Leonardo Senatore (Stanford)

On the EFT of LSS in the Dark Sector

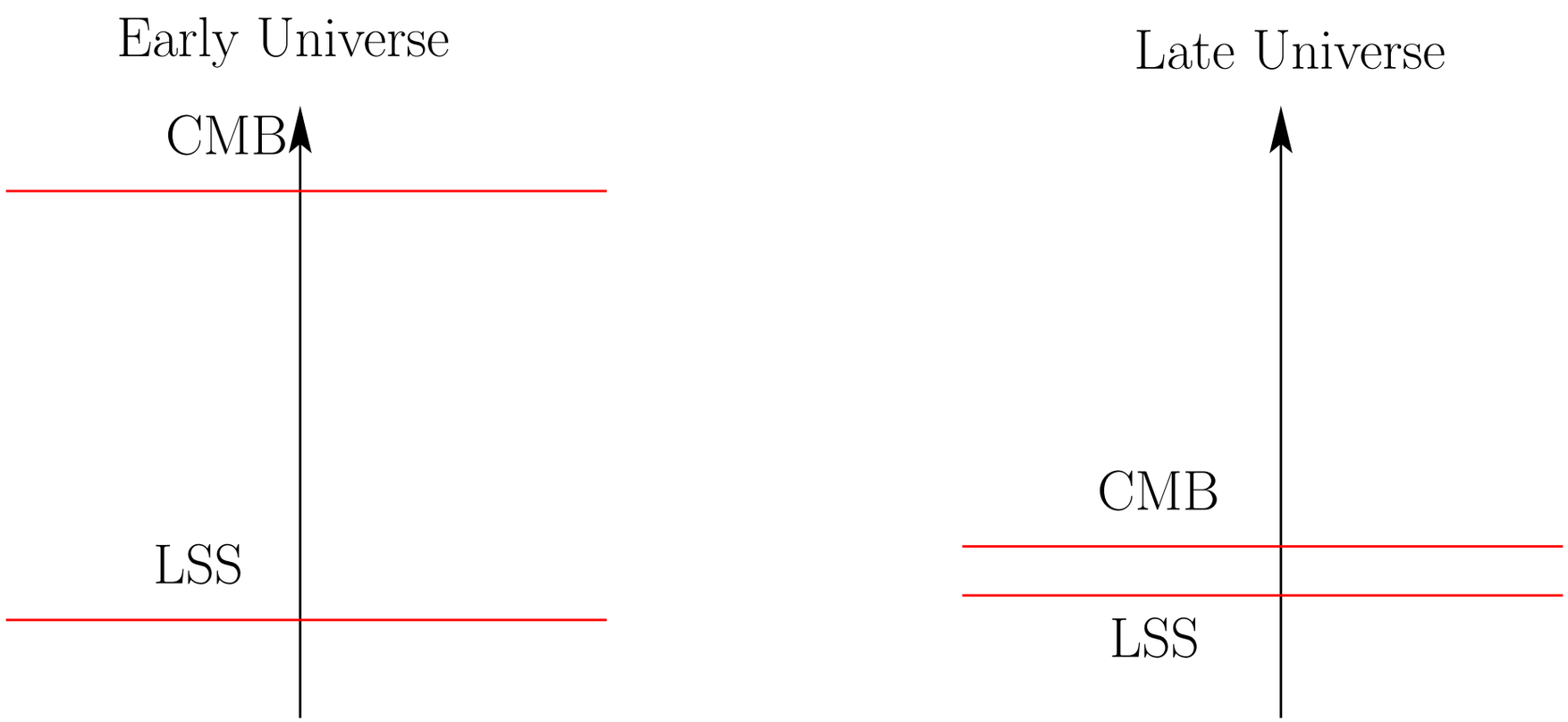
The Gathering Storm

The Gathering Storm

– After the completion of the Planck satellite, no guaranteed very large improvement is expected from measurements of the primordial CMB

– How to we continue to explore the beginning of the universe?

– LSS (directly or through CMB) will be the leading next probe. But where do we stand:



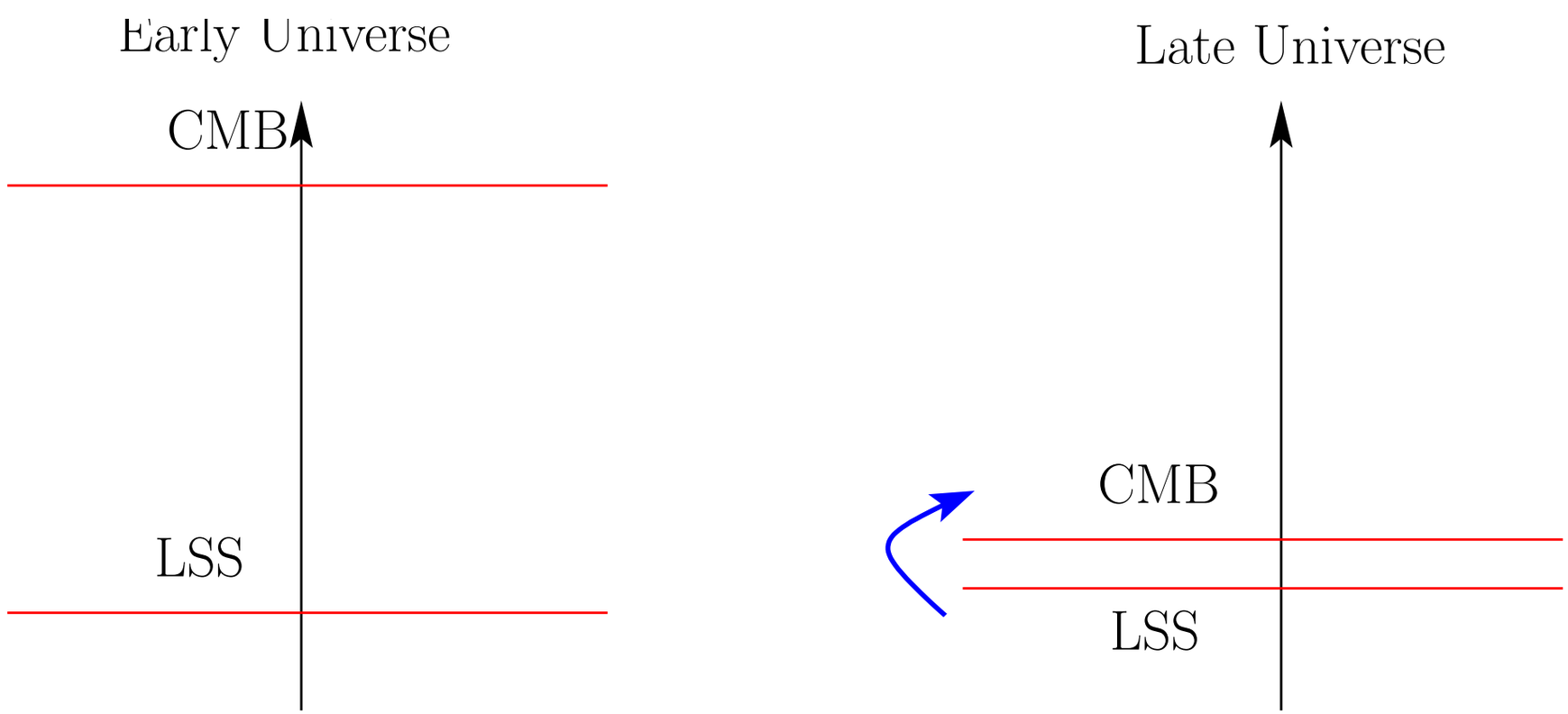
– If you are interested in the physics of the late time universe, such as dark energy or astrophysics, you are fine: a small jump is enough.

The Gathering Storm

– After the completion of the Planck satellite, no guaranteed very large improvement is expected from measurements of the primordial CMB

– How to we continue to explore the beginning of the universe?

– LSS (directly or through CMB) will be the leading next probe. But where do we stand:



– If you are interested in the physics of the late time universe, such as dark energy or astrophysics, you are fine: a small jump is enough.

The Gathering Storm

– But the precision of the CMB and the heroes such as the WMAP and Planck teams, have allowed Cosmology to be part not just of *astrophysics*, but also of the so-called *fundamental sciences*, such as quantum gravity, BSM, etc..

– If we want that to continue to belong to this group, we need to make this happen:

– a huge jump is required

– We have to do it, either with sims or analytics. I will present an analytic approach.

– In both cases, to claim discoveries, we better trust our predictions.

The Gathering Storm

– But the precision of the CMB and the heroes such as the WMAP and Planck teams, have allowed Cosmology to be part not just of *astrophysics*, but also of the so-called *fundamental sciences*, such as quantum gravity, BSM, etc..

– If we want that to continue to belong to this group, we need to make this happen:

– a huge jump is required

– We have to do it, either with sims or analytics. I will present an analytic approach.

– In both cases, to claim discoveries, we better trust our predictions.

The Situation is Grievous

I have nothing to offer but blood, toil, tears and sweat.

Winston Churchill
End of Battle of France, 1940

The Situation is Grievous

I have nothing to offer but blood, toil, tears and sweat.

Winston Churchill
in the movie Dunkirk

The Complete Story

A long, lonely journey

– Dark Matter & Baryons

– Galaxies

– Redshift space

– Of course, none of this would have been possible without the precedent work of people like Bernardeau, Bond, Kaiser, Matsubara, MacDonald, Refregier, Scheth, Scoccimarro, Seljak, White…

– But the EFTofLSS provides a rigorous, convergent formalism to the true answer for $k \ll k_{\rm NL}$

- for this reason, it is unique (up to change of variables)

– With it, we are not trying to answer *Astrophysics* questions (for which the astro-models are enough and you should keep using them). Our purpose is to continue the journey that allowed us to make F*undamental Physics* out of cosmology

- because of this, we have to be very rigorous

Galaxies Statistics

alone **1406**
with Lewandowsky *et al* **1512**
with Perko, et al. **1610**

Galaxies in the EFTofLSS

- The nature of Galaxies is very complicated. If we change the electron mass, the number density of galaxies changes (galaxies are UV sensitive objects).
- So practically impossible to predict

$$n_{\rm gal}(\vec{x},t) = f_{\text{very complicated}}\left[\{H(t'),\Omega_{\rm dm}(t'),\ldots,\rho_{\rm dm}(x',t'),\rho_b(x',t'),\ldots,m_e,m_p,g_{ew},\ldots\}\big|_{\text{on past light cone}}\right]$$

- However, if we are interested only on *long-wavelength* properties of $n_{\rm gal}(t)_k$, we realize that the only objects carrying non trivial space dependence are the fluctuating fields, which, *at long-wavelengths*, are small $\Rightarrow$ we can Taylor expand $f_{\text{very complicated}}$

Galaxies in the EFTofLSS

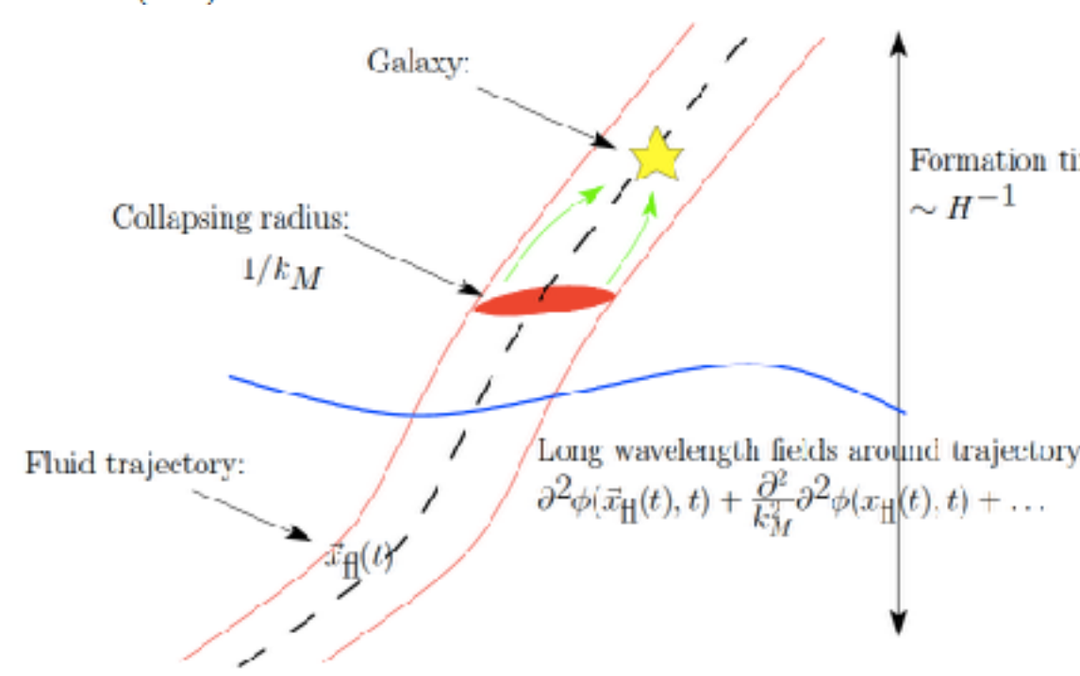
Senatore **1406**

- Therefore

$$n_{\rm gal}(\vec{x},t) = f_{\text{very complicated}}\left[\{H(t'),\Omega_{\rm dm}(t'),\ldots,\rho_{\rm dm}(x',t'),\rho_b(x',t'),\ldots,m_e,m_p,g_{ew},\ldots\}|_{\text{on past light cone}}\right]$$

$$\Downarrow \quad \text{Taylor Expansion}$$

$$\begin{aligned}\delta_M(\vec{x},t) \simeq \int^t dt'\; H(t')\; \Bigg[& \bar{c}_{\partial^2\phi}(t,t')\,\frac{\partial^2\phi(\vec{x}_{\rm fl},t')}{H(t')^2} \\ & + \bar{c}_{\partial_i v^i}(t,t')\,\frac{\partial_i v^i(\vec{x}_{\rm fl},t')}{H(t')} + \bar{c}_{\partial_i\partial_j\phi\partial^i\partial^j\phi}(t,t')\,\frac{\partial_i\partial_j\phi(\vec{x}_{\rm fl},t')}{H(t')^2}\frac{\partial^i\partial^j\phi(\vec{x}_{\rm fl},t')}{H(t')^2} + \ldots \\ & + \bar{c}_{\epsilon}(t,t')\,\epsilon(\vec{x}_{\rm fl},t') + \bar{c}_{\epsilon\partial^2\phi}(t,t')\,\epsilon(\vec{x}_{\rm fl},t')\,\frac{\partial^2\phi(\vec{x}_{\rm fl},t')}{H(t')^2} + \ldots \\ & + \bar{c}_{\partial^4\phi}(t,t')\,\frac{\partial^2_{x_{\rm fl}}}{{k_{\rm M}}^2}\frac{\partial^2\phi(\vec{x}_{\rm fl},t')}{H(t')^2} + \ldots \Bigg].\end{aligned}$$



- where $c_i(t,t') = \dfrac{\delta\, n_M(t,k)}{\delta\, \partial^2\phi(t',k)}$
- all terms allowed by symmetries are present
- all physical effects are included

with Angulo, Fasiello, Vlah **1503**
Schmidt **1602**

Baryons

- Since the dynamics of baryons is described by an EFTofLSS with two species, the galaxies depends on these two fields

$$
\begin{aligned}
\delta_h(\mathbf{x},t) \simeq \int^t dt'\, H(t') \Bigg[& \bar{c}_{\partial^2\phi}(t,t')\, \frac{\partial^2\phi(\mathbf{x}_{\text{fl}},t')}{H(t')^2} + \bar{c}_{\delta_b}(t,t')\, w_b\, \delta_b(\mathbf{x}_{\text{fl}b}) \\
& + \bar{c}_{\partial_i v_c^i}(t,t')\, w_c\, \frac{\partial_i v_c^i(\mathbf{x}_{\text{fl,c}},t')}{H(t')} + \bar{c}_{\partial_i v_b^i}(t,t')\, w_b\, \frac{\partial_i v_b^i(\mathbf{x}_{\text{fl,b}},t')}{H(t')} \\
& + \bar{c}_{\partial_i\partial_j\phi\partial^i\partial^j\phi}(t,t')\, \frac{\partial_i\partial_j\phi(\mathbf{x}_{\text{fl}},t')}{H(t')^2} \frac{\partial^i\partial^j\phi(\mathbf{x}_{\text{fl}},t')}{H(t')^2} + \dots \\
& + \bar{c}_{\epsilon_c}(t,t')\, w_c\, \epsilon_c(\mathbf{x}_{\text{fl,c}},t') + \bar{c}_{\epsilon_b}(t,t')\, w_b\, \epsilon_b(\mathbf{x}_{\text{fl,b}},t') \\
& + \bar{c}_{\epsilon_c\partial^2\phi}(t,t')\, w_c\, c_c(\mathbf{x}_{\text{fl,c}},t') \frac{\partial^2\phi(\mathbf{x}_{\text{fl}},t')}{H(t')^2} + \bar{c}_{\epsilon_b\partial^2\phi}(t,t')\, w_b\, c_b(\mathbf{x}_{\text{fl,b}},t') \frac{\partial^2\phi(\mathbf{x}_{\text{fl}},t')}{H(t')^2} \dots \\
& + \bar{c}_{\partial^4\phi}(t,t')\, \frac{\partial^2_{x_{\text{fl}}}}{{k_{\text{M}}}^2} \frac{\partial^2\phi(\mathbf{x}_{\text{fl}},t')}{H(t')^2} + \sum_{\sigma,\sigma'=b,c} w_\sigma\, v^i_{\sigma,\text{CM}}(\mathbf{x}_{\text{fl}\sigma},t') \frac{\partial_i\delta_{\sigma'}(\mathbf{x}_{\text{fl}\sigma'},t')}{H} + \dots \Bigg] ,
\end{aligned}
$$

- Notice
  - doubling of bias parameters, but weighted by w_b
  - presence of velocity in center of mass (so Compton effect already included)

Primordial Non-Gaussianities

with Angulo, Fasiello, Vlah **1503**
Assassi et al **1506,** Assassi et al **1509,**
with Lewandowsky *et al* **1512**

- In the case of primordial non-Gaussianity, short-mode collapse is controlled not just by the dynamical effects of the long modes, but also by their coupled initial conditions.

- $\Longrightarrow$ Short modes initial conditions are sensitive to the squeezed limit $k_L \ll k_S$,

$$\zeta(\mathbf{k}_S) \simeq \zeta_g(\mathbf{k}_S) + f_{\rm NL}\left(\frac{k_L}{k_S}\right)^{\alpha} \zeta_g(\mathbf{k}_S - \mathbf{k}_L)\zeta_g(\mathbf{k}_L)\ ,$$

$$\Longrightarrow\ \delta^{(1)}(\mathbf{k}_S, t_{\rm in}) \simeq \delta_g(\mathbf{k}_S) + f_{\rm NL}\tilde{\phi}(\mathbf{k}_L, t_{\rm in})\delta_g(\mathbf{k}_S - \mathbf{k}_L, t_{\rm in})\ ,$$

$$\tilde{\phi}(k_L, t_{\rm in}) \sim \frac{1}{T(k)}\left(\frac{k_L}{k_S}\right)^{\alpha} \delta_g(k_L, t_{\rm in})$$

- $\Longrightarrow\ \langle\delta_s^2\rangle_l(\mathbf{x}_{\rm in}, t_{\rm in}) \supset \langle\delta_s^2\rangle_0(t_{\rm in}) f_{\rm NL}\tilde{\phi}(\mathbf{x}_{\rm in}, t_{\rm in})$

$$\Longrightarrow\ \delta_h(\mathbf{x}, t) \simeq f_{\rm NL}\,\tilde{\phi}(\mathbf{x}_{\rm fl}(t, t_{\rm in}), t_{\rm in}) \int^t dt'\ H(t')\ \Big[\bar{c}^{\,\tilde{\phi}}(t, t') + \bar{c}^{\,\tilde{\phi}}_{\partial^2\phi}(t, t')\ \frac{\partial^2\phi(\mathbf{x}_{\rm fl}, t')}{H(t')^2}$$

$$+\bar{c}^{\,\tilde{\phi}}_{\partial_i v^i}(t, t')\ \frac{\partial_i v^i(\mathbf{x}_{\rm fl}, t')}{H(t')} + \bar{c}^{\,\tilde{\phi}}_{\partial_i\partial_j\phi\partial^i\partial^j\phi}(t, t')\ \frac{\partial_i\partial_j\phi(\mathbf{x}_{\rm fl}, t')}{H(t')^2}\frac{\partial^i\partial^j\phi(\mathbf{x}_{\rm fl}, t')}{H(t')^2} +$$

$$+\bar{c}^{\,\tilde{\phi}}_{\epsilon}(t, t')\ \epsilon(\mathbf{x}_{\rm fl}, t') + \bar{c}^{\,\tilde{\phi}}_{\epsilon\partial^2\phi}(t, t')\ \epsilon(\mathbf{x}_{\rm fl}, t')\frac{\partial^2\phi(\mathbf{x}_{\rm fl}, t')}{H(t')^2} + \dots$$

$$+\bar{c}^{\,\tilde{\phi}}_{\partial^4\phi}(t, t')\ \frac{\partial^2_{x_{\rm fl}}}{{k_{\rm M}}^2}\frac{\partial^2\phi(\mathbf{x}_{\rm fl}, t')}{H(t')^2} + \dots\Big]$$

$$+{f_{\rm NL}}^2\ \tilde{\phi}(\mathbf{x}_{\rm fl}(t, t_{\rm in}), t_{\rm in})^2 \int^t dt'\ H(t')\ \Big[\bar{c}^{\,\tilde{\phi}^2}(t, t') + \bar{c}^{\,\tilde{\phi}^2}_{\partial^2\phi}(t, t')\ \frac{\partial^2\phi(\mathbf{x}_{\rm fl}, t')}{H(t')^2} + \dots\Big] +$$

Novel functional form
non-compatible with GR

Redshift space

with Zaldarriaga **1409**
with Lewandowsky *et al* **1512**

Counterterms

with Zaldarriaga **1409**
with Lewandowsky *et al* **1512**

- Need for counterterms (expectation value on short modes)

$$\delta_r(\vec{k}) \simeq \delta(\vec{k}) - i\frac{k_z}{aH}v_z(\vec{k}) + \frac{i^2}{2}\left(\frac{k_z}{aH}\right)^2 [v_z^2]_{\vec{k}} - \frac{i^3}{3!}\left(\frac{k_z}{aH}\right)^3 [v_z^3]_{\vec{k}} + \dots$$

$$[v_z^2]_{R,\vec{k}} = [v_z^2]_{\vec{k}} + \left(\frac{aH}{k^r_{\rm NL}}\right)^2 \left[c_{11}\delta_D^{(3)}(\vec{k}) + \left(c_{12} + c_{13}\mu^2\right)\delta(\vec{k})\right]$$

expectation value

response

- Baryons, Primordial NG included

- Now, all pieces ingredients are prepared.

The Final Piece:
Galaxies in Redshift Space

with Perko, Jennings, Wechsler **1610**

Galaxies in Redshift space in the EFTofLSS

- We have all the ingredients:

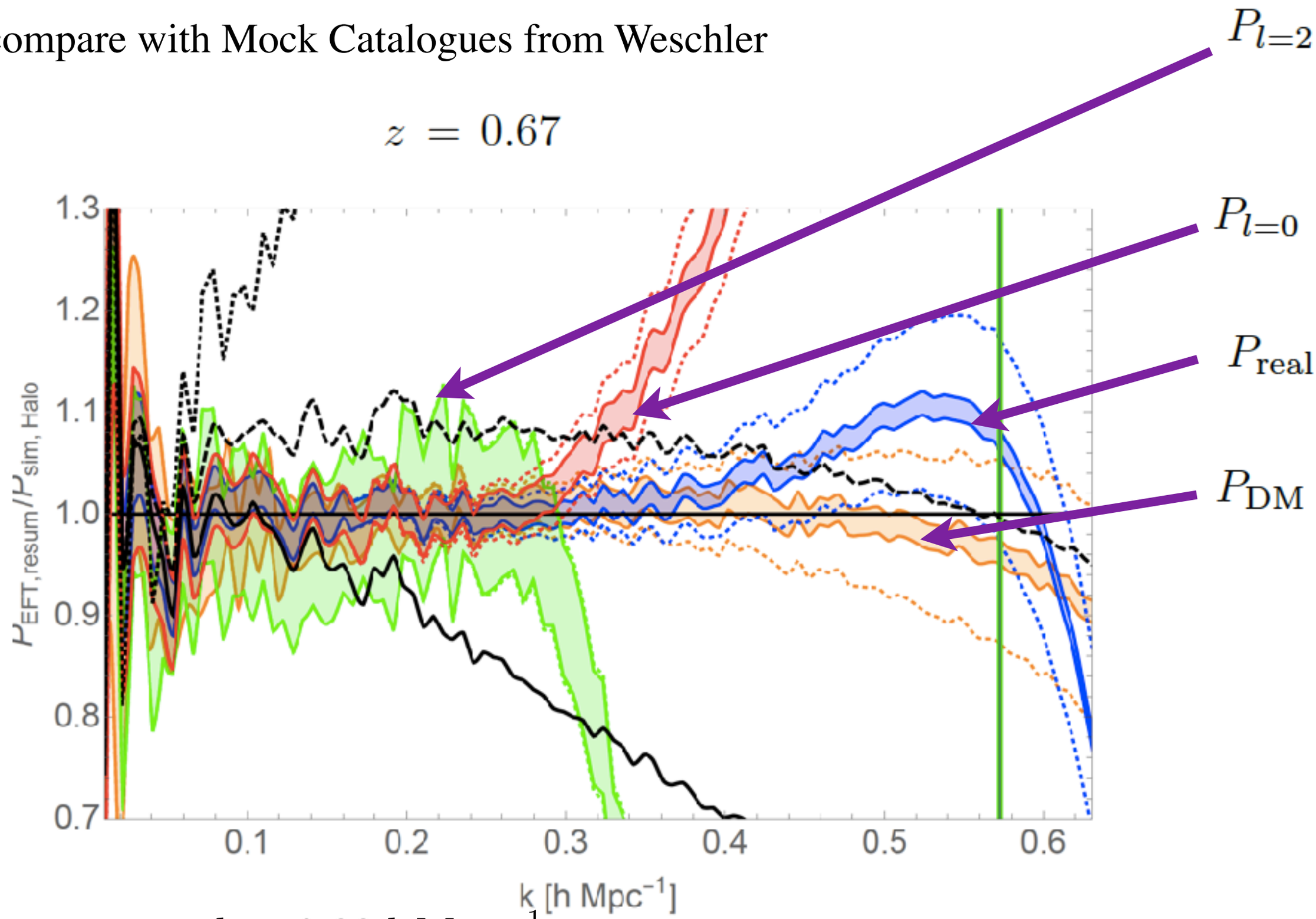
$$\delta_{h,r}(\vec{k}) = \delta_h(\vec{x}) + \int d^3x \; e^{-i\vec{k}\cdot\vec{x}} \left(\exp\left(-i \frac{k_z}{aH} v_{h,z}(\vec{x}) \right) - 1 \right) (1 + \delta_h(\vec{x}))$$

$$\delta_{h,r}(\vec{k}) = \delta_h(\vec{k}) - i\frac{k_z}{aH} v_{h,z}(\vec{k}) + \frac{i^2}{2}\left(\frac{k_z}{aH}\right)^2 [v_{h,z}^2]_{\vec{k}} - \frac{i^3}{3!}\left(\frac{k_z}{aH}\right)^3 [v_{h,z}^3]_{\vec{k}} - i\frac{k_z}{aH}[v_{h,z}\delta_h]_{\vec{k}}$$

- renormalize them as for dark matter

Results

- One-loop prediction from the EFTofLSS
- We compare with Mock Catalogues from Weschler

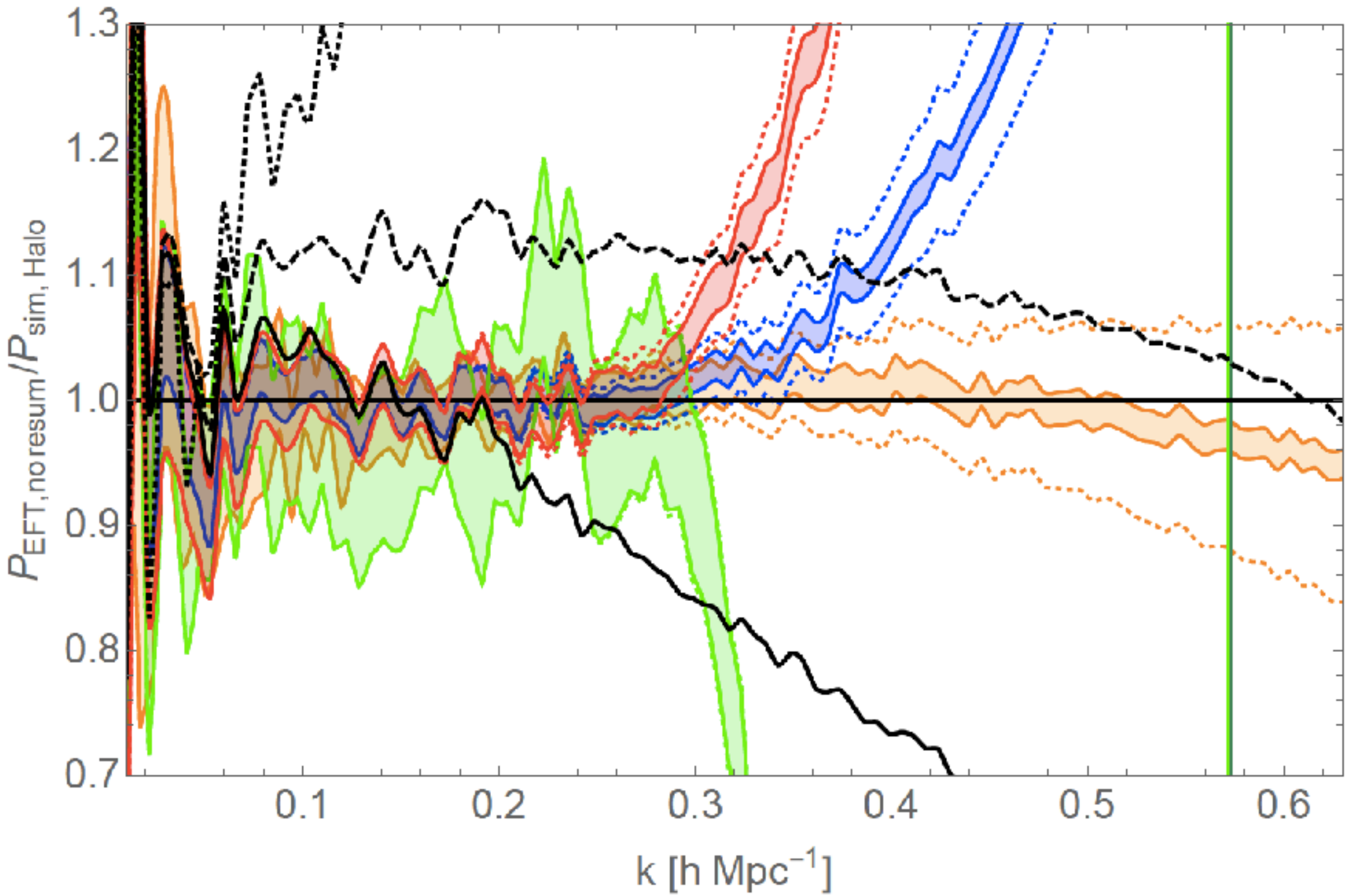


- It matches until $k \simeq 0.33\, h\,\mathrm{Mpc}^{-1}$!!

Comments on Results

- IR-resummation done, it works, and it is very important

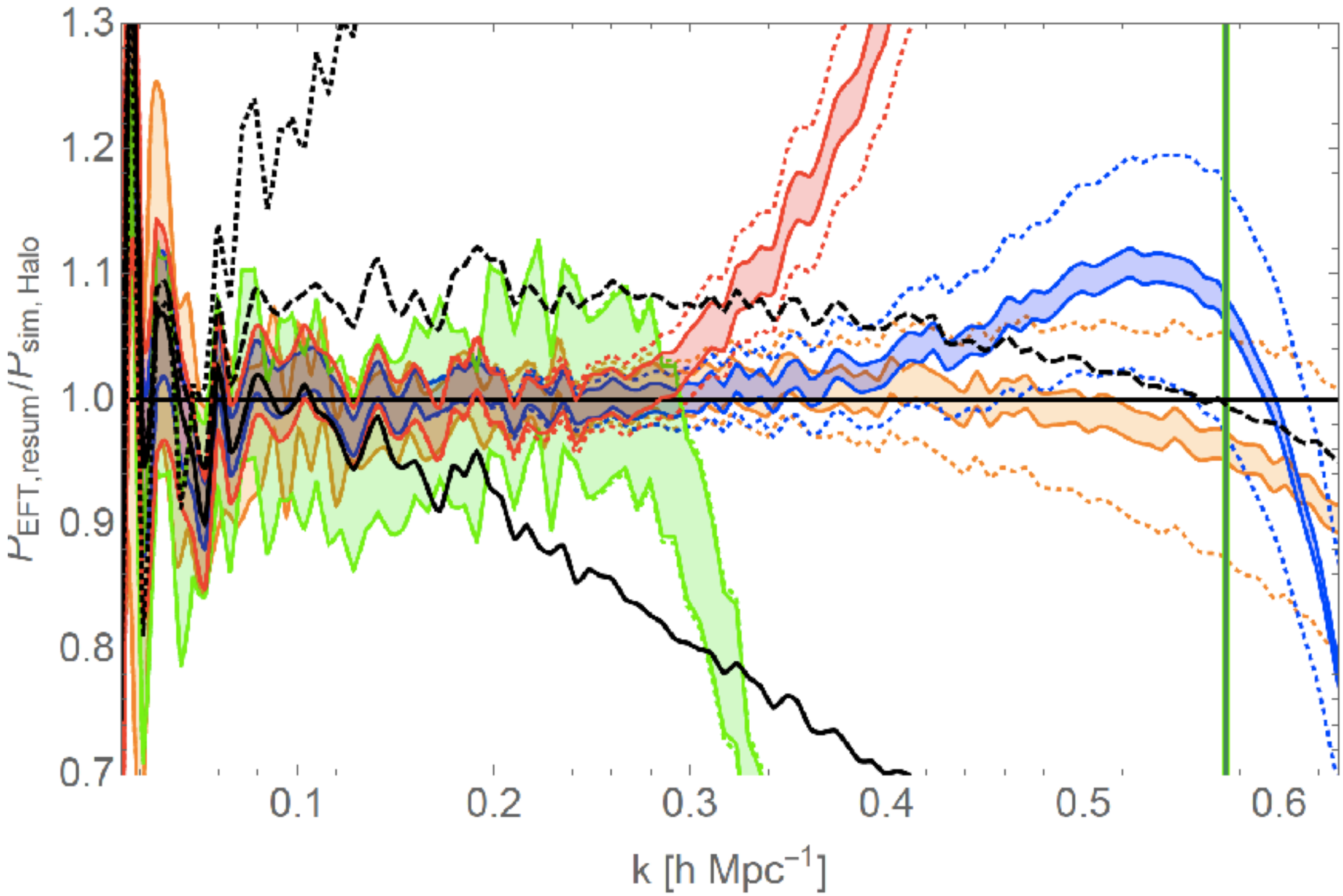
with Zaldarriaga **1406**
with Lewandowsky *et al* **1512**



Comments on Results

- IR-resummation done, it works, and it is very important

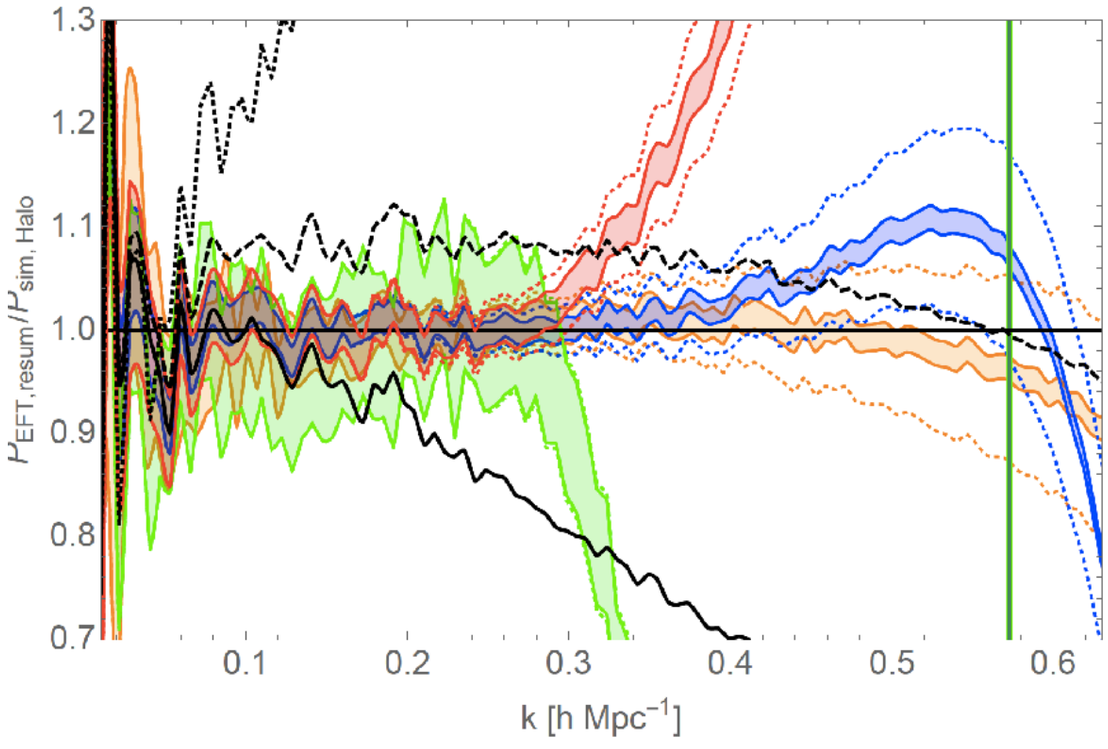
with Zaldarriaga **1406**
with Lewandowsky *et al* **1512**



Comments on Results

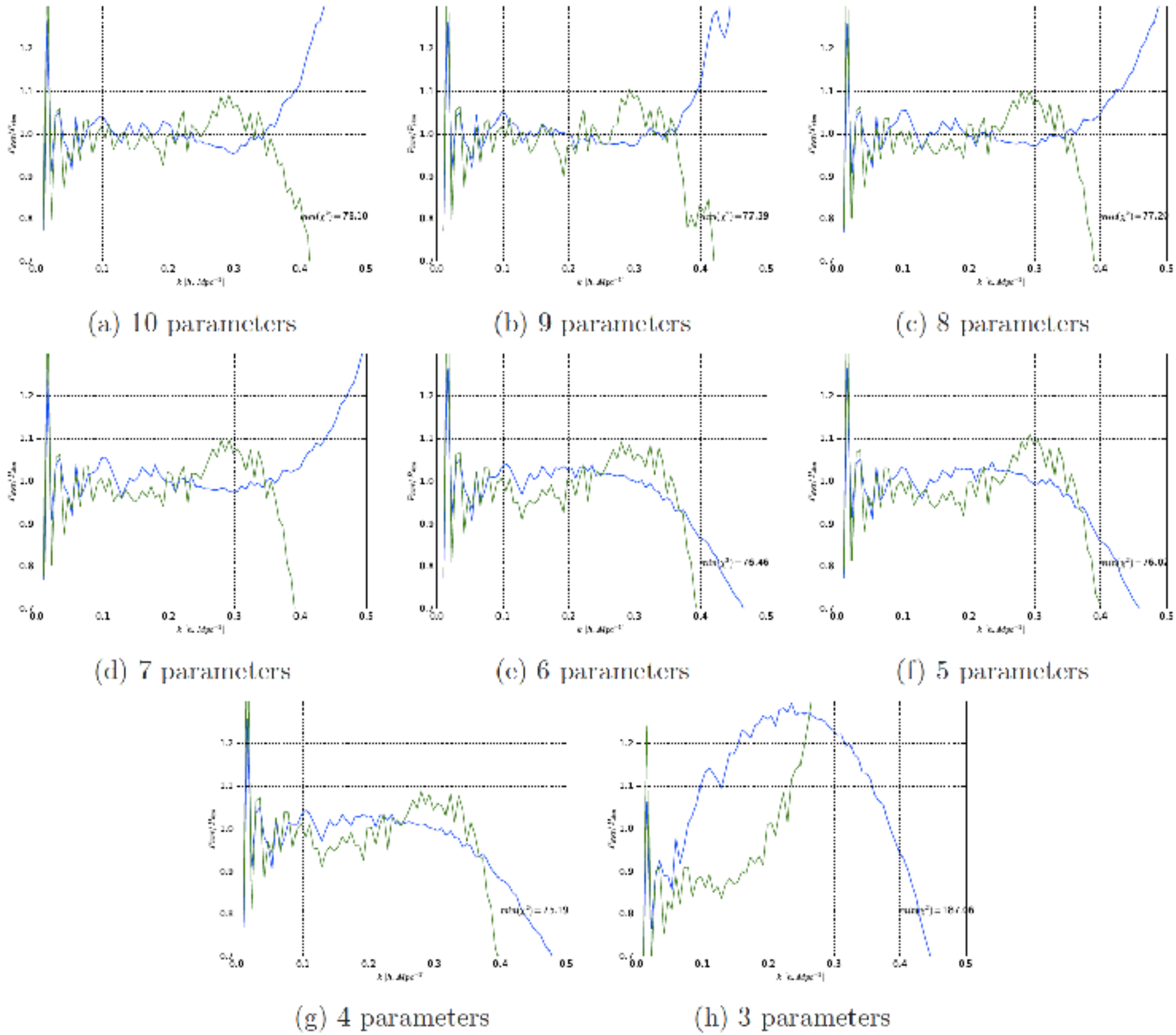
- 10 parameters used, but will be reduced somewhat after optimization
- Less parameters and probably higher k reach that the renown models (and we obtained this much earlier)
- This answer yesterday's question if comparison with Galaxies has been made
- We have done the fit including theory errors, which are available for the EFTofLSS (and the plot is quite unchanged)

- So we are ready to analyze SDSS data!!



How many parameters?

- We can drop out data-degenerate parameters. We obtain equivalent fit with just 4 or parameters!
- Consistent with
- cov. eigenvalues



(a) 10 parameters (b) 9 parameters (c) 8 parameters

(d) 7 parameters (e) 6 parameters (f) 5 parameters

(g) 4 parameters (h) 3 parameters

The EFTofLSS extended to Massive Neutrinos

with Zaldarriaga **1707**

Neutrinos in LSS

- Shall we detect neutrinos in LSS? Will people outside LSS believe it?

- Let us try to formulate a rigorous and satisfactory perturbative approach.

- Dark matter is described as an effective fluid-like system up to $k \lesssim k_{\rm NL}$, as $k_{\rm fs,\,dm} \simeq k_{\rm NL}$

- Neutrinos have different velocities, and most of them have a free streaming length longer than the non-linear scale $k_{\rm fs} \ll k_{\rm NL}$

- In order to have a description that works up to $k_{\rm fs} \ll k \ll k_{\rm NL}$, we need to describe neutrinos with their Boltzmann equation

Set-up equations

- Boltzmann equation for Neutrinos

$$\frac{\partial f(\vec{x},\vec{v},\tau)}{d\tau} + \frac{v^i}{a}\frac{\partial f(\vec{x},\vec{v},\tau)}{\partial x^i} - a\frac{\partial \Phi}{\partial x^i}\frac{\partial f(\vec{x},\vec{v},\tau)}{\partial v^i} = 0 \ ,$$

- Effective fluid-like system for dark matter

$$\frac{\partial^2}{a^2}\Phi = \frac{3}{2}H_0^2\frac{a_0^3}{a^3}\left(\Omega_{\mathrm{dm},0}\delta_{\mathrm{dm}} + \Omega_{\nu,0}\delta_\nu\right) \simeq \frac{3}{2}H_0^2\frac{a_0^3}{a^3}\Omega_{\mathrm{NR},0}\left(\delta_{\mathrm{dm}} + f_\nu\left(\delta_\nu - \delta_{\mathrm{dm}}\right)\right)$$

$$\frac{1}{a}\frac{\partial \delta_{\mathrm{dm}}}{\partial \tau} + \frac{1}{a\rho_{\mathrm{dm}}^{(0)}}\partial_i \pi_{\mathrm{dm}}^i = 0 \ ,$$

$$\frac{1}{a}\frac{\partial \pi_{\mathrm{dm}}^i}{\partial \tau} + 4H\pi_{\mathrm{dm}}^i + \frac{1}{a}\partial_j\left(\frac{\pi_{\mathrm{dm}}^i\pi_{\mathrm{dm}}^j}{\rho_{\mathrm{dm}}^{(0)}(1+\delta_{\mathrm{dm}})}\right) + \frac{1}{a}\rho_{\mathrm{dm}}^{(0)}(1+\delta_{\mathrm{dm}})\partial^i\Phi = -\frac{1}{a}\partial_j\tau^{ij} \ .$$

- Example of quantities:

$$\rho(\vec{x},t) = \rho_{\mathrm{dm}}(\vec{x},t) + \rho_\nu(\vec{x},t) \quad \Rightarrow \quad \delta = (1-f_\nu)\delta_{\mathrm{dm}} + f_\nu\,\delta_\nu = \delta_{\mathrm{dm}} + f_\nu\left(\delta_\nu - \delta_{\mathrm{dm}}\right) \ ,$$

$$\langle \delta(\vec{k},t)\,\delta(\vec{k}',t)\rangle = \left(P_{\mathrm{dm,\,dm}}(k) + 2f_\nu\,P_{\mathrm{diff,\,dm}}(k)\right)$$

$$\delta_{\mathrm{diff}} = \delta_\nu - \delta_{\mathrm{dm}}$$

Set-up equations

- Boltzmann equation for Neutrinos

$$\frac{\partial f(\vec{x},\vec{v},\tau)}{d\tau} + \frac{v^i}{a}\frac{\partial f(\vec{x},\vec{v},\tau)}{\partial x^i} - a\frac{\partial \Phi}{\partial x^i}\frac{\partial f(\vec{x},\vec{v},\tau)}{\partial v^i} = 0 \ ,$$

- Effective fluid-like system for dark matter

$$\frac{\partial^2}{a^2}\Phi = \frac{3}{2}H_0^2\frac{a_0^3}{a^3}\left(\Omega_{\mathrm{dm},0}\delta_{\mathrm{dm}} + \Omega_{\nu,0}\delta_\nu\right) \simeq \frac{3}{2}H_0^2\frac{a_0^3}{a^3}\Omega_{\mathrm{NR},0}\left(\delta_{\mathrm{dm}} + f_\nu\left(\delta_\nu - \delta_{\mathrm{dm}}\right)\right)$$

$$\frac{1}{a}\frac{\partial \delta_{\mathrm{dm}}}{\partial \tau} + \frac{1}{a\rho_{\mathrm{dm}}^{(0)}}\partial_i \pi_{\mathrm{dm}}^i = 0 \ ,$$

$$\frac{1}{a}\frac{\partial \pi_{\mathrm{dm}}^i}{\partial \tau} + 4H\pi_{\mathrm{dm}}^i + \frac{1}{a}\partial_j\left(\frac{\pi_{\mathrm{dm}}^i\pi_{\mathrm{dm}}^j}{\rho_{\mathrm{dm}}^{(0)}(1+\delta_{\mathrm{dm}})}\right) + \frac{1}{a}\rho_{\mathrm{dm}}^{(0)}(1+\delta_{\mathrm{dm}})\partial^i\Phi = -\frac{1}{a}\partial_j\tau^{ij} \ .$$

- Example of quantities:

$$\rho(\vec{x},t) = \rho_{\mathrm{dm}}(\vec{x},t) + \rho_\nu(\vec{x},t) \quad \Rightarrow \quad \delta = (1-f_\nu)\delta_{\mathrm{dm}} + f_\nu\,\delta_\nu = \delta_{\mathrm{dm}} + f_\nu\left(\delta_\nu - \delta_{\mathrm{dm}}\right) \ ,$$

$$\langle \delta(\vec{k},t)\,\delta(\vec{k}',t)\rangle = \left(P_{\mathrm{dm,\,dm}}(k) + 2f_\nu\,P_{\mathrm{diff,\,dm}}(k)\right)$$

$$\delta_{\mathrm{diff}} = \delta_\nu - \delta_{\mathrm{dm}}$$

Set-up equations

- Boltzmann equation for Neutrinos

$$\frac{\partial f(\vec{x},\vec{v},\tau)}{d\tau} + \frac{v^i}{a}\frac{\partial f(\vec{x},\vec{v},\tau)}{\partial x^i} - a\frac{\partial \Phi}{\partial x^i}\frac{\partial f(\vec{x},\vec{v},\tau)}{\partial v^i} = 0 \ ,$$

- Effective fluid-like system for dark matter

$$\frac{\partial^2}{a^2}\Phi = \frac{3}{2}H_0^2\frac{a_0^3}{a^3}\left(\Omega_{\mathrm{dm},0}\delta_{\mathrm{dm}} + \Omega_{\nu,0}\delta_\nu\right) \simeq \frac{3}{2}H_0^2\frac{a_0^3}{a^3}\Omega_{\mathrm{NR},0}\left(\delta_{\mathrm{dm}} + f_\nu\left(\delta_\nu - \delta_{\mathrm{dm}}\right)\right)$$

$$\frac{1}{a}\frac{\partial \delta_{\mathrm{dm}}}{\partial \tau} + \frac{1}{a\rho_{\mathrm{dm}}^{(0)}}\partial_i \pi_{\mathrm{dm}}^i = 0 \ ,$$

$$\frac{1}{a}\frac{\partial \pi_{\mathrm{dm}}^i}{\partial \tau} + 4H\pi_{\mathrm{dm}}^i + \frac{1}{a}\partial_j\left(\frac{\pi_{\mathrm{dm}}^i\pi_{\mathrm{dm}}^j}{\rho_{\mathrm{dm}}^{(0)}(1+\delta_{\mathrm{dm}})}\right) + \frac{1}{a}\rho_{\mathrm{dm}}^{(0)}(1+\delta_{\mathrm{dm}})\partial^i\Phi = -\frac{1}{a}\partial_j\tau^{ij} \ .$$

- Example of quantities:

$$\rho(\vec{x},t) = \rho_{\mathrm{dm}}(\vec{x},t) + \rho_\nu(\vec{x},t) \quad \Rightarrow \quad \delta = (1-f_\nu)\delta_{\mathrm{dm}} + f_\nu\,\delta_\nu = \delta_{\mathrm{dm}} + f_\nu\left(\delta_\nu - \delta_{\mathrm{dm}}\right) \ ,$$

$$\langle \delta(\vec{k},t)\,\delta(\vec{k}',t)\rangle = \left(P_{\mathrm{dm,\,dm}}(k) + 2f_\nu P_{\mathrm{diff,\,dm}}(k)\right)$$

$$\delta_{\mathrm{diff}} = \delta_\nu - \delta_{\mathrm{dm}}$$

Perturbative Solution

- We solve at leading order in $f_\nu \ll 1 \Rightarrow$ neutrinos move in external gravitational field
- Expansion parameter: consider the time Δt to cross a region of size $L \sim 1/k$.

$$x(t) \quad x(t_{\rm in}) \sim \int^t dt_1 \left(v \quad \int^{t_1} dt_2\, \partial\Phi \left(x \quad \int^{t_2} dt_3 \left(v \quad \int^{t_3} dt_4\, \partial\Phi\, (x \quad \ldots) \right) \right) \right)$$
$$\sim v\Delta t + \partial\Phi\, (\Delta t)^2 + \partial^2\Phi\, v\, (\Delta t)^3 + \ldots\,.$$

- Corrections go as $$\frac{\delta x^{(2)}}{\delta x^{(1)}} \sim \frac{\partial\Phi\, \Delta t}{v}$$

- Clearly $\Delta t \sim \mathrm{Min}[1/H, 1/(kv)]$, so we have fast and slow neutrinos according if they have enough time to cross a region. If they are slow, $\Delta t \sim 1/H$ and they do not cross the whole region but only a region $1/k_{\rm fs}$, and we have:

$$\left.\frac{\delta x^{(2)}}{\delta x^{(1)}}\right|_{k_{\rm fs} \gtrsim k} \sim \frac{\partial^2\Phi}{k_{\rm fs}} \cdot \frac{1}{vH} \sim \frac{\partial^2\Phi}{H^2} \sim \delta(k) \quad \text{for} \quad k_{\rm fs} \gtrsim k$$

- Fast Neutrinos, instead: $$\left.\frac{\delta x^{(2)}}{\delta x^{(1)}}\right|_{k_{\rm fs} \lesssim k} \sim \frac{\partial\Phi}{v} \cdot \frac{1}{vk} \sim \frac{\Phi}{v^2} \sim \delta(k) \left(\frac{k_{\rm fs}}{k}\right)^2 , \quad \text{for} \quad k_{\rm fs} \lesssim k$$

- For fast neutrinos, expansion parameter $\ll 1$ at all k's!

Perturbative Solution

- $$f^{[n]}(\vec{k},\vec{v},\tau) = \int_0^\tau d\tau'\, G_R(\tau,\tau';\vec{v},k)\left[a(\tau')\left(\frac{\partial}{\partial x^i}\Phi(\vec{x},\tau')\right)\frac{\partial}{\partial v^i}f^{[n-1]}(\vec{x},\vec{v},\tau')\right]_{\vec{k}}$$

- First order:

$$f^{[1]}(\vec{k},\vec{v},s) = \int_{s_i}^{s} ds'\, a(s')^2\, e^{-i\,\vec{k}\cdot\vec{v}\,(s-s')}\, i\frac{\vec{v}}{v}\cdot\vec{k}\,\Phi(\vec{k},s')\frac{\partial f^{[0]}(v)}{\partial v}$$

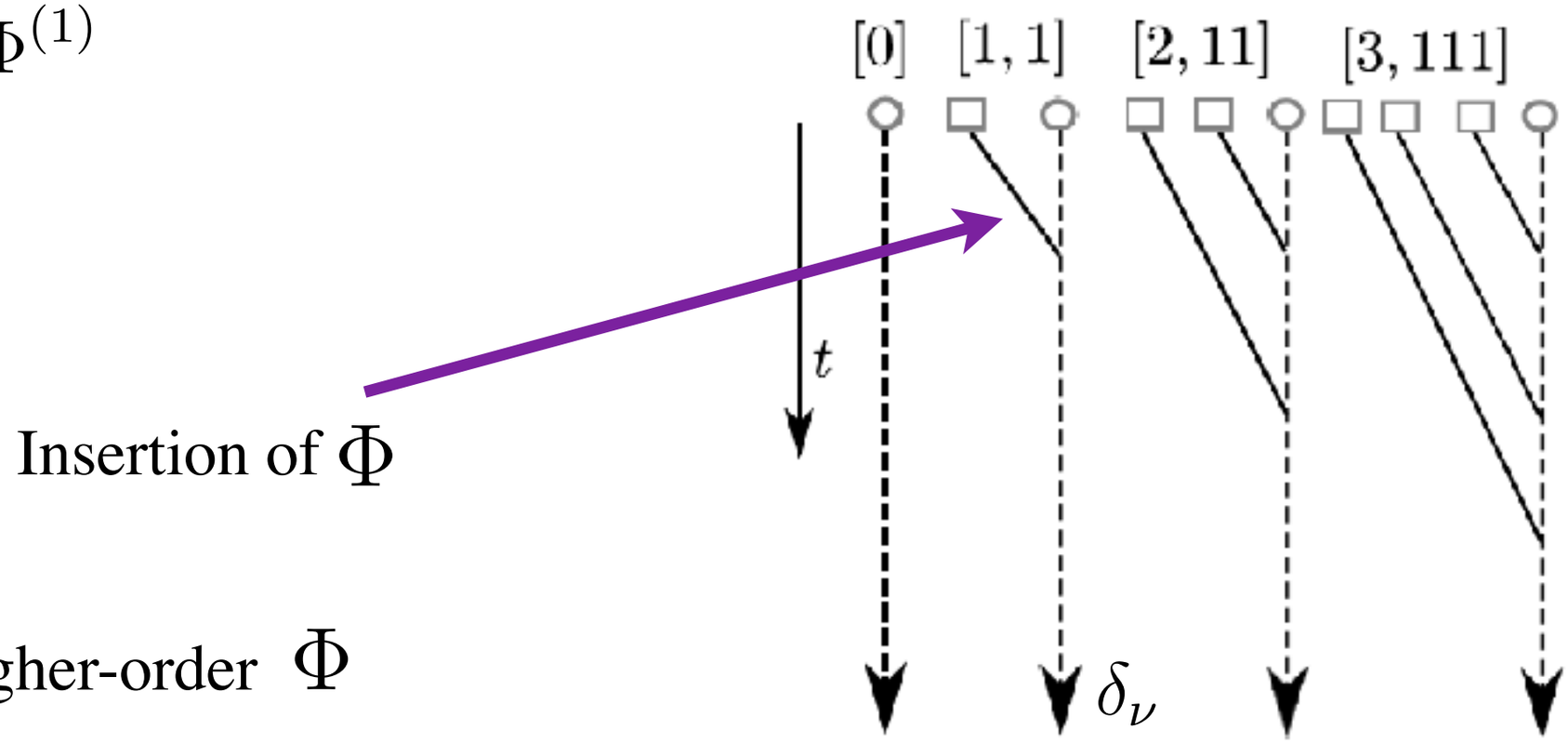
- Second order:

$$f^{[2]}(\vec{k},\vec{v},s) = \int\frac{d^3q_1}{(2\pi)^3}\int\frac{d^3q_2}{(2\pi)^3}\,(2\pi)^3\delta_D^{(3)}(\vec{k}-\vec{q}_1-\vec{q}_2)$$
$$\int_0^s ds_1\, a(s_1)^2\, e^{-i\,(\vec{q}_1+\vec{q}_2)\cdot\vec{v}\,(s-s_1)}\, i\, q_1^{i_1}\Phi(\vec{q}_1,s_1)$$
$$\frac{\partial}{\partial v^{i_1}}\int_0^{s_1} ds_2\, a(s_2)^2\, e^{-i\,\vec{q}_2\cdot\vec{v}\,(s_1-s_2)}\, i\, q_2^{i_2}\,\Phi(\vec{q}_2,s_2)\frac{\partial f^{[0]}(v)}{\partial v^{i_2}}$$

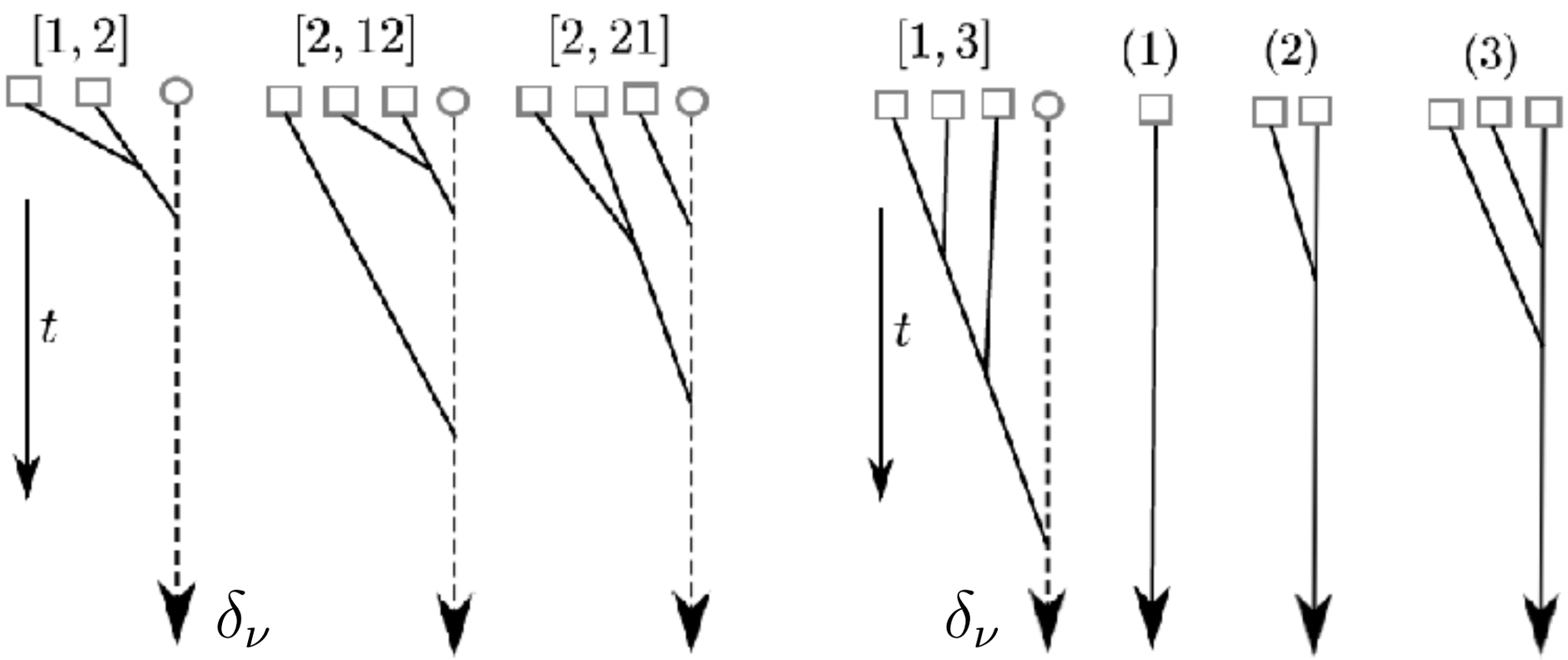
- Every insertion scales as $v_{\rm NL}/v \ll 1$ for fast neutrinos, and as $\delta(q)$ for slow ones.
- Notice that expression is sensitive to product of high wavenumber fields: $q_1, q_2 \gg k$
  - These part of phase space is not under control: need to be renormalized

Diagrammatic Representation

- Solutions with $\Phi^{(1)}$



- Solutions with higher-order Φ



Renormalization of Local Product

- We need to renormalize the local product of fields we are sensitive to. High wavenumber means local in space.

$$[\delta(\vec{x}_1,\tau_1)\,\delta(\vec{x}_2,\tau_2)]_{\rm R} = [\delta(\vec{x}_1,\tau_1)]_{\rm R}\,[\delta(\vec{x}_2,\tau_2)]_{\rm R} + [\delta(\vec{x}_1,\tau_1)\,\delta(\vec{x}_2,\tau_2)]_{\rm C}\ ,$$

$$\downarrow$$

$$[\delta(\vec{x}_1,t_1)\delta(\vec{x}_2,t_2)]_{\rm C} = \delta_D^{(3)}(\vec{x}_1-\vec{x}_2)\ \ f_{\text{very complicated}}\,(\text{everything},\ldots,\partial_i\partial_j\Phi(x)\ \ldots)|_{\text{past light cone}}$$

$=$ at long wavelengths we can Taylor expand $=$

$$[\delta(\vec{x}_1,\tau_1)\delta(\vec{x}_2,\tau_2)]_{\rm C} = \int^{\rm Max(\tau_1,\tau_2)} d\tau'\ \Big[C_1^{(2)}(\tau_1,\tau_2,\tau') + \epsilon^{(2)}_{\rm stoch,1}(\tau_1,\tau_2,\vec{x}_{\rm fl}(\vec{x}_1,\tau_1,\tau'),\tau') +$$

$$\partial^i\partial^j\Phi(\vec{x}_{\rm fl}(\vec{x}_1,\tau,\tau'),\tau')\left(C_2^{(2)}(\tau,\tau')\delta^{ij} + C_3^{(2)}(\tau,\tau')\frac{\partial_i\partial_j}{\partial^2}\right) + \qquad ($$

$$\epsilon^{(2),ij}_{\rm stoch,2}(\tau_1,\tau_2,\vec{x}_{\rm fl}(\vec{x}_1,\tau_1,\tau'),\tau')\frac{\partial_i\partial_j}{\partial^2} + \ldots\Big]\ \ \frac{1}{k_{\rm NL}^3}\,\delta_D^{(3)}\left(\vec{x}_{\rm fl}(\vec{x}_1,\tau_1,\tau') - \vec{x}_{\rm fl}(\vec{x}_2,\tau_2,\tau')\right)\ .$$

- These counterterms are able to fix any UV dependence of the perturbative diagrams

Renormalization of Local Product

- We need to renormalize the local product of fields we are sensitive to. High wavenumber means local in space.

$$[\delta(\vec{x}_1,\tau_1)\,\delta(\vec{x}_2,\tau_2)]_{\rm R} = [\delta(\vec{x}_1,\tau_1)]_{\rm R}\,[\delta(\vec{x}_2,\tau_2)]_{\rm R} + [\delta(\vec{x}_1,\tau_1)\,\delta(\vec{x}_2,\tau_2)]_{\rm C}\ ,$$

$$[\delta(\vec{x}_1,t_1)\delta(\vec{x}_2,t_2)]_{\rm C} = \delta_D^{(3)}(\vec{x}_1-\vec{x}_2)\ \ f_{\text{very complicated}}\,(\text{everything},\ldots,\partial_i\partial_j\Phi(x)\ \ldots)|_{\text{past light cone}}$$

$=$ at long wavelengths we can Taylor expand $=$

Expectation value

Responses

$$[\delta(\vec{x}_1,\tau_1)\delta(\vec{x}_2,\tau_2)]_{\rm C} = \int^{\rm Max(\tau_1,\tau_2)} d\tau'\ \Big[C_1^{(2)}(\tau_1,\tau_2,\tau') + \epsilon^{(2)}_{\rm stoch,1}(\tau_1,\tau_2,\vec{x}_{\rm fl}(\vec{x}_1,\tau_1,\tau'),\tau') +$$

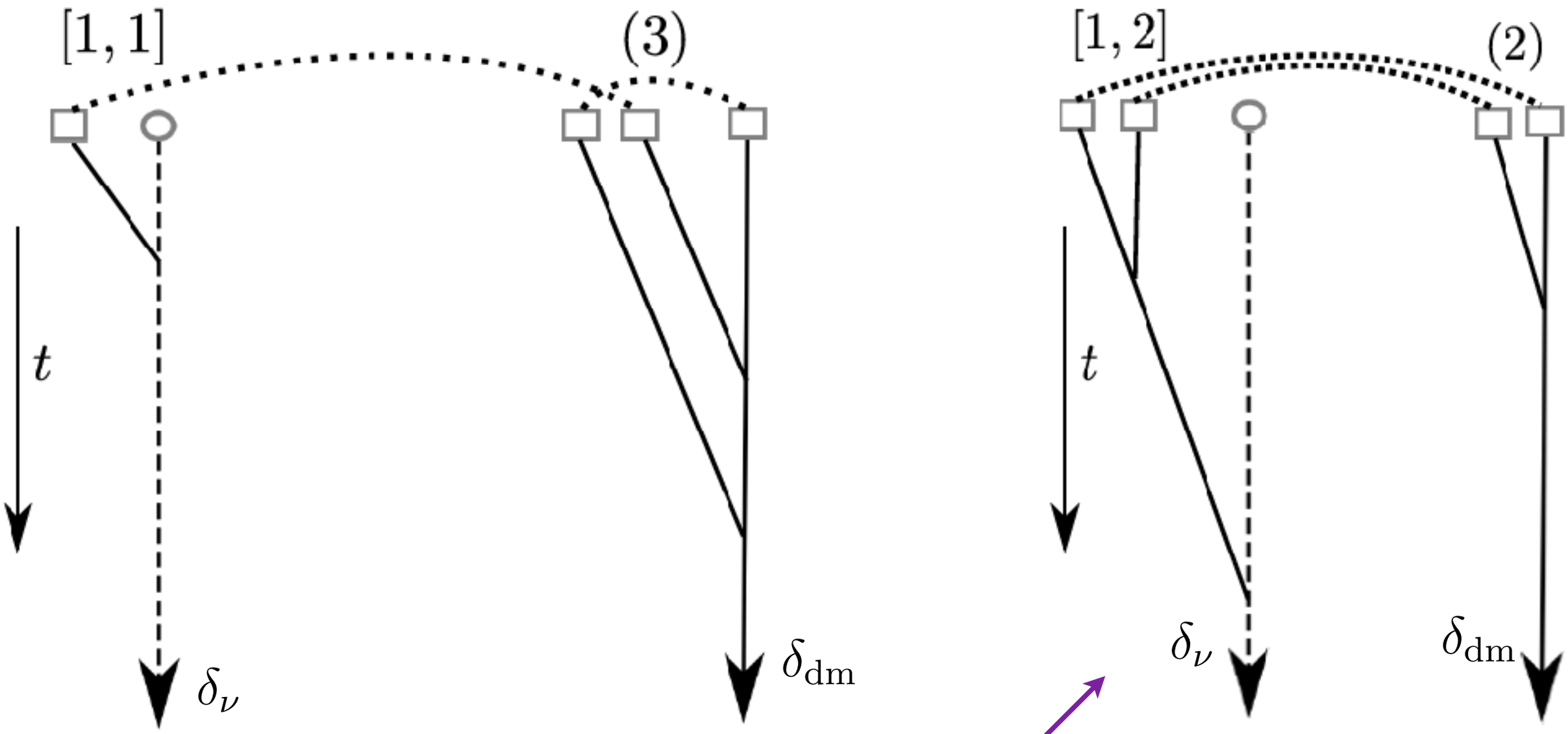
$$\partial^i\partial^j\Phi(\vec{x}_{\rm fl}(\vec{x}_1,\tau,\tau'),\tau')\left(C_2^{(2)}(\tau,\tau')\delta^{ij} + C_3^{(2)}(\tau,\tau')\frac{\partial_i\partial_j}{\partial^2}\right) + \qquad ($$

$$\epsilon^{(2),ij}_{\rm stoch,2}(\tau_1,\tau_2,\vec{x}_{\rm fl}(\vec{x}_1,\tau_1,\tau'),\tau')\frac{\partial_i\partial_j}{\partial^2} + \ldots\Big]\ \ \frac{1}{k_{\rm NL}^3}\,\delta_D^{(3)}\left(\vec{x}_{\rm fl}(\vec{x}_1,\tau_1,\tau') - \vec{x}_{\rm fl}(\vec{x}_2,\tau_2,\tau')\right)\ .$$

- These counterterms fix any UV dependence of the perturbative diagrams.

Diagrams for $P_{\mathrm{diff,\,dm}}$

- Evaluate diagrams. Examples:



$$P^{[1,2]\,|\,(2)}_{\mathrm{diff,\,dm,\,(2,2)}}(k,a_0) - 2\int^{a_0} da_1 \left(\frac{3}{2}\right)\Omega_{\mathrm{dm}}(a_1)\mathcal{H}(a_1)D(a_0)^2 D(a_1)^2\,(s-s(a_1))\tilde{f}^{[0]}(\vec{k}(s-s(a_1)))$$
$$\int \frac{d^3q}{(2\pi)^3}\; P_{\mathrm{dm},11}(|\vec{k}-\vec{q}|,a_0)P_{\mathrm{dm},11}(q,a_0)\; F_2(-\vec{q},-\vec{k}+\vec{q})\; F_2(\vec{q},\vec{k}-\vec{q})\;. \qquad (4.3)$$

Diagrams for $P_{\mathrm{dm,\,dm}}$

- Neutrinos affect gravitational potential that then affect dynamics of dark matter

$$\frac{\partial^2}{a^2}\Phi = \frac{3}{2}H_0^2\frac{a_0^3}{a^3}\left(\Omega_{\mathrm{dm},0}\delta_{\mathrm{dm}} + \Omega_{\nu,0}\delta_\nu\right) \simeq \frac{3}{2}H_0^2\frac{a_0^3}{a^3}\Omega_{\mathrm{NR},0}\left(\delta_{\mathrm{dm}} + f_\nu\left(\delta_\nu - \delta_{\mathrm{dm}}\right)\right)$$

$$\frac{1}{a}\frac{\partial\delta_{\mathrm{dm}}}{\partial a} + \frac{1}{a}\theta_{\mathrm{dm}} + \frac{1}{a}\partial_i\left(\delta_{\mathrm{dm}}v^i_{\mathrm{dm}}\right) = 0\ ,$$

$$\frac{1}{a}\frac{\partial\theta_{\mathrm{dm}}}{\partial a} + H\theta_{\mathrm{dm}} + \frac{1}{a}\partial^2\Phi + \frac{1}{a}\partial_i\left(v^j_{\mathrm{dm}}\partial_j v^i_{\mathrm{dm}}\right) = 0\ .$$

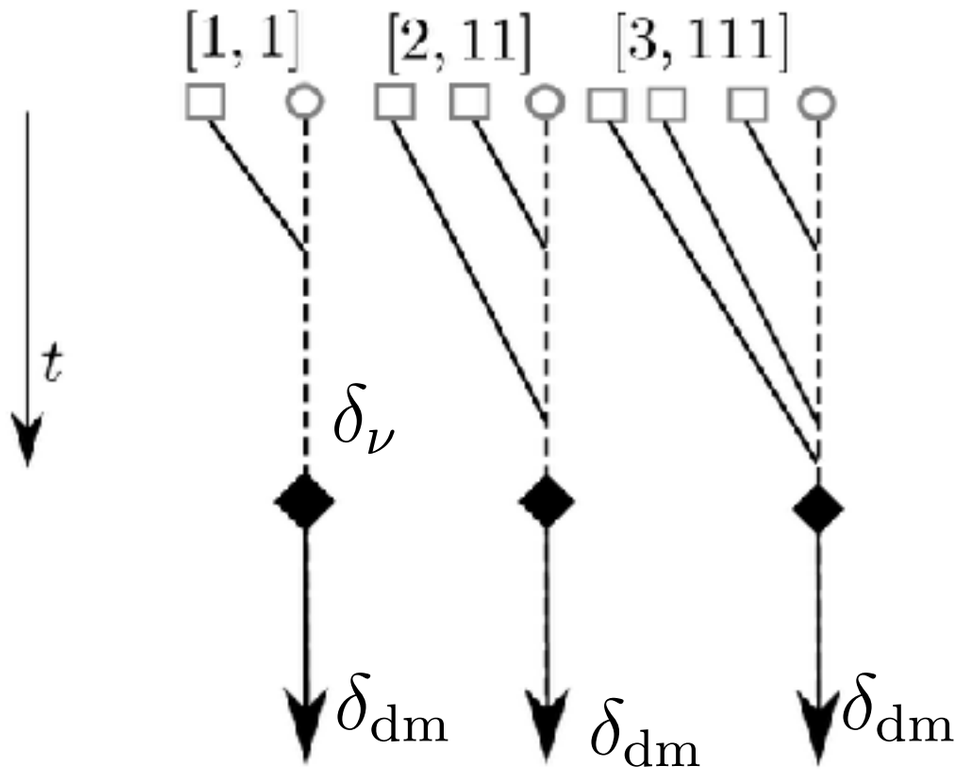
- Leads to

$$\delta^{(n)}_{\mathrm{dm}\leftarrow\nu}(\vec{k},a) = f_\nu\int^a d\tilde{a}\ G_2^\delta(a,\tilde{a})\ \frac{3}{2}\frac{\Omega_{\mathrm{dm}}(\tilde{a})}{f_g(\tilde{a})}\delta^{(n)}_\nu(\vec{k},\tilde{a})$$

Diagrams for $P_{\mathrm{dm,dm}}$

- .

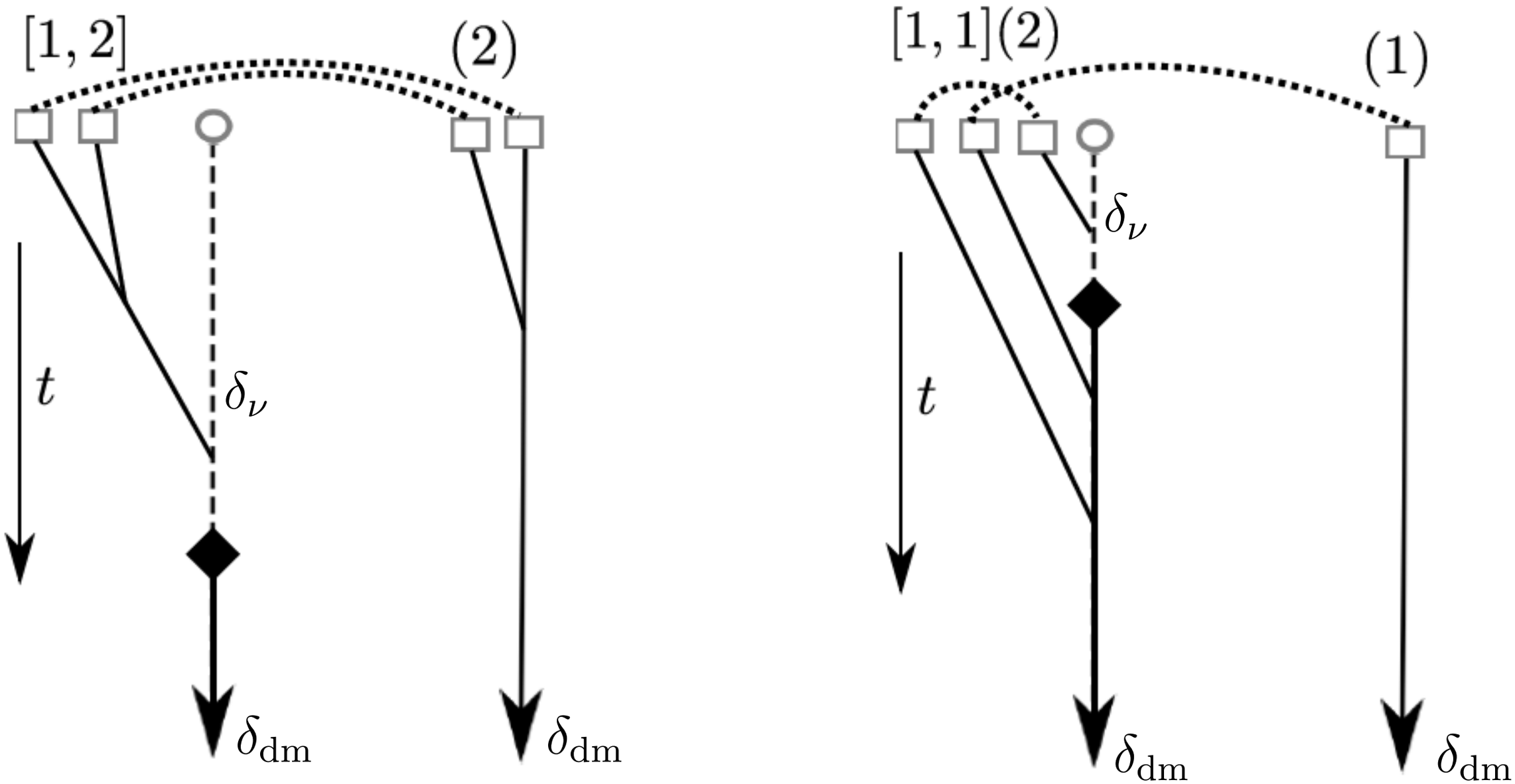
$$\delta^{(n)}_{\mathrm{dm}\leftarrow\nu}(\vec{k},a) = f_\nu \int^a d\tilde{a}\; G^\delta_2(a,\tilde{a})\; \frac{3}{2}\frac{\Omega_{\mathrm{dm}}(\tilde{a})}{f_g(\tilde{a})}\delta^{(n)}_\nu(\vec{k},\tilde{a})$$



- We can also form higher order solution by using gravitational interactions or non-linear gravitational field
  - technically, one has to solve for exact time-dependence with k-dependent growth factor.

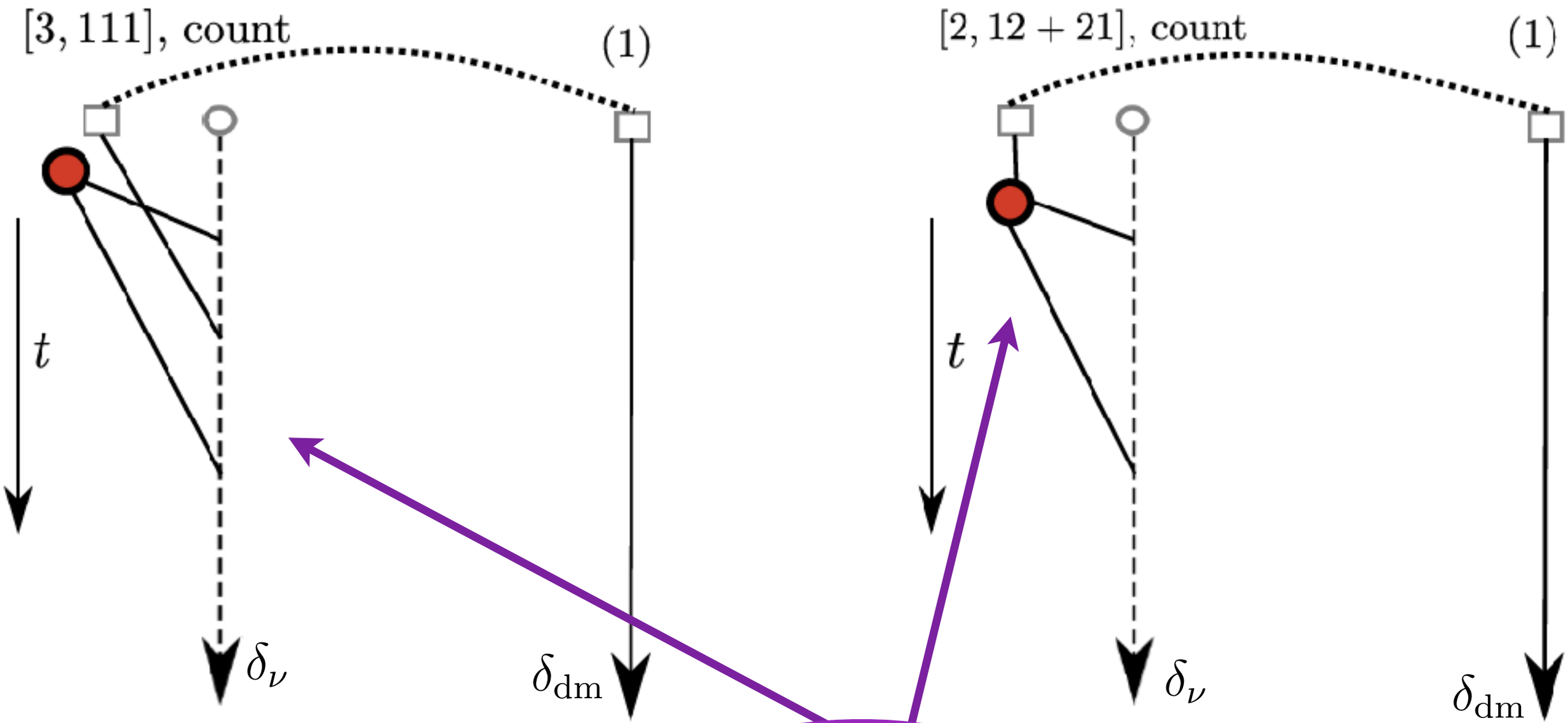
Diagrams for $P_{\mathrm{dm,\,dm}}$

- Examples of diagrams



Counterterms for $P_{\mathrm{diff,\,dm}}$

- Examples of diagrams: simply written modifying the no-counterterm diagrams



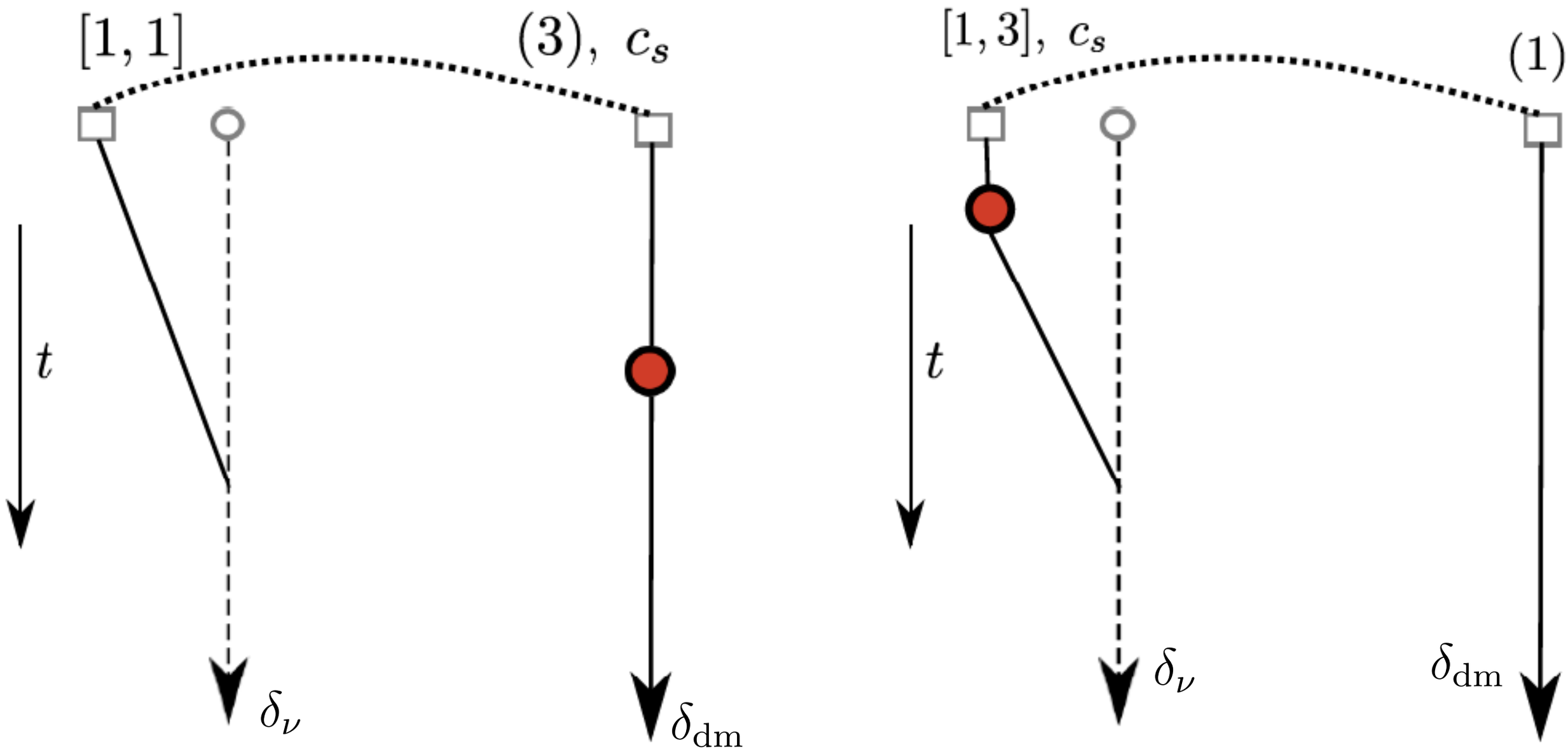
$$[\delta(\vec{x}_1,\tau_1)\delta(\vec{x}_2,\tau_2)]_{\mathrm{C}} = \int^{\mathrm{Max}(\tau_1,\tau_2)} d\tau' \Big[C_1^{(2)}(\tau_1,\tau_2,\tau') + \epsilon^{(2)}_{\mathrm{stoch},1}(\tau_1,\tau_2,\vec{x}_{\mathrm{fl}}(\vec{x}_1,\tau_1,\tau'),\tau') +$$

$$\partial^i\partial^j\Phi(\vec{x}_{\mathrm{fl}}(\vec{x}_1,\tau,\tau'),\tau')\left(C_2^{(2)}(\tau,\tau')\delta^{ij} + C_3^{(2)}(\tau,\tau')\frac{\partial_i\partial_j}{\partial^2}\right) + \qquad ($$

$$\epsilon^{(2),ij}_{\mathrm{stoch},2}(\tau_1,\tau_2,\vec{x}_{\mathrm{fl}}(\vec{x}_1,\tau_1,\tau'),\tau')\frac{\partial_i\partial_j}{\partial^2} + \ldots\Big] \frac{1}{k_{\mathrm{NL}}^3}\,\delta_D^{(3)}\left(\vec{x}_{\mathrm{fl}}(\vec{x}_1,\tau_1,\tau') - \vec{x}_{\mathrm{fl}}(\vec{x}_2,\tau_2,\tau')\right) \ .$$

Counterterms for $P_{\mathrm{diff,\,dm}}$

- Of course we need to use the `correct' dark matter potential: for example



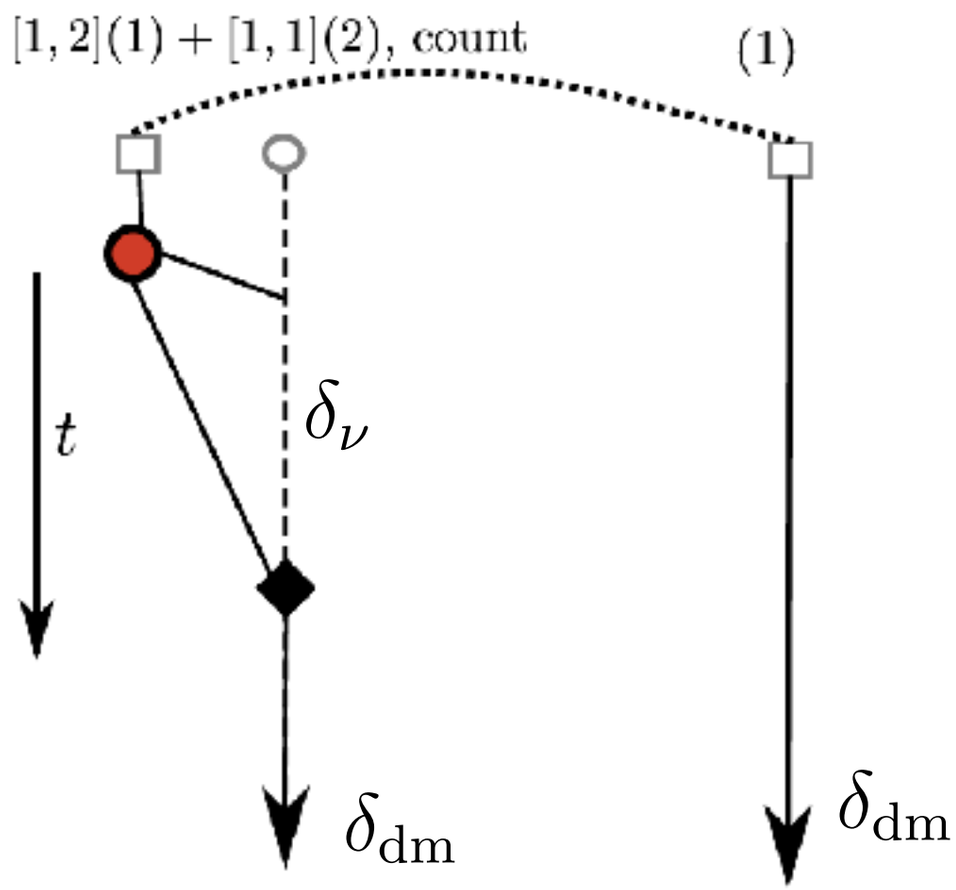
Counterterms for $P_{\mathrm{dm,\,dm}}$

- Modification to Poisson equation modifies Euler equation

$$\dot{\pi}^i_{\mathrm{dm}} + 4H\pi^i_{\mathrm{dm}} + \frac{1}{a}\partial_j\begin{pmatrix}\pi^i_{\mathrm{dm}}\pi^j_{\mathrm{dm}} \\ \rho^{(0)}_{\mathrm{dm}}(1+\delta_{\mathrm{dm}})\end{pmatrix} + \frac{3}{2}H_0^2\frac{a_0^3}{a^2}\Omega_{\mathrm{NR},0}\rho^{(0)}_{\mathrm{dm}}(1+\delta_{\mathrm{dm}})\frac{\partial^i}{\partial^2}\left(\delta_{\mathrm{dm}} + f_\nu\left(\delta_\nu - \delta_{\mathrm{dm}}\right)\right) = 0\ .$$

New Local Product of Fields: renormalize with expressions used before.

The $\delta_{\mathrm{dm}}\partial_i\delta_{\mathrm{dm}}$ has same functional form as usual c_s counterterm



Log-enhancement

- Some diagrams receive an enhancement proportional to $\log(a_{\text{equality}}) \sim 8$

- This comes from the effect of linear $f_\nu \; \delta^{(1)}_{\text{diff}}$ onto the growth of $\delta^{(1)}_{\text{dm}}$: DM grows where only a fraction $(1 - f_\nu)$ of non-relativistic matter clusters:

$$\delta^{(1)}_{\text{dm}}(a) \propto D^{1+f_\nu} \sim D(a) \cdot (1 + f_\nu \log(a))$$

- However, keeping only these diagrams is 30% off, as there are many diagrams, so, this gives us an order of magnitude of the effect, but not a good truncation.

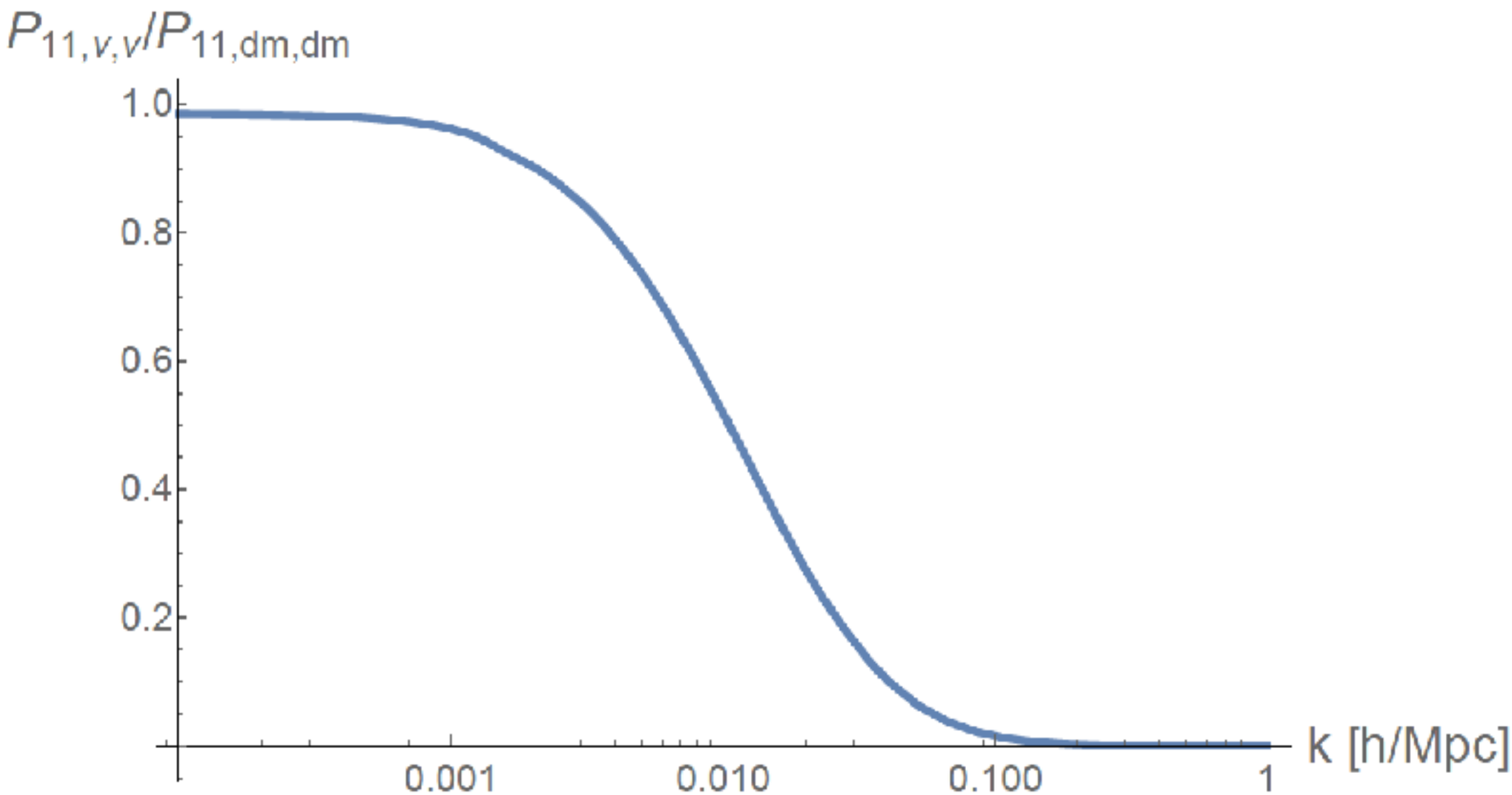
Counterterms for Slow Neutrinos

- For fast neutrinos, each insertion of a non-linear mode is a suppression.
- Instead, for slow neutrinos, the same insertion is unsuppressed. We can account for this by adding an additional counterterm for slow neutrinos.
  - Since slow neutrinos $k_{\rm fs} \gtrsim k_{\rm NL}$, they behave like dark matter.
  - $\Rightarrow$The counterterm is the same as the dark matter one, apart for the coefficient:

$$P_{\text{counter,slow } \nu'\text{s}} = f_{\nu,\,\text{slow}} c_s^2 P_{11}(k)$$

Results

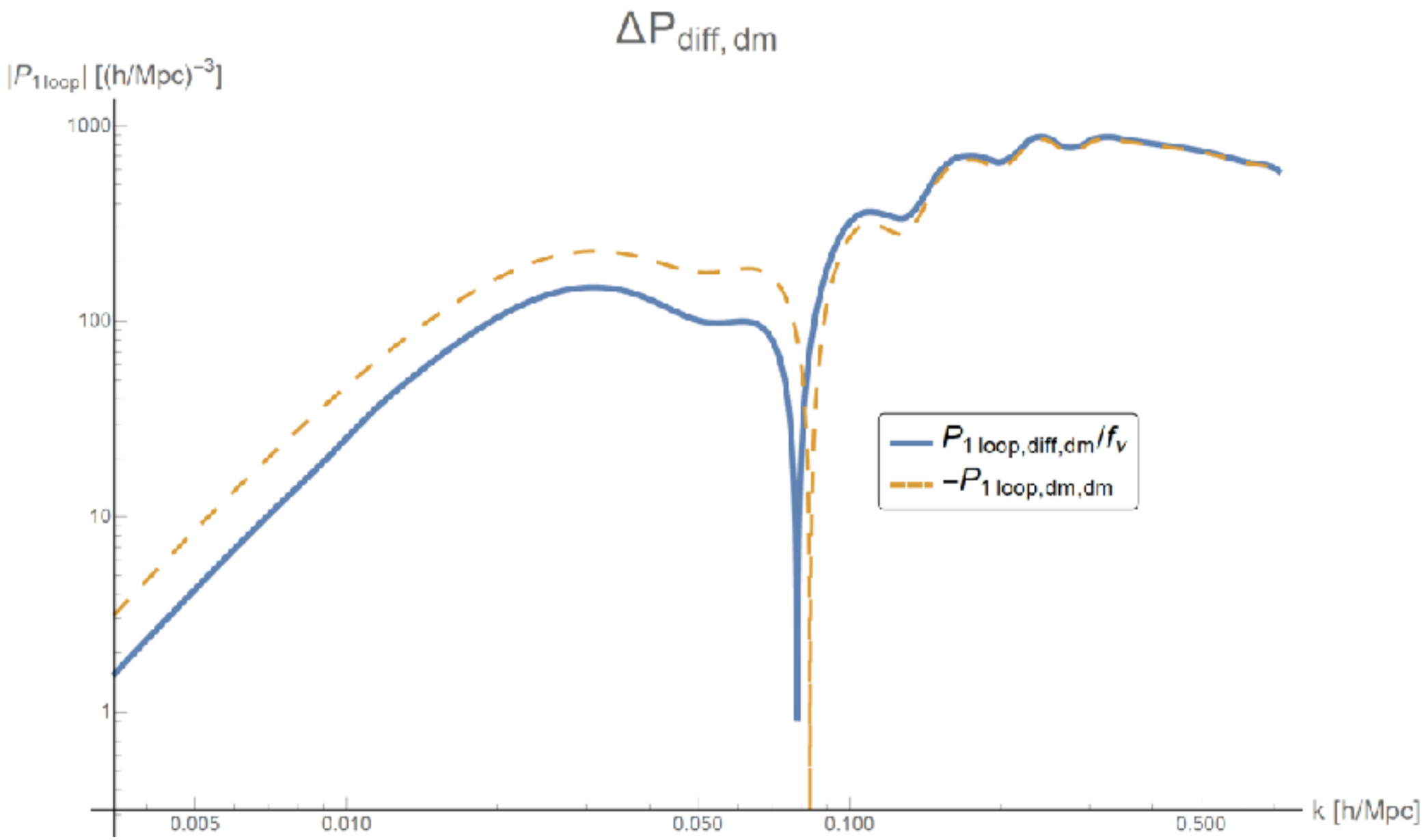
- Linear Transfer Function



- Suppression for $k \gtrsim k_{\mathrm{fs}}$, equal to dark matter for $k \lesssim k_{\mathrm{fs}}$

Results: $P_{\mathrm{diff,\,dm}}$

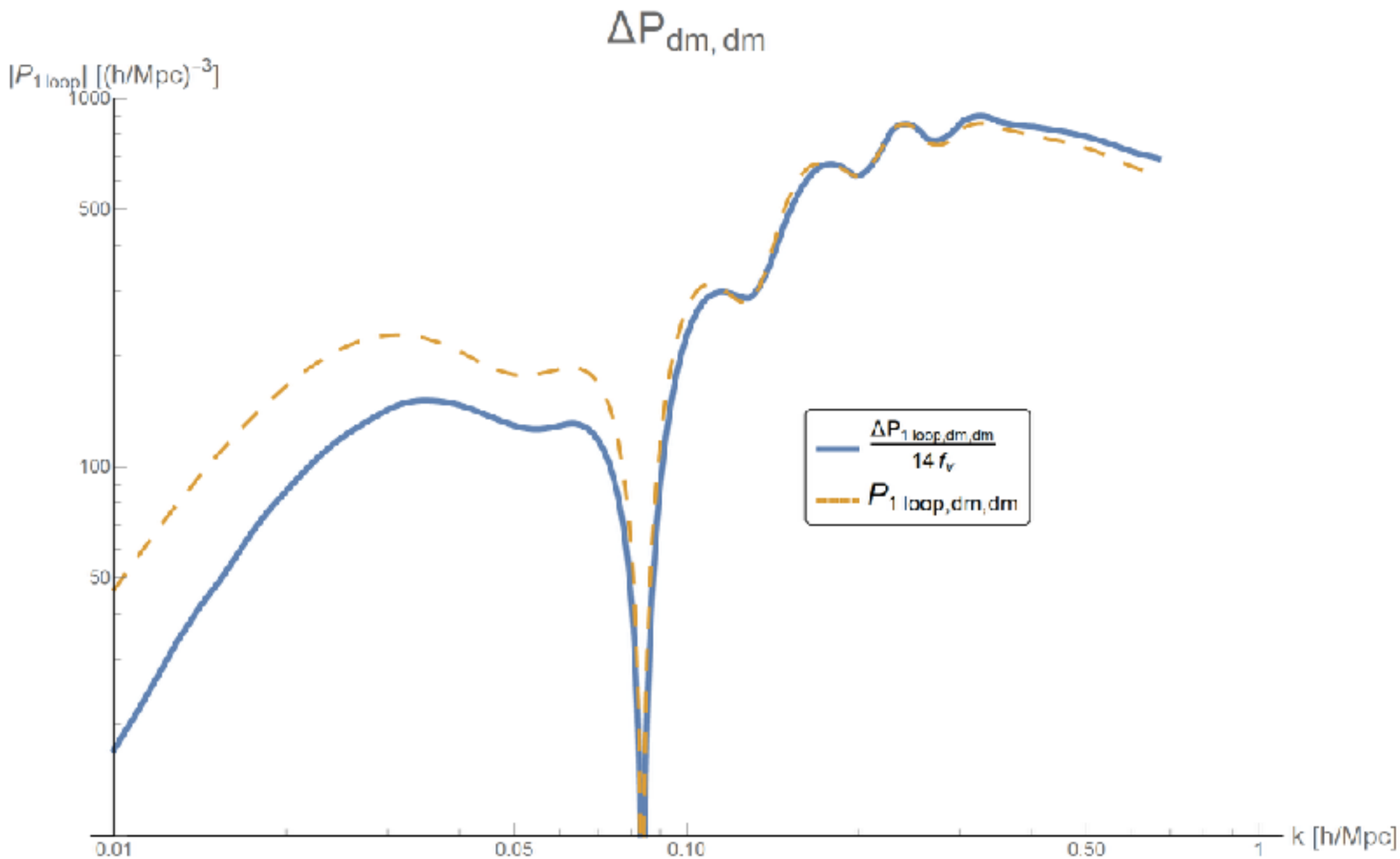
- Ordinary Diagrams



- $\sim -f_\nu P_{\mathrm{dm,\,dm}}$ for $k \gtrsim k_{\mathrm{fs}}$, $-\frac{1}{2} f_\nu P_{\mathrm{dm,\,dm}}$ otherwise
  - $k \lesssim k_{\mathrm{fs}}$, $\delta^{(1)}_{\mathrm{diff}} \simeq 0$ but not all diagrams give zero, as in some diagrams the contribution is dominated by UV where $\delta^{(1)}_{\mathrm{diff}} \simeq \delta^{(1)}_{\mathrm{dm}}$. We we do not get zero.

Results: $P_{\mathrm{dm,\,dm}}$

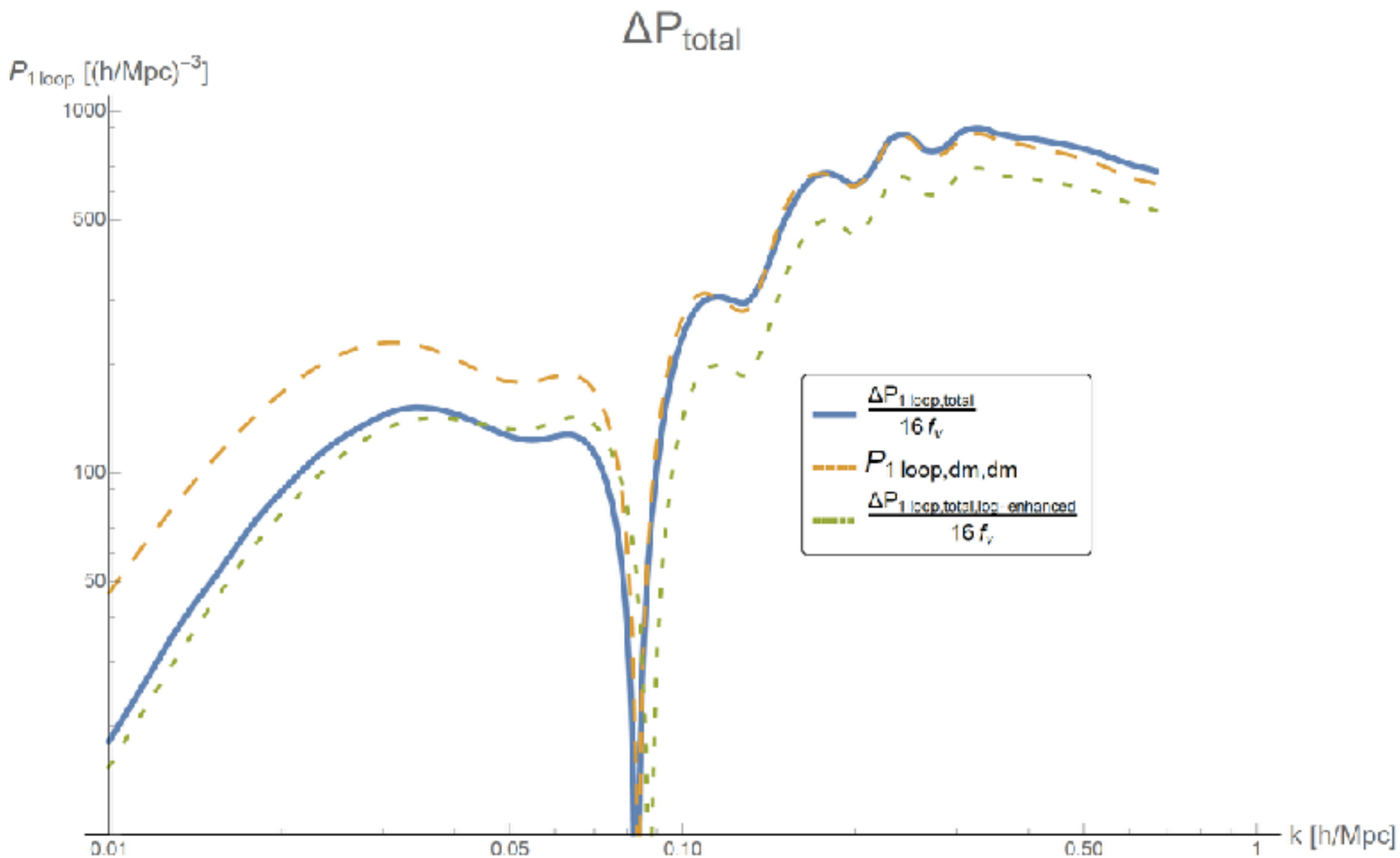
- Ordinary Diagrams



- $\sim -14 f_\nu P_{\mathrm{dm,\,dm}}$ for $k \gtrsim k_{\mathrm{fs}}$, $\sim -7 f_\nu P_{\mathrm{dm,\,dm}}$ otherwise
  - same reason

Results: ΔP_{total}

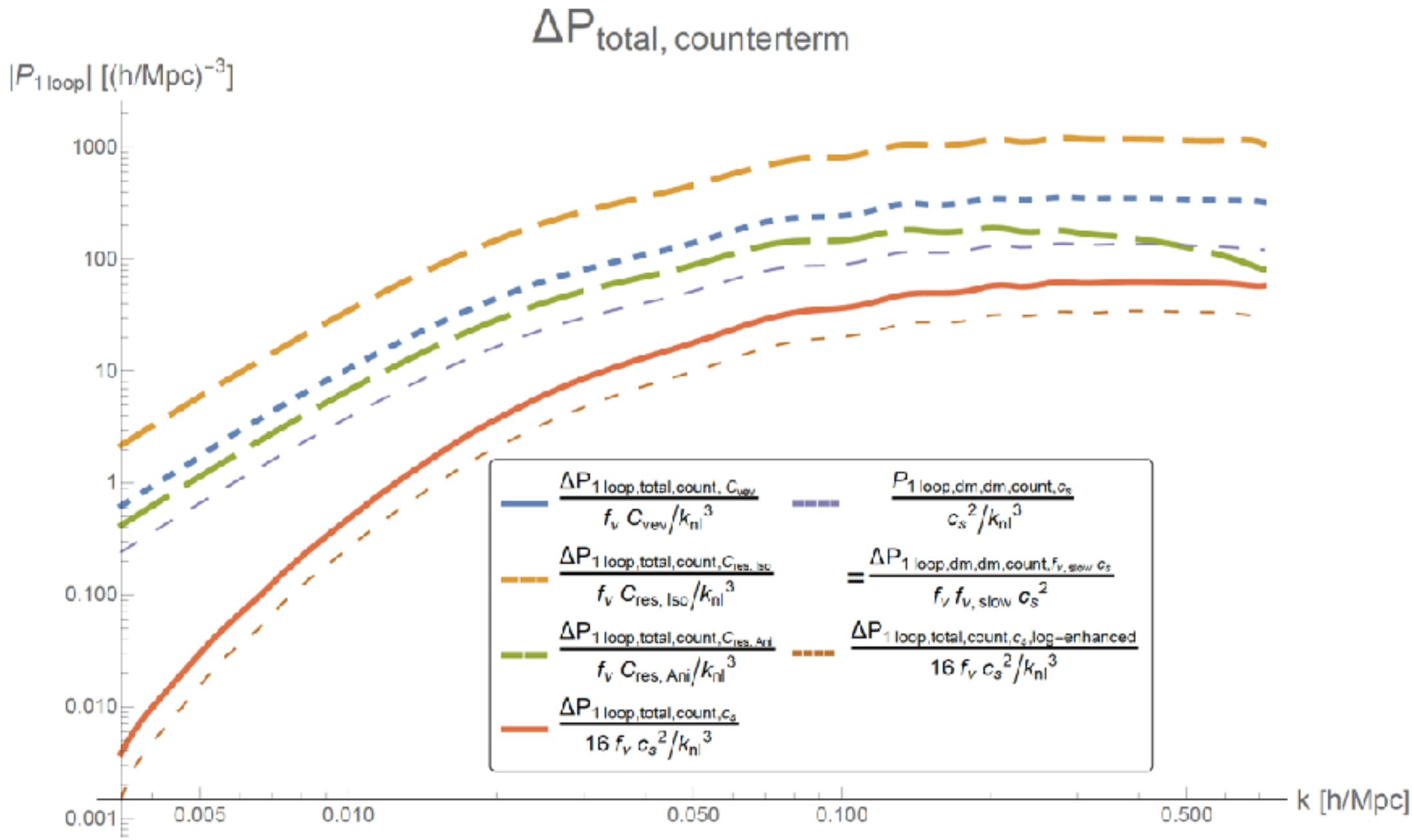
- Ordinary Diagrams



- $\sim \quad -16 f_\nu P_{\text{dm, dm}}$ for $k \gtrsim k_{\text{fs}}$, $\sim -8 f_\nu P_{\text{dm, dm}}$ otherwise
  - $k \lesssim k_{\text{fs}}$, $\delta^{(1)}_{\text{diff}} \simeq 0$ but not all diagrams give zero, as in some diagrams the contribution is dominated by UV where $\delta^{(1)}_{\text{diff}} \simeq \delta^{(1)}_{\text{dm}}$. So we do not get zero.

Results: ΔP_{total} counter

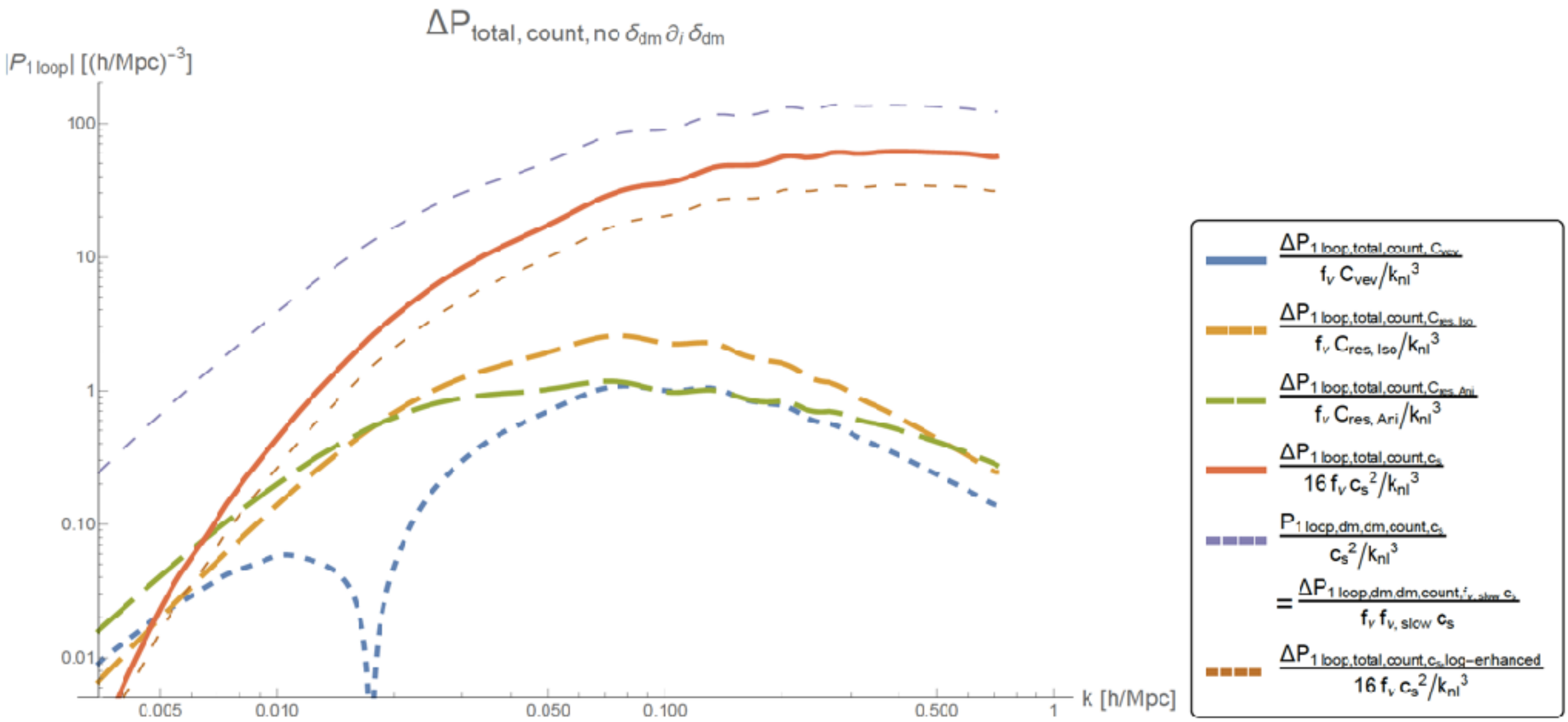
- Counterterm Diagrams



- They are sizable. But the largest of them are either determined by c_s or come from the counterterms of $\delta_{\text{dm}}\partial_i\delta_{\text{dm}}$, whose functional form is the one of the c_s counterterm

Results: ΔP_{total}

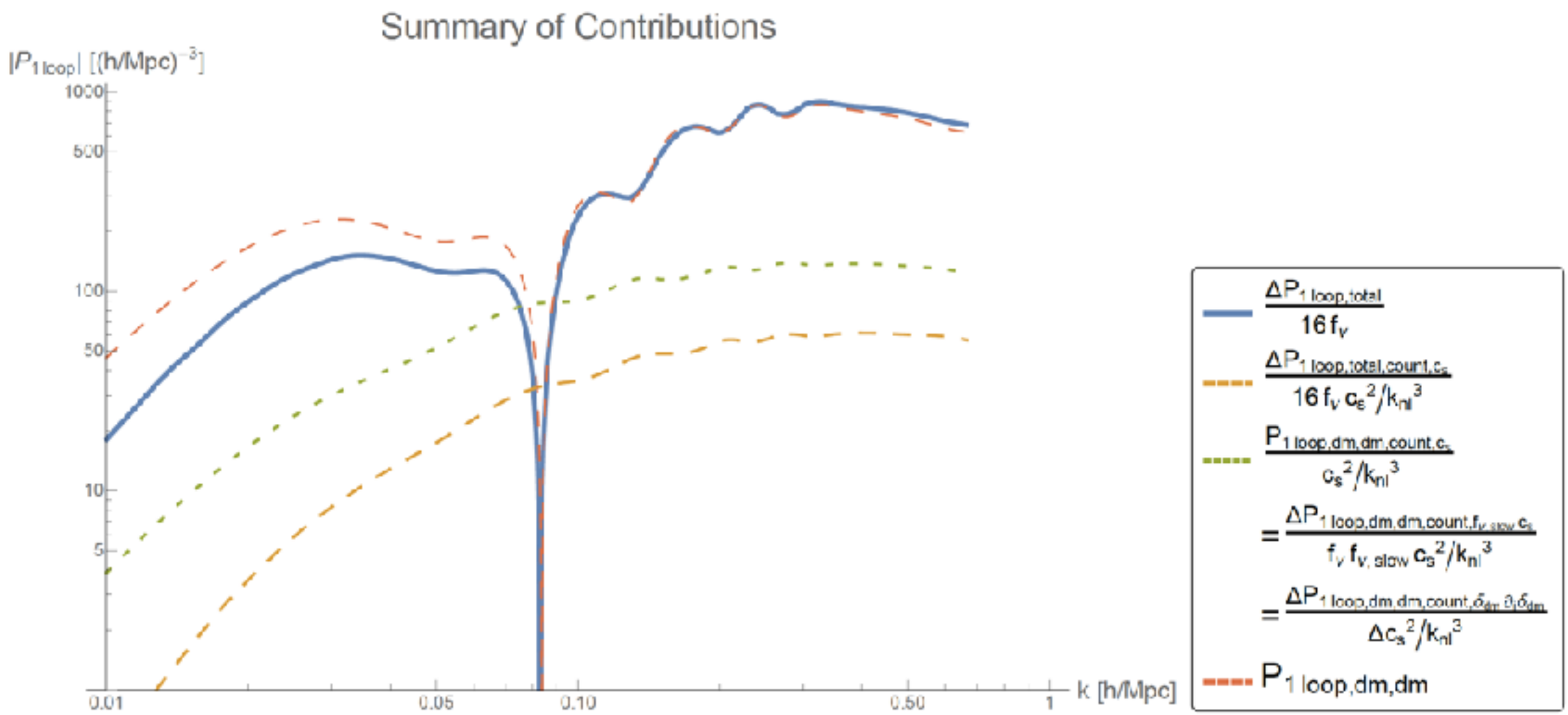
- Counterterm Diagrams: removing c_s -degenerate contributions



- What is left is quite smaller, and can be neglected to a first approximation.
  - smallness is because for fast neutrinos: contribution scales as $\frac{1}{{k_{\rm NL}}^2}\left(\frac{k_{\rm fs}}{k_{\rm NL}}\right)$

Summary

- Summary plot: the effect is large! but we can compute it! as we developed the EFT
  - solve Boltzmann equation with added suitable counterterms.



- A leading calculation just involves a redefinition of the same parameter:

$$c_s^2 \to c_s^2 + f_\nu \Delta c_s^2 + f_\nu f_{\nu,\mathrm{slow}} c_s^2$$

- But functional form of counterterm is different

Dark Energy in the EFTofLSS

with Lewandowsky and Maleknejad **JCAP2017**

Dark Energy in the EFTofLSS

- Let us assume that dark energy is associated to a state that breaks spontaneously time diff.. In this case it is described by the Effective Field Theory for Dark Energy

$$S_G = \int d^4x\sqrt{-g}\left\{\frac{M_*^2}{2}f(t)R - \Lambda(t) - c(t)g^{00}\right\} + S_{DE}^{(2)}$$

with Creminelli, Luty, Nicolis **0606**
Creminelli, D'Amico, Norena, Vernizzi **0811**
Giubitosi, Piazza, Vernizzi **0912**
…

- A lot of work was done at linear level

see **EFTCAMB**

- Now that we know how to do computations for dark matter, we can compute the effect of dark energy on the quasi linear regime

- One of the most interesting regime is clustering dark energy

$$c_s \to 0 \;\&\; \delta_Q^{(1)} \sim (1+w)\delta_{\rm dm}^{(1)}$$

see for ex. with Creminelli, D'amico, Vernizzi **0911**

Dark Energy in the EFTofLSS

- This system has been already studied with the old perturbation theory,

Sefusatti and Vernizzi **JCAP2011**

- but never using the EFTofLSS (which is the only correct one)
  - So, it needs to be done
  - In doing so, we will find a couple of subtle issues
- We will also describe how they affect biased tracers.

- Since $c_s \simeq 0$, the isocurvature mode decays, and Dark Energy and Dark Matter become comoving (apart for counterterms), and there is only the adiabatic mode.
- Usually, the resulting adiabatic mode, is described assuming a fluid like equation.

$$a\mathcal{H}\delta_A' + C(a)\theta = -\alpha\delta_A\theta$$

$$a\mathcal{H}\theta' + \mathcal{H}\theta + \frac{3}{2}\mathcal{H}_0^2\frac{a_0}{a}\Omega_m\delta_A = -\beta\theta\theta + \text{counterterms}$$

$$C(a) = 1 + (1+w)\frac{\Omega_Q}{\Omega_m}\left(\frac{a}{a_0}\right)$$

- However, it is interesting to derive these equations in terms of π , as in general fluid equations do not apply

Dark Energy in the EFTofLSS

- The system:
  - DE: $$\int d^4x\sqrt{-g}\left[\frac{M_{\rm Pl}^2}{2}R-\Lambda(t)-c(t)g_{\rm u}^{00}+\frac{M_2^4(t)}{2}(\delta g_{\rm u}^{00})^2\right]$$
  - DM: $$M_2^4(t)\sim\frac{\rho_Q+p_Q}{c^2}$$

$$\dot\delta_m+\frac{1}{a}\partial_i((1+\delta_m)v_m^i)=0$$

$$\partial_i\dot v_m^i+H\partial_i v_m^i+\frac{1}{a}\partial_i(v_m^j\partial_j v_m^i)+\frac{1}{a}\partial^2\Phi=-\frac{1}{a}\partial_i\left(\frac{1}{\rho_m}\partial_j\tau^{ij}\right)_s+\frac{1}{a}\partial_i\gamma_s^i$$

- Notice the presence of the force, as for the baryon case

with Lewandwski, Perko, **JCAP2015**

Dark Energy in the EFTofLSS: linear level

- At linear level we have:
  - π equation:

$$\frac{1}{a^3 M_2^4}\frac{d}{dt}\left\{a^3 M_2^4(\dot{\pi}-\Phi)\right\} = c_s^2 a^{-2}\partial^2\pi$$

  - $\Rightarrow \quad \dot{\pi}-\Phi \sim c_s^2\,\partial^2\Phi/H^2 \quad \Rightarrow$ Limit $c_s \to 0$ must be taken with care

$$\Rightarrow \quad \partial_i\pi \sim \partial_i\Phi/H \sim v^i \quad \Rightarrow$$ Comoving

  - Poisson equation:

$$a^{-2}\partial^2\Psi = \frac{3}{2}\frac{\Omega_{m,0}\mathcal{H}_0^2 a_0}{a^3}\left(\delta_m + \frac{4M_2^4}{\bar{\rho}_m}(\dot{\pi}-\Phi)\right)$$

$$\Rightarrow \quad \delta_A = \delta_m + \frac{4a^3 M_2^4}{a_0^3\,\bar{\rho}_{m,0}}(\dot{\pi}-\Phi)$$

Good $c_s \to 0$ limit

- Linear equations

$$\dot{\delta}_A = \dot{\delta}_m + \frac{4}{a_0^3\,\bar{\rho}_{m,0}}\frac{d}{dt}\left\{a^3 M_2^4(\dot{\pi}-\Phi)\right\}$$
$$= -\frac{1}{a}\theta_m + \frac{4a^3 M_2^4}{a_0^3\,\bar{\rho}_{m,0}}c_s^2 a^{-2}\partial^2\pi = -\frac{1}{a}C(a)\theta_m\,;$$

$$\Rightarrow \quad \dot{\delta}_A + \frac{1}{a}C(a)\theta_A = 0$$
$$\dot{\theta}_A + H\theta_A + \frac{3}{2}\frac{\Omega_{m,0}\mathcal{H}_0^2 a_0}{a^2}\delta_A = 0\,.$$

Dark Energy in the EFTofLSS: non-linear level

- At linear level we have:
  - π equation: $$\nabla_\mu \frac{\delta \mathcal{L}}{\delta\, \partial_\mu \pi} = \frac{\delta \mathcal{L}}{\delta \pi}, \quad \Rightarrow \delta g_{\rm u}^{00} \propto c_s^2 \text{ at all orders}$$
  - . T^{00} $$\Rightarrow \quad \delta_A = \delta_m - \frac{2a^{-3}}{\bar{\rho}_m} \frac{\delta \sqrt{-g}\, \mathcal{L}}{\delta g^{00}}, \quad \Rightarrow \quad \delta_A = \delta_m - \frac{2}{\bar{\rho}_m} M_2^4 \delta g_{\rm u}^{00}$$
  - π equation $\Rightarrow$ NR-limit $$-\frac{2}{a^3}\partial_t \left(a^3 M_2^4\, \delta g_{\rm u}^{00}\right) = -2a^{-2}\partial_i \left(\partial_i \pi \left(-c + M_2^4 \delta g_{\rm u}^{00}\right)\right)$$
  - . $\Rightarrow$ $$\dot{\delta}_A = \dot{\delta}_m - \frac{2}{\rho_{m,0}} \partial_t \left(a^3 M_2^4\, \delta g_{\rm u}^{00}\right) = -\frac{1}{a} C(a) \theta_A - \frac{1}{a} \partial_i \left(\delta_A v_A^i\right) \ ,$$
  - $\Rightarrow$ Total equations: $$\dot{\delta}_A + \frac{1}{a} C(a) \theta_A = -\frac{1}{a} \partial_i \left(\delta_A v_A^i\right)$$ $$\dot{\theta}_A + H \theta_A + \frac{3}{2} \frac{\Omega_{m,0} \mathcal{H}_0^2 a_0}{a^2} \delta_A = -\frac{1}{a} \partial_i \left(v_A^j \partial_j v_A^i\right)$$

Dark Energy in the EFTofLSS: counterterms

- If $c_s^2 \ll c_{s,\text{dark matter}}$,we need to adjust the counterterm for dark matter:

$$-\left(\frac{1}{\rho_m}\partial_j\tau^{ij}\right)_s(a,\vec{x}) + \gamma_s^i(a,\vec{x}) =$$
$$\int da'\Big[\kappa^{(1)}(a,a')\,\partial^i\partial^2\Phi(a',\vec{x}_{\text{fl}}(\vec{x};a,a')) + \kappa^{(2)}(a,a')\,\frac{1}{H}\partial^i\partial_j v_m^j(a',\vec{x}_{\text{fl}}(\vec{x};a,a'))$$
$$+\kappa_1^{(\text{stoch.})}(a,a')\bar{\Delta}^i_{\text{stoch.}}(a',\vec{x}_{\text{fl}}(\vec{x};a,a'))\ldots\Big]\,.$$

- Notice the stochastic term: $\langle\Delta^i_{\text{stoch.}}(\vec{k})\Delta^j_{\text{stoch.}}(\vec{k}')\rangle = \frac{(2\pi)^3}{k_{\text{NL}}^3}\delta(\vec{k}+\vec{k}')C^{(1)}\delta^{ij} + \ldots,$

- Otherwise, they will start moving differently

Perturbative Solutions

- We solve:

$$\dot{\delta}_A + \frac{1}{a} C(a)\theta_A = 0$$

$$\dot{\theta}_A + H\theta_A + \frac{3}{2}\frac{\Omega_{m,0}\mathcal{H}_0^2 a_0}{a^2}\delta_A = 0 \ .$$

- Perturbatively: $\delta_{\vec{k}}(a) = \sum_{n=1}^{\infty} \delta_{\vec{k}}^{(n)}(a) + \delta_{\vec{k}}^{(ct)}(a)$ and $\Theta_{\vec{k}}(a) = \sum_{n=1}^{\infty} \Theta_{\vec{k}}^{(n)}(a) + \Theta_{\vec{k}}^{(ct)}(a)$,

$$\Rightarrow \quad a\delta_{\vec{k}}^{(n)\prime} - f_+\Theta_{\vec{k}}^{(n)} = \frac{f_+}{C}\sum_{m=1}^{n-1}\int \frac{d^3q}{(2\pi)^3}\alpha(\vec{q},\vec{k}-\vec{q})\Theta_{\vec{q}}^{(m)}\delta_{\vec{k}-\vec{q}}^{(n-m)},$$

$$a\Theta_{\vec{k}}^{(n)\prime} - f_+\Theta_{\vec{k}}^{(n)} - \frac{f_-}{f_+}(\Theta_{\vec{k}}^{(n)} - \delta_{\vec{k}}^{(n)}) = \frac{f_+}{C}\sum_{m=1}^{n-1}\int \frac{d^3q}{(2\pi)^3}\beta(\vec{q},\vec{k}-\vec{q})\Theta_{\vec{q}}^{(m)}\Theta_{\vec{k}-\vec{q}}^{(n-m)},$$

- Solve using Green's function (very simple)

$$\delta_{\vec{k}}^{(n)} = \int_0^a d\tilde{a}\left(G_1^{\delta}(a,\tilde{a})S_1^{(n)}(\tilde{a},\vec{k}) + G_2^{\delta}(a,\tilde{a})S_2^{(n)}(\tilde{a},\vec{k})\right) ,$$

$$\Theta_{\vec{k}}^{(n)} = \int_0^a d\tilde{a}\left(G_1^{\Theta}(a,\tilde{a})S_1^{(n)}(\tilde{a},\vec{k}) + G_2^{\Theta}(a,\tilde{a})S_2^{(n)}(\tilde{a},\vec{k})\right) ,$$

Perturbative Solutions

- We solve

$$\delta^{(n)}_{\vec{k}} = \int_0^a d\tilde{a}\left(G^{\delta}_1(a,\tilde{a}) S^{(n)}_1(\tilde{a},\vec{k}) + G^{\delta}_2(a,\tilde{a}) S^{(n)}_2(\tilde{a},\vec{k}) \right) ,$$

$$\Theta^{(n)}_{\vec{k}} = \int_0^a d\tilde{a}\left(G^{\Theta}_1(a,\tilde{a}) S^{(n)}_1(\tilde{a},\vec{k}) + G^{\Theta}_2(a,\tilde{a}) S^{(n)}_2(\tilde{a},\vec{k}) \right) ,$$

- We get:

$$\delta^{(3)}_{\vec{k}}(a) = \iint \frac{d^3p}{(2\pi)^3}\frac{d^3q}{(2\pi)^3}\Big(\alpha^{\sigma}(\vec{k},\vec{p},\vec{q})\mathcal{U}^{\delta}_{\sigma}(a) + \beta^{\sigma}(\vec{k},\vec{p},\vec{q})\mathcal{V}^{\delta}_{\sigma 2}(a) + \gamma^{\sigma}(\vec{k},\vec{p},\vec{q})\mathcal{V}^{\delta}_{\sigma 1}(a)\Big)\delta^{\rm in}_{\vec{k}-\vec{p}}\delta^{\rm in}_{\vec{p}-\vec{q}}\delta^{\rm in}_{\vec{q}} ,$$

$$\Theta^{(3)}_{\vec{k}}(a) = \iint \frac{d^3p}{(2\pi)^3}\frac{d^3q}{(2\pi)^3}\Big(\alpha^{\sigma}(\vec{k},\vec{p},\vec{q})\mathcal{U}^{\Theta}_{\sigma}(a) + \beta^{\sigma}(\vec{k},\vec{p},\vec{q})\mathcal{V}^{\Theta}_{\sigma 2}(a) + \gamma^{\sigma}(\vec{k},\vec{p},\vec{q})\mathcal{V}^{\Theta}_{\sigma 1}(a)\Big)\delta^{\rm in}_{\vec{k}-\vec{p}}\delta^{\rm in}_{\vec{p}-\vec{q}}\delta^{\rm in}_{\vec{q}} ,$$

- where

$$\mathcal{U}^{\delta}_{\sigma}(a) = \int_0^1 \frac{f_+(\tilde{a})D_+(\tilde{a})}{C(\tilde{a})D_+(a_i)}\mathcal{G}^{\delta}_{\sigma}(\tilde{a})G^{\delta}_1(a,\tilde{a})d\tilde{a},$$

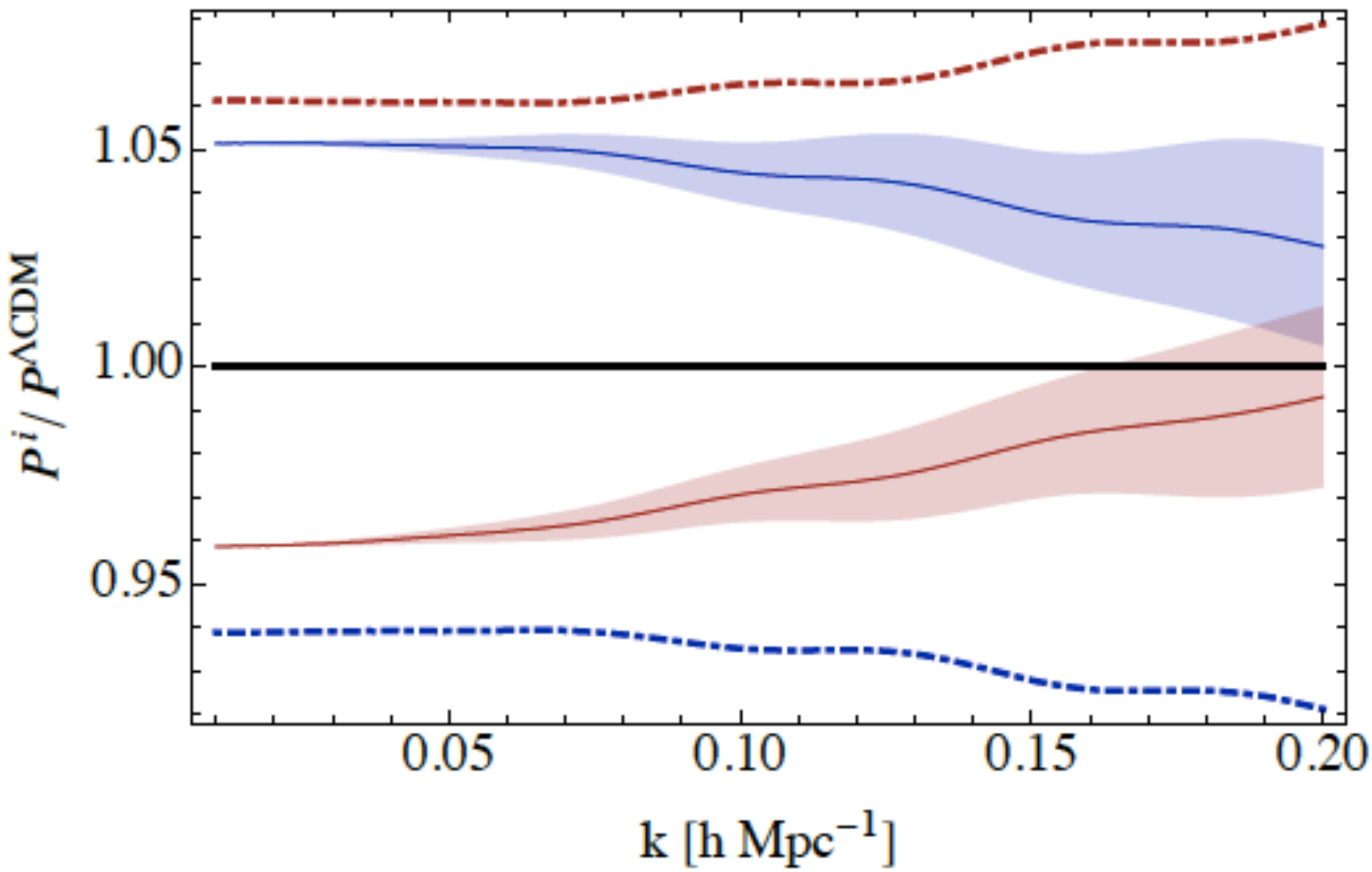
$$\mathcal{U}^{\Theta}_{\sigma}(a) = \int_0^1 \frac{f_+(\tilde{a})D_+(\tilde{a})}{C(\tilde{a})D_+(a_i)}\mathcal{G}^{\delta}_{\sigma}(\tilde{a})G^{\Theta}_1(a,\tilde{a})d\tilde{a},$$

$$\mathcal{V}^{\delta}_{\sigma\tilde{\sigma}}(a) = \int_0^1 \frac{f_+(\tilde{a})D_+(\tilde{a})}{C(\tilde{a})D_+(a_i)}\mathcal{G}^{\Theta}_{\sigma}(\tilde{a})G^{\delta}_{\tilde{\sigma}}(a,\tilde{a})d\tilde{a},$$

$$\mathcal{V}^{\Theta}_{\sigma\tilde{\sigma}}(a) = \int_0^1 \frac{f_+(\tilde{a})D_+(\tilde{a})}{C(\tilde{a})D_+(a_i)}\mathcal{G}^{\Theta}_{\sigma}(\tilde{a})G^{\Theta}_{\tilde{\sigma}}(a,\tilde{a})d\tilde{a}.$$

Results:

- .



Biased Tracers

with Angulo, Fasiello, Vlah **1503**
Schmidt **1602**

- Very similar to the Baryon paper:

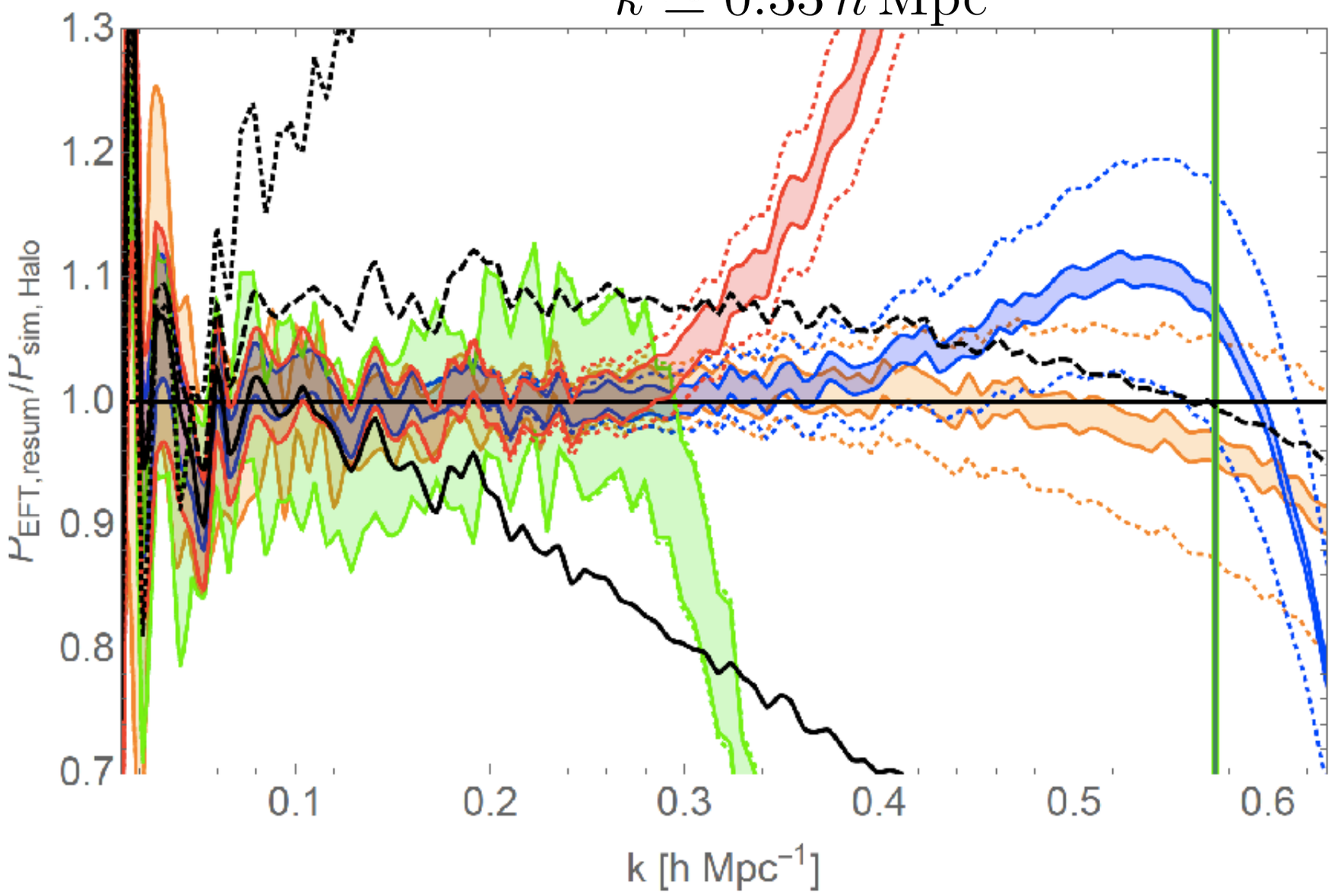
$$
\begin{aligned}
\delta_h(\vec{x},t) \simeq \int^t dt' H(t') \Bigg(& \bar{c}_{\partial^2\phi}(t,t') \frac{\partial^2\phi(\vec{x}_{\mathrm{fl}},t')}{H(t')^2} + \bar{c}_{\delta_D}(t,t')\Omega_D(t')\delta_D(\vec{x}_{\mathrm{fl}},t') \\
& + \bar{c}_{\partial_i v^i}(t,t') \frac{\partial_i v^i_A(\vec{x}_{\mathrm{fl}},t')}{H(t')} + \bar{c}_{\partial_i v^i_{\mathrm{rel.}}}(t,t') \frac{\partial_i v^i_{\mathrm{rel.}}(\vec{x}_{\mathrm{fl}},t')}{H(t')} \\
& + \bar{c}_{\partial_i\partial_j\phi\partial^i\partial^j\phi}(t,t') \frac{\partial_i\partial_j\phi(\vec{x}_{\mathrm{fl}},t')}{H(t')^2} \frac{\partial^i\partial^j\phi(\vec{x}_{\mathrm{fl}},t')}{H(t')^2} + \dots \\
& + \bar{c}_{\epsilon_m}(t,t')\,\Omega_m(t')\,\epsilon_m(\vec{x},t') + \bar{c}_{\epsilon_D}(t,t')\,\Omega_D(t')\,\epsilon_D(\vec{x},t') \\
& + \left[\bar{c}_{\epsilon_m\partial^2\phi}(t,t')\,\Omega_m(t')\,\epsilon_m(\vec{x}_{\mathrm{fl}},t') + \bar{c}_{\epsilon_D\partial^2\phi}(t,t')\,\Omega_D(t')\,\epsilon_D(\vec{x}_{\mathrm{fl}},t')\right] \frac{\partial^2\phi(\vec{x},t')}{H(t')^2} + \dots \\
& + \bar{c}_{\partial^4\phi}(t,t') \frac{\partial^2_{x_{\mathrm{fl}}}}{k_M^2} \frac{\partial^2\phi(\vec{x},t')}{H(t')^2} + \dots \Bigg) ,
\end{aligned}
$$

- Very roughly: double the terms

Summary

- The EFTofLSS is ready for data analysis
  - 10 (or probably 4) coefficients fit the galaxies in redshift space up to

$$k \simeq 0.33\, h\,\mathrm{Mpc}^{-1}$$



- We are doing the SDSS analysis!
- We are including everything