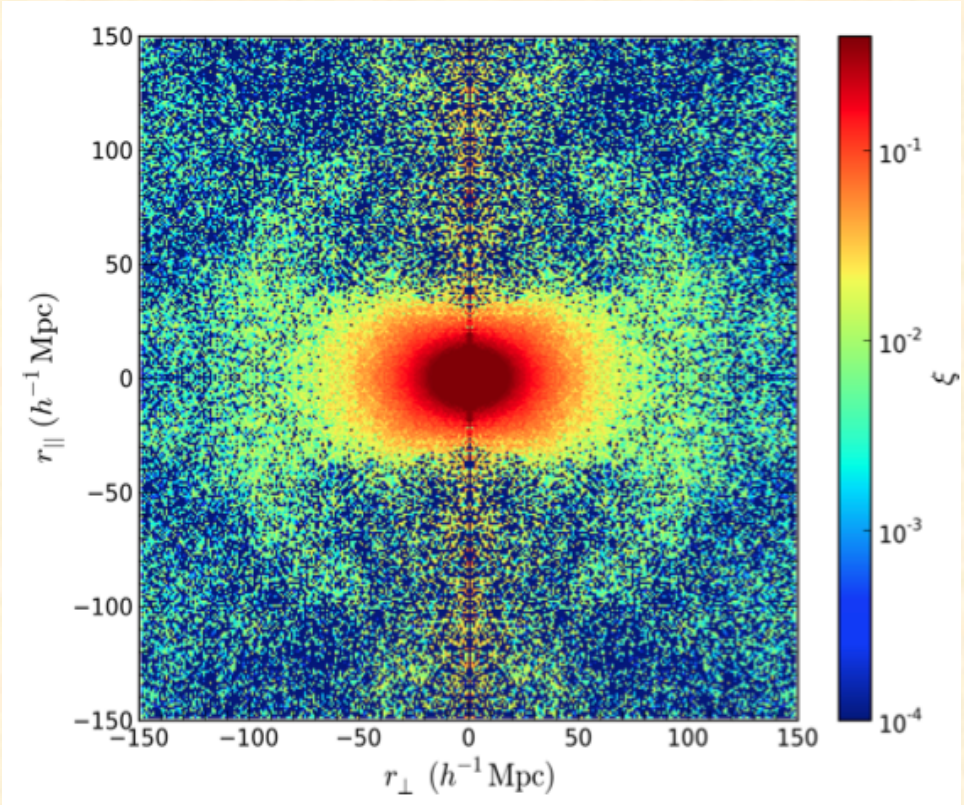
Redshift-Space Distortions 101

Etienne Burtin, CEA-Saclay

(originally prepared for SDSS Collaboration Meeting
Madison, Wisconsin – June 29, 2016)



2-Pts correlation function

BOSS – LRG

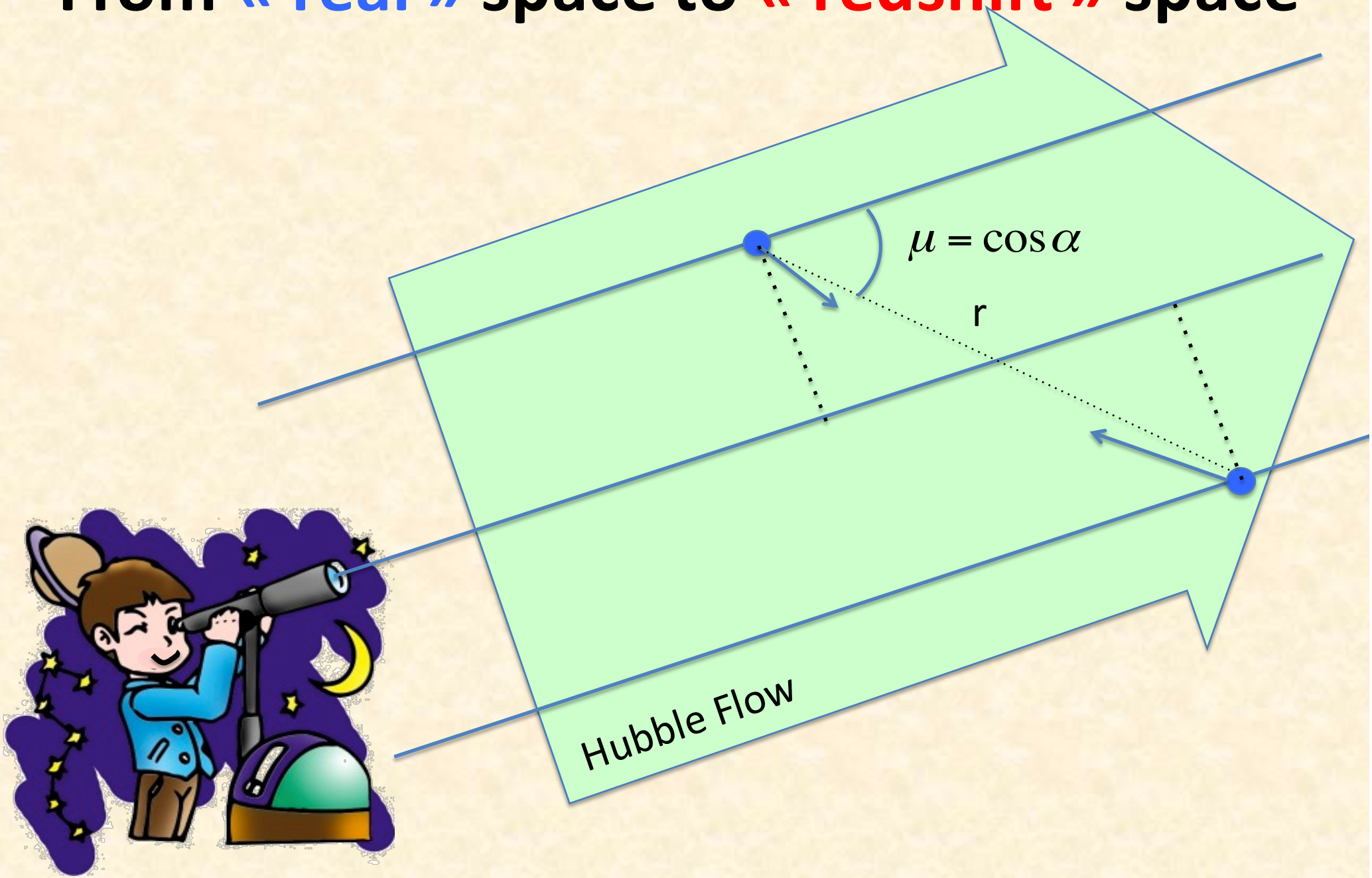
Samushia et al. (2014)

Why is it like that ?

What can we learn for it ?

Kaiser (1987)
Hamilton (1995)

From « real » space to « redshift » space



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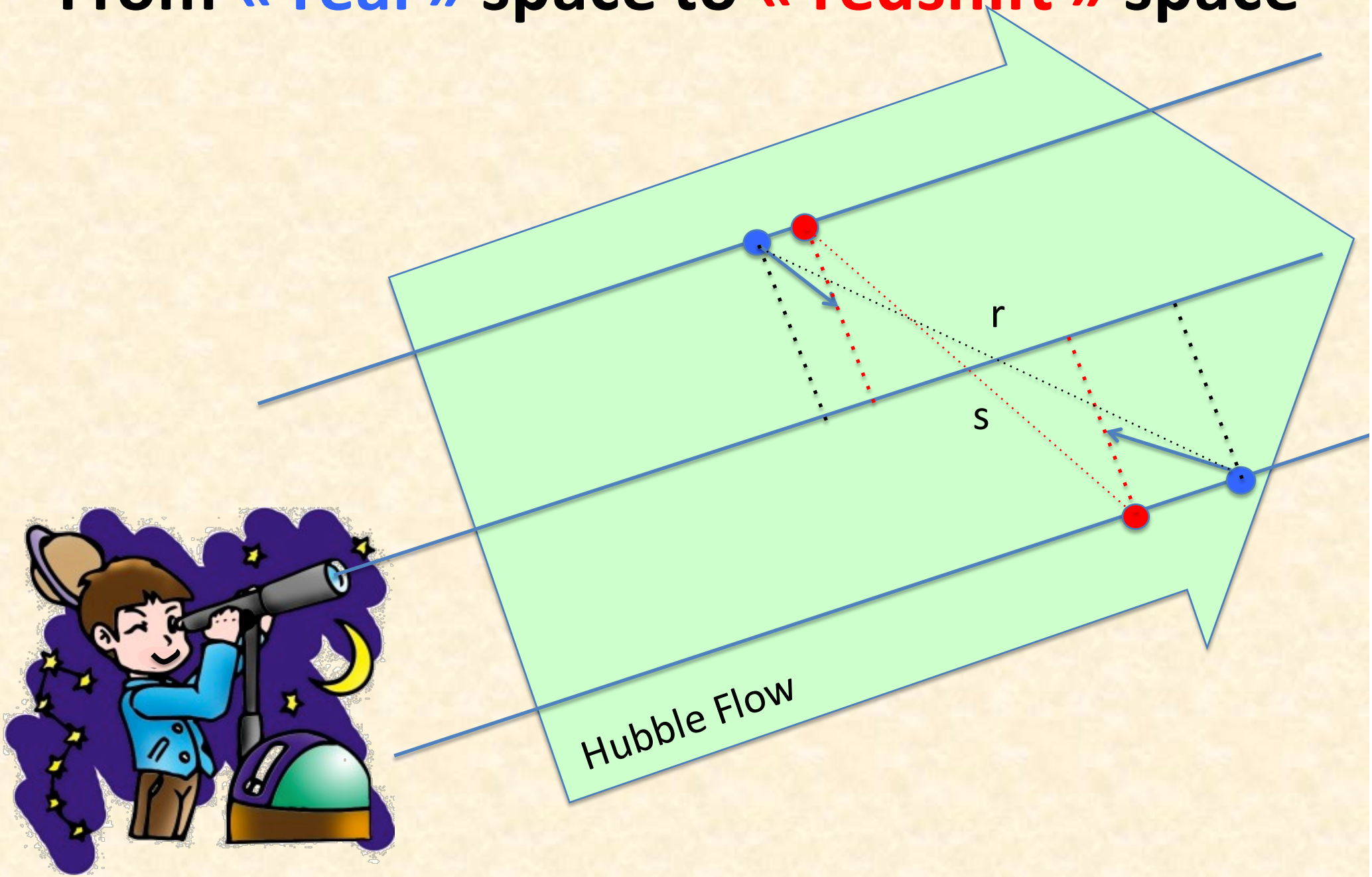
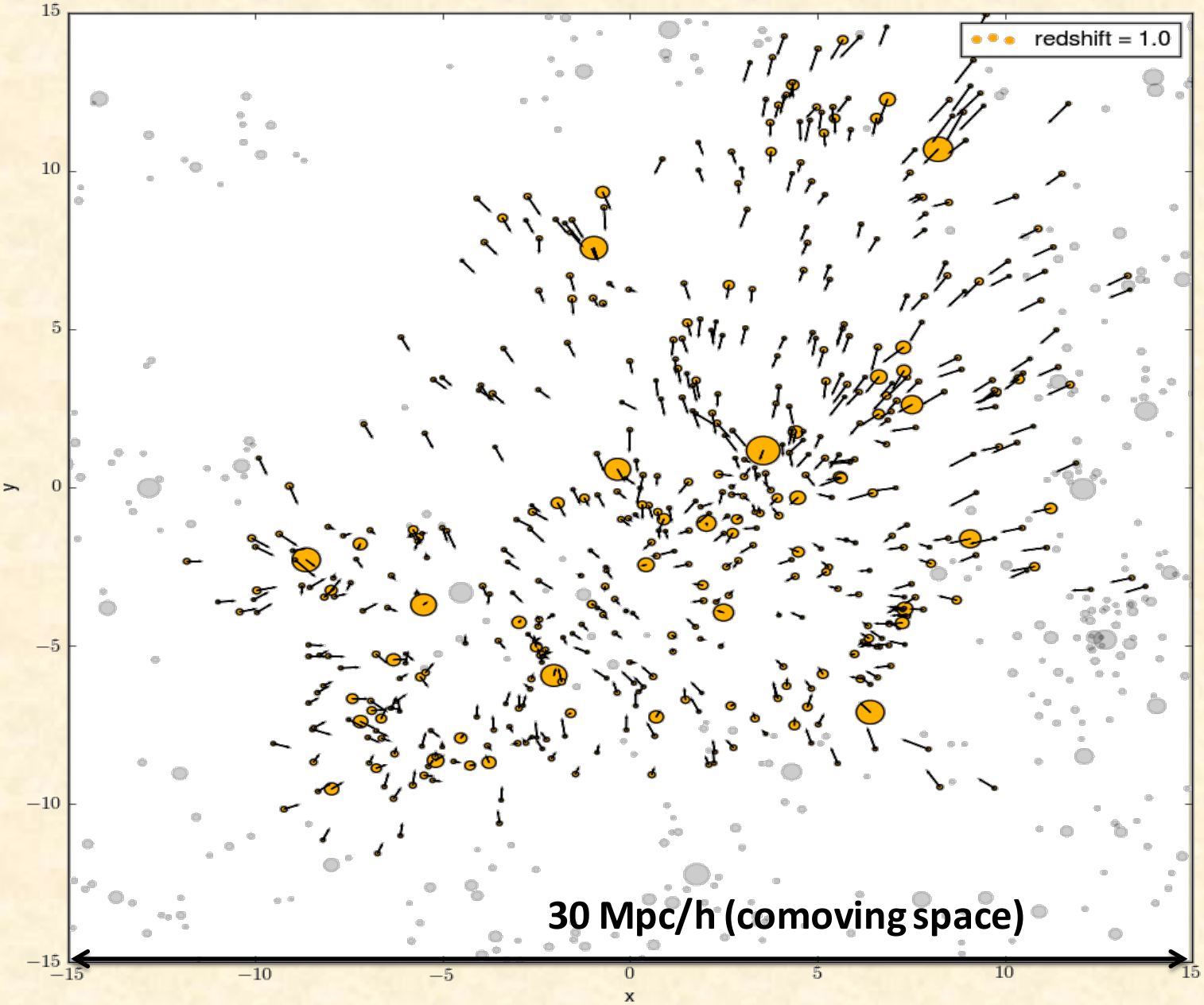
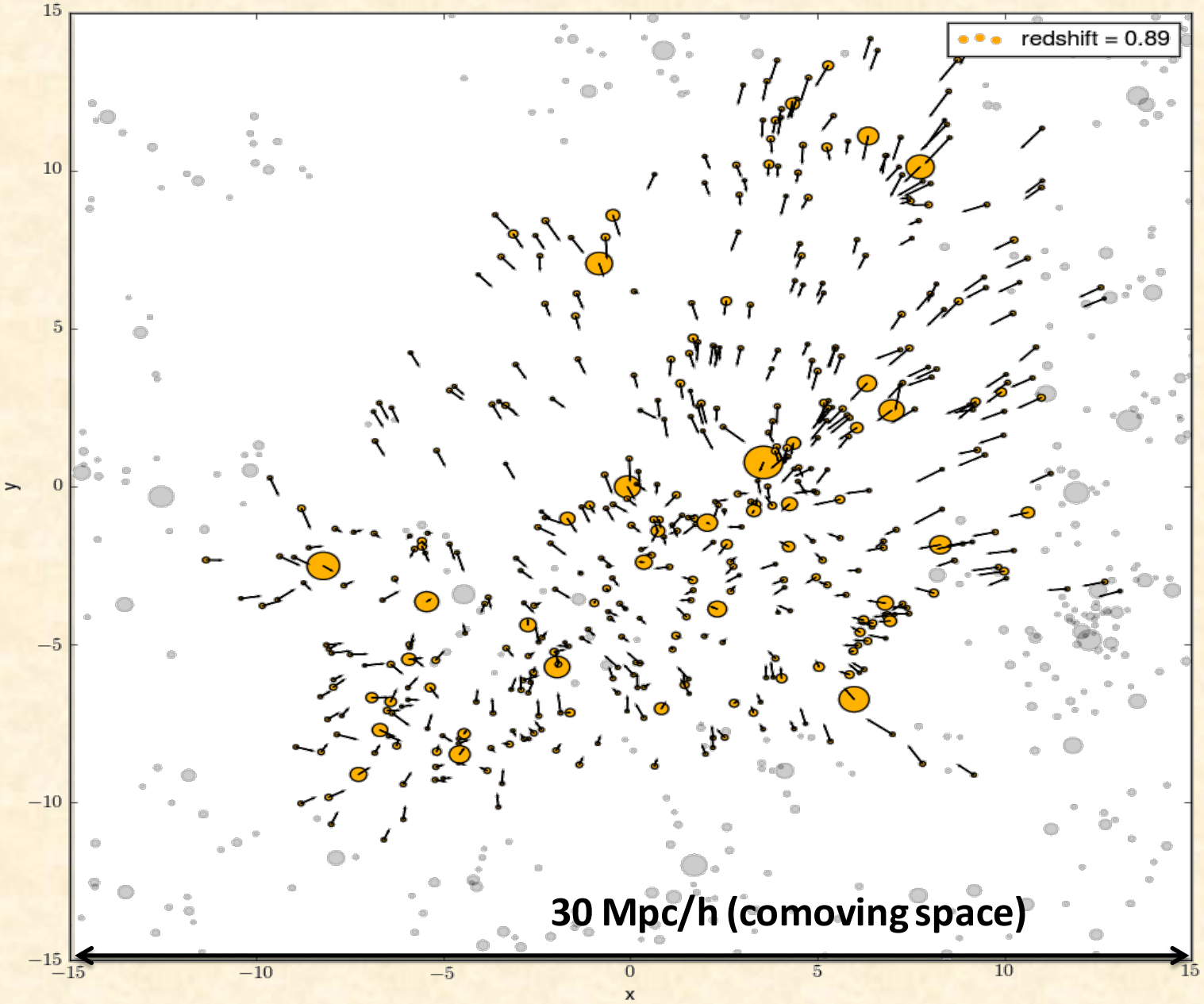


Coherent Infall



From MultiDark
N-Body simulation
Dark Matter only
$M_{halo}>10^{12}M_{sun}$
Klypin et al. (2014)

Coherent Infall



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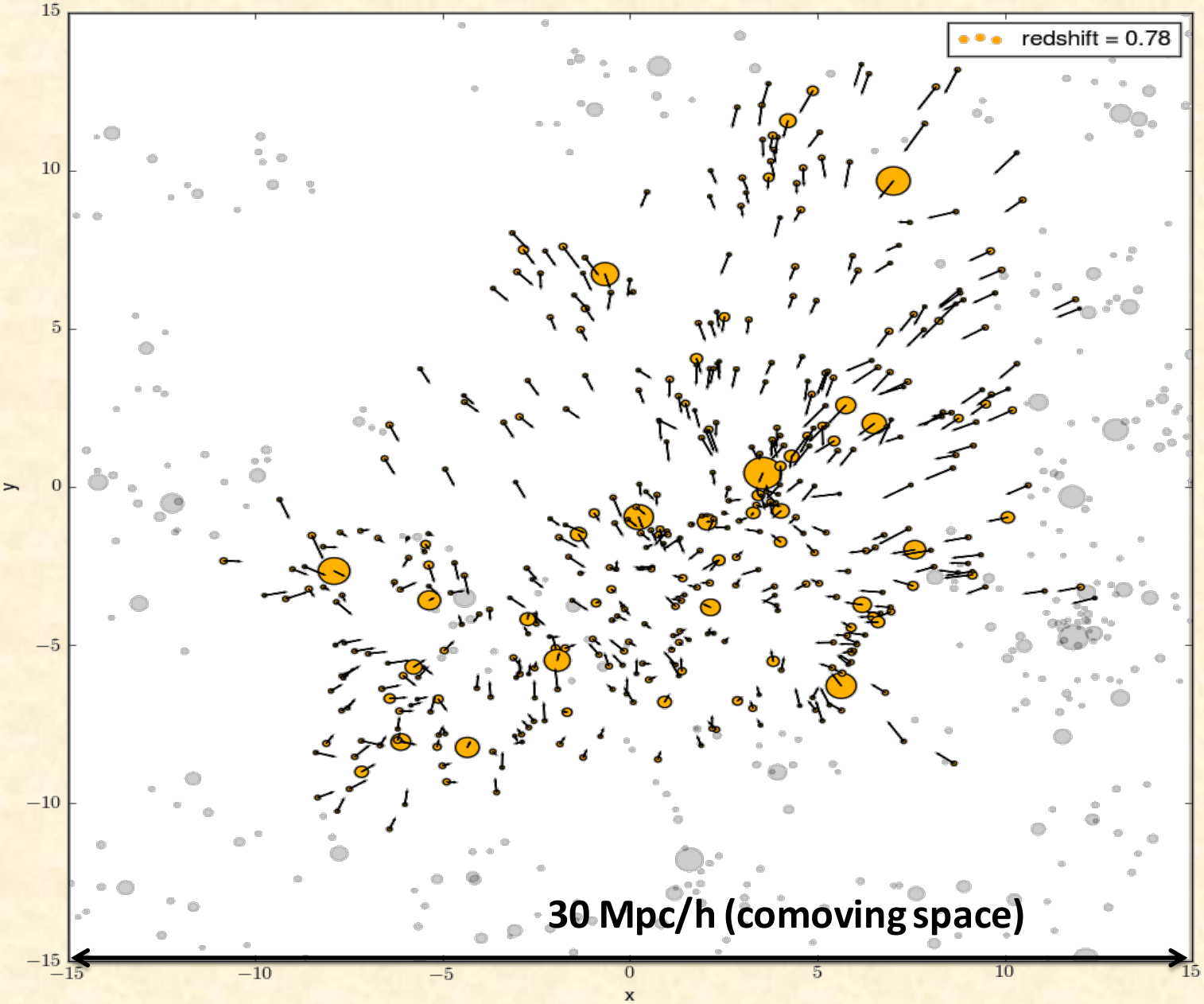

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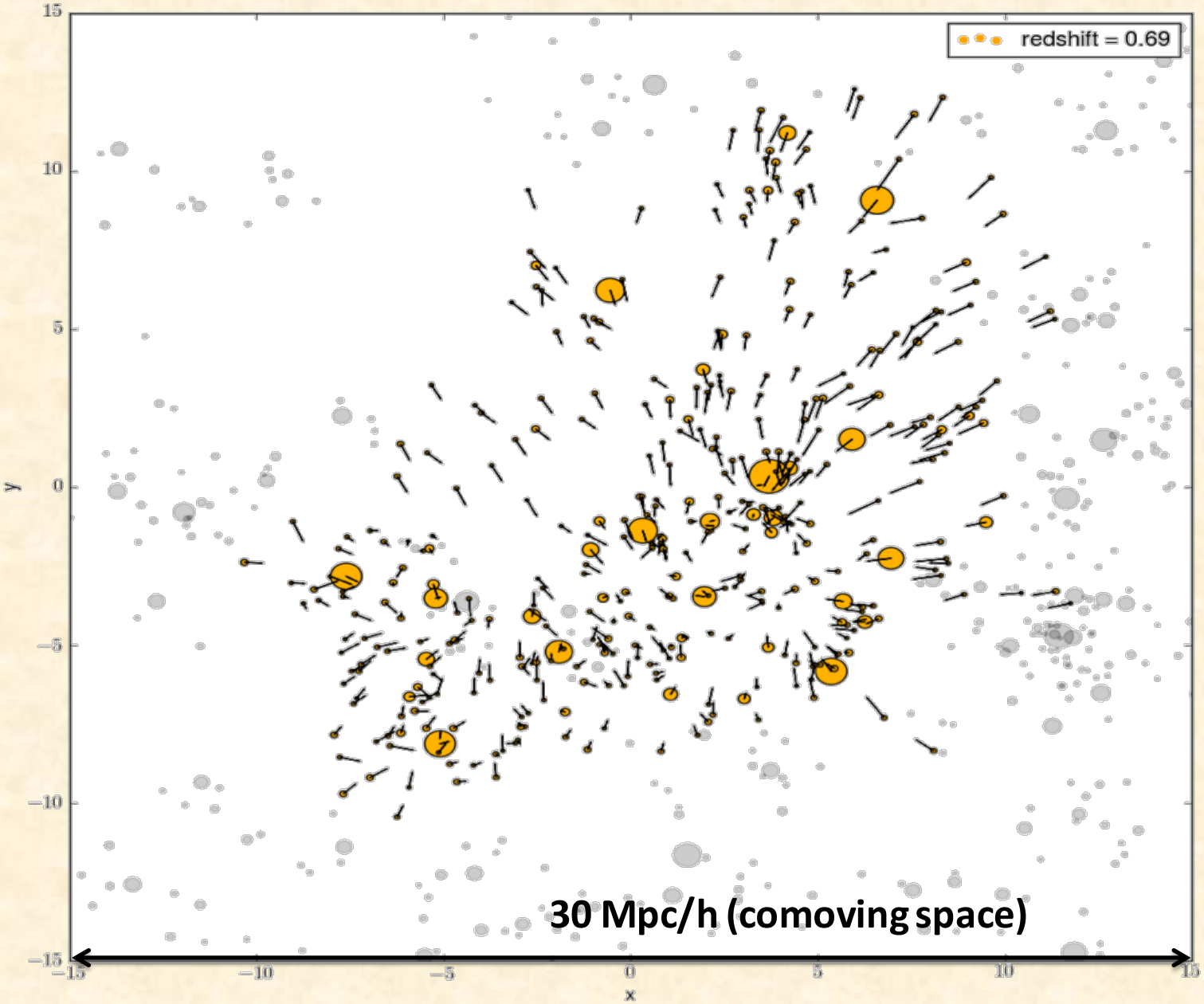

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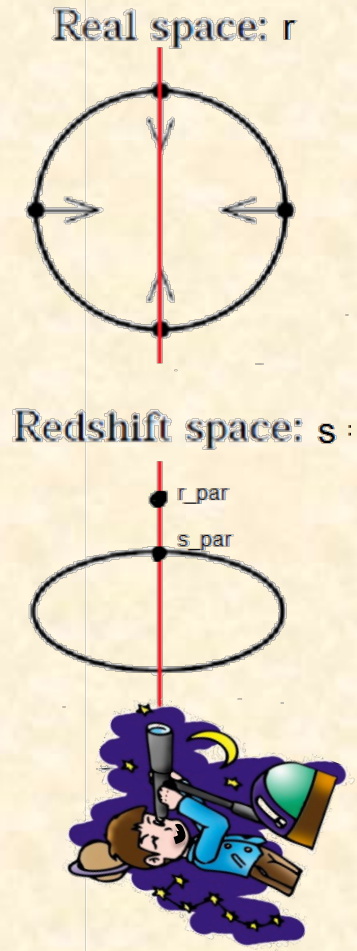

Growth of structures in linear theory

Density perturbation : $\delta = \frac{\Delta\rho}{\rho} << 1$

Evolution of δ described by:

- Mass conservation (continuity equation)
- Momentum conservation (Euler equation)
- Matter-gravitational potential relation (Poisson equation)

$\Rightarrow$ Linearized equation gives :

$$\ddot{\delta} + 2H\dot{\delta} - 4\pi G\bar{\rho}\delta = 0$$

$\Rightarrow$ 2 solutions

$$\delta(t) = \delta_+ D_+(t) + \delta_- D_-(t)$$

Linear growth rate of structures f :

$$f(a) \equiv \frac{d\ln(D_+(a))}{d\ln(a)}$$

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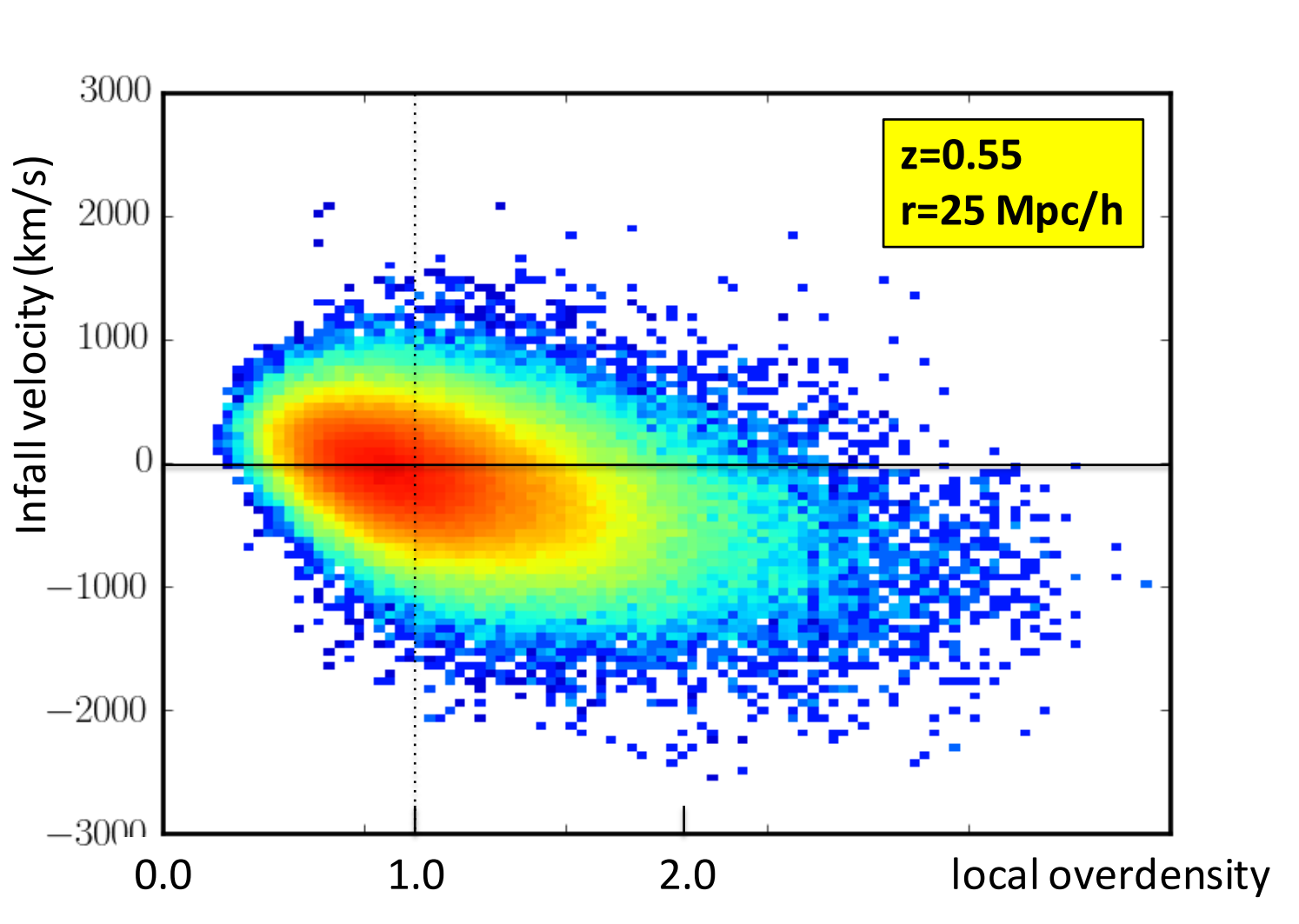
Linked to the divergence of the velocity field :

$$\nabla \cdot v = -f\delta$$

in General Relativity :

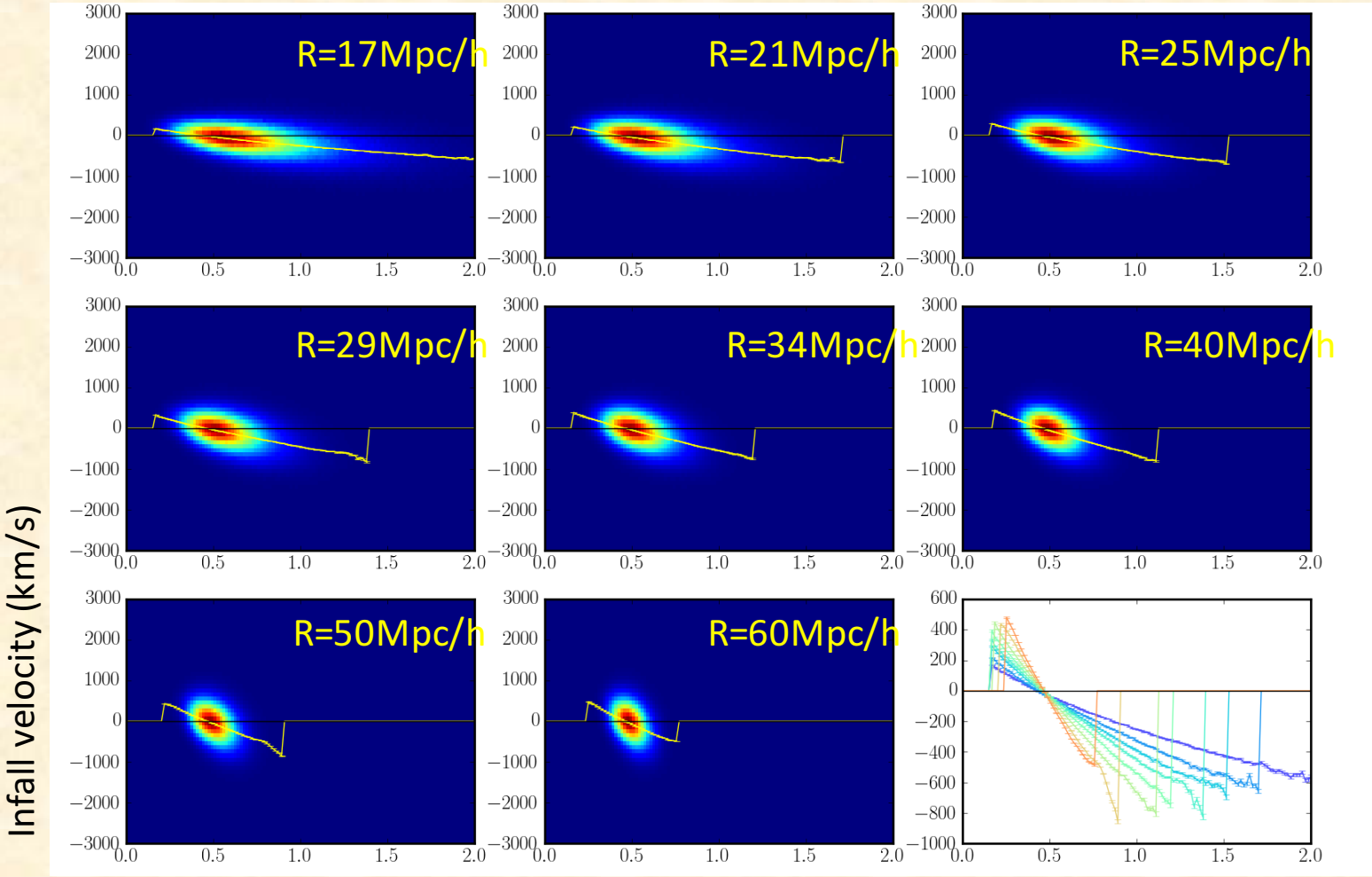
$$f(a) = \Omega_m(a)^{\gamma=0.55}$$

Pair-wise velocity distribution

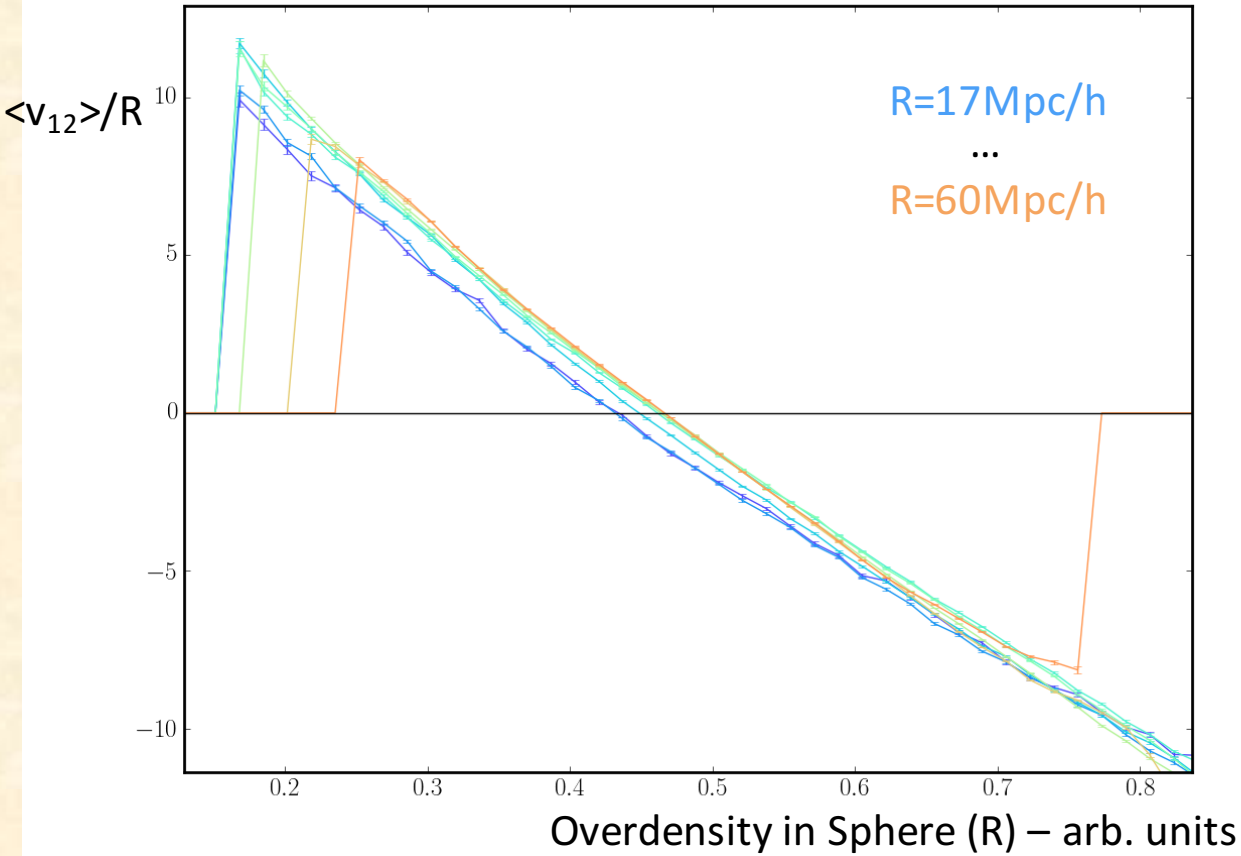


Scoccimaro (2004)
Tinker (2007)

Infall velocity vs local overdensity

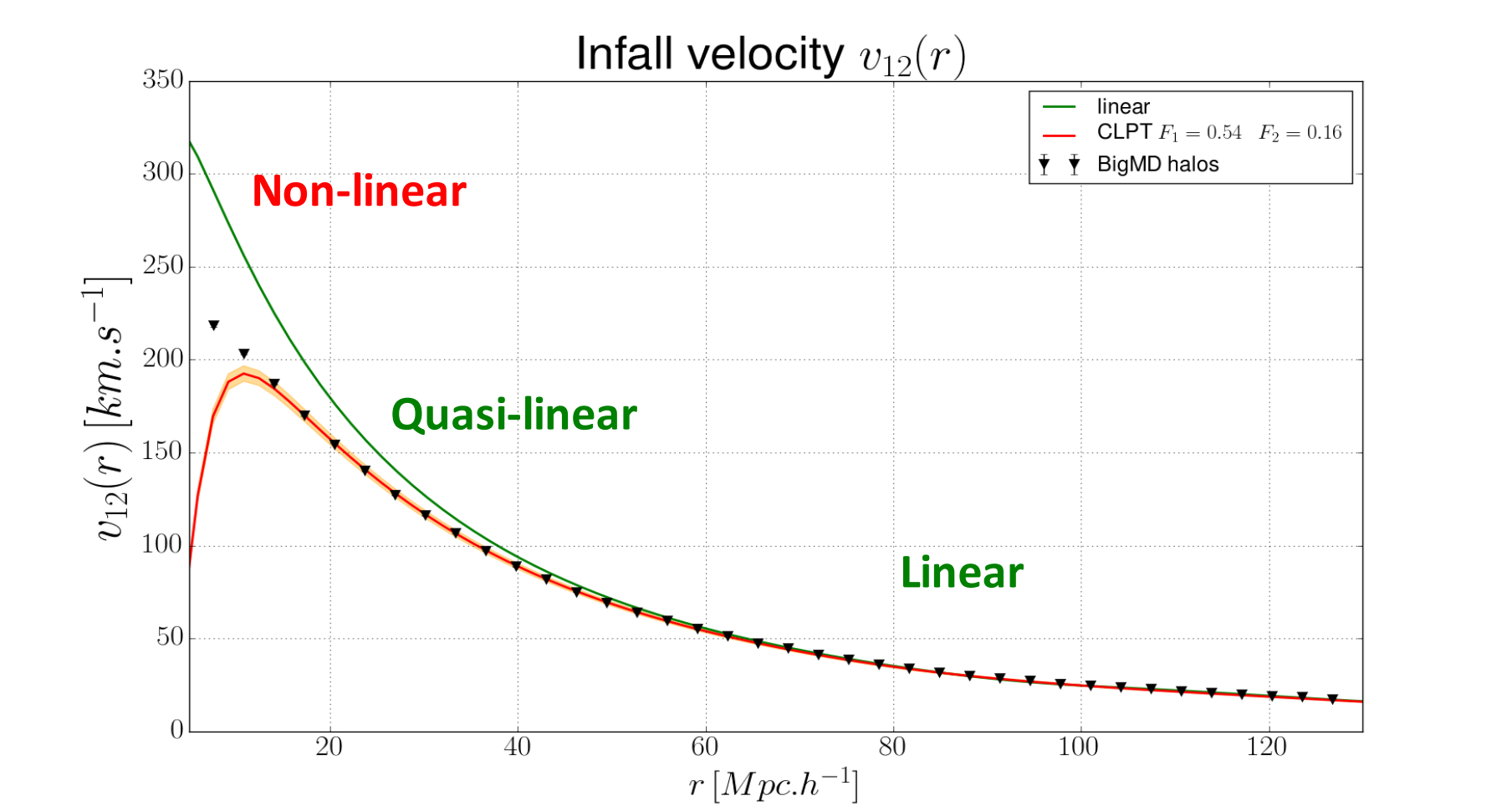


Velocities normalized by scale R



In Peebles(1980), for Clusters, Spherical collapse:
$v/R=-1/3\ H * f * \delta$

Pair-wise velocity distribution



$$v_{12} = -\frac{fb}{\pi^2}\int dk\, k\, P(k)\, j_1(kr)$$

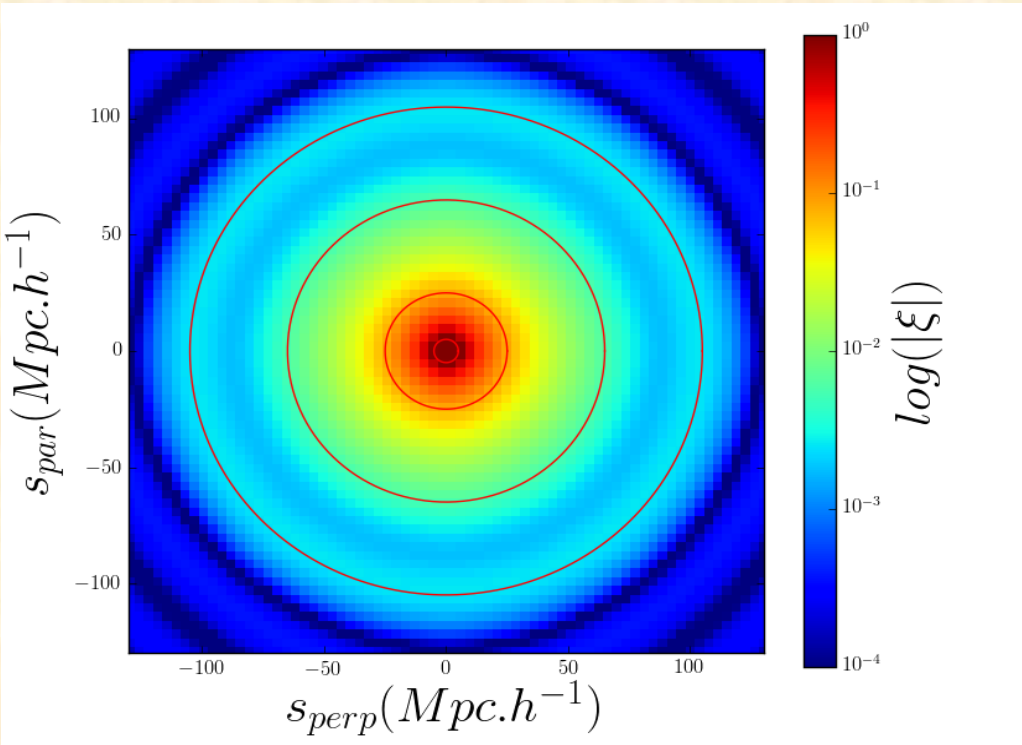
Anisotropic 2-Pts correlation function

Gaussian Streaming model :

Real space (model)

Redshift space

$$1+\xi(s_{\parallel},s_{\perp}) = \int dr_{\parallel}\left[1+\xi(r)\right]G(s_{\parallel}-r_{\parallel},\mu\cdot v_{12}(r),\sigma_{12}(r,\mu))$$



Fisher (1995)
Reid & White (2011)

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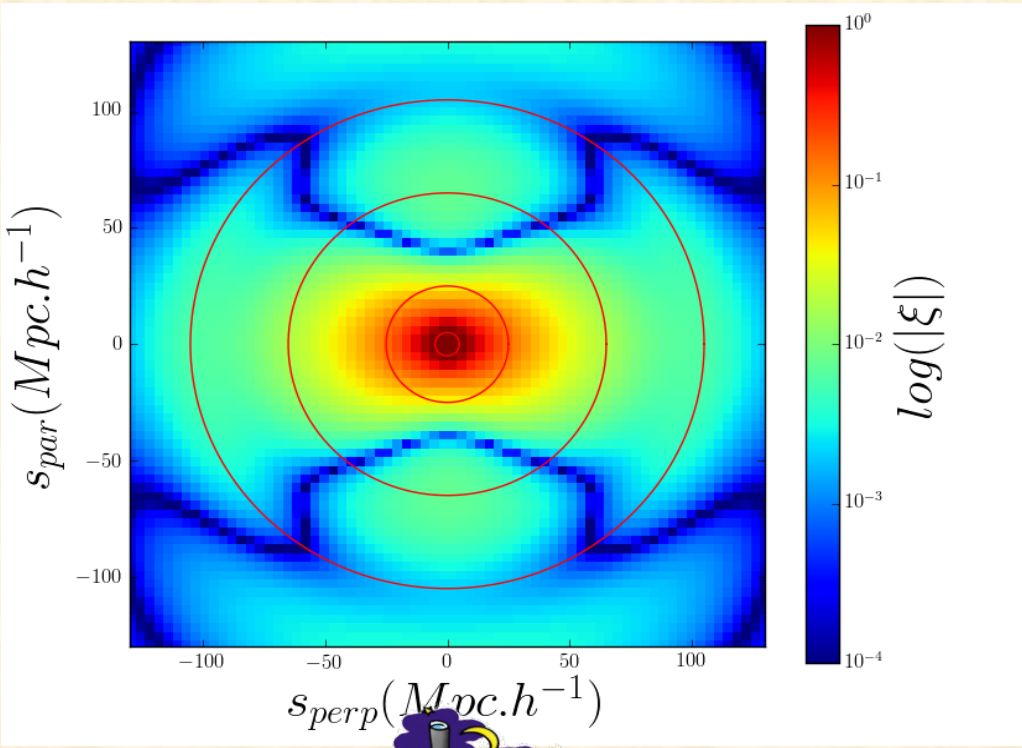

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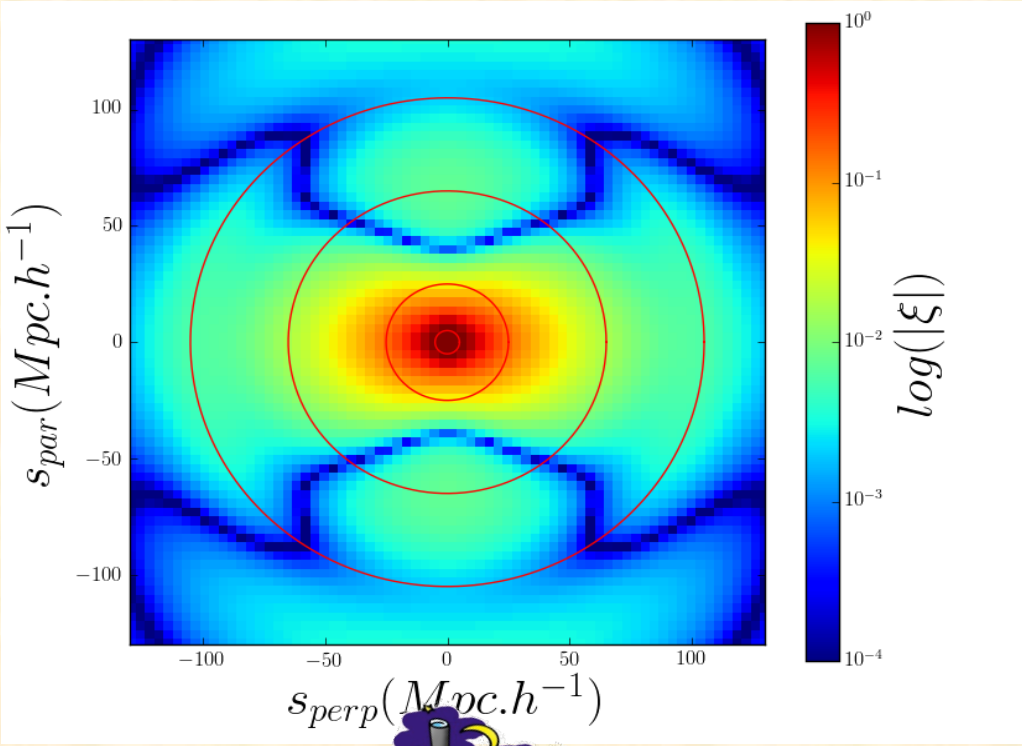
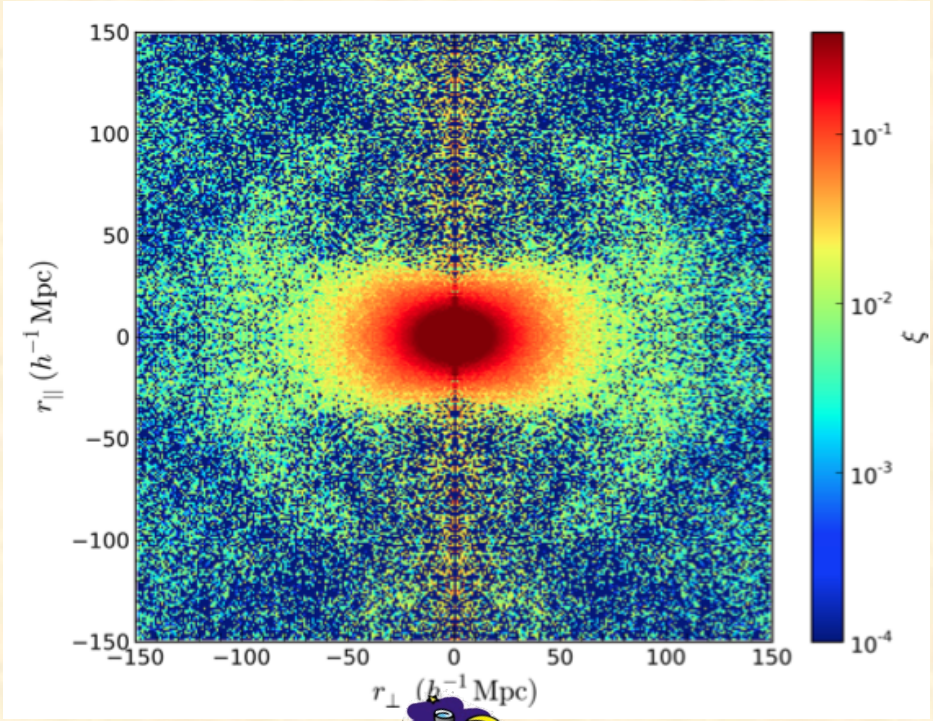




BOSS – LRG

Samushia et al. (2014)

Legendre multipoles decomposition

$$\xi(r) = \frac{2l+1}{2}\int \xi(r,\mu)L_l(\mu)d\mu$$

Only *l*=0,2,4 are non zero in linear theory

Biased tracers of matter :

$$\xi_{tr}(r) = b^2\xi_m(r)$$

Monopole (l=0):

$$\xi_{tr,0}(s) = (b^2 + \tfrac{2}{3}bf + \tfrac{1}{5}f^2)\xi_m(r)$$

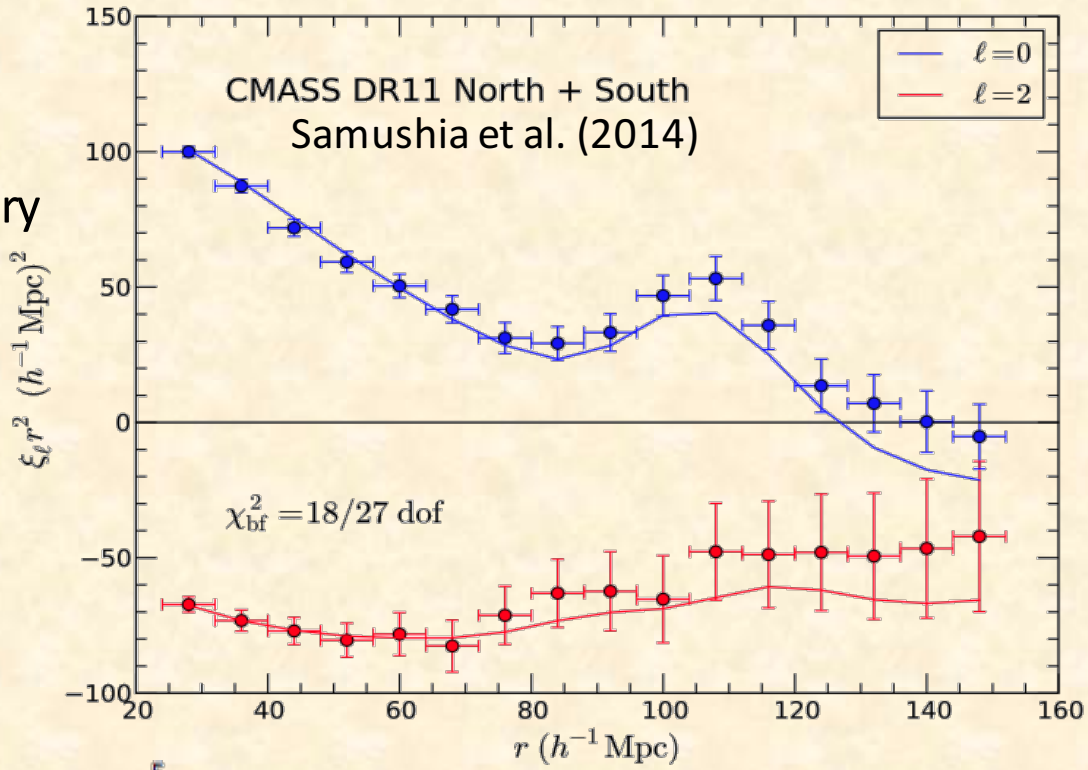
Quadrupole (l=2):

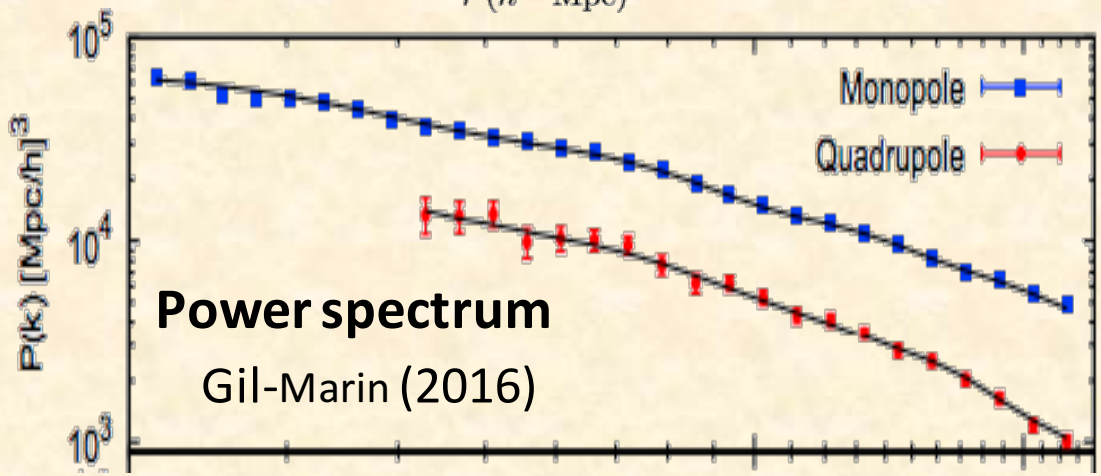
$$\xi_{tr,2}(s) = (\tfrac{4}{3}bf + \tfrac{4}{7}f^2)\xi_m'(r)$$

Hexadecapole (l=4):

$$\xi_{tr,4}(s) = \tfrac{8}{35}f^2\xi_m''(r)$$

Percival & White (2008)



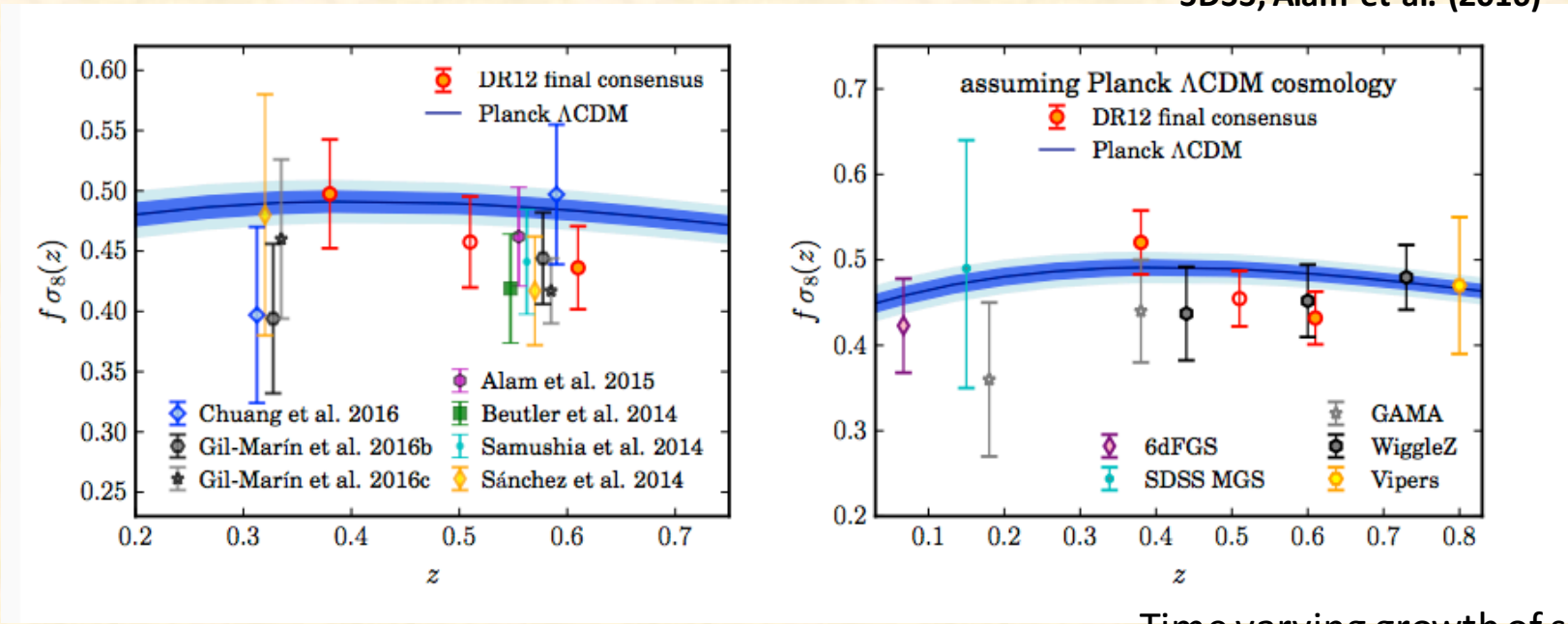


Power spectrum

Gil-Marin (2016)

Growth of structures

SDSS, Alam et al. (2016)

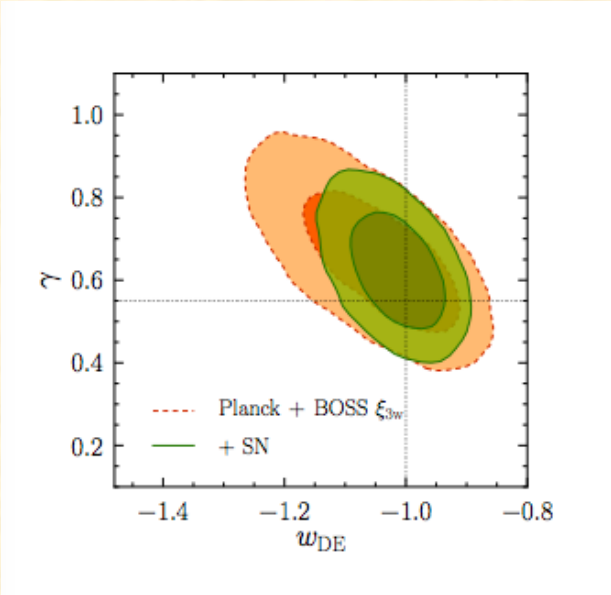


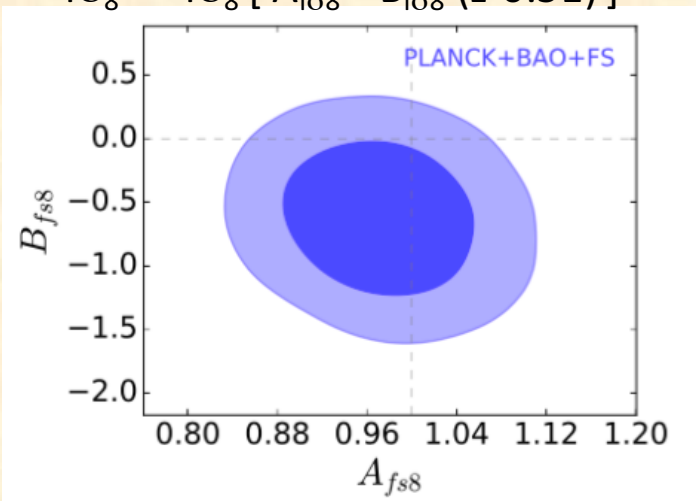
Time varying growth of structures :

$f\sigma_8 \rightarrow f\sigma_8 [A_{f\sigma 8} + B_{f\sigma 8} (z-0.51)]$

Sanchez et al. (2016)

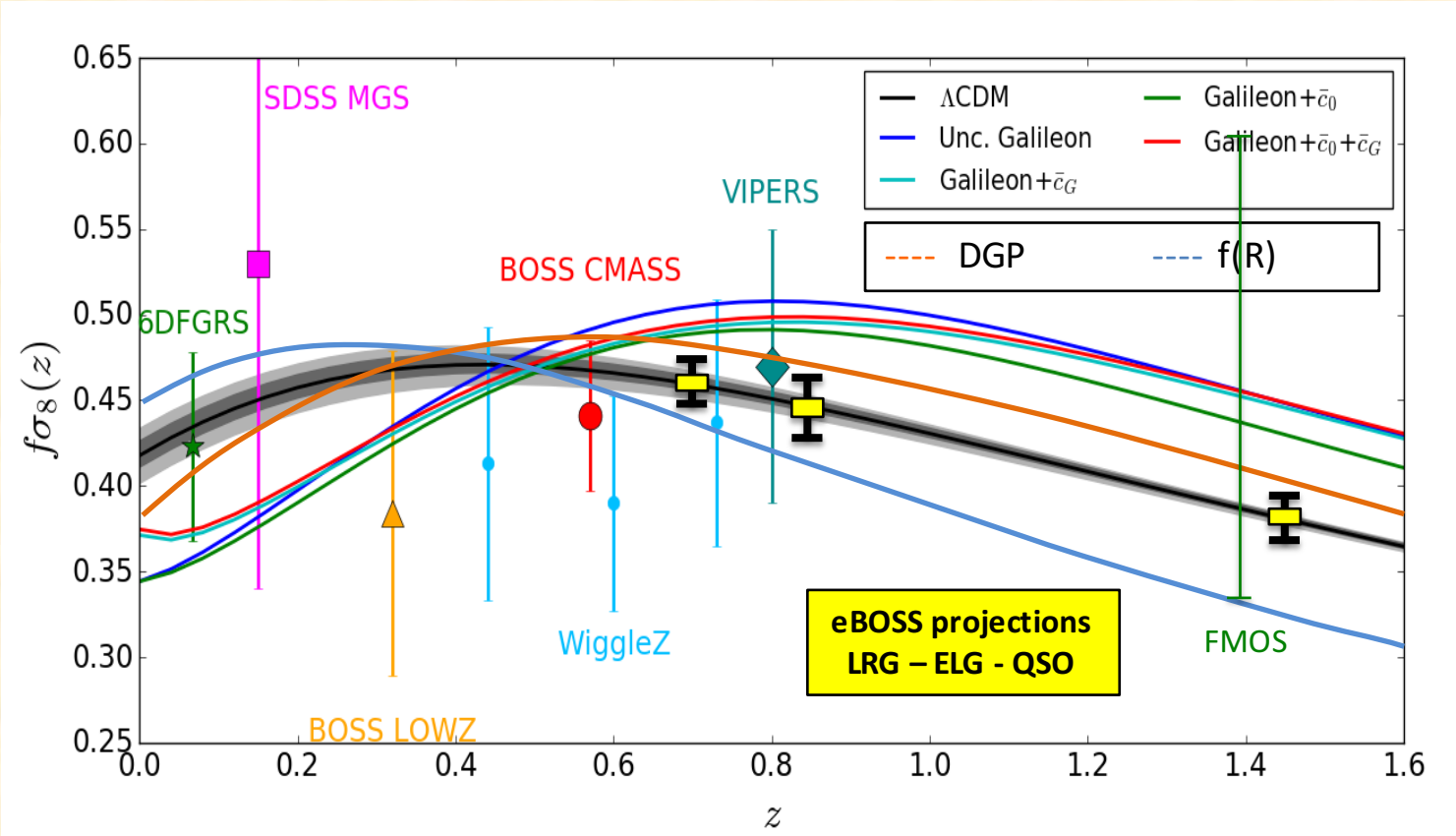
$f=\Omega_m(z)^\gamma$





Sensitivity of f*σ_8 measurements

σ_8 : Normalisation of matter density fluctuations



Galileon model from J. Neveu et al., (1605.02627)

Sensitivity of f*σ_8 measurements

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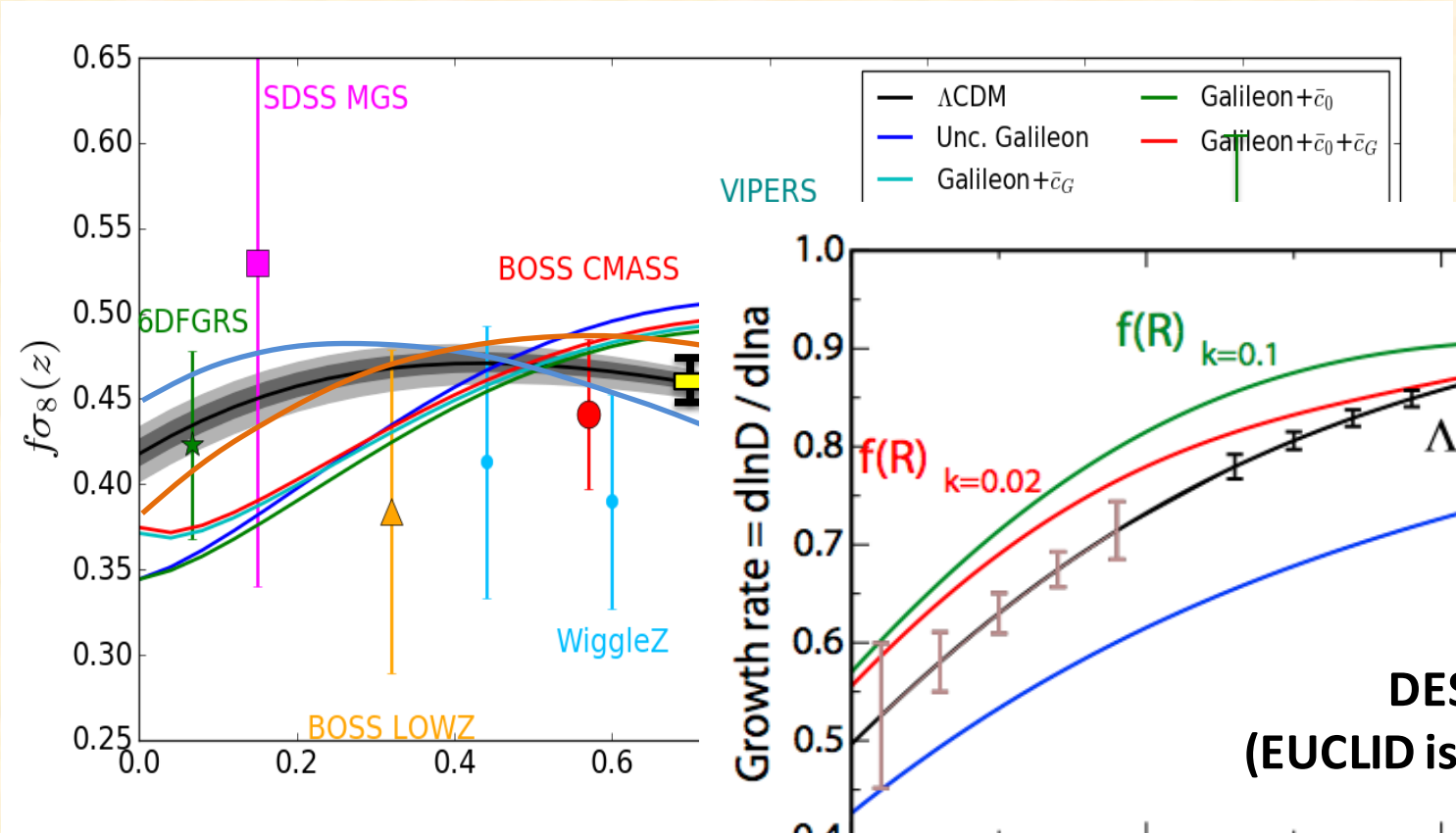
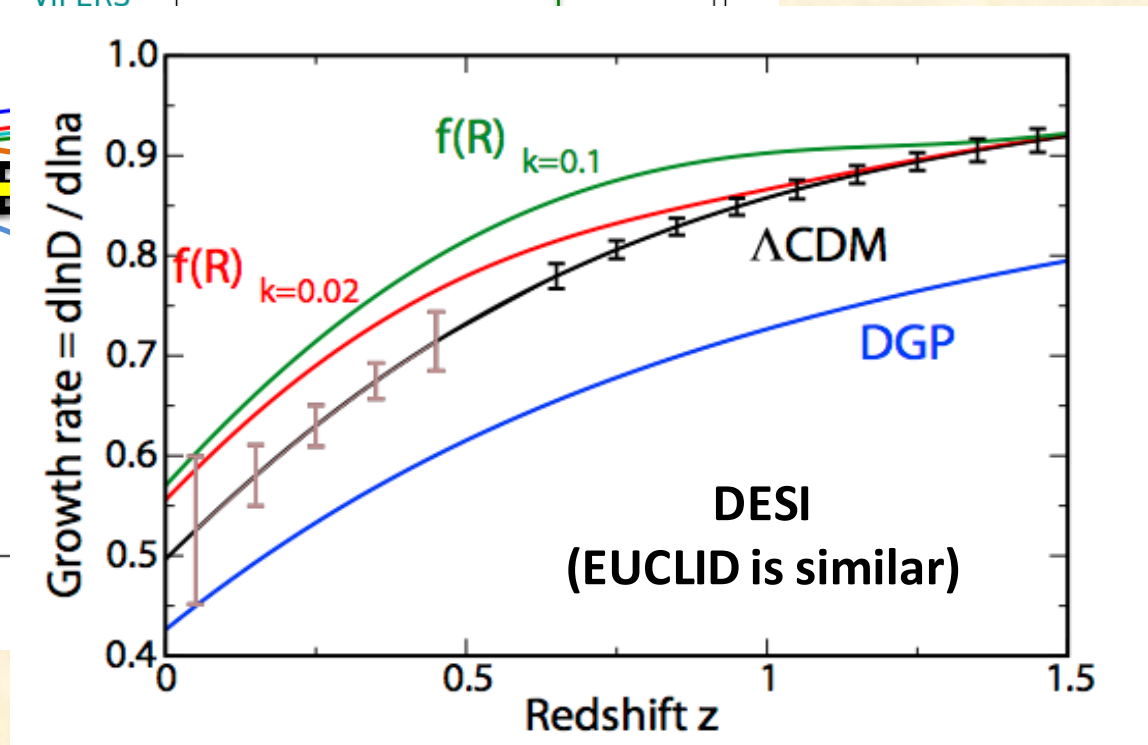
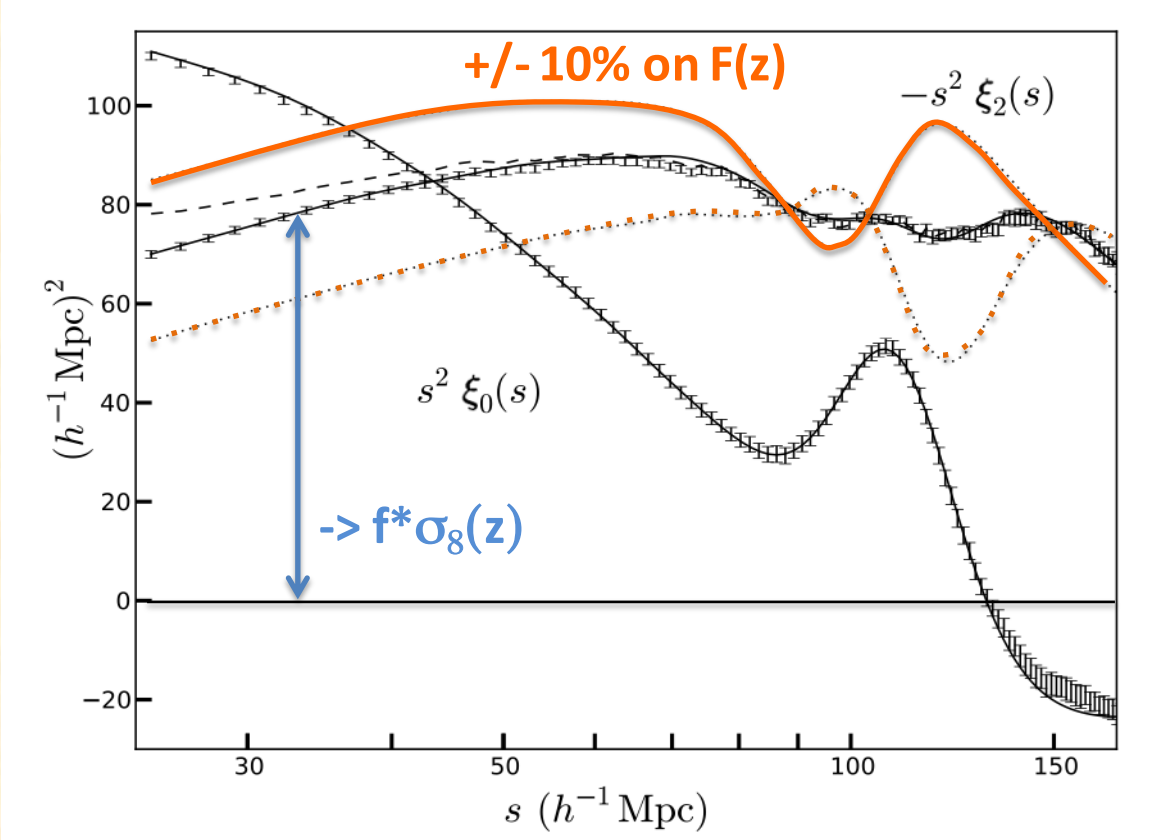




Other sources of distortions : Alcock-Paczinski Effect

When the data is analyzed with a different cosmology than the real one



Reid & White (2011)

BAO peak sensitive to :

$$D_V(z) \equiv \left((1+z)^2 D_A^2(z) \frac{cz}{H(z)} \right)^{1/3}$$

Quadrupole sensitive to :

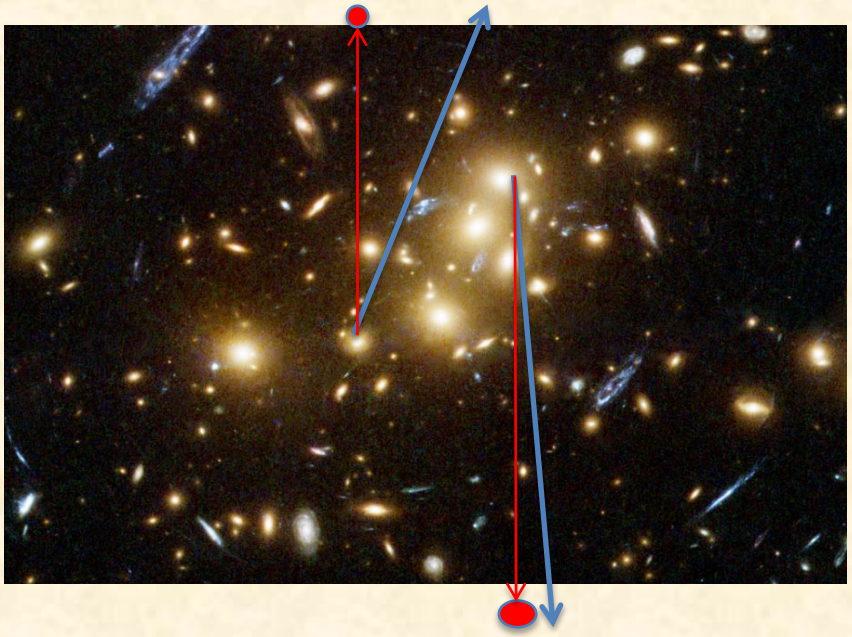
$$F(z) \equiv (1+z) D_A(z) H(z) / c$$

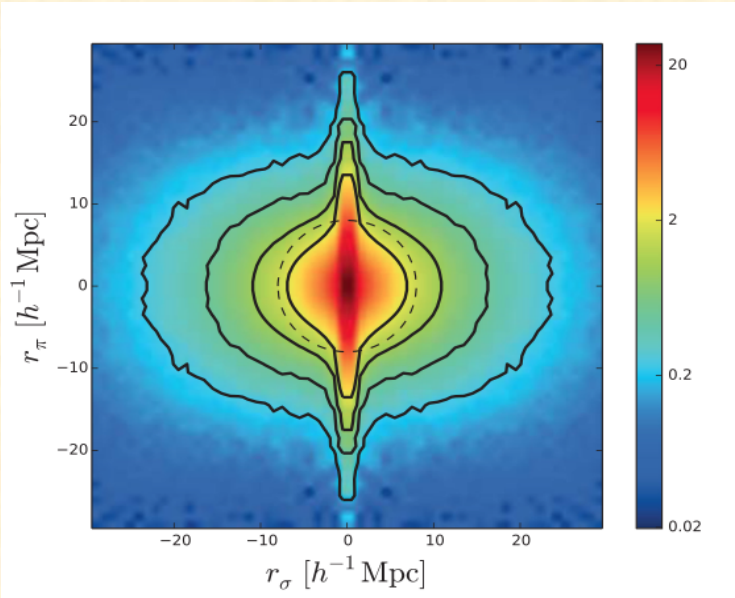
Full RSD analysis :
⇒ Recognized as a key probe of cosmology

Other sources of distortions : Finger-of-God and redshift errors

Bound galaxies within clusters

=> Dispersion of velocities



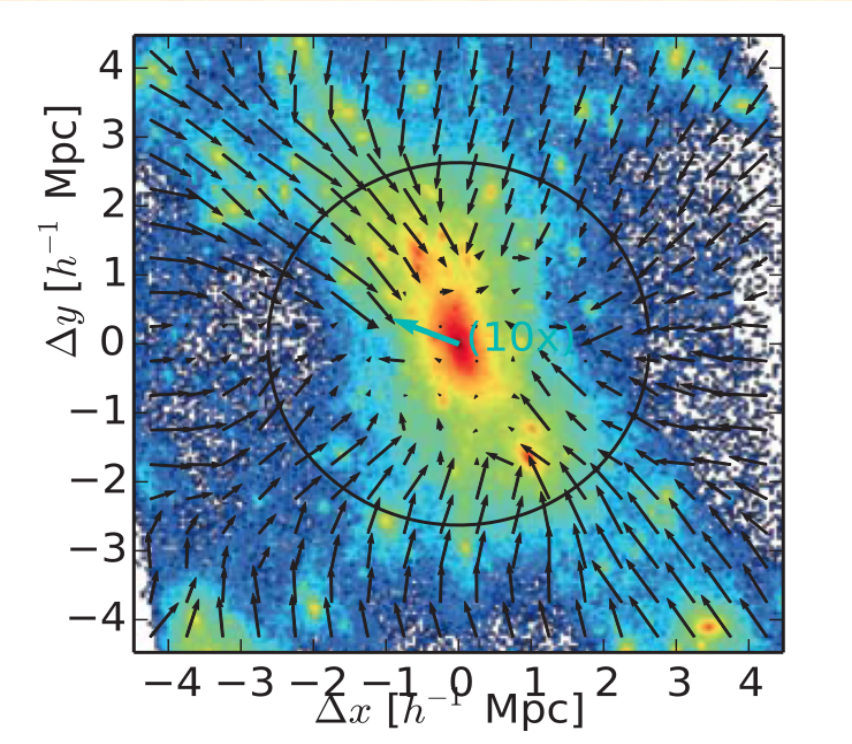


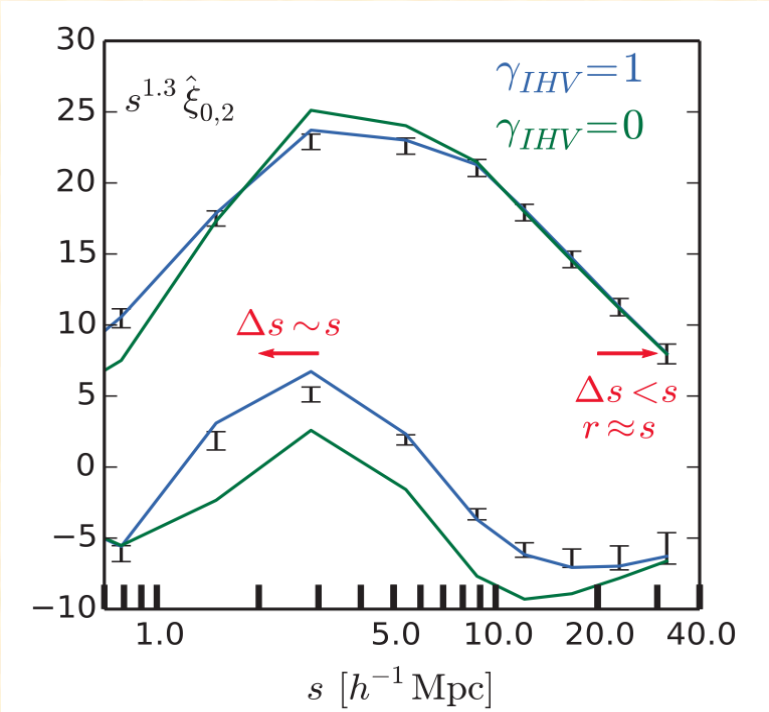
Redshift errors

=> Dispersion of velocities

Not only a small scale effect

Small Scales





Reid et al (2014)

⇒ Gain on precision : 2.5 … but model dependent

SnowPac 2016 conference on small scales :
⇒ Consensus that precision cosmology cannot go below 3 Mpc

Formalisme

Transformation from real space to redshift space:

$$s = x - u_z(x)\hat{z}$$

Hubble law

Peculiar velocity along the line of sight (z)

Jacobien : $\frac{d^3s}{d^3r} = (1+\frac{u_z}{z})^2(1+\frac{du_z}{dz})$

Use mass conservation : $(1+\delta_s)d^3s = (1+\delta_r)d^3r$

$$1+\delta_g^s = (1+\delta_g)(1+\frac{du_z}{dz})^{-1}$$

$$\Rightarrow \delta_g^s = \delta_g - \frac{du_z}{dz} - \delta_g\frac{du_z}{dz} + (\frac{du_z}{dz})^2 + \ldots$$

$$\theta \equiv \nabla \cdot u \qquad u_z = \partial / \partial z \nabla^{-2}\theta$$

In Fourier space: $(\partial / \partial z)^2 \nabla^{-2} = k_z^2 / k^2 = \mu^2 \qquad (= \cos^2\alpha)$

$$\boxed{\delta_g^s(k) = \delta_g(k) - \mu^2\theta(k) - \mu^2\left[\delta_g \otimes \theta\right](k) + \mu^4\left[\theta \otimes \theta\right](k) + \cdots}$$

Linear approximation

$$P_g^s(k) = <(\delta_g(k) - \mu^2\theta(k))^2> = P_{gg}(k) - 2\mu^2 P_{g\theta}(k) + \mu^4 P_{\theta\theta}(k)$$

In linear theory: $\delta_g(k) = b\delta_{mass}(k)$

b is the bias

QSO (z=2)	b=3
CMASS (z=0.57)	b=1.5
Ly a (z=2.3)	b=0.2

$$\theta(k) = -f\delta_{mass}(k) \qquad f \equiv \frac{d\ln D}{d\ln a} \cong \Omega_m^{0.6}(z)$$

f is the growth rate

$$P_g^s(k) = (b^2 + 2\mu^2 fb + \mu^4 f^2)P_{mass}(k)$$

(Kaiser formula)

$$= b^2(1 + 2\mu^2\beta + \mu^4\beta^2)P_{mass}(k) \qquad \beta = f/b$$

Full model - Taruya et al. (2010):

$$P^s(k,\mu) = D(k\mu\sigma_{12})\Big(b^2 P_{\delta\delta}(k) + 2fb\mu^2 P_{\delta\theta}(k) + f^2\mu^4 P_{\theta\theta}(k) + \\ + C_A(k,\mu,f,b) + C_B(k\,u,f,b)\Big), \qquad (30)$$

Small Scales

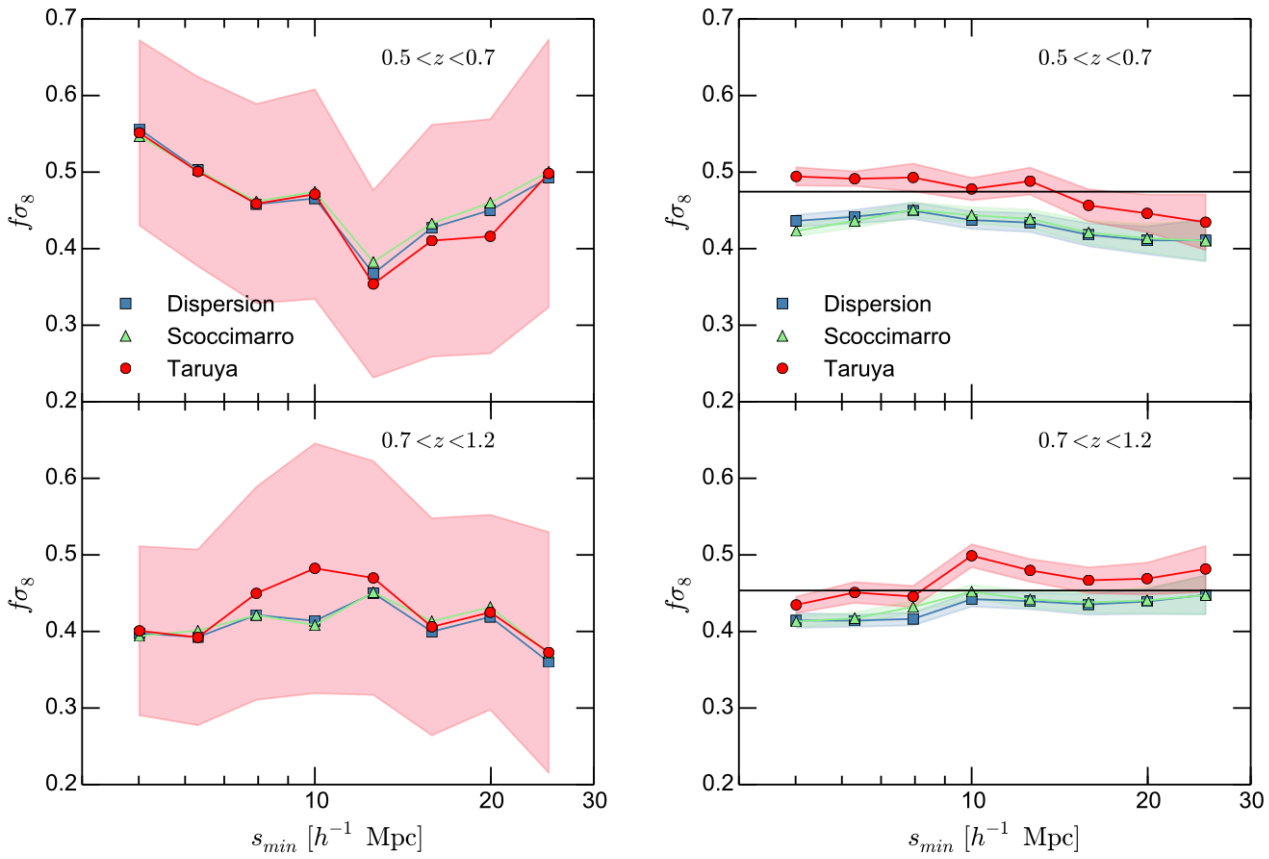


Fig. 17. Left panels: the measured values of $f\sigma_8$ from the VIPERS survey in the two redshift bins, using the TNS model in its optimal set-up that we derived in section 8 (double damping factor: free lorentzian + fixed gaussian), as a function of the minimum fitting scale $s_{\rm min}$. The maximum fitting scale $s_{\rm max}$ is always fixed at $50\,h^{-1}$Mpc. The shaded area gives the statistical error at each $s_{\rm min}$ for the TNS model, as derived from the mocks. Right panels: The same measurements performed on the average of the mocks, i.e. plotting the results of Fig. 15 but showing explicitly the recovered values of $f\sigma_8$, to ease comparison to the data results on the left.

Pezzota et al, 1612.05645