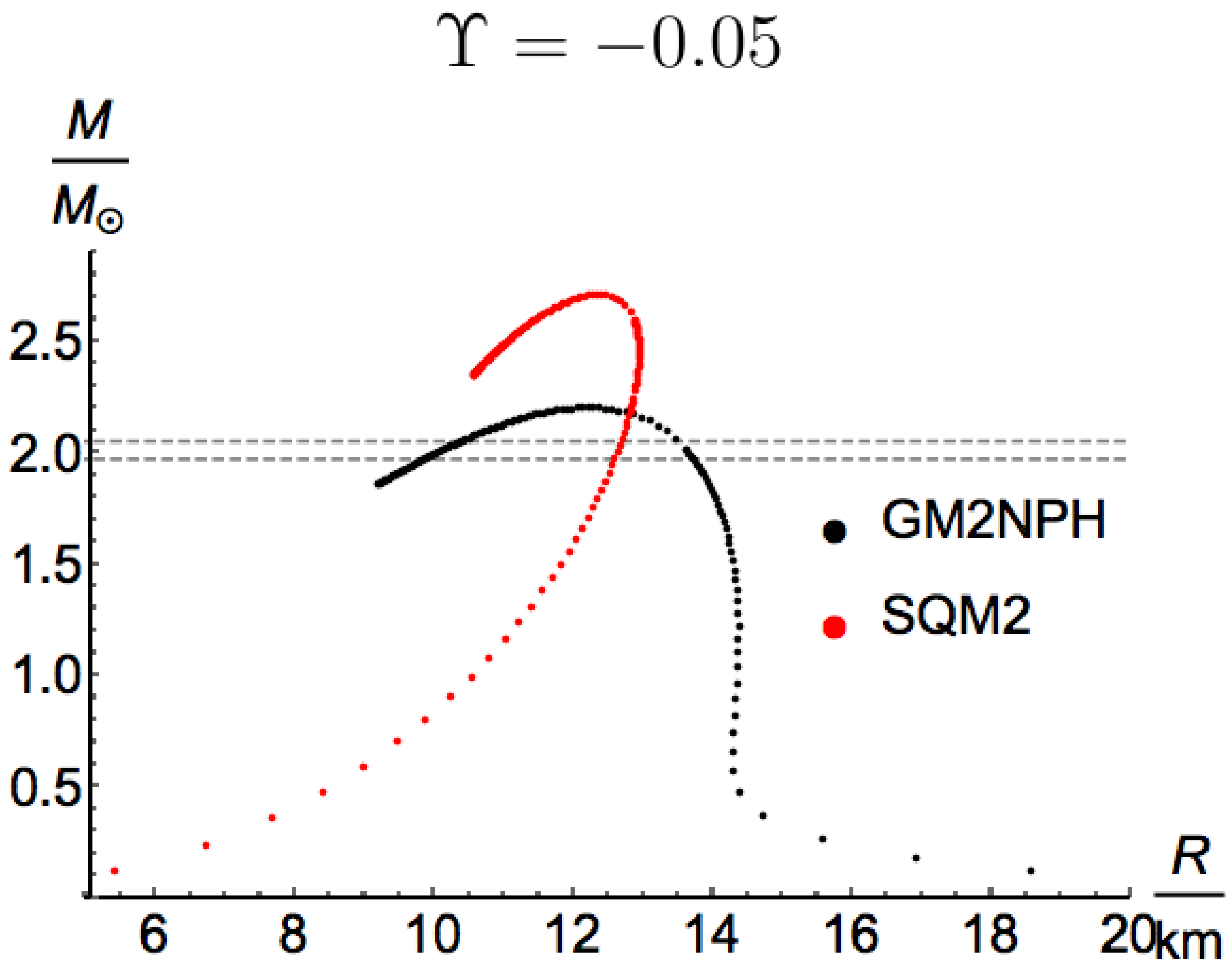
Hyperon and quark stars





- Hyperon stars
- Quarks stars

Conclusions

- Beyond Horndeski theories can give masses larger than 2 solar masses
- The $\bar{I} - \mathcal{C}$ persists in bH theories. There is a difference from GR relation, which provides a robust state-independent of the theory
- The hyperon puzzle is solved in bH theories