The hair of compact objects in Horndeski theory

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1. Horndeski theories
2. The no-hair theorem for shift-symmetric Horndeski theories
3. Breaking the hypotheses

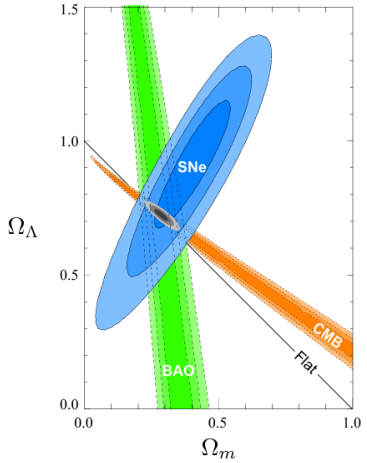
Modifying gravity: why?

1. Cosmological regime

2. High curvature regime; Establishing benchmarks

Modifying gravity: why?

1. Cosmological regime



2. High curvature regime; Establishing benchmarks

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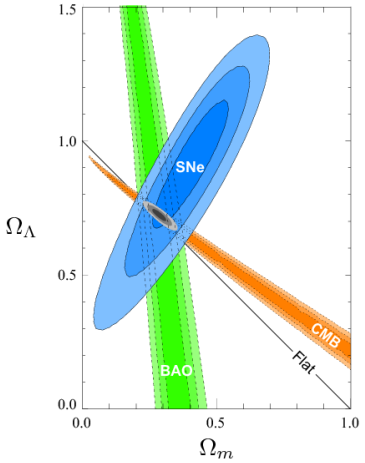
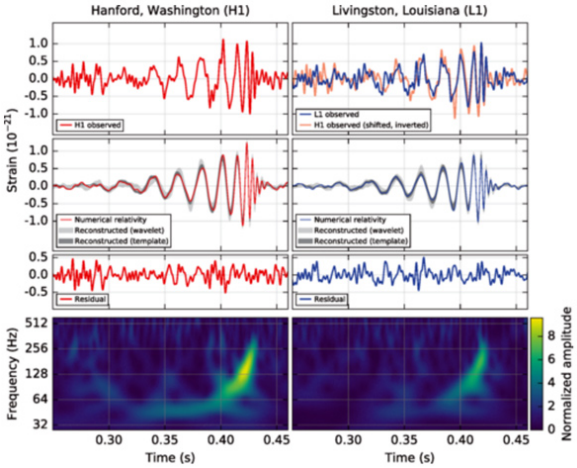


2. High curvature regime; Establishing benchmarks



Modifying gravity: how?

Horndeski theory in 4D

Horndeski '74

$$S_{\rm H} = \int {\rm d}^4x \mathcal{L}_{\rm H}(g_{\mu\nu}, g_{\mu\nu,i_1}, \ldots, g_{\mu\nu,i_1\ldots i_p};\ \phi, \phi_{,i_1}, \ldots, \phi_{,i_1\ldots i_q})$$

$$\Downarrow \quad \frac{1}{\sqrt{-g}} \frac{\delta\ \cdot}{\delta g_{\mu\nu}}$$

$$\mathcal{E}^{\mu\nu}(g_{\mu\nu}, g_{\mu\nu,i_1}, g_{\mu\nu,i_1,i_2};\ \phi, \phi_{,i_1}, \phi_{,i_1,i_2}) = 0$$

Modifying gravity: how?

Shift-symmetric Horndeski action

$$S_{\rm H} = \int \sqrt{-g}\, {\rm d}^4x \left(\mathcal{L}_2 + \mathcal{L}_3 + \mathcal{L}_4 + \mathcal{L}_5\right)$$

$$\begin{aligned}
\mathcal{L}_2 &= G_2(X), \\
\mathcal{L}_3 &= -G_3(X)\Box\phi, \\
\mathcal{L}_4 &= G_4(X)R + G_{4X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right], \\
\mathcal{L}_5 &= G_5(X)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi - \frac{1}{6}G_{5X}\left[(\Box\phi)^3 - 3\Box\phi(\nabla_\mu\nabla_\nu\phi)^2\right. \\
&\qquad \left. + 2(\nabla_\mu\nabla_\nu\phi)^3\right],
\end{aligned}$$

where $X = -\partial^\mu\phi\partial_\mu\phi/2$

Shift-symmetry

Action invariant under $\phi \to \phi + C$

Noether current

$$J^\mu = \frac{1}{\sqrt{-g}} \frac{\delta S[\phi]}{\delta(\partial_\mu \phi)}, \text{ such that } \nabla_\mu J^\mu = 0$$

1 Horndeski theories

2 The no-hair theorem for shift-symmetric Horndeski theories

3 Breaking the hypotheses

The theorem

Assumptions

1. Spherical symmetry and staticity of the scalar field and the metric
2. Asymptotic flatness with a constant scalar field at spatial infinity
3. Finite norm of the current J^2
4. Canonical kinetic term $X \subseteq G_2$, and analyticity of the action around a trivial scalar field configuration

Theorem Hui and Nicolis '12; AL, Babichev, Charmousis '17

The solutions are identical to GR, with constant ϕ

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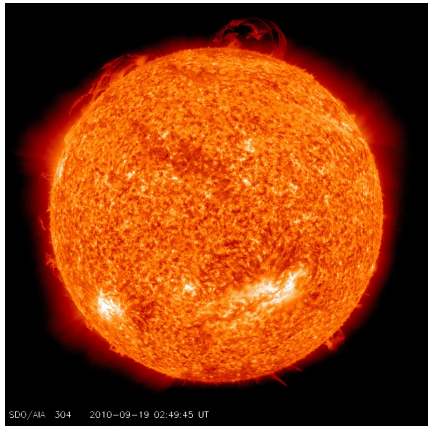
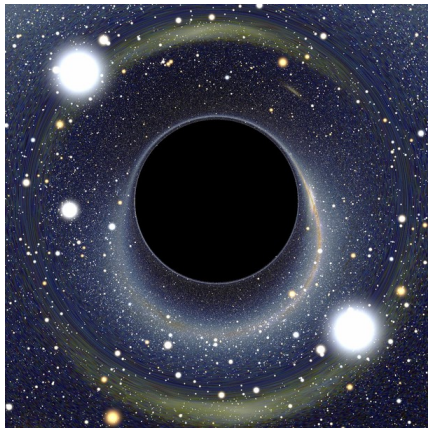
Step 1: No influx

Spherically symmetric and static ansatz

$$\mathrm{d}s^2 = -h(r)\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{f(r)} + r^2(\mathrm{d}\theta^2 + \sin^2\theta \mathrm{d}\phi^2), \qquad \phi = \phi(r)$$

Non-vanishing f: either stars or black hole exteriors





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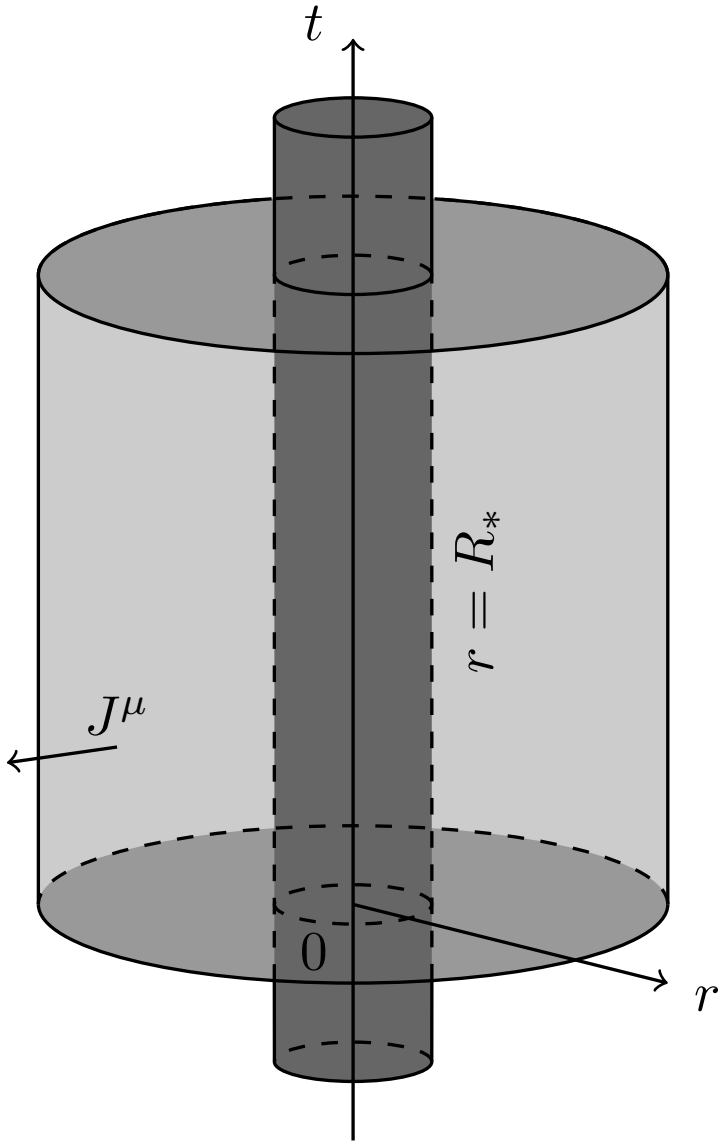

$$0 = \nabla_\mu J^\mu = \partial_\mu(\sqrt{-g}J^\mu)$$

Hence,

$$J^r = \frac{Q}{r^2}\sqrt{\frac{f}{h}}$$

$$0 = J^2 = J_\mu J^\mu = \frac{Q^2}{hr^4}$$

Step 1: No influx

$$J^2 = \frac{Q^2}{hr^4}$$

Stars

Origin of spherical coordinates is a spacetime point

Black holes

Presence of a horizon. Black holes no-hair theorem

Therefore $Q = 0$ and $J^r = 0$: no influx

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Step 2: $J^r = 0$ + assumptions 2/4 $\Rightarrow \phi' = 0$

Dependence of J^r on the scalar field

$$J^r = \phi' \mathcal{J}(\phi', f, h'/h, r)$$

with $\mathcal{J}$ regular around $\phi = \text{constant}$.

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- Then, progressively dialing the radius inwards, $\mathcal{J} = -f \neq 0$
- One cannot leave the trivial branch $\phi' = 0$ before reaching a horizon (or the origin)

Step 2: $J^r = 0$ + assumptions 2/4 $\Rightarrow \phi' = 0$

Dependence of J^r on the scalar field

$$J^r = \phi' \mathcal{J}(\phi', f, h'/h, r)$$

with $\mathcal{J}$ regular around $\phi = \text{constant}$.

$\phi' = 0$ everywhere. Solutions identical to GR

Complementary result: emission of gravitational waves

Gravitational waves and star binaries Barausse and Yagi '15

- Very similar assumptions, without staticity
- No modification with respect to GR at leading PN order

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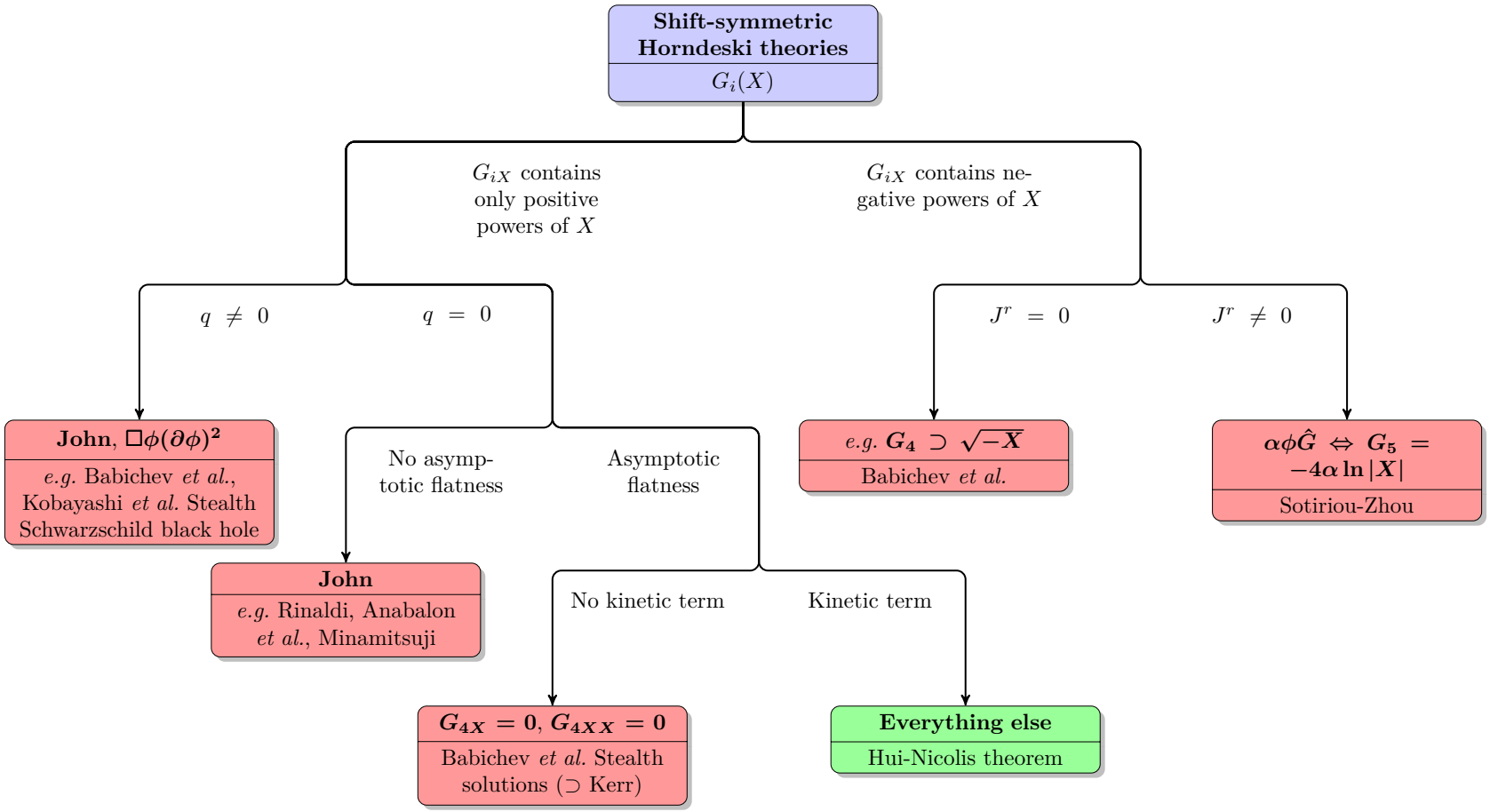
A classification of hairy solutions



Breaking assumption 1: $\phi(t,r) = qt + \psi(r)$

Example: John Lagrangian Babichev and Charmousis '14

$$S = \int \mathrm{d}^4x\sqrt{-g}\,(\zeta R + \beta G^{\mu\nu}\partial_\mu\phi\partial_\nu\phi)$$

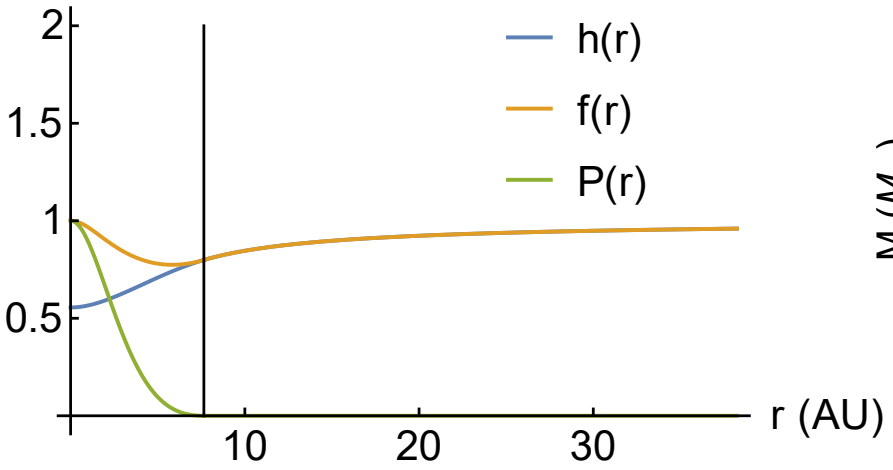
- Stealth Schwarzschild solutions
  $f = h = 1 - \dfrac{2M}{r},\ X(r) = -\dfrac{q^2}{2}$
- Ultra soft hair: $T^{\phi}_{\mu\nu} = 0$

Breaking assumption 1: $\phi(t,r) = qt + \psi(r)$

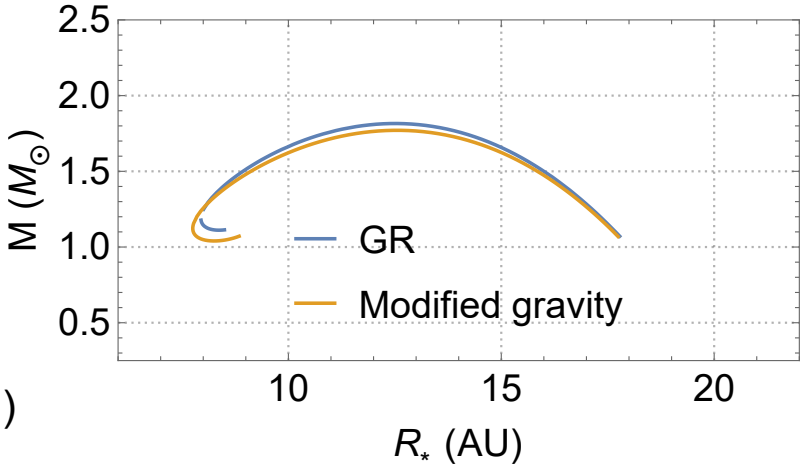
Perfect fluid stars — Cisterna *et al.* '15

$$S = \int \mathrm{d}^4x\sqrt{-g}(\zeta R + \beta G^{\mu\nu}\partial_\mu\phi\partial_\nu\phi) + S_{\mathrm{PF}}$$

Matching with the stealth Schwarzschild solution outside



Metric and pressure profile



Mass-radius relation

Breaking assumption 2: de Sitter solutions

John+... Rinaldi '12

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[R - 2\Lambda + \left(-\frac{1}{2}g^{\mu\nu} + \beta G^{\mu\nu}\right)\partial_\mu\phi\partial_\nu\phi\right]$$

- de Sitter asymptotics
- Interior solution?

Breaking assumption 3: Infinite J^2

Coupling to Gauss-Bonnet density Sotiriou and Zhou '14

$$S = \int \mathrm{d}^4x\sqrt{-g}\left(\zeta R - \frac{1}{2}\partial^\mu\phi\partial_\mu\phi + \alpha\phi\hat{G}\right)$$

with $\hat{G} = R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - 4R_{\mu\nu}R^{\mu\nu} + R^2$

- Divergent J^2 at horizon
- Regular metric under some conditions
- Can such a solution be produced by a dynamical collapse?

Breaking assumption 4: non-analytic G_i functions

Keeping static ϕ + Asymptotic flatness:

[AL, Babichev, Charmousis '17]

- $G_2 \supset \sqrt{-X}$
- $G_3 \supset \ln|X|$
- $G_4 \supset \sqrt{-X}$
- $G_5 \supset \ln|X|$
- $F_4 \supset (-X)^{-3/2}$
- $F_5 \supset X^{-2}$

Reissner-Nordström-like solution

$$f(r) = h(r) = 1 - \frac{\mu}{r} - \frac{\beta^2}{2\zeta\eta r^2}$$

$$X(r) = -\frac{\beta^2}{\eta^2 r^4}$$

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Keeping static ϕ + Asymptotic flatness:

[AL, Babichev, Charmousis '17]

- $G_2 \supset \sqrt{-X}$
- $G_3 \supset \ln|X|$
- $G_4 = \beta\sqrt{-X} + \zeta, \; G_2 = \eta X$
- $G_5 \supset \ln|X|$
- $F_4 \supset (-X)^{-3/2}$
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Conclusions

- No non-trivial scalar solutions under reasonable assumptions
- Classification of more exotic models: where to look for solutions or not
- Extension to stationary solutions?