excellent review Burrage + Sakstein 1709.09071.

1

Screening Mechanisms in Modified Gravity Theories

- Let me start by considering chameleon-type theories.

General scalar-tensor theory with coupling to matter.

Einstein frame

$$S = \int d^4x \sqrt{-g} \left[R + (\partial_\mu \phi)^2 + V(\phi) \right].$$

$$+ S_m \left[\Psi_i , \tilde{g}_{\mu\nu}(\phi) \right]$$

here $\tilde{g}_{\mu\nu}(\phi) = A^2(\phi)\, g_{\mu\nu}$

- matter is coupled to the Jordan frame metric. The coupling function, $A(\phi)$, results in deviations from GR. The particle moves on geodesics governed by $\tilde{g}_{\mu\nu}(\phi)$, not $g_{\mu\nu}$.

Geodesic eq^n

$$\ddot{x}^i + \Gamma^i_{\cdot\cdot} = - \frac{\beta(\phi)}{M_{pl}} \nabla^i \phi .$$

where $\beta(\phi) = M_{pl} \frac{d \ln A(\phi)}{d\phi}$ is the coupling.

Since christoffell $\Gamma^i_{00} = \partial^i \Phi_N$,

where Φ_N is Newton potential, there is a fifth force

$$\boxed{\vec{F}_5 = -\frac{\beta(\phi)}{M_{pl}} \vec{\nabla} \phi}$$

Now consider the equation of motion for ϕ

$$\Box \phi = +\frac{\partial V(\phi)}{\partial \phi} - \frac{\beta(\phi)}{M_{pl}} T^m.$$

where $T = g_{\mu\nu} T^{\mu\nu}$ (matter)

$T^{\mu\nu} = 2/\sqrt{g}\ \delta S_m / \delta g_{\mu\nu}$ - energy mom. tensor in Einstein frame.

We can rewrite this as

$$\Box \phi = \frac{dV(\phi)}{d\phi} + \frac{\beta(\phi)}{M_{pl}} \rho_m \equiv \frac{dV_{eff}(\phi)}{d\phi}.$$

where $V_{eff}(\phi) = V(\phi) + \rho \ln A(\phi)$

+ this effective potential governs the dynamics of ϕ, not just $V(\phi)$.

There are 3 models with this type of behaviour. They screen the 5th force in a couple of different ways.

Consider a spherical object of mass M, radius R in a medium of density ρ_0

$$\phi = \phi_0 + \delta\phi \qquad \text{where } \phi_0 \text{ is the min of } \phi \text{ e } \phi_0(\rho_0).$$

$$\therefore \nabla^2 \delta\phi - m^2_{eff}(\phi_0)\,\delta\phi = \frac{\beta(\phi_0)}{M_{pl}}\rho(r)$$

$$m^2_{eff} = V''_{eff}(\phi)$$

Scalar field outside source is

$$\delta\phi = \frac{\beta(\phi_0)}{4\pi M_{pl}} \frac{f(M,R)}{r} e^{-m_{eff}r}.$$

model dependent f^n.

for pt source $f(M,R) = M$.

There are 3 ways to suppress the effect of the scalar (i.e 5th force)

1) $m_{eff} r \gg 1$ – force is short range

2) Coupling to matter $\beta(\phi_0)$ small

3) not all of the mass sources the scalar field.

The chameleon mechanism uses 1).
Symmetron + dilaton use 2).

Chameleon Mechanism – Khoury + Weltman

astroph 0309300 / 0309411
Brax et al 0408415

consider a potential

$$V(\phi) = \frac{\Lambda^{n+4}}{\phi^n}, \quad A(\phi) = e^{\beta\phi/M_{Pl}}$$

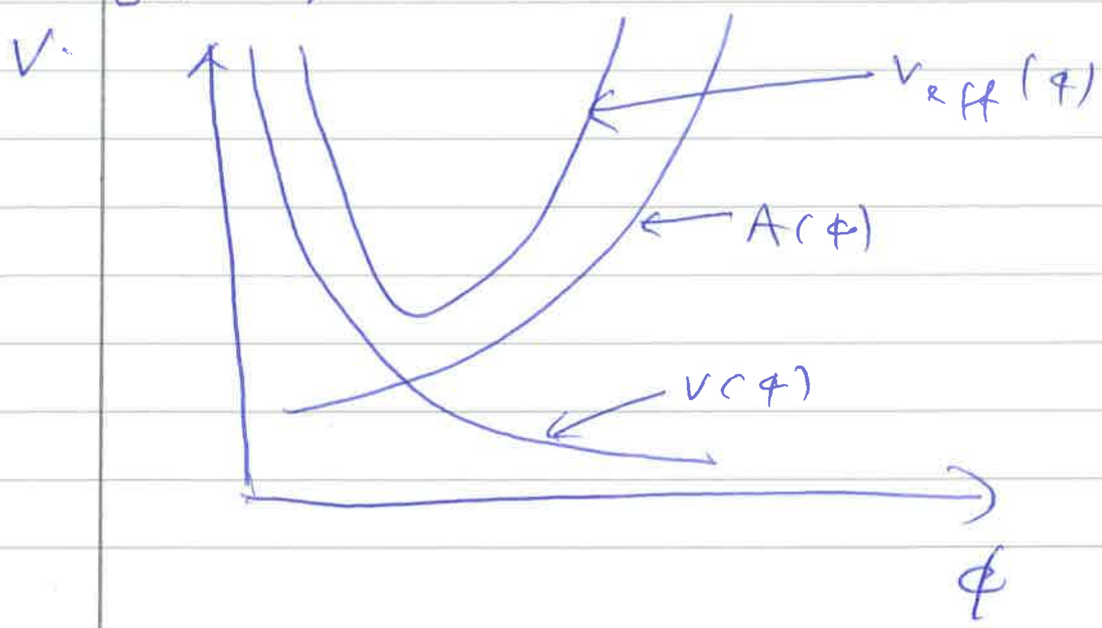
$$\therefore V_{eff} = \frac{\Lambda^{n+4}}{\phi^n} + \rho\beta\frac{\phi}{M_{Pl}}$$

usually $\beta = O(1)$ i.e. coupling gravitational strength, but can take β larger.

$$\phi_{min}(\rho) = \left(\frac{n M_{Pl}\Lambda^{4+n}}{\beta\rho}\right)^{1/n+1}$$

$$\text{\& } m^2_{eff} = V''_{eff}(\phi) = n(n+1)\Lambda^{n+4}\left(\frac{\rho\beta}{n M_{Pl}\Lambda^{4+n}}\right)^{\frac{n+2}{n+1}}$$

with $n > -1$



unfortunately the potential doesn't self-accelerate. We could take

$$V(\phi) = \Lambda^4 \exp(\Lambda^4/\phi^n)$$

(Brax et al. ~~Khoury et~~ al)

$$\sim \Lambda^4 + \frac{\Lambda^{4+n}}{\phi^n}.$$

, take $\Lambda = \Lambda_{DE} = 2.4\ \text{meV}$.

The mass being $m_{eff}(\rho)$ is not enough to screen the 5th force. We also need the thin shell effect.

Consider a spherical object of mass m, radius R, density ρ_1 embedded in a medium of density ρ_2

(e.g. moon in the atmosphere of solar system)

Far away from object the field is at min of $V_{eff}(\rho_2)$ $\phi(r) = \phi_{min}(\rho_2)$, ~~ins~~

deep inside the body $\phi = \phi_{min}(\rho_1)$, $V'_{eff}(\phi) = 0$

In general

$$\nabla^2\phi = \frac{1}{r^2}\frac{d}{dr}\left(r^2 \frac{d\phi}{dr}\right) = \frac{dV}{\partial\phi} + \frac{\beta(\phi)}{m_{pl}}\rho$$

We solve for $\phi_{r<R}$, $\phi_{r>R}$ + ϕ'

PTO

5'

<u>computation</u>

let $\phi = \phi_0 + \delta\phi$

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d\phi}{dr}\right) = \underbrace{m_0^2\,\delta\phi}_{\text{ignore : mis}} + \frac{\beta(\phi_0)}{M_{pl}}\,\delta\rho$$

$\Rightarrow$ Poisson eqn

If field sits close to minimum of V_{eff} at centre & remains up to r_s, then rolls to asymptotic value ϕ_∞

No fifth force inside $r < r_s$ – screening radius.

Outside r_s

$$\frac{d\phi}{dr} = \frac{\beta}{4\pi M_{pl}}\cdot\frac{M(r) - M(r_s)}{r^2} \quad (*)$$

$$M(r) = \int_0^r 4\pi r'^2\,\delta\rho(r')\,dr' \qquad M = M(R)$$

calc r_s integrate $(*)$

$$\phi_0 - \phi_s = \frac{\beta(\phi)\,M(r_s)}{4\pi M_{pl}\,r_s} + \int_{r_s}^{\infty}\frac{\beta\,M(r')}{4\pi r'^2}\,dr'$$

$\beta\,(M(R) - M(r_s))$ is effective scalar charge.

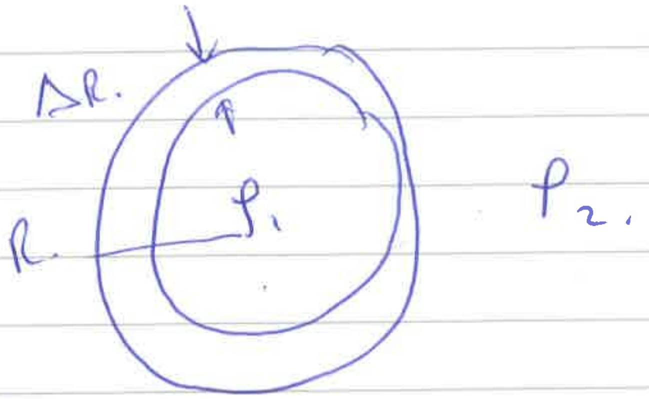
self screening param $\chi = \dfrac{\phi_0}{2\beta M_{pl}}$

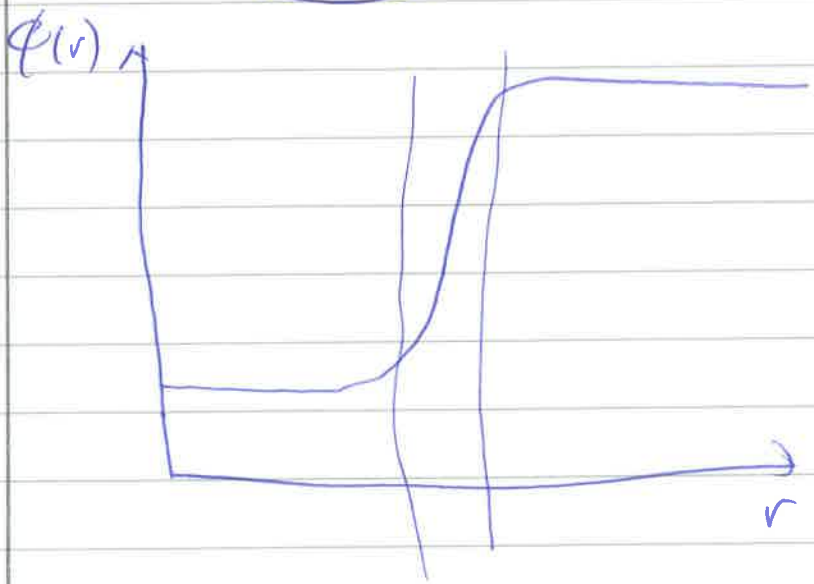
$\boxed{\chi \ll \Phi \text{ fully screened.}}$

$$\equiv -\Phi_N(r_s) - r_s\,\Phi_N'(r_s)$$

metal at $r = R$. — this is w/s D.E.

If $\rho_1 \gg \rho_2$ one finds a thin shell





$$F_5 \propto \nabla\phi \simeq \nabla \left(\frac{\Delta R}{R}\right)$$

e.g. see Tom Waterhouse astro-ph/0611816

Brax, van de Bruck, Davis, Shaw

f1

<u>f(R) models</u> + <u>chams</u> 0806.2075

$$S = \frac{1}{16\pi G} \int d^4x \sqrt{-g} \left[R + f(R) + S_m \right]$$

we can rewrite this as scalar-tensor theory via

$$\phi = -\frac{\beta}{2} m_{pl} \ln[1 + f'(\phi)]$$

<u>where $\beta = \sqrt{1/6}$</u>

$$V(\phi) = m_{pl}^2 \left(\frac{R f'(\phi) - f(\phi)}{(1 + f'(\phi))^2} \right)$$

$$\tilde{g}_{\mu\nu} = A^2(\phi) g_{\mu\nu}$$

$$A^2(\phi) = 1 + df/d\phi.$$

$$\Rightarrow S = \int d^4x \sqrt{g} \left[\frac{R}{16\pi G} - \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) \right] + S_m \left[e^{\sqrt{2/3}\, \phi / m_{pl}} \right]$$

$\boxed{\text{Hu-Sawicki} \quad 0705.1158.}$

$$f(R) = -a \frac{\mu^2}{1 + (R/\mu)^{-b}}$$

$a, b > 0$

f2

$R \gg \mu$

$$\Rightarrow \quad f(R) = -a\mu^2 + a\mu^2\left(\frac{R}{\mu^2}\right)^{-b}$$

$$\equiv \quad -\frac{2\Lambda_0^4}{M_{pl}^2} - \frac{f_{R_0}}{n}\frac{R_0^{n+1}}{R^n}.$$

Tests constrain f_{R_0}.