Reconstructing Horndeski models from the effective field theory of dark energy

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The model space

- Large model space of different theories, even within scalar-tensor theories.
- Efficiently computing observational consequences of each theory is challenging.
- Effective Field Theory of Dark Energy gives an efficient exploration of the model space.

Horndeski Theory

- Add a scalar field into Einstein's gravity.
- Most general, local, Lorentz covariant, four-dimensional scalar-tensor theory with second order equations of motion.
- Dilaton from string theory, compactified internal spaces, extra dimensions, brane worlds...

Horndeski Theory

$$S = \sum_{i=2}^{5} \int d^4x \sqrt{-g} \mathcal{L}_i \,,$$

$$\begin{aligned}
\mathcal{L}_2 &\equiv G_2(\phi, X)\,, \\
\mathcal{L}_3 &\equiv G_3(\phi, X)\Box\phi\,, \\
\mathcal{L}_4 &\equiv G_4(\phi, X)R \\
&\quad -2G_{4X}(\phi, X)\left[(\Box\phi)^2 - (\nabla^\mu\nabla^\nu\phi)(\nabla_\mu\nabla_\nu\phi)\right]\,, \\
\mathcal{L}_5 &\equiv G_5(\phi, X)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi \\
&\quad +\frac{1}{3}G_{5X}(\phi, X)\left[(\Box\phi)^3 - 3(\Box\phi)(\nabla_\mu\nabla_\nu\phi)(\nabla^\mu\nabla^\nu\phi)\right. \\
&\quad \left. +2(\nabla_\mu\nabla_\nu\phi)(\nabla^\sigma\nabla^\nu\phi)(\nabla_\sigma\nabla^\mu\phi)\right]\,,
\end{aligned}$$

where $X = g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$.

Effective field theory of Dark Energy (EFTofDE)

- Generalized and efficient approach to testing different theories.
- Unitary gauge: ADM decomposition with $\phi = tM_*^2$. Then $X = (-1 + \delta g^{00})M_*^2$.

$$S = S^{(0,1)} + S^{(2)} + S_M[g_{\mu\nu}, \psi]\,,$$
$$S^{(0,1)} = \frac{M_*^2}{2}\int d^4x\sqrt{-g}\,[\Omega(t)R]$$

- $\Omega(t) = 1 \Rightarrow$ Einstein gravity.

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- $\Gamma = M_*^2 \Rightarrow$ Quintessence.

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S^{(2)} &= \int d^4x\sqrt{-g}\left[\frac{1}{2}M_2^4(t)(\delta g^{00})^2 - \frac{1}{2}\bar{M}_1^3(t)\delta K\delta g^{00}\right. \\
&\quad \left. -\bar{M}_2^2(t)\left(\delta K^2 - \delta K^{\mu\nu}\delta K_{\mu\nu} - \frac{1}{2}\delta R^{(3)}\delta g^{00}\right)\right]\,,
\end{aligned}
$$

- Ghost condensate, DGP, khronon, kinetic braiding, k-essence, galileon, Horndeski...
- With $H(t)$ that's now only seven free functions.

Alternative description of EFT functions

- Can go to another basis for the EFT functions, $\alpha_M, \alpha_B, \alpha_K, \alpha_T$, which have a more physical motivation.
- Expansion history $H(t)$ fixed. Looking for probes of modified gravity at the level of linear perturbations.
- Two descriptions related via a linear transformation e.g.

$$\Omega = \frac{M^2}{M_*^2} c_T^2 \ , \ \bar{M}_2^2 = -\frac{1}{2} M^2 \alpha_T \ ,$$

$$\bar{M}_1^3 = M^2 \left[H\alpha_M c_T^2 + \dot{\alpha}_T - 2H\alpha_B \right] \ .$$

Reconstructing Covariant theories

- The EFT functions Ω, Γ, $\bar{M}_2^2$ etc are phenomenological functions.
- Given constraints on the phenomenological EFT functions, what can we say about the space of covariant theories?

Reconstructing Covariant theories

- The EFT functions Ω, Γ, $\bar{M}_2^2$ etc are phenomenological functions.
- Given constraints on the phenomenological EFT functions, what can we say about the space of covariant theories?
- Reconstruct the class of covariant Horndeski models that are equivalent at the level of the background and linear perturbations.

Reconstructing Covariant theories

- Class of Horndeski theories that correspond to the EFT of DE at the level of background and linear perturbations.

$$
\begin{aligned}
G_2(\phi, X) &= -\,M_*^2 U(\phi) - \frac{1}{2} M_*^2 Z(\phi) X + a_2(\phi) X^2 \\
&\quad + \Delta G_2\,, \\
G_3(\phi, X) &= b_0(\phi) + b_1(\phi) X + \Delta G_3\,, \\
G_4(\phi, X) &= \frac{1}{2} M_*^2 F(\phi) + c_1(\phi) X + \Delta G_4\,, \\
G_5(\phi, X) &= \Delta G_5\,,
\end{aligned}
$$

The Reconstruction

- For example

$$U(\phi) = \Lambda + \frac{\Gamma}{2} - \frac{M_2^4}{2M_*^2} - \frac{9H\bar{M}_1^3}{8M_*^2} - \frac{(\bar{M}_1^3)'}{8} + \frac{M_*^2(\bar{M}_2^2)''}{4} + \dots ,$$

$$Z(\phi) = \frac{\Gamma}{M_*^4} - \frac{2M_2^4}{M_*^6} - \frac{3H\bar{M}_1^3}{2M_*^6} + \frac{(\bar{M}_1^3)'}{2M_*^4} - \frac{(\bar{M}_2^2)''}{M_*^2} + \dots ,$$

$$F(\phi) = \Omega + \frac{\bar{M}_2^2}{M_*^2}\ , \ c_1(\phi) = \frac{\bar{M}_2^2}{2M_*^4} .$$

Similar expressions for other terms in the reconstructed action.

Non-linear corrections

- Such a reconstruction cannot be unique. Each ΔG_i quantifies the non-linear corrections that one can make to move to an action that is degenerate at background and linear level.

$$\Delta G_{2,3} = \sum_{n>2} \xi_n^{(2,3)}(\phi) \left(1 + \frac{X}{M_*^4}\right)^n ,$$

$$\Delta G_{4,5} = \sum_{n>3} \xi_n^{(4,5)}(\phi) \left(1 + \frac{X}{M_*^4}\right)^n .$$

- Each $\xi_n^{(i)}(\phi)$ is a free function of ϕ.

Derivation - Zeroth and first order

- Note the correspondence at the linear level

$$\delta g^{00} = 1 + X/M_*^4 \,.$$

- Starting from the background and first order action with $\Omega = 1$ we find that

$$S^{(0,1)}_{\Omega=1} = \int d^4x\sqrt{-g}\left\{\frac{M_*^2}{2}R - M_*^2\Lambda(\phi) - \frac{M_*^2}{2}\Gamma(\phi) - \frac{\Gamma(\phi)}{2M_*^2}X\right\} .$$

- This corresponds to a quintessence model in non-canonical form. Can perform a field re-definition to make it canonical.

Derivation - Quadratic order

- Choose a term involving $X^n \Box\phi$. For simplicity choose $n = 1$. In the unitary gauge this becomes

$$M_*^{-6}\ell_3(\phi)X\Box\phi = \left[\dot{\ell}_3(t) - 3\ell_3(t)H\right] g^{00} - \ell_3(t)\delta g^{00}\delta K \\ - 3\ell_3(t)H + \frac{3H}{4}\ell_3(t)(\delta g^{00})^2 \\ - \frac{1}{4}\dot{\ell}(t)(\delta g^{00})^2\,. \qquad (1)$$

- Move all terms apart from $\delta g^{00}\delta K$ to the left hand side, and make replacement $\delta g^{00} = 1 + X/M_*^4$.

Derivation - Quadratic order

- Identify $\ell_3(t) \equiv \frac{1}{2}\bar{M}_1^3(t)M_*^{-6}$. One obtains the action

$$S = \int d^4x\sqrt{-g}\left\{\frac{9H\bar{M}_1^3}{8} + \frac{M_*^2(\bar{M}_1^3)'}{8} + \frac{\bar{M}_1^3}{2M_*^6}X\Box\phi \right.$$
$$\left. + \left[\frac{3H\bar{M}_1^3}{4M_*^4} - \frac{(\bar{M}_1^3)'}{4M_*^2}\right]X + \left[\frac{(\bar{M}_1^3)'}{8M_*^6} - \frac{3H\bar{M}_1^3}{8M_*^8}\right]X^2\right\}, \qquad (2)$$

- This action is constructed such that it reduces to $-\frac{1}{2}\bar{M}_1^3(t)\delta g^{00}\delta K$ in the unitary gauge. All of the background and linear contributions cancel.

Examples

Assume the EFT functions are measured as

$$M_*^2\Gamma(t) = 4M_2^4(t) = 3H(t)\bar{M}_1^3(t) = -\lambda H(t)\,,$$
$$\Omega(t) = \exp(-2M_* t)\,,$$

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The covariant action that this corresponds to:

$$\mathcal{L} = \frac{M_*^2}{2} e^{-2\phi/M_*} R - \frac{r_c^2}{M_*} X \Box\phi + \mathcal{L}_M\,,$$

with $\lambda = 6M_*^5 r_c^2$.

Restricting model space with reconstruction

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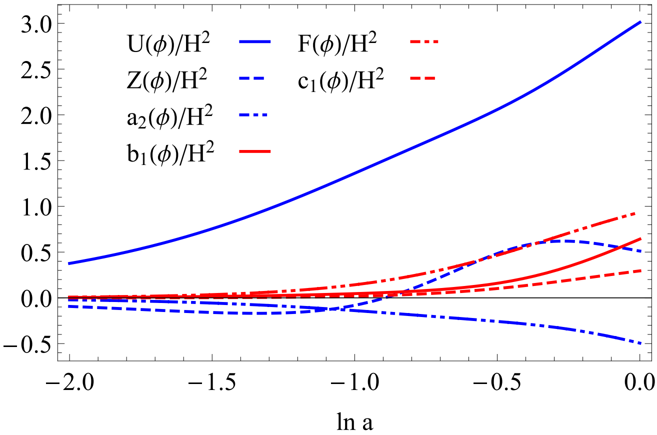
- Using the reconstruction we find that this implies $c_1(\phi) = 0$.
- On linear scales $G_4(\phi, X) \rightarrow G_4(\phi)$.

Plots

- Can reconstruct particular parametrizations, for example $\alpha_i(t) = c_i \Omega_\Lambda(t)$. Choose observationally constrained values (Bellini et al 2016), ensuring stability $c_s^2 > 0$, $c_T^2 > 0$ etc



In Prep.

Summary

- EFTofDE provides a generalized and efficient exploration of the parameter space.
- Provided a reconstruction from EFT back to the space of manifestly covariant theories, e.g. $U(\phi)$ 1705.09290.
- Can explore the theory space probed by different parametrizations of the EFT functions.
- Are the theories described by EFT guaranteed to have an Einstein gravity limit?
- Long term aim: connect observables to theories.

$$\mu(a,k)\,,\gamma(a,k) \to \int d^4x\sqrt{-g}\,\{???\}$$

Stability conditions

- There exist certain conditions within EFT that need to be satisfied in order for the theory to be theoretically stable. E.g. no ghost or gradient instabilities.
- Can one find a parametrization of the EFT functions that correspond directly to a stable theory.
- The reconstructed action within this space of theories is then guaranteed to be theoretically stable.

Plots

Screening conditions

- Are the theories described by the EFT of dark energy guaranteed to have an Einstein limit.

- Construct a covariant scalar-tensor theory that leads to weak gravity.