More on the Effective Theory of Dark Energy

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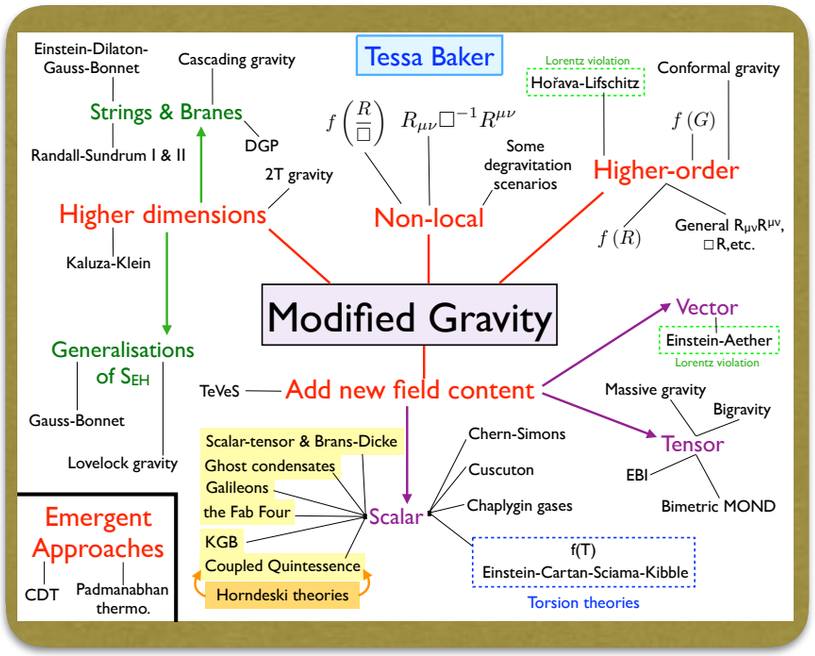
DARKMOD, Gif-sur-Yvette, 25 September 2017

Summary

- Effective Theory of Dark Energy
- Phenomenology beyond Horndeski
- Viability of Degenerate Higher-Order Scalar-Tensor theories
- Conclusion

Models

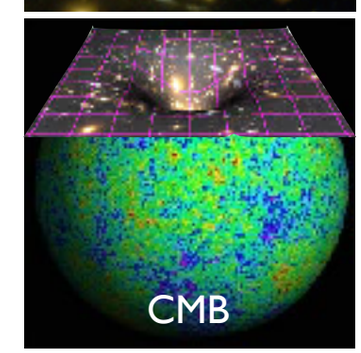


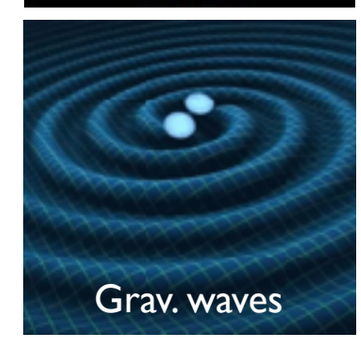
ETofDE

$$\alpha_K(t),\ \alpha_B(t),\ \alpha_M(t),$$
$$\alpha_T(t),\ \alpha_H(t), \ldots$$

Bridge models and observations in a minimal and systematic way

Observations





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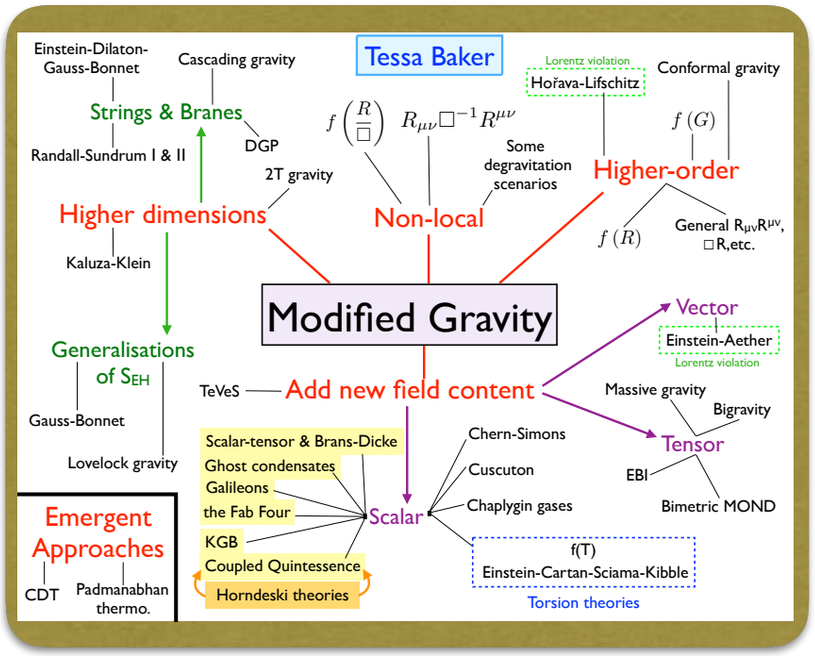

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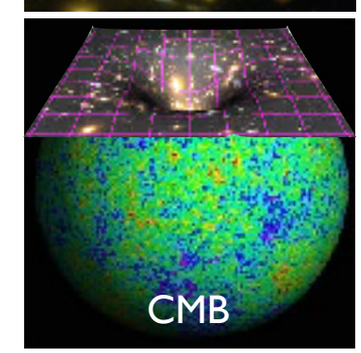

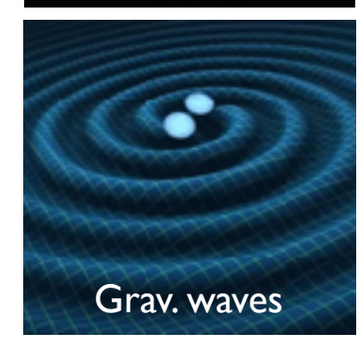

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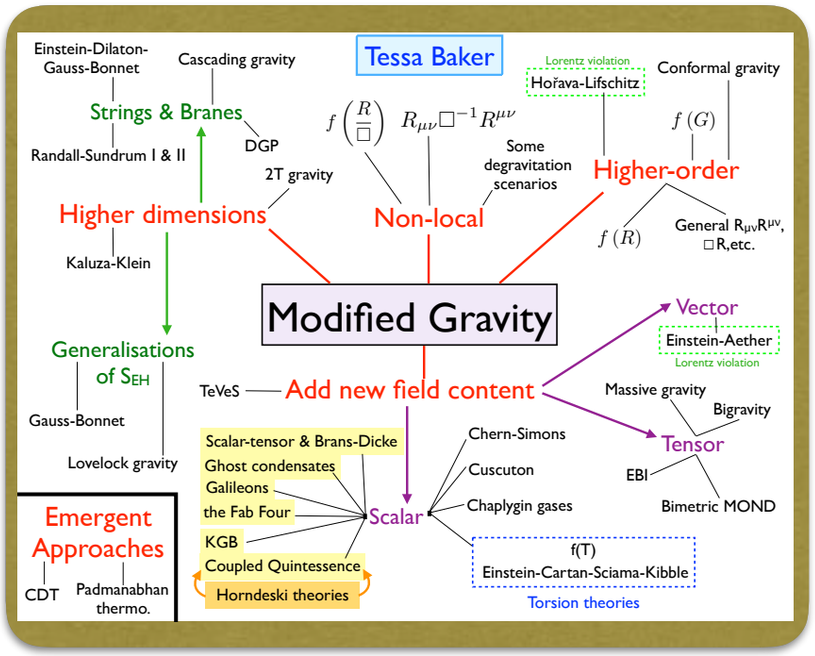
Bridge models and observations in a minimal and systematic way

Theoretically motivated: locality, causality, unitarity, stability, etc…

Observationally motivated: efficiency, now implemented in Einstein-Boltzmann codes

Models



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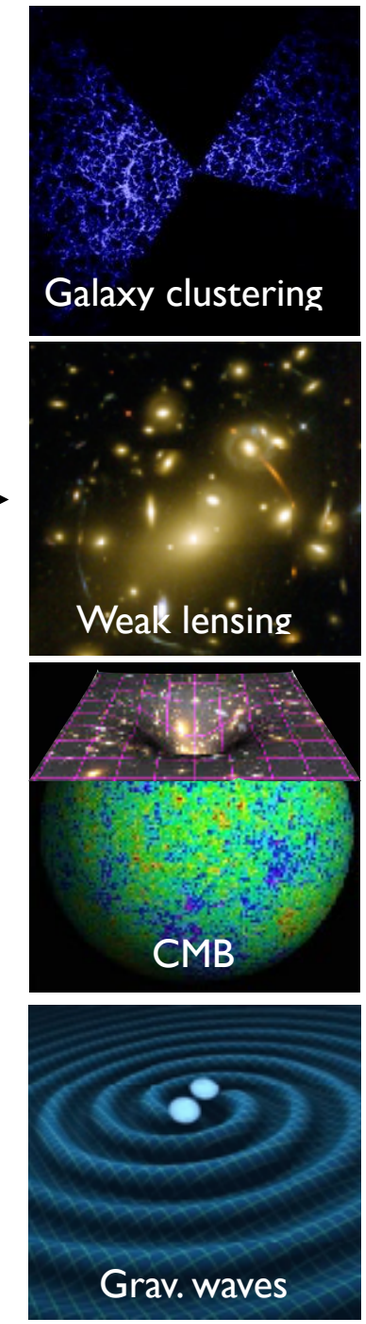


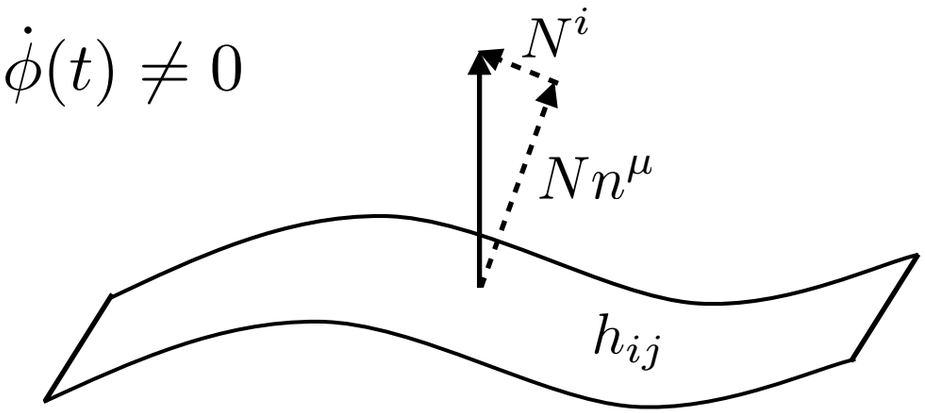
Bridge models and observations in a minimal and systematic way

Analogous approaches, further developments and extensions:

Alonso, Baker, Bellini, Bloomfield, Hu, Ferreira, Frusciante, Kennedy, Lagos, Lewandowski, Lombriser, Motohashi, Raveri, Sawicki, Silvestri, Skordis, Zumalacarregui and many more…

Constructing the action

- Use metric quantities in uniform scalar field slicing $\dot{\phi}(t) \neq 0$



  - ADM decomposition

$$ds^2 = -N^2 dt^2 + h_{ij}(N^i dt + dx^i)(N^j dt + dx^j)$$

- Lagrangian contains all possible scalars under spatial diffs, **ordered by number of perturbations and derivatives** Cheung et al. 07

$$S = \int d^4x \sqrt{-g} L[t; N, K^i_j, {}^{(3)}R^i_j, \ldots]$$

Lapse	N	$\sim \dot{\phi}$	$(\partial\phi)^2 = -\dot{\phi}_0^2(t)/N^2$
Extrinsic curvature	K_{ij}	$\sim \partial_t g_{ij}$	$K_{ij} = \frac{1}{2N}(\dot{h}_{ij} - \nabla_i N_j - \nabla_j N_i)$
Intrinsic curvature	${}^{(3)}R_{ij}$	$\sim \partial^2 g_{ij}$	

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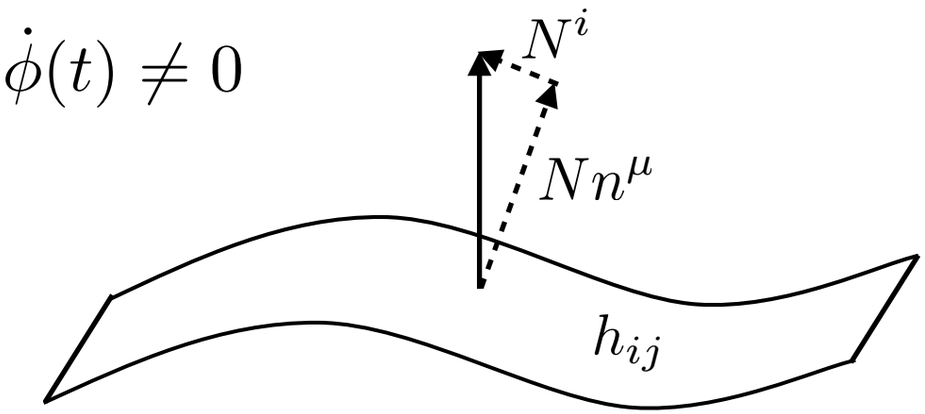

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- Expand the action

$$\delta N \equiv N - 1 \,, \qquad \delta K_{ij} \equiv K_{ij} - H h_{ij} \,, \qquad {}^{(3)}R_{ij}$$

$$L(N, K^i_j, R^i_j, \ldots) = \bar{L} + L_N \delta N + \frac{\partial L}{\partial K^i_j} \delta K^i_j + \frac{\partial L}{\partial R^i_j} \delta R^i_j + L^{(2)} + \ldots$$

Building blocks of linear perts

$$S^{(2)} = \int d^4x\sqrt{h}\frac{M^2}{2}\left[\delta K_i^j \delta K_j^i - \delta K^2 + {}^{(3)}R + \delta N {}^{(3)}R + \sum_i \alpha_i(t)\mathcal{O}_i^{(2)}(\delta N, \delta K, \ldots)\right]$$

Gleyzes, Langlois, Piazza, FV 13 (see also Bloomfield 13)

- New operators describe deviations from GR (ΛCDM)
- Time dependent couplings (functions α_i): expansion around FRW background
- Functions $\alpha_i(t)$ independent of background evolution $H(t) = \dot{a}/a$
  - we fit to data $H(t)$ and $\alpha_i(t)$ (agnostic of their time dependence and parametrization)

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Notation of Bellini, Sawicki 14 for the alphas

α_i	α_K	α_B	α_M	α_T
$\mathcal{O}_i^{(2)}$	δN^2	$\delta N \delta K$	$\frac{dM^2}{d\ln a}$	${}^{(3)}R$
quintessence, k-essence	✓			
Cubic Galileon	✓	✓		
JBD, f(R)	✓	✓	✓	
Horndeski	✓	✓	✓	✓

Horndeski theories

Covariantization of the Galileons. Most general Lorentz-invariant scalar-tensor theories with 2nd-order EOM. **No extra modes**: 1 scalar + 2 tensor polarisations

Horndeski 73, Deffayet et al. 11

$$\mathcal{L}_{\rm H}^{(2)} = G_2(\phi, X) \qquad X = \nabla_\mu\phi\nabla^\mu\phi$$

$$\mathcal{L}_{\rm H}^{(3)} = G_3(\phi, X)\Box\phi$$

$$\mathcal{L}_{\rm H}^{(4)} = G_4(\phi, X)R - 2G_{4,X}(\phi, X)\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right]$$

$$\mathcal{L}_{\rm H}^{(5)} = G_5(\phi, X)G^{\mu\nu}\nabla_\mu\nabla_\nu\phi + \frac{1}{3}G_{5,X}(\phi, X)\left[(\Box\phi)^3 - 3\Box\phi(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3\right]$$

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4 functions of time instead of 4 functions of ϕ, $(\partial\phi)^2$; minimal number of parameters

Beyond Horndeski

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Adding the following combination does not change the number of DOF

$$\mathcal{L}_{\rm GLPV} = F_4(\phi, X)\epsilon^{\mu\nu\rho}{}_\sigma\epsilon^{\mu'\nu'\rho'\sigma}\nabla_\mu\phi\nabla_{\mu'}\phi(\nabla_\nu\nabla_{\nu'}\phi)(\nabla_\rho\nabla_{\rho'}\phi)$$
$$+ F_5(\phi, X)\epsilon^{\mu\nu\rho\sigma}\epsilon^{\mu'\nu'\rho'\sigma'}\nabla_\mu\phi\nabla_{\mu'}\phi(\nabla_\nu\nabla_{\nu'}\phi)(\nabla_\rho\nabla_{\rho'}\phi)(\nabla_\sigma\nabla_{\sigma'}\phi)$$

with $XG_{5,X}F_4 = 3F_5\left(G_4 - 2XG_{4,X}\right)$

Gleyzes, Langlois, Piazza, FV 14;
(see also Zumalacarregui, Garcia-Bellido 13)

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Structure of Horndeski + BH closed by invertible metric redefinitions of the type

$$g_{\mu\nu} \;\rightarrow\; C(\phi)g_{\mu\nu} + D(\phi, X)\partial_\mu\phi\partial_\nu\phi$$

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Change of frame

$$S^{(2)} = \int d^4x\sqrt{h}\frac{M^2}{2}\left[\delta K_i^j \delta K_j^i - \delta K^2 + {}^{(3)}R + \delta N {}^{(3)}R + \sum_i \alpha_i(t)\mathcal{O}_i^{(2)}(\delta N, \delta K, \ldots)\right]$$

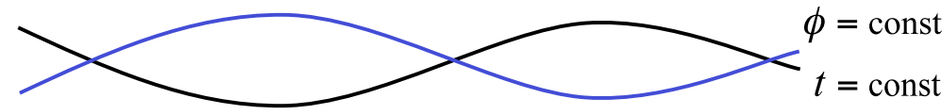
	OLD SCHOOL SCALAR TENSOR	HORNDESKI	BEYOND HORNDESKI
OPERATORS	$\alpha_{\rm K}, \alpha_{\rm B}, \alpha_{\rm M}$	$\alpha_{\rm K}, \alpha_{\rm B}, \alpha_{\rm M}, \alpha_{\rm T}$	$\alpha_{\rm K}, \alpha_{\rm B}, \alpha_{\rm M}, \alpha_{\rm T}, \alpha_{\rm H}$
COUPLING	$\tilde{g}_{\mu\nu} = C(\phi)g_{\mu\nu}$	$\tilde{g}_{\mu\nu} = C(\phi)g_{\mu\nu} + D(\phi)\partial_\mu\phi\partial_\nu\phi$	$\tilde{g}_{\mu\nu} = C(\phi)g_{\mu\nu} + D(\phi, X)\partial_\mu\phi\partial_\nu\phi$

Bettoni, Liberati 13; Gleyzes, Langlois, Piazza, FV 14;

Beyond Horndeski $\xrightarrow[D(X)]{}$ Horndeski

Beyond Horndeski phenomenology

Undo unitary gauge: $t \to t + \pi(t, \vec{x})$



Newtonian gauge (scalar flucts): $dt^2 = -(1 + 2\Phi)dt^2 + a^2(t)(1 - 2\Psi)d\vec{x}^2$

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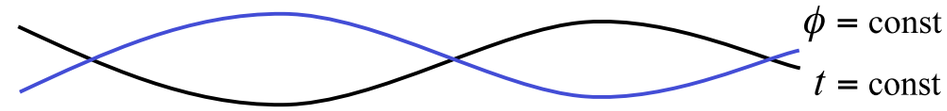

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Higher derivative term in Jordan frame:

$$\mathcal{L}^{(\mathrm{Jord})} = \mathcal{L}_{\mathrm{Horn}}(\Phi, \Psi) + 2M^2\alpha_H \nabla\dot{\pi}\nabla\Psi - \Phi\delta\rho_{\mathrm{m}}$$

Can be de-mixed by going to Horndeski frame: $\Phi \;\to\; \Phi_{\mathrm{H}} = \Phi - \alpha_H\dot{\pi}$

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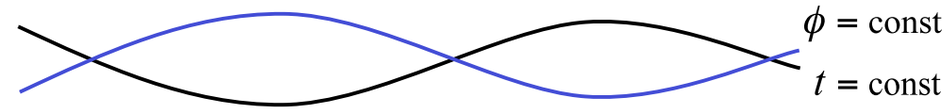

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Matter/dark energy dispersion relation:

$$(\omega^2 - c_s^2k^2)(\omega^2 - c_{\mathrm{m}}^2k^2) = \lambda^2\omega^2k^2$$

Gleyzes, Langlois, Piazza, FV 14
De Felice, Koyama, Tsujikawa 15

Frame invariant parameter $\lambda^2 \sim \alpha_H^2$

Quasi-static limit

Quasi-static approximations — valid on scales $k \gg aHc_s^{-1}$. Sawicki, Bellini '15
E.g., for surveys such as Euclid $c_s \gtrsim 0.1$.

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Modified Poisson equation:

$$\nabla^2\Phi = \frac{3}{2}H^2\Omega_{\rm m}(1+\Upsilon_\Phi)\delta_{\rm m}$$

fifth force

Horndeski theories: $\Upsilon_\Phi \geq 0$

Ex., braiding: $\Upsilon = \dfrac{2\alpha_B^2}{\alpha c_s^2}$

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matter mixing

Kobayashi, Watanabe, Yamauchi 15

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Beyond Horndeski: $\gamma \neq 0$

Using continuity and Euler equations:

$$\ddot{\delta}_m + (2H+\gamma)\dot{\delta}_m = \frac{3}{2}H^2\Omega_m(1+\Upsilon_\Phi)\delta_m$$

D'Amico, Mancarella, FV 16

Einstein-Boltzmann solvers

Full Einstein-Boltzmann solver:

$$\frac{df_I}{d\eta} = C_I[f_I] \ , \quad I = \gamma, \nu, b, \mathrm{CDM}$$

$$\frac{\delta S^{(2)}}{\delta \pi} = 0 \qquad \& \qquad G_{ij}^{\mathrm{modified}} = 8\pi G \sum_I T_{ij}^{(I)}$$

- EFTCAMB (from CMBFAST) (Hu, Raveri, Frusciante, Silvestri et al.)
- hi_class (from CLASS) (Zumalacarregui, Bellini, Sawicki et al.)
- COOP (indep. code, Zhiqi Huang) (with D'Amico, Huang and Mancarella)
- LVDM-CLASS (from CLASS) (Blas, Ivanov, Sibiryakov)
- others …

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Parametrization: $$H = H_0\sqrt{\Omega_{\mathrm{m},0}a^{-3} + 1 - \Omega_{\mathrm{m},0}} \qquad \alpha_I(t) = \alpha_{I,0}\,\frac{1-\Omega_{\mathrm{m}}(t)}{1-\Omega_{\mathrm{m},0}}$$

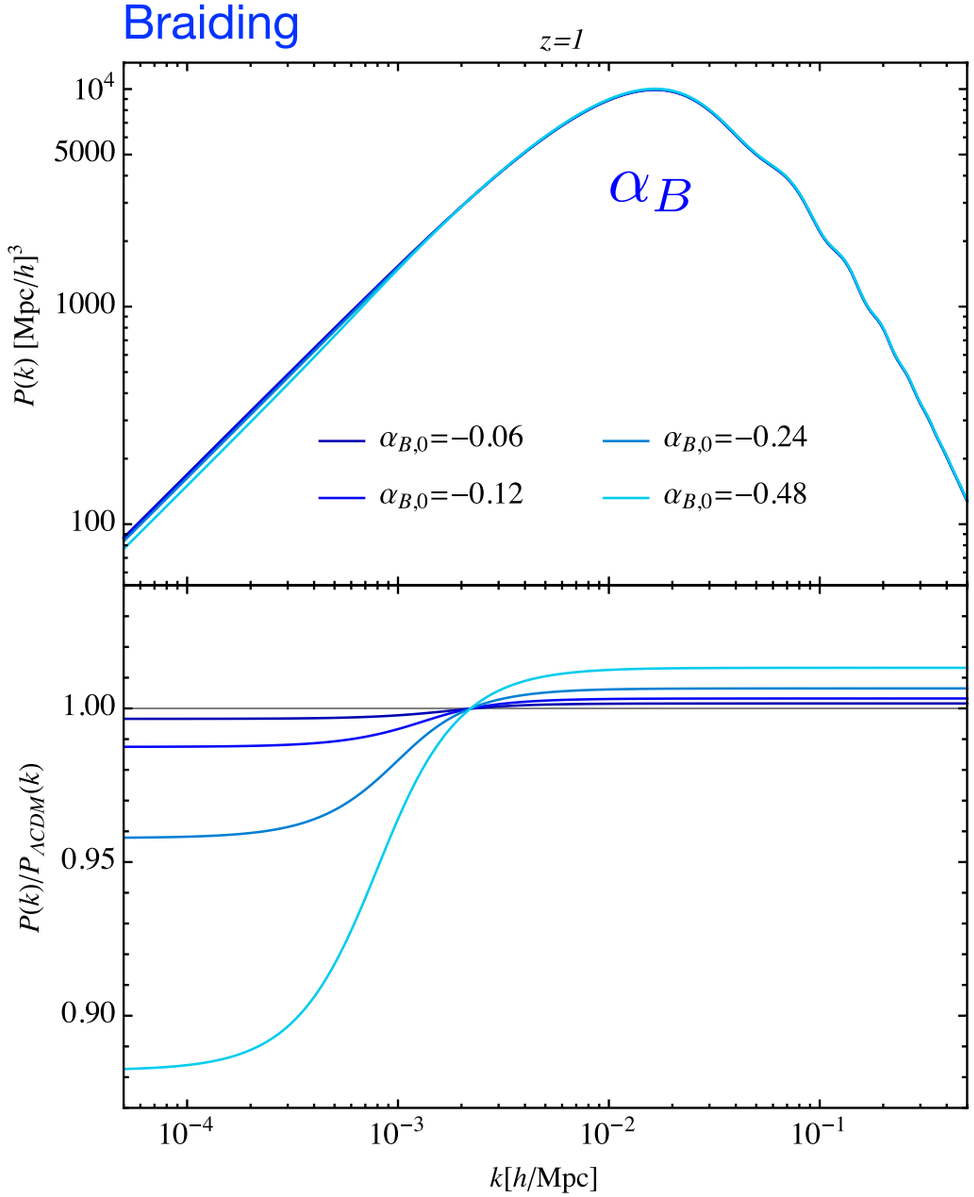
Two case studies:

- Braiding $\alpha_B \neq 0$. Stability implies: $\alpha_B < 0$
- Beyond Horndeski $\alpha_H \neq 0$. Stability implies: $\alpha_H > 0$

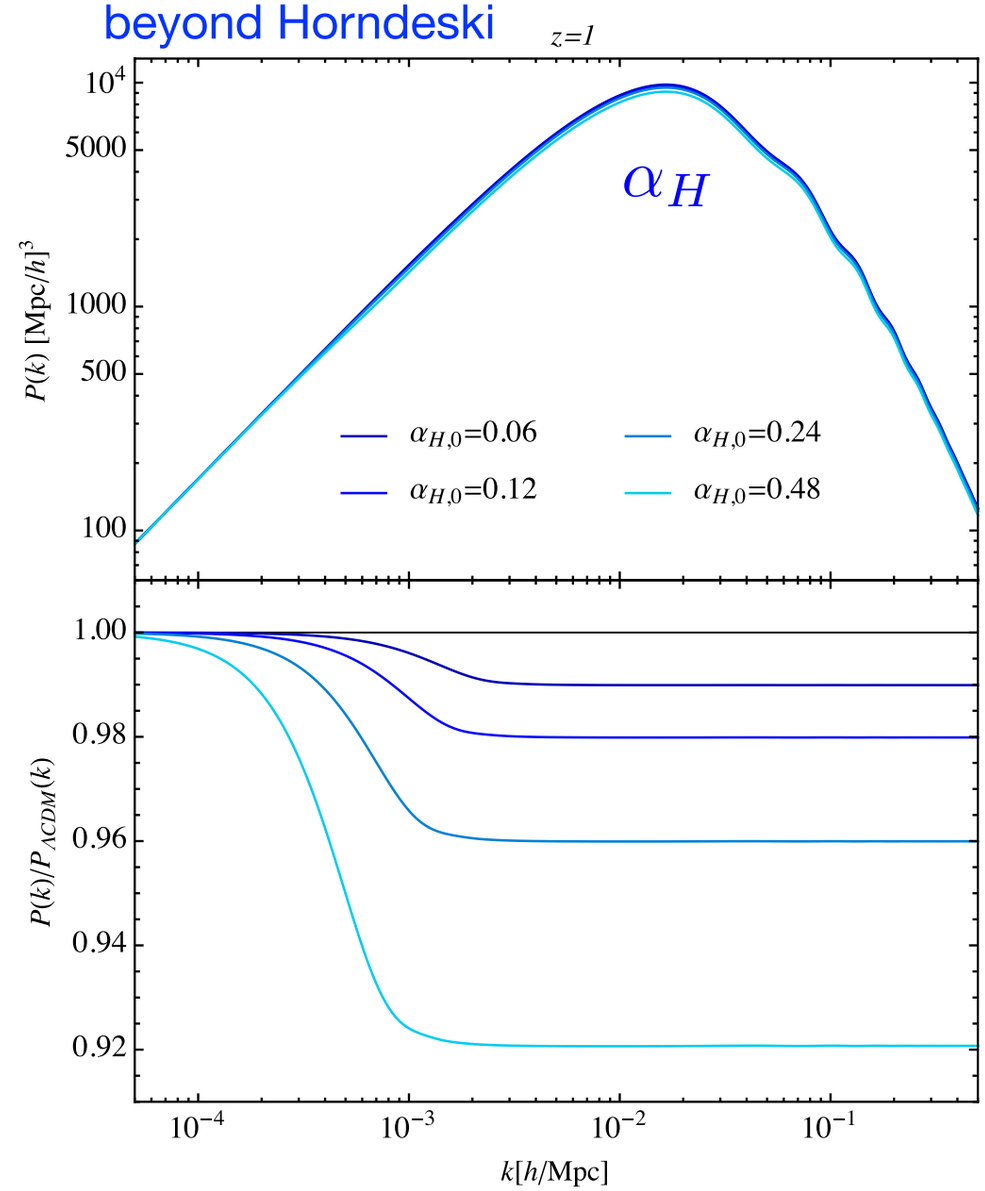
Power spectrum

D'Amico, Mancarella, FV 16

Braiding



beyond Horndeski



$\Upsilon_\Phi > 0\,, \quad \gamma = 0$

$\Upsilon_\Phi = -\gamma < 0$

$$\ddot{\delta}_m + (2H + \gamma)\dot{\delta}_m = \frac{3}{2}H^2\Omega_m(1 + \Upsilon_\Phi)\delta_m$$

Planck/small scale tension

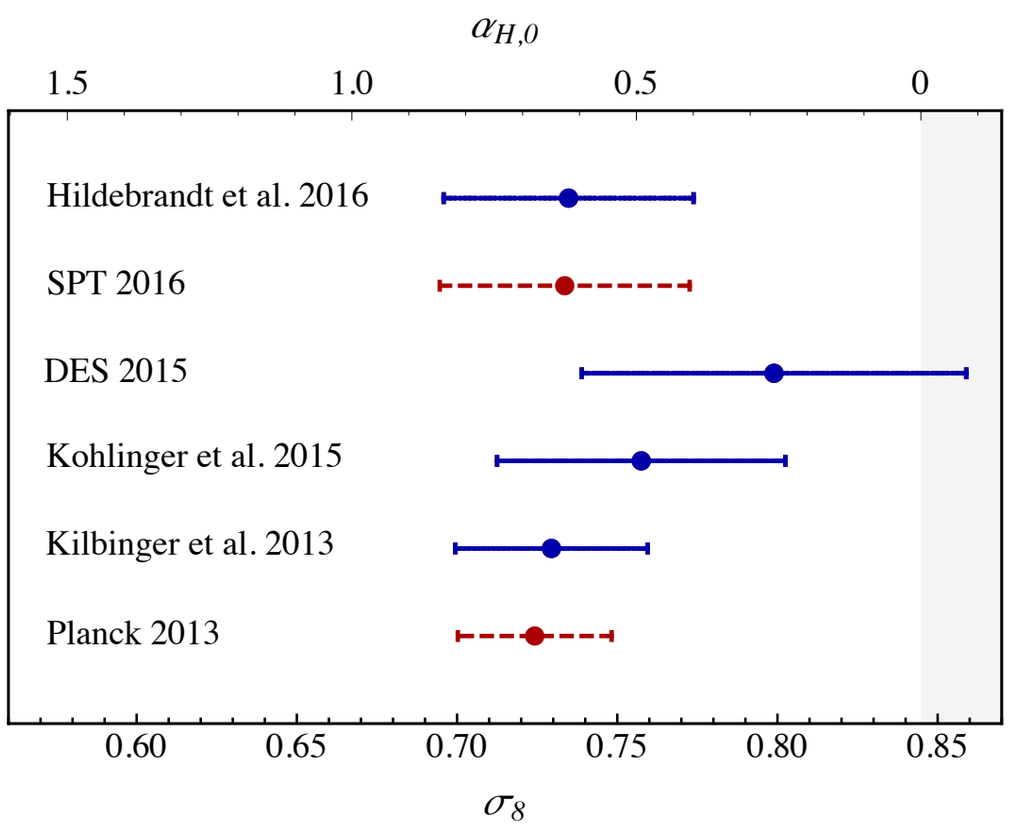
- Lensing observations by the CFHTLenS and KiDS persistently discordant with Planck satellite measurements (~2σ)

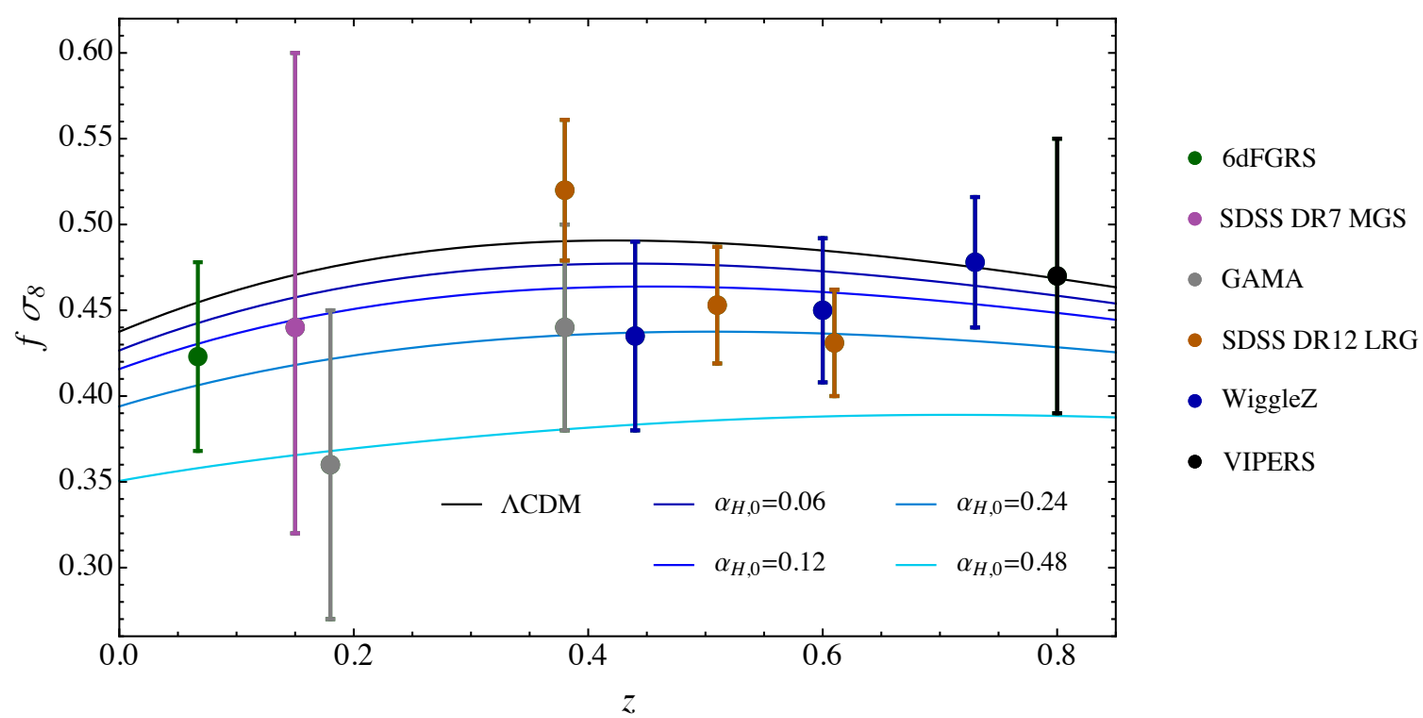
Ade et al. ’14; MacCrann et al. ’15; Ade et al. ‘16;
Köhlinger et al. ’15; Joudaki et al. ‘16
Hildebrandt et al. ’16; Joudaki et al. ‘16

- Similar trend observed in cluster counts

Ade et al. ’13, ’15; de Haan et al. ‘16

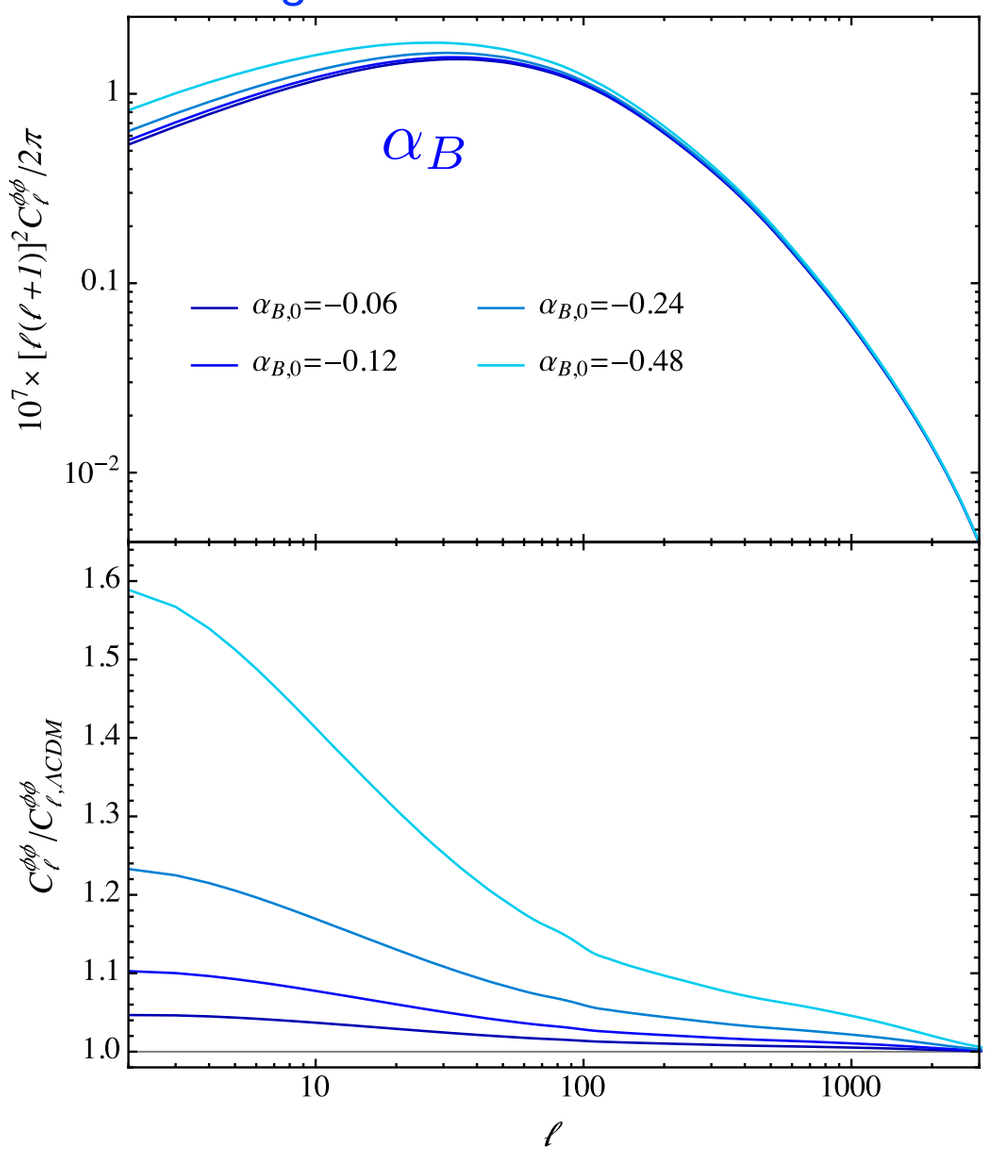
- Beyond Horndeski term relaxes the tension:



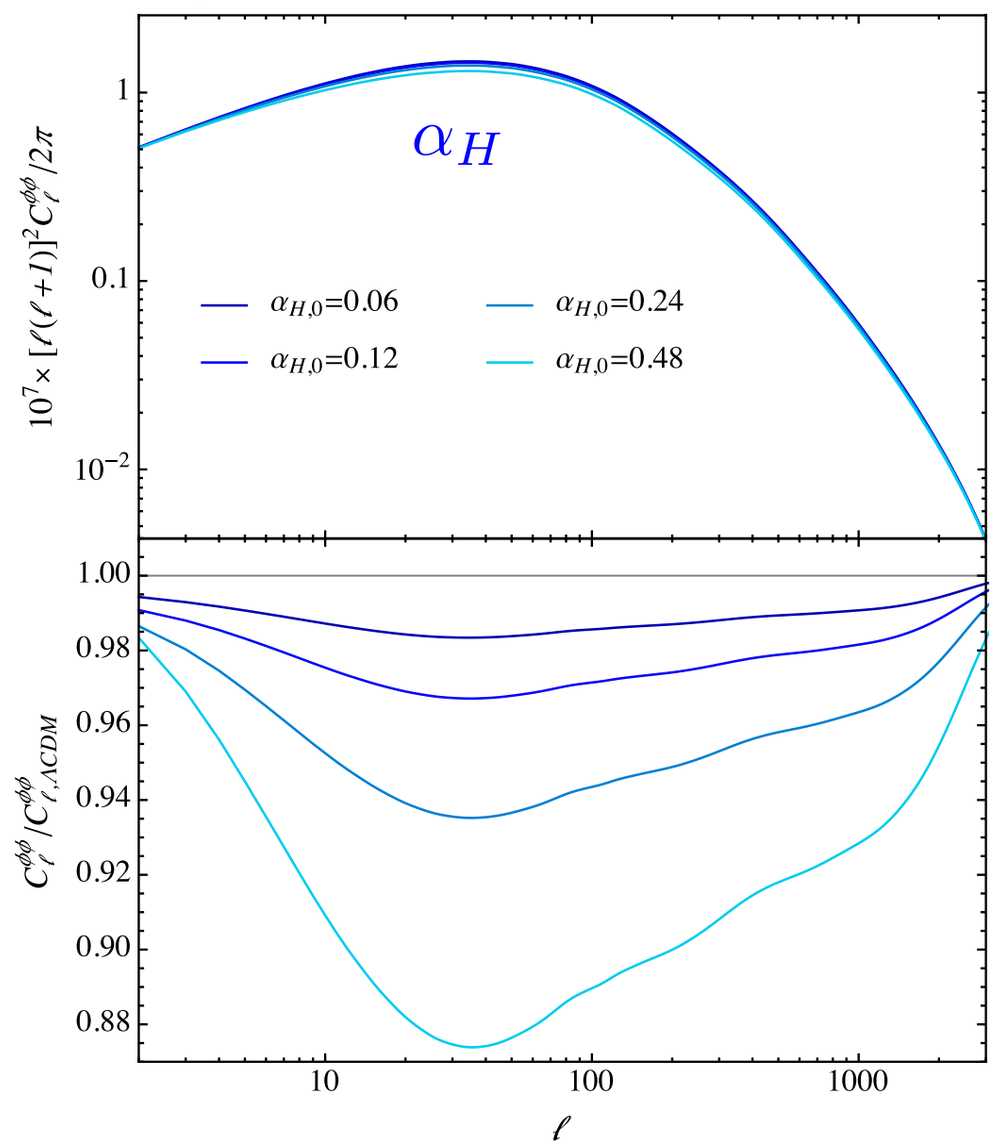


CMB lensing

Braiding



beyond Horndeski



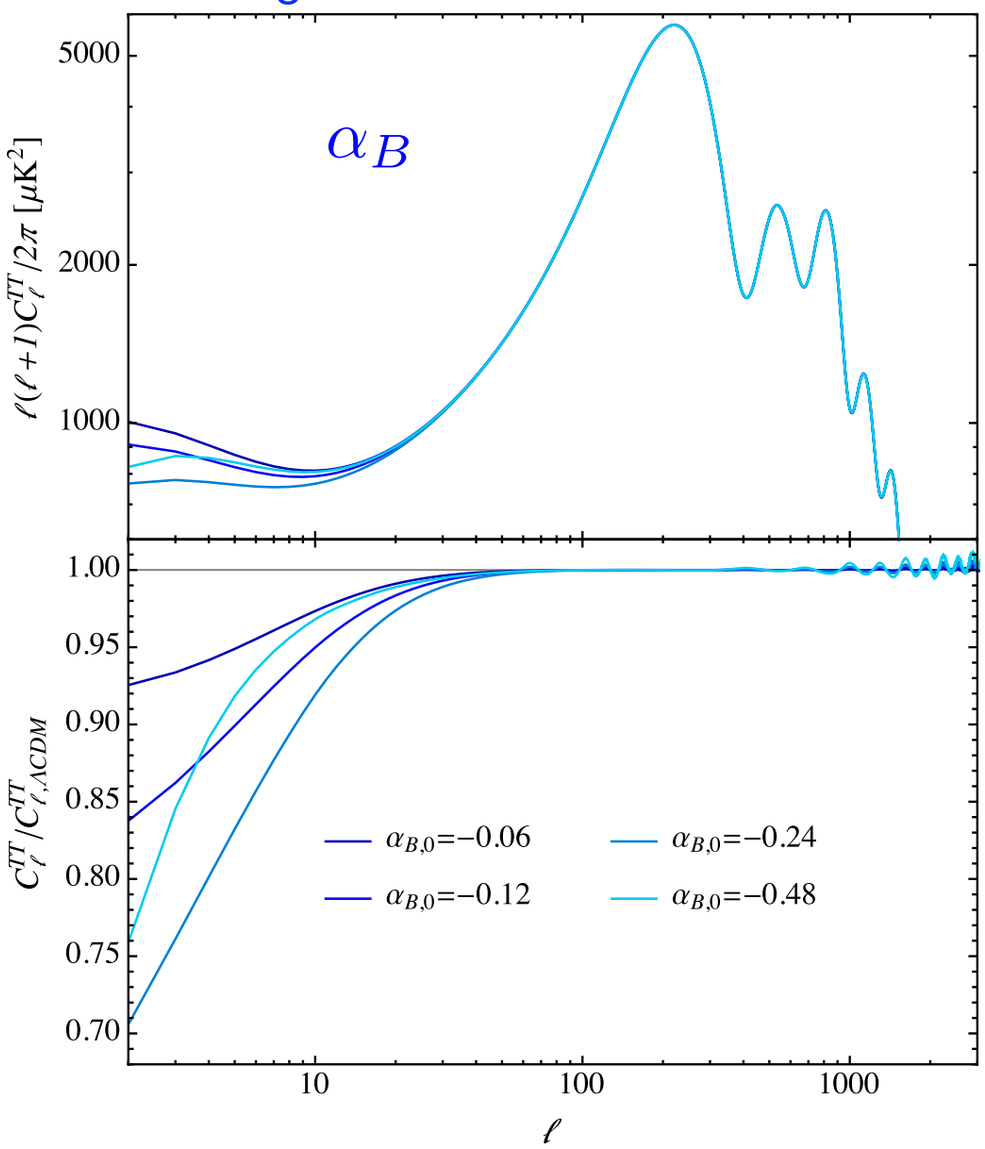
$\Upsilon_{\rm lens} > 0\,, \quad \gamma = 0$

$\Upsilon_{\rm lens} < 0\,, \quad \gamma > 0$

$$\nabla^2(\Psi+\Phi) = 8\pi G(1+\Upsilon_{\rm lens} - \frac{2}{3}\gamma f)a^2\rho_{\rm m}\delta_{\rm m} \qquad f \equiv \frac{d\ln\delta_{\rm m}}{d\ln a}$$

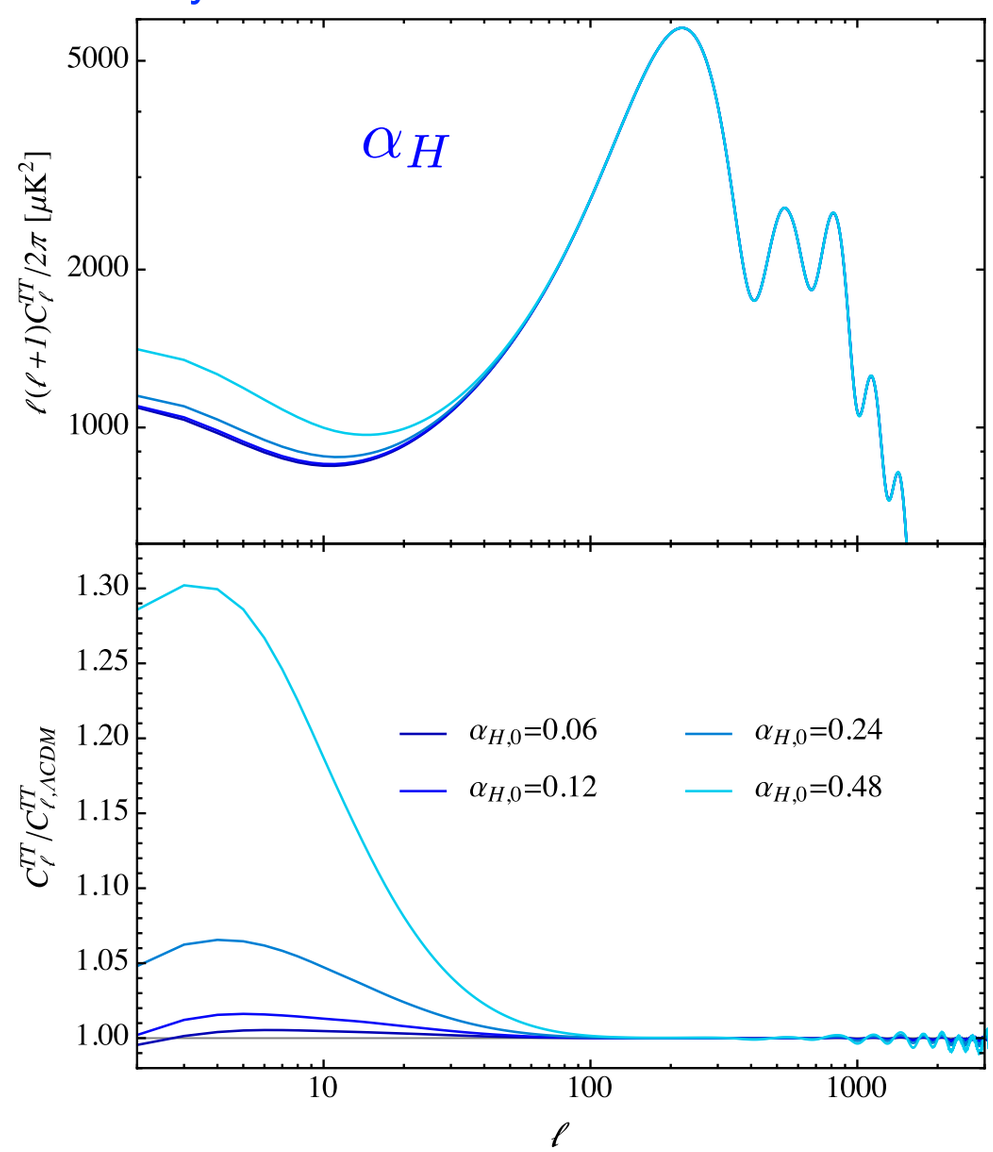
CMB

Braiding



$$\frac{d\ln(1+\Upsilon_{\rm lens})}{d\ln a} > 0$$

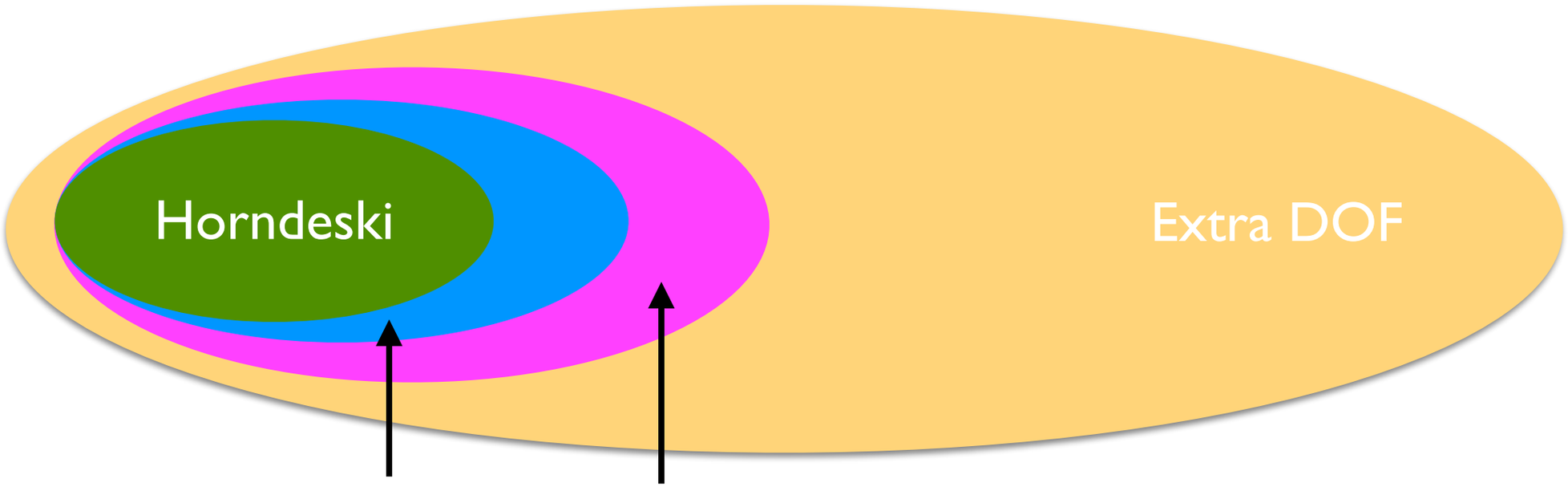
beyond Horndeski



$$\frac{d\ln(1+\Upsilon_{\rm lens})}{d\ln a} < 0$$

$$\frac{d\ln(\Psi+\Phi)}{d\ln a} \approx f - 1 + \frac{d\ln(1+\Upsilon_{\rm lens})}{d\ln a}$$

Higher-Order Degenerate theories



beyond Horndeski

Zumalacarregui, Garcia-Bellido 13
Gleyzes, Langlois, Piazza, FV 14;

More degenerate theories:
DHOST/EST

Langlois, Noui 15, 16;
Crisostomi, Koyama, Tasinato 16
De Rham, Matas 16
Motohashi et al 16; Klein & Roest 16 for multifields

Degenerate Higher-Order Scalar-Tensor theories or Extended Scalar Tensor theories

- Quartic theories:

$$\mathcal{L}^{(2)}_{\rm DHOST} = f_2(\phi, X)^{(4)}R + \sum_i^5 C_i^{\mu\nu\rho\sigma}(\phi, \nabla_\alpha\phi)\nabla_\mu\nabla_\nu\phi\nabla_\rho\nabla_\sigma\phi$$

- 3 degeneracy conditions (7 classes) associated to second-class constraints

- Extended to <u>Quadratic + Cubic</u> theories: 9 classes for cubic, 7 + 9 + 25 combinations of quadratic and cubic theories

Higher-Order theories

$$S^{(2)} = \int d^4x\sqrt{h}\frac{M^2}{2}\left[\delta K^j_i \delta K^i_j - \delta K^2 + {}^{(3)}R + \delta N {}^{(3)}R + \sum_i \alpha_i(t)\mathcal{O}^{(2)}_i(\delta N, \delta K, \ldots)\right]$$

All quadratic operators **up to two** derivatives

Langlois, Mancarella, Noui, FV 17

α_i	α_K	α_B	α_M	α_T	α_H	α_L	β_1	β_2	β_3
$\mathcal{O}^{(2)}_i$	δN^2	$\delta N \delta K$	$\frac{dM^2}{d\ln a}$	${}^{(3)}R$	$\delta N {}^{(3)}R$	δK^2	$\delta \dot{N}^2$	$\delta \dot{N} \delta K$	$(\partial_i \delta N)^2$
quintessence, k-essence	✓								
Cubic Galileon	✓	✓							
JBD, f(R)	✓	✓	✓						
Horndeski	✓	✓	✓	✓					
Beyond Horndeski	✓	✓	✓	✓	✓				
DHOST/EST theories	✓	✓	✓	✓	✓	✓	✓	✓	✓

Higher-Order theories

$$S^{(2)} = \int d^4x\sqrt{h}\frac{M^2}{2}\left[\delta K_i^j \delta K_j^i - \delta K^2 + {}^{(3)}R + \delta N\, {}^{(3)}R + \sum_i \alpha_i(t)\mathcal{O}_i^{(2)}(\delta N, \delta K, \ldots)\right]$$

All quadratic operators **up to two** derivatives

Langlois, Mancarella, Noui, FV 17

α_i	α_K	α_B	α_M	α_T	α_H	α_L	β_1	β_2	β_3
$\mathcal{O}_i^{(2)}$	δN^2	$\delta N \delta K$	$\frac{dM^2}{d\ln a}$	${}^{(3)}R$	$\delta N\, {}^{(3)}R$	δK^2	$\delta \dot{N}^2$	$\delta \dot{N} \delta K$	$(\partial_i \delta N)^2$

- Generic scalar dispersion relation: $\mathcal{E}_1\omega^4 + \mathcal{E}_2\omega^2 k^2 + \mathcal{E}_3\omega^2 + \mathcal{E}_4 k^4 + \mathcal{E}_5 k^2 = 0$

- **Two types** of degeneracy conditions lead to $\omega^2 - c_s^2 k^2 = 0$

$$\mathcal{C}_{\rm I}: \quad \alpha_L = 0\,, \qquad \beta_2 = f_2(\beta_1)\,, \qquad \beta_3 = f_3(\beta_1)$$

$$\mathcal{C}_{\rm II}: \quad \beta_1 = f_1(\alpha_T, \alpha_H, \alpha_L)\,, \quad \beta_2 = f_2(\alpha_T, \alpha_H, \alpha_L)\,, \quad \beta_3 = f_3(\alpha_T, \alpha_H, \alpha_L)$$

$$c_s^2 \propto -c_T^2$$ ruled out!

Higher-Order theories

$$S^{(2)} = \int d^4x\sqrt{h}\frac{M^2}{2}\left[\delta K^j_i \delta K^i_j - \delta K^2 + {}^{(3)}R + \delta N {}^{(3)}R + \sum_i \alpha_i(t)\mathcal{O}^{(2)}_i(\delta N, \delta K, \ldots)\right]$$

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	OLD SCHOOL SCALAR TENSOR	HORNDESKI	BEYOND HORNDESKI	DHOST
OPERATORS	$\alpha_{\rm K}, \alpha_{\rm B}, \alpha_{\rm M}$	$\alpha_{\rm K}, \alpha_{\rm B}, \alpha_{\rm M}, \alpha_{\rm T}$	$\alpha_{\rm K}, \alpha_{\rm B}, \alpha_{\rm M}, \alpha_{\rm T}, \alpha_{\rm H}$	$\alpha_{\rm K}, \alpha_{\rm B}, \alpha_{\rm M}, \alpha_{\rm T}, \alpha_{\rm H}, \alpha_{\rm L}, \beta_1, \beta_2, \beta_3$
COUPLING	$\tilde{g}_{\mu\nu} = C(\phi) g_{\mu\nu}$	$\tilde{g}_{\mu\nu} = C(\phi) g_{\mu\nu} + D(\phi)\partial_\mu\phi\partial_\nu\phi$	$\tilde{g}_{\mu\nu} = C(\phi) g_{\mu\nu} + D(\phi, X)\partial_\mu\phi\partial_\nu\phi$	$\tilde{g}_{\mu\nu} = C(\phi, X) g_{\mu\nu} + D(\phi, X)\partial_\mu\phi\partial_\nu\phi$

Higher-Order theories

$$S^{(2)} = \int d^4x\sqrt{h}\frac{M^2}{2}\left[\delta K_i^j \delta K_j^i - \delta K^2 + {}^{(3)}R + \delta N\, {}^{(3)}R + \sum_i \alpha_i(t)\mathcal{O}_i^{(2)}(\delta N, \delta K, \ldots)\right]$$

All quadratic operators **up to two** derivatives

Langlois, Mancarella, Noui, FV 17

α_i	α_K	α_B	α_M	α_T	α_H	α_L	β_1	β_2	β_3
$\mathcal{O}_i^{(2)}$	δN^2	$\delta N \delta K$	$\frac{dM^2}{d\ln a}$	${}^{(3)}R$	$\delta N\, {}^{(3)}R$	δK^2	$\delta \dot{N}^2$	$\delta \dot{N} \delta K$	$(\partial_i \delta N)^2$

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- **Two types** of degeneracy conditions lead to $\omega^2 - c_s^2 k^2 = 0$

$$\mathcal{C}_{\rm I}: \quad \alpha_L = 0\,, \qquad \beta_2 = f_2(\beta_1)\,, \qquad \beta_3 = f_3(\beta_1)$$

- Class $\mathcal{C}_{\rm I}$ can be brought to **Horndeski frame**: $\alpha_H = 0,\ \beta_J = 0$

$$\text{DHOST I} \xrightarrow[C(X)]{} \text{Beyond Horndeski} \xrightarrow[D(X)]{} \text{Horndeski}$$

- Changing frame changes matter couplings (Horndeski vs Jordan): **Matter matters**!

Conclusions

- Effective Theory of Dark Energy
- Fifth force in beyond Horndeski can be repulsive and slow down structure formation on short scales
- Use Einstein-Boltzmann code to study its large-scale phenomenology
- Effective description includes higher-order degenerate scalar-tensor theories
- Of the many degenerate existing classes only two survive. Simplifying power of an effective description