Towards a unified description of theories with a single scalar degree of freedom

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The unified description

Suppose we have

Generic actions describing gravity ($g_{\mu\nu}$) coupled to

- Scalar (ϕ)
- Vector (A^{μ})
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I don't think so

Scalar-Tensor Theories

EFT of Dark Energy gives us the action!
If up to second derivatives we have linearized Horndeski!

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Kineticity: α_{K}

- standard perfect-fluid structures
- not present in archetypal "modified gravity" models

Braiding: α_{B}

- mixes the kinetic terms of the scalar and metric
- responsible for all the new scale-dependence

Planck mass run-rate: α_{M}

- $\frac{\mathrm{d}}{\mathrm{d}t} M_*^2$. M_*^2 effective Planck mass
- anisotropic stress

Tensor speed excess: α_{T}

- determines the speed of gravitational waves
- anisotropic stress

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You choose

- Fields
- Gauge symmetries
- Number of derivatives

The rest is done by your PC!

Vector-Tensor Theories

- Fields: $g_{\mu\nu}$, A^{μ} plus matter φ
- Gauge symmetries: linear diff invariance
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SVT Decomposition

$$ds^2 = a^2(\tau)\left[-\left(1-\delta g_{00}\right)d\tau^2 + 2\delta g_{0i}d\tau dx^i + \delta g_{ij}dx^i dx^j\right]$$

$$\delta g_{00} = 2\psi$$
$$\delta g_{0i} = \partial_i B + S_i$$
$$\delta g_{ij} = 2\phi\delta_{ij} + \partial_i\partial_j E + \partial_i F_j + \partial_j F_i + h_{ij}$$

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$\boxed{A^\mu = (\bar{A} + \delta A_0,\, \delta A_i)}$ with $\delta A_0 = \alpha_0$ and $\delta A_i = \partial_i\alpha + \alpha_i$

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$$\boxed{\varphi = \bar{\varphi} + \delta\varphi}$$

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- Generic action

$$S = \int d^3x d\tau \, \mathcal{L}\left(g_{00}, g_{0i}, g_{ij}, A^0, A^i\right) + S_m$$

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- Quadratic Lagrangian

$$\mathcal{L}^{(2)} = \mathcal{L}_{g_{00}g_{00}} \delta g_{00}^2 + \mathcal{L}_{g_{00}\ddot{g}_{00}} \delta g_{00} \delta \ddot{g}_{00} + \mathcal{L}_{\dot{A}^i \dot{A}_i} \delta \dot{A}^i \delta \dot{A}_i + \ldots$$

- Quadratic Lagrangian in SVT

$$\mathcal{L}^{(2)} = \mathcal{L}_{g_{00}g_{00}} \psi^2 + \mathcal{L}_{g_{00}\ddot{g}_{00}} \psi \ddot{\psi} + \mathcal{L}_{\dot{A}^i \dot{A}_i} (\partial_i \dot{\alpha} \partial^i \dot{\alpha} + \dot{\alpha}_i \dot{\alpha}^i) + \ldots$$

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Linear coordinate transformation

$$\hat{\delta g}_{\mu\nu} = \delta g_{\mu\nu} + \partial_\alpha g_{\mu\nu}\xi^\alpha + g_{\mu\alpha}\partial_\nu\xi^\alpha + \partial_\mu g_{\alpha\nu}\xi^\alpha$$
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$$\xi^\alpha = (\pi,\, \partial^i\varepsilon + \gamma^i)$$

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$$\hat{\mathcal{L}^{(2)}} = \mathcal{L}^{(2)}$$

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$$\Rightarrow \quad \frac{\delta\hat{\mathcal{L}^{(2)}}}{\delta\pi} = \frac{\delta\hat{\mathcal{L}^{(2)}}}{\delta\varepsilon} = \frac{\delta\hat{\mathcal{L}^{(2)}}}{\delta\gamma^i} = 0$$

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Noether constraints

$$\mathcal{L}_{g_{00}g_{00}} = 0$$

$$\mathcal{L}_{A_0A_0} + \mathcal{L}_{A_0\dot{A}_0} = 0$$

$$\ldots$$

Minimal number of free functions of time

Results and (Unification?)

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Unification: OK only for practical purposes!

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Thank you!