Black holes in scalar tensor and vector tensor theories

LPT Orsay, CNRS

Dark Energy and Modified-Gravity cosmologies: DARKMOD

Galileons/Horndeski [Horndeski 1973]

What is the most general scalar-tensor theory

with second order field equations [Horndeski 1973]?

Horndeski has shown that the most general action with this property is

$$S_H = \int d^4x \sqrt{-g}\,(L_2 + L_3 + L_4 + L_5)$$

$$L_2 = K(\phi, X),$$
$$L_3 = -G_3(\phi, X)\Box\phi,$$
$$L_4 = G_4(\phi, X)R + G_{4X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right],$$
$$L_5 = G_5(\phi, X)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi - \frac{G_{5X}}{6}\left[(\Box\phi)^3 - 3\Box\phi(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3\right]$$

the G_i are free functions of ϕ and $X \equiv -\frac{1}{2}\nabla^\mu\phi\nabla_\mu\phi$ and $G_{iX} \equiv \partial G_i/\partial X$.

- In fact same action as covariant Galileons [Deffayet, Esposito-Farese, Vikman]. Galileons are scalars with Galilean symmetry for flat spacetime.

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- Examples: $G_4 = 1 \longrightarrow R$.
  $G_4 = X \longrightarrow G^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi$.
  $G_3 = X \longrightarrow$ "DGP" term, $(\nabla\phi)^2\Box\phi$
  $G_5 = lnX \longrightarrow$ gives GB term, $\hat{G} = R^{\mu\nu\alpha\beta}R_{\mu\nu\alpha\beta} - 4R^{\mu\nu}R_{\mu\nu} + R^2$
  Action is unique modulo integration by parts.

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- Horndeski theory admits self accelerating vacua with a non trivial scalar field in de Sitter spacetime. A subset of Horndeski theory self tunes the cosmological constant.

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- Generically ST or SA vacua acquire a non trivial scalar field with flat or de Sitter metric.

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- This brings up the issue of time dependance which will be crucial for black holes.

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-For an arbitrary cosmological constant that is allowed to change in time (as a step function...)

-Without fine tuning the parameters of the theory.

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-Admitting flat space time solution with a non trivial scalar
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\begin{aligned}
\mathcal{L}_{john} &= \sqrt{-g}V_{john}(\phi)G^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi \\
\mathcal{L}_{paul} &= \sqrt{-g}V_{paul}(\phi)(*R*)^{\mu\nu\alpha\beta}\nabla_\mu\phi\nabla_\alpha\phi\nabla_\nu\nabla_\beta\phi \\
\mathcal{L}_{george} &= \sqrt{-g}V_{george}(\phi)R \\
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Fab 4 terms

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Fab 4 terms

- All are scalar-curvature interaction terms stemming from Lovelock theory. They are the unique interaction terms yielding second order field equations.
- Theory depends on 4 arbitrary potentials $V = V_{fab4}(\phi)$.
- Fab 4 terms can self-tune the cosmological constant for flat spacetime. At the absence of curvature Fab 4 terms drop out.
- Adding a standard kinetic term self tunes to de Sitter [Gubitosy, Linder]

Example self tuning solution

Consider a slowly varying scalar field in the presence of an arbitrary cc in a time evolving universe,

- Flat spacetime: Milne metric $ds^2 = -dT^2 + T^2\left(\frac{d\chi^2}{1+\chi^2} + \chi^2 d\Omega^2\right)$...
- For simplicity take analytic expansion:

$$V_{john} = C_j \,, V_{paul} = C_p \,, V_{george} = C_g + C_g^1\,\phi \,, V_{ringo} = C_r + C_r^1\,\phi - \frac{1}{4}C_j\,\phi^2$$

- Friedmann equation reads,

$$c_j(\dot\phi H)^2 - c_p(\dot\phi H)^3 - c_g^1(\dot\phi H) + \rho_\Lambda = 0$$

  with matter source $\rho_\Lambda = \Lambda$, vacuum cosmological constant. Note that $\dot\phi H$ appear as homogeneous powers of time...
- Hence since $H = 1/t$ for Milne, taking $\phi = \phi_0 + \phi_1 T^2$ gives $c_j(\phi_1)^2 - c_p(\phi_1)^3 - c_g^1(\phi_1) + \rho_\Lambda = 0$ an algebraic constraint.
- Integration constant ϕ_1 is fixed by the cosmological constant for arbitrary values of the theory potentials.
- Going to spherically symmetric coords scalar is space and time dependent! Same holds for De Sitter self tuning...
- Non trivial vacua inherently demand a time dependent scalar!

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the G_i are free functions of ϕ and $X \equiv -\frac{1}{2}\nabla^\mu\phi\nabla_\mu\phi$ and $G_{iX} \equiv \partial G_i/\partial X$.

- Horndeski theory includes Shift symmetric theories where G_i's depend only on X and $\phi \to \phi + c$.
  Associated with the symmetry there is a Noether current, J^μ which is conserved $\nabla_\mu J^\mu = 0$.
  Presence of this symmetry permits a very general no hair argument

So far...

- Even for a static spherically symmetric spacetime scalar field is to be time dependent if we are going to be in a non trivial branch of solutions
- Shift symmetric Horndeski theory provides a conserved Noether current.

Black holes have no hair [recent review Herdeiro and Radu 2015]

During gravitational collapse...
Black holes eat or expel surrounding matter
their stationary phase is characterized by a limited number of charges
and no details
black holes are bald...

No hair arguments/theorems dictate under some reasonable hypotheses that adding degrees of freedom lead to singular solutions...

For example in vanilla scalar-tensor theories black hole solutions are GR black holes with constant scalar.

Warning : beyond GR Birkhoff's theorem is not valid.
Spherical symmetry thus does not guarantee staticity.
Scalar tensor black holes radiate monopole gravity waves.
There is no reason for metric and scalar not to radiate for spherical symmetry
Let us now see a classical example of a hairy solution...

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Example: BBMB solution

- Consider a conformally coupled scalar field ϕ:

$$S[g_{\mu\nu},\phi,\psi] = \int_{\mathcal{M}} \sqrt{-g}\left(\frac{R}{16\pi G} - \frac{1}{2}\partial_\alpha\phi\partial^\alpha\phi - \frac{1}{12}R\phi^2\right)\mathrm{d}^4x + S_m[g_{\mu\nu},\psi]$$

- **Invariance of the EOM of ϕ under the conformal transformation**

$$\begin{cases} g_{\alpha\beta} \mapsto \tilde{g}_{\alpha\beta} = \Omega^2 g_{\alpha\beta} \\ \phi \mapsto \tilde{\phi} = \Omega^{-1}\phi \end{cases}$$

- There exists a black hole geometry with non-trivial scalar field and secondary black hole hair.
The BBMB solution [N. Bocharova et al.-70 , J. Bekenstein-74]

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- Static and spherically symmetric solution

$$ds^2 = -\left(1 - \frac{m}{r}\right)^2 dt^2 + \frac{dr^2}{\left(1 - \frac{m}{r}\right)^2} + r^2 \left(d\theta^2 + \sin^2\theta d\varphi^2\right)$$

with **secondary** scalar hair

$$\boxed{\phi = \sqrt{\frac{3}{4\pi G}} \frac{m}{r - m}}$$

- Geometry is that of an extremal RN.
Problem:The scalar field is **unbounded** at $(r = m)$.
- A cosmological constant can cure this; [MTZ] family of solutions
- Secondary hair black hole

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The BBMB solution [N. Bocharova et al.-70 , J. Bekenstein-74]

- Static and spherically symmetric solution

$$ds^2 = -\left(1-\frac{m}{r}\right)^2 dt^2 + \frac{dr^2}{\left(1-\frac{m}{r}\right)^2} + r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right)$$

with **secondary** scalar hair

$$\boxed{\phi = \sqrt{\frac{3}{4\pi G}}\frac{m}{r-m}}$$

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Summary so far

- Vacua in Horndeski can be non trivial. Non trivial vacua lead to time dependent scalars even for flat spacetime.
- Time independence for spherical symmetry is not guaranteed. We dont have Birkhoff's theorem in scalar tensor theories
- No hair theorems are not valid for time dependent spacetimes.

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No hair [Hui, Nicolis] [Sotiriou, Zhou] [Babichev, CC, Lehébel]

Static no hair theorem

Consider shift symmetric Horndeski theory with G_2, G_3, G_4, G_5 arbitrary functions of X. We have a Noether current J^μ which is conserved, $\nabla_\mu J^\mu = 0$.

We now suppose that:

1. spacetime and scalar are spherically symmetric and static,
$$ds^2 = -h(r)dt^2 + \frac{dr^2}{f(r)} + r^2 dK^2, \ \phi = \phi(r)$$
2. spacetime is asymptotically flat, $\phi' \to 0$ as $r \to \infty$ and the norm of the current J^2 is finite on the horizon,
3. there is a canonical kinetic term X in the action,
4. and the G_i functions are such that their X-derivatives contain only positive or zero powers of X.

Under these hypotheses, ϕ is constant and thus the only black hole solution is locally isometric to Schwarzschild.

Most interesting part of no go theorem: Breaking any of these hypotheses leads to black hole solutions!

Theorem can be extended for star solutions. [Lehébel et al.]

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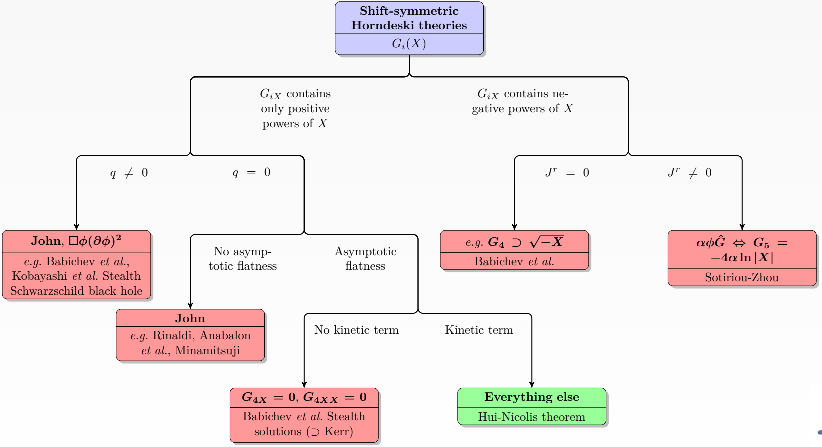
Hair versus no hair [figure: Lehébel]



Introducing time dependence, $q \neq 0$

Spherical symmetry certainly does not impose staticity. In fact no hair theorems may be pointing out to an inconsistency in this direction.

- Furthermore, for self accelerating or self tuning solutions one has a time dependence for the scalar in FRW coordinates
- In spherical symmetry this leads to a time and radially depending scalar already for flat spacetime.
- So let us allow time dependence for the scalar as a first step while keeping for a static and spherically symmetric spacetime.

But is it consistent with respect to the field equations $\mathcal{E}_\phi = 0$, and $\mathcal{E}_{\mu\nu} = 0$ to do so?

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The question of time dependence, $qt + \psi(r)$

Consistency theorem [Babichev, CC, Hassaine]

Consider an arbitrary shift symmetric Horndeski theory and a scalar-metric ansatz with $q \neq 0$. The unique solution to the scalar field equation $\mathcal{E}_\phi = 0$ and the "matter flow" metric equation $\mathcal{E}_{tr} = 0$ is given by $J^r = 0$.

- We are killing two birds with one stone.
- The current now reads, $J^\mu J_\mu = -h(J^t)^2 + (J^r)^2/f$ and is regular. Time dependence renders no hair theorem irrelevant.
- Given the higher order nature of Horndeski theory this theorem basically tells us that if $\phi = qt + \psi(r)$ then there exist $\phi' \neq 0$ solutions to the field equations.
- One can prove for some theories that if $\phi = \phi(t, r)$ then the only compatible ϕ are $\phi = qt + \psi(r)$ and also $\phi = \phi_1(r^2 - t^2)$ for flat spacetime (Fab 4 self tuning solution)

General solution

Consider, $L = R - \eta(\partial\phi)^2 + \beta G^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - 2\Lambda$ For static and spherically symmetric spacetime.

The general solution of theory L for static and spherically symmetric metric and $\phi = \phi(t, r)$ is given as a solution to the following third order algebraic equation with respect to $\sqrt{k(r)}$:

$$(q\beta)^2\left(\kappa + \frac{r^2}{2\beta}\right)^2 - \left(2\kappa + (1 - 2\beta\Lambda)\frac{r^2}{2\beta}\right)k(r) + C_0 k^{3/2}(r) = 0$$

All metric and scalar functions given with respect to k.

For general shift symmetric G_2, G_4 the result can be extended, [Kobayashi, Tanahashi]

Let us now give some specific examples for the different cases...

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Scalar with constant velocity $q \neq 0$

Consider the action,

$$S = \int d^4x \sqrt{-g} \left[\zeta R - 2\Lambda - \eta (\partial\phi)^2 + \beta G^{\mu\nu} \partial_\mu \phi \partial_\nu \phi \right] \ldots,$$

- Scalar field equation and conservation of current,

$$\nabla_\mu J^\mu = 0, \quad J^\mu = (\eta g^{\mu\nu} - \beta G^{\mu\nu}) \partial_\nu \phi.$$

- Take $ds^2 = -h(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2$, and $\phi = \phi(t, r)$ then
- $\phi = \psi + qt$ while $\mathcal{E}_{tr} = -\frac{q^2 J^r}{f} \longrightarrow J^r = 0$ solves both equations...
- $\beta G^{rr} - \eta g^{rr} = 0$ ie. $f = \frac{(\beta + \eta r^2) h}{\beta (rh)'}$ or $\phi' = 0$

For a higher order theory $J^r = 0$ does not necessarily imply $\phi = const.$

$J^r = 0$ means that we kill primary hair since, $\nabla_\mu J^\mu = 0 \rightarrow \sqrt{-g}(\beta G^{rr} - \eta g^{rr}) \partial_r \phi = c$

- We now solve for the remaining field eqs...

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Solving the remaining EoM

- From (rr)-component get ψ'

$$\psi' = \pm \frac{\sqrt{r}}{h(\beta + \eta r^2)} \left(q^2 \beta(\beta + \eta r^2) h' - \frac{\zeta\eta + \beta\Lambda}{2} (h^2 r^2)' \right)^{1/2} .$$

- and finally (tt)-component gives $h(r)$ via,

$$h(r) = -\frac{\mu}{r} + \frac{1}{r} \int \frac{k(r)}{\beta + \eta r^2} dr,$$

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Asymptotically flat limit : $\Lambda = 0$, $\eta = 0$

- Consider $S = \int d^4x\sqrt{-g}\,[\zeta R + \beta G^{\mu\nu}\partial_\mu\phi\partial_\nu\phi]$
- Algebraic equation to solve: $q^2\beta^3 - 2\zeta\beta k + C_0 k^{3/2} = 0 \rightarrow k = constant!$
- $f(r) = h(r) = 1 - \mu/r$
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- Consider $v = t + \int (fh)^{-1/2}dr$ then $ds^2 = -hdv^2 + 2\sqrt{h/f}\,dvdr + r^2d\Omega^2$
  Regular chart for horizon, EF coordinates
- $\phi_+ = q\left[v - r + 2\sqrt{\mu r} - 2\mu\log\left(\sqrt{\frac{r}{\mu}} + 1\right)\right] + \text{const}$
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Schwarzschild geometry with a non-trivial regular scalar field.

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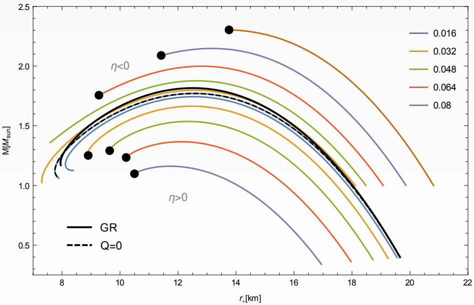
Star solutions [Cisterna, Delsate, Rinaldi], [Maselli, Silva, Minamitsuji, Berti]

- Consider $S = \int d^4x\sqrt{-g}\,[\zeta R + \beta G^{\mu\nu}\partial_\mu\phi\partial_\nu\phi]$
- Take stealth solution for exterior and consider PF matter for interior with ρ and P that does not couple to scalar.
- $J^r = 0$, and therefore $G^{rr} = 0$ which effects star interior.
- For fixed star radius $\beta > 0$ ($\beta < 0$) gives heavier (lighter) stars than GR.
- No GR limit for $q \to 0$



Star solutions [Cisterna, Delsate, Rinaldi], [Maselli, Silva, Minamitsuji, Berti]

- Consider $S = \int d^4x\sqrt{-g}\,[\zeta R + \beta G^{\mu\nu}\partial_\mu\phi\partial_\nu\phi]$
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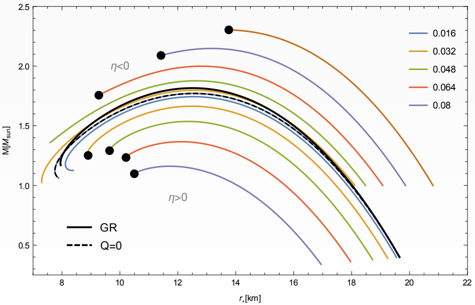

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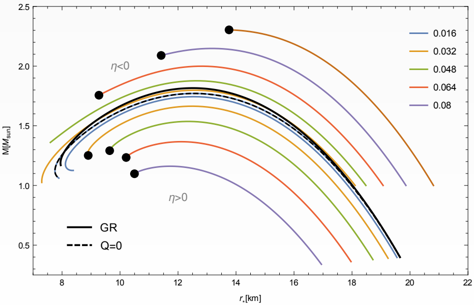

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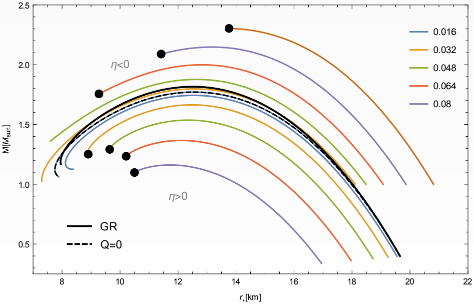

So far...

- Horndeski theory admits ST and SA vacua that naturally lead to a softly time dependent scalar.
- Shift symmetry permits the existence of a no hair theorem valid for static configurations that allows to construct hairy black holes.
- Need more work on time dependent metrics.
- Higher order Horndeski terms permit novel branches of solutions
- Time dependent scalars permit regularity on the black hole horizon
- We constructed a simple stealth Schwarzschild black hole that leads to well defined star solutions distinct from GR

Self tuning de Sitter black hole

- $$S = \int d^4x \sqrt{-g} \left[\zeta R - 2\Lambda - \eta (\partial\phi)^2 + \beta G^{\mu\nu} \partial_\mu \phi \partial_\nu \phi \right]$$

  ... $q^2\beta(\beta + \eta r^2)^2 - \left(2\zeta\beta + (\zeta\eta - \beta\Lambda) r^2\right) k + C_0 k^{3/2} = 0$
- $f = h = 1 - \frac{\mu}{r} + \frac{\eta}{3\beta} r^2$ de Sitter Schwarzschild!
- $\psi' = \pm \frac{q}{h}\sqrt{1-h}$ and $\phi(t, r) = q\, t + \psi(r)$
- The effective cosmological constant is not the vacuum cosmological constant. In fact,
- Self tuning relation : $q^2\eta = \Lambda - \Lambda_{eff} > 0$
- Hence for any $\Lambda > \Lambda_{eff}$ fixes q, integration constant.
- where $\Lambda_{eff} = -\frac{\eta}{\beta}$ is fixed by effective theory.
- Solution hides vacuum cosmological constant leaving a smaller effective cosmological constant [Gubitosi, Linder]

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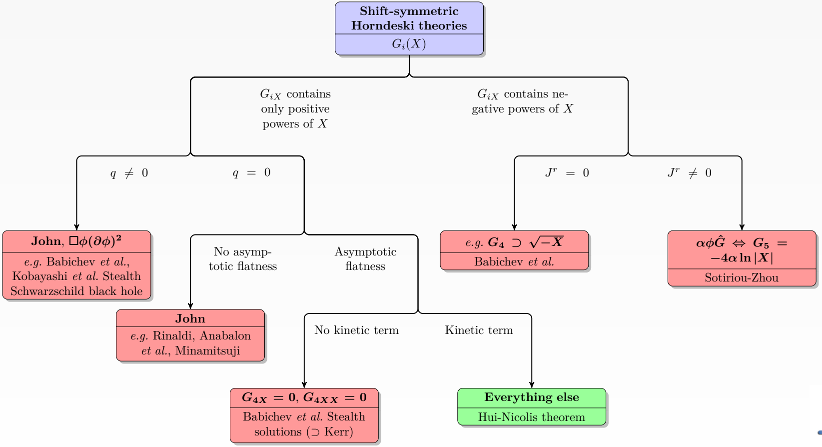
Hair versus no hair [Lehébel]



The special case of the Gauss-Bonnet invariant

[Sotiriou, Zhou] [Duncan et.al] [Mavromatos et.al]

The Gauss-Bonnet term, $\hat{G} = R^{\mu\nu\alpha\beta}R_{\mu\nu\alpha\beta} - 4R^{\mu\nu}R_{\mu\nu} + R^2$, is a topological invariant in 4 dimensions.
Variation with respect to the metric gives the 4 dim Lovelock identity, $H_{\mu\nu} = -2P_{\mu cde}R_{\nu}{}^{cde} + \frac{g_{\mu\nu}}{2}\hat{G} = 0$. If we couple to scalar then $\phi\hat{G}$ ceases to be trivial.
It can be obtained in Horndeski theory via $G_5 \sim \ln X$
The theory

$$\mathcal{L}^{\mathrm{GB}} = \frac{R}{2} - \frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi + \alpha\phi\hat{G}$$

is non trivial and shift symmetric. Here, $\hat{G}$ (is independent of ϕ) and acts as a source to the scalar which cannot be set to zero.

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- $\Box\phi + \alpha\hat{G} = 0$
- Numerical solution can be found where the scalar and mass integration constants are fixed so that the solution is regular at the horizon.

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- $\Box\phi + \alpha\hat{G} = 0$
- The mass of the black hole has a minimal size fixed by the GB coupling α. The singularity is attained at positive r.

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- The solution has infinite current norm at the horizon because $J^r \neq 0$

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- $\Box\phi + \alpha\hat{G} = 0$
- Solutions with $q \neq 0$ and regular Noether current are in a different branch and are singular.

So far...

- For $q \neq 0$ we can find solutions analytically for G_2, G_4 and otherwise numerically
- For $q = 0$ we need to source the scalar field equation breaking one of the hypotheses of the theorem [Babichev, CC, Lehébel]
- For generic Horndeski we can use KK of known Lovelock solutions to construct black holes [CC, Gouteraux, Kiritsis]
- Slow rotation gives identical correction to GR. Stationary solutions not known except for stealth Kerr...
- In dense matter regions how does scalar couple to matter? Neutron stars etc...

Conformally coupled scalar field

- Consider a conformally coupled scalar field ϕ revisited:

$$S[g_{\mu\nu},\phi,\psi] = \int_{\mathcal{M}} \sqrt{-g}\left(\frac{R}{16\pi G} - \frac{1}{2}\partial_\alpha\phi\partial^\alpha\phi - \frac{1}{12}R\phi^2\right)\mathrm{d}^4x + S_m[g_{\mu\nu},\psi]$$

- **Invariance of the EOM of ϕ under the conformal transformation**

$$\begin{cases} g_{\alpha\beta} \mapsto \tilde{g}_{\alpha\beta} = \Omega^2 g_{\alpha\beta} \\ \phi \mapsto \tilde{\phi} = \Omega^{-1}\phi \end{cases}$$

- There exists a black hole geometry with non-trivial scalar field and secondary black hole hair.
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BBMB completion [CC, Kolyvaris, Papantonopoulos and Tsoukalas]

- We would like to combine the above properties in order to obtain a hairy black hole.
- Consider the following action, $S(g_{\mu\nu},\phi,\psi) = S_0 + S_1$ where
- $$S_0 = \int dx^4 \sqrt{-g}\,\left[\zeta R + \eta\left(-\frac{1}{2}(\partial\phi)^2 - \frac{1}{12}\phi^2 R\right)\right]$$
  and
- $$S_1 = \int dx^4 \sqrt{-g}\,\left(\beta G_{\mu\nu}\nabla^\mu\Psi\nabla^\nu\Psi - \gamma T^{BBMB}_{\mu\nu}\nabla^\mu\Psi\nabla^\nu\Psi\right),$$
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$$f(r) = h(r) = 1 - \frac{m}{r} + \frac{\gamma c_0^2}{12\beta r^2} \ ,$$

$$\phi(r) = \frac{c_0}{r} \ ,$$

$$\psi'(r) = \pm q \frac{\sqrt{mr - \frac{\gamma c_0^2}{12\beta}}}{r\, h(r)},$$

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- Scalar charge c_0 playing similar role to EM charge in RN
Galileon Ψ regular on the future horizon

$$\psi = qv - q\int \frac{dr}{1 \pm \sqrt{1 - h(r)}}$$

A vector tensor interaction

- Consider the following completion to Einstein Maxwell theory [Horndeski],

$$S[g,A]=\int\sqrt{-g}\,d^4x\left[R-2\Lambda-\frac{1}{4}\mathcal{F}^2+\gamma\mathcal{F}_{\mu\nu}\mathcal{F}_{\rho\sigma}P^{\mu\nu\rho\sigma}\right].$$

- The additional vector-curvature interaction is due to Horndeski and corrects EM in curved spacetime.
- The theory has $U(1)$ symmetry
- In flat spacetime we have Maxwell equations
- Unique such term with second order eom

The above theory obeys Birkhoff's theorem

but breaks EM duality. The solutions have unusual asymptotics akin to Lifschitz or conical spacetimes

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$$S[g,A] = \int \sqrt{-g}\, d^4x \left[R - 2\Lambda - \frac{1}{4}\mathcal{F}^2 + \gamma \mathcal{F}_{\mu\nu}\mathcal{F}_{\rho\sigma}P^{\mu\nu\rho\sigma} \right].$$

- The additional vector-curvature interaction is due to Horndeski and corrects EM in curved spacetime.
- The theory has $U(1)$ symmetry
- In flat spacetime we have Maxwell equations
- Unique such term with second order eom

The above theory obeys Birkhoff' s theorem

but breaks EM duality. The solutions have unusual asymptotics akin to Lifschitz or conical spacetimes

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Einstein Proca theory

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$$S[g, A] = \int \sqrt{-g}\, d^4x \left[R - 2\Lambda - \frac{1}{4}\mathcal{F}^2 - \frac{\mu^2}{2} A^2 \right].$$

- Proca field is "Maxwell field" with mass μ and no longer has U(1) gauge symmetry.
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A modified Proca theory [Babichev, CC, Hassaine]

- Consider the following tensor-vector theory,

$$S[g,A] = \int \sqrt{-g}\, d^4x \left[R - 2\Lambda - \frac{1}{4}\mathcal{F}^2 + \beta G_{\mu\nu} A^\mu A^\nu \right].$$

- For $\beta \neq 0$ we have a modified Maxwell theory of effective mass μ with an additional gravity-vector interaction term $G_{\mu\nu} A^\mu A^\nu$.
- Here, mass feeds in to the photon at strong curvature. In flat spacetime we have Maxwell equations and the field here can still be a Maxwell field.
- It could modify predictions for cosmological magnetic fields. It is a well defined modification of gravity.

From scalar tensor to a vector-tensor theory

- Putting it all together,

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- So the velocity charge q is now replaced by an electric potential function $q \to a(r)$ whereas $\nabla_\mu \psi \to \chi$
- χ is gauge freedom for Maxwell ($\mu = 0$) but is not gauge otherwise. So we cannot discard it!
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$$H^\nu := \nabla_\mu (F^{\mu\nu}) - \mu^2 A^\nu + 2\beta A_\mu G^{\mu\nu} = 0$$

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Solving the field equations

Consider spacetime,

$$ds^2 = -h(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2_{2,\kappa}$$

As before we solve for f and χ... We then make the substitution,

$$h(r) = -\frac{2M}{r} + \frac{1}{r}\int \frac{k(r)}{\mu^2 r^2 + 2\beta\kappa} dr$$

yielding at the end,

$$\left[\frac{(\mu^2 r^2 + 2\beta\kappa)(r\ a)'}{\sqrt{k(r)}}\right]' = (1-4\beta)a(r)\left[\frac{(\mu^2 r^2 + 2\beta\kappa)}{\sqrt{k(r)}}\right]'$$

$$C_1 k^{3/2} - k\left[2\beta\kappa + r^2\left(\frac{\mu^2}{2} - \beta\Lambda\right)\right] + \frac{1}{8}\left(\mu^2 r^2 + 2\beta\kappa\right)^2\left[[(ra)']^2 - (1-4\beta)(a^2 r)'\right] = 0$$

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Example solutions: Solitons for $\beta = 1/4$ and adS asymptotics

- Integration constants, C_1, Q, Q_2, M
- One can find the general solution for $\beta = 1/4$ and $C_1 = 0$ with spherical symmetry
- Regular asymptotics akin to adS spacetime and asymptotically flat solutions ($\mu = \Lambda = 0$).
- adS solitons

Example solutions: $\beta = 1/4$

- Integration constants, C_1, Q, Q_2, M
- Fixing C_1, AdS asymptotics. Similar solution to adS Sch.
- Coulomb charge Q
- Q_2 is like a Proca charge acting as effective curvature.
- Regular soliton solution for $M = 0$ with adS asymptotics.
- The additional "John" term regularizes asymptotics

$$h(r) = \frac{2\mu^2}{3} r^2 + \frac{2Q_2^2\mu^2 - 6\mu^2 - \Lambda}{\Lambda - 2\mu^2} - \frac{2M}{r} + \frac{(Q_2^2\mu^2 - 2\mu^2 - \Lambda)^2}{\sqrt{2}\mu(\Lambda - 2\mu^2)^2} \frac{\arctan\left(\sqrt{2}r\mu\right)}{r},$$

$$f(r) = h(r)\left(\frac{r^2 + \frac{1}{2\mu^2}}{r^2 + \frac{Q_2^2 - 4}{2\left(\Lambda - 2\mu^2\right)}}\right)^2 ., \quad a(r) = \frac{Q}{r} + \#Q_2 + \#\frac{\arctan r}{r}$$

Conclusions

- Starting from a no hair theorem we have seen how to construct hairy black holes.
- Similar theorem exists for neutron stars.
- Using Lovelock solutions we can construct black holes in Horndeski theory.
- Many questions about stability of solutions; staticity of spacetime quite unclear
- Higher order terms essential for novel branches of black holes
- One can construct solutions with EM fields and black hole solutions with primary hair by adding additional scalar fields
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Slowly rotating solutions [Maselli, Silva, Minamitsuji, Berti]

Using the Hartle Thorne perturbative approximation in which frame-dragging is assumed linear in angular velocity

$$\mathrm{d}s^2 = -h(r)\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{f(r)} + r^2(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\varphi^2) - 2\omega(r)r^2\sin^2\theta\mathrm{d}t\mathrm{d}\varphi,$$

We get an ode to linear order:

$$2(1-\beta X)\left[\omega'' + \frac{\omega'}{2}\left(\frac{f'}{f} + \frac{8}{r} - \frac{h'}{h}\right)\right] - 2\beta X'\omega' = 0$$

which agrees with GR for X constant.
What happens for $X \neq$ const.
We can integrate once,

$$(1-\beta X)\omega' = \frac{C_1\sqrt{k}}{r^4(1+\frac{r^2}{2\beta})}$$

but, one can show by using remaining field equations that correction is always identical to GR [Lehébel].

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