Black holes in massive gravity

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OUTLINE

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- Classes of solutions
- Solutions with a source
- Black holes
- Perturbations and (in)stability of black holes
- Conclusions

INTRODUCTION

Why Bi-gravity?

Reasons to study bi-gravity

- ❖ Cosmological constant problems
- ❖ Theoretical curiosity
- ❖ Benchmark for testing General Relativity
- ❖ Dark matter from bi-gravity

Old problems of massive gravity

1. Physical ghost [Boulware&Deser'72]

2. Extra propagating degrees of freedom.
It is difficult to pass basic Solar system gravity tests.
vDVZ discontinuity [van Dam&Veltman'70, Zakharov'70]

Fierz-Pauli massive gravity

Expand the Einstein-Hilbert action:

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

$$S_{GR} = M_P^2 \int d^4x \sqrt{-g} R = \int d^4x \left(-\frac{1}{2} h^{\mu\nu} \mathcal{E}^{\alpha\beta}_{\mu\nu} h_{\alpha\beta} \right) + \mathcal{O}(h^3)$$

$$\mathcal{E}^{\alpha\beta}_{\mu\nu} h_{\alpha\beta} = -\frac{1}{2}\partial_\mu\partial_\nu h - \frac{1}{2}\Box h_{\mu\nu} + \frac{1}{2}\partial_\rho\partial_\mu h^\rho_\nu + \frac{1}{2}\partial_\rho\partial_\nu h^\rho_\mu - \frac{1}{2}\eta_{\mu\nu}(\partial^\rho\partial^\sigma h_{\rho\sigma} - \Box h)$$

2 massless polarisations of graviton

$$x^\alpha \to x^\alpha + \xi^\alpha, \; h_{\mu\nu} = -\xi_{\mu;\nu} - \xi_{\nu;\mu}$$

Fierz-Pauli massive gravity

Fierz-Pauli action:

[Fierz&Pauli'39]

$$S_{PF} = M_P^2 \int d^4x \left[-\frac{1}{2} h^{\mu\nu} \mathcal{E}_{\mu\nu}^{\alpha\beta} h_{\alpha\beta} - \frac{1}{4} m^2 \left(h_{\mu\nu} h^{\mu\nu} - h^2 \right) \right]$$

Linearized Einstein-Hilbert term

mass term

$$\cancel{x^\alpha \to x^\alpha + \xi^\alpha, \; h_{\mu\nu} = -\xi_{\mu;\nu} - \xi_{\nu;\mu}}$$

5 healthy degrees of freedom (because of a particular choice of the potential, h=0)

for a generic mass term 6 d.o.f., one is necessary Ostrogradski ghost

Non-linear massive gravity

Extra degrees of freedom

New degrees of freedom due to the broken diff invariance =>

Change of predictions, i.e. Solar system tests

Non-linear massive gravity

Non-linear completion ?

Introduce an extra metric to construct a mass term
(in order to contract indices)

$g_{\mu\nu}$:physical metric, matter couples to it

$f_{\mu\nu}$:an extra metric (may be dynamical or fixed)

Construct a potential, which is
invariant under diffeomorphism (common for two metrics)
+ some technical conditions

Non-linear massive gravity

potential for metric

building block: $\mathbf{g^{-1}f}$

$$S_{int}^{(2)} \equiv -\frac{1}{8}m^2 M_P^2 \int d^4x \,\sqrt{-f}\; H_{\mu\nu}H_{\sigma\tau}\left(f^{\mu\sigma}f^{\nu\tau} - f^{\mu\nu}f^{\sigma\tau}\right)$$ [Boulware & Deser'72

$$S_{int}^{(3)} \equiv -\frac{1}{8}m^2 M_P^2 \int d^4x \,\sqrt{-g}\; H_{\mu\nu}H_{\sigma\tau}\left(g^{\mu\sigma}g^{\nu\tau} - g^{\mu\nu}g^{\sigma\tau}\right)$$ [Arkani-Hamed et al'03]

where $H_{\mu\nu} = g_{\mu\nu} - f_{\mu\nu}$

Vainshtein mechanism

screens extra degree of freedom

Non-linear effects restore General Relativity
close to the source
due to the non-linear effects

[Vainshtein'72]
[EB, Deffayet, Ziour'09'10]

Problem 2 is solved

Non-linear massive gravity

Boulware-Deser ghost

Generically there are two propagating scalars: one is a ghost !

[Boulware & Deser'72]

dRGT massive gravity

special case of non-linear massive gravity

Massive gravity without Boulware-Deser ghost

[de Rham, Gabadadze, Tolley'10'11, Hassan & Rosen'12]
+many other works

$$Y = \sqrt{\mathbf{g}^{-1}\mathbf{f}}$$

g is physical metric;
f is fixed (flat) or extra dynamical metric.

$$S = M_P^2 \int d^4x \sqrt{-g}\left[R[g] - m^2 \sum_{k=0}^{k=4} \beta_k e_k(Y)\right] + \kappa M_P^2 \int d^4x \sqrt{-f}\,\mathcal{R}[f]$$

$$e_0 = 1,\ e_1 = [X],\ e_2 = \frac{1}{2}\left([X]^2 - [X^2]\right),\ e_3 = \frac{1}{6}\left([X]^3 - 3[X][X^2] + 2[X^3]\right),$$
$$e_4 = \frac{1}{24}\left([X]^4 - 6[X]^2[X^2] + 3[X^2]^2 + 8[X][X^3] - 6[X^4]\right)$$

Equations of motion

$$G^{\mu}{}_{\nu} = m^2 \left(T^{\mu}{}_{\nu} + \Lambda_g \delta^{\mu}_{\nu}\right) + \frac{T^{\mu}{}_{\nu(matter)}}{M_P^2}$$

$$\mathcal{G}^{\mu}{}_{\nu} = m^2 \left(\frac{\sqrt{-g}}{\sqrt{-f}} \frac{\mathcal{T}^{\mu}{}_{\nu}}{\kappa} + \Lambda_f \delta^{\mu}_{\nu}\right)$$

Variation of mass term
Non-derivative coupling of the two metrics

$G_{\mu\nu}$ is the Einstein tensor for metric $g_{\mu\nu}$

$\mathcal{G}_{\mu\nu}$ is the Einstein tensor for metric $f_{\mu\nu}$

Spherical symmetry and beyond

Two types of (static) spherically symmetric solutions

Bi-diagonal: When two metrics can be put in the diagonal form simultaneously.

Non Bi-diagonal: When this is not the case

A “no-go theorem” for bi-diagonal black holes

[Deffayet, Jacobson’11]

Spherically symmetric solutions

Bi-diagonal

✦ Solutions with source.

Vainshtein mechanism: GR is restored, tiny modification of GR

✦ Black holes

- Bi-diagonal solutions: the two metrics are GR-like and equal or proportional (horizons coinside).
- hairy black holes (numerics), non-GR

✦ Stability of black holes

- Black holes are unstable (mild tachyon-like instability)

Non Bi-diagonal

✦ Solutions with source.

GR is perfectly restored

✦ Black holes

- Non bi-diagonal solutions: the two metrics are GR-like and not proportional (horizons may not coinside).

✦ Stability of black holes

- Black holes are stable

✦ Rotating solutions

- Two GR-like equal metrics

✦ Rotating solutions

- Two GR-like non-equal metrics

Solutions with a source

Vainshtein mechanism in bi-gravity

Weak-field approximation [EB,Deffayet,Ziour'10]
Vainshtein mechanism in bi-gravity [Volkov'12] [EB,Crisostomi'13]

$$ds^2 = -e^{\nu} dt^2 + e^{\lambda} dr^2 + r^2 d\Omega^2,$$

$$df^2 = -e^{n} dt^2 + e^{l} \left(r + r\mu\right)'^2 dr^2 + \left(r + r\mu\right)^2 d\Omega^2$$

$$\{\lambda,\, \nu,\, l,\, n\} \ll 1, \quad \{r\,\lambda',\, r\,\nu',\, r\,l',\, r\,n'\} \ll 1$$

Linearized Einstein equations for metric g

Linearized Einstein equations for metric f

Inside the Compton length $r \ll 1/m$

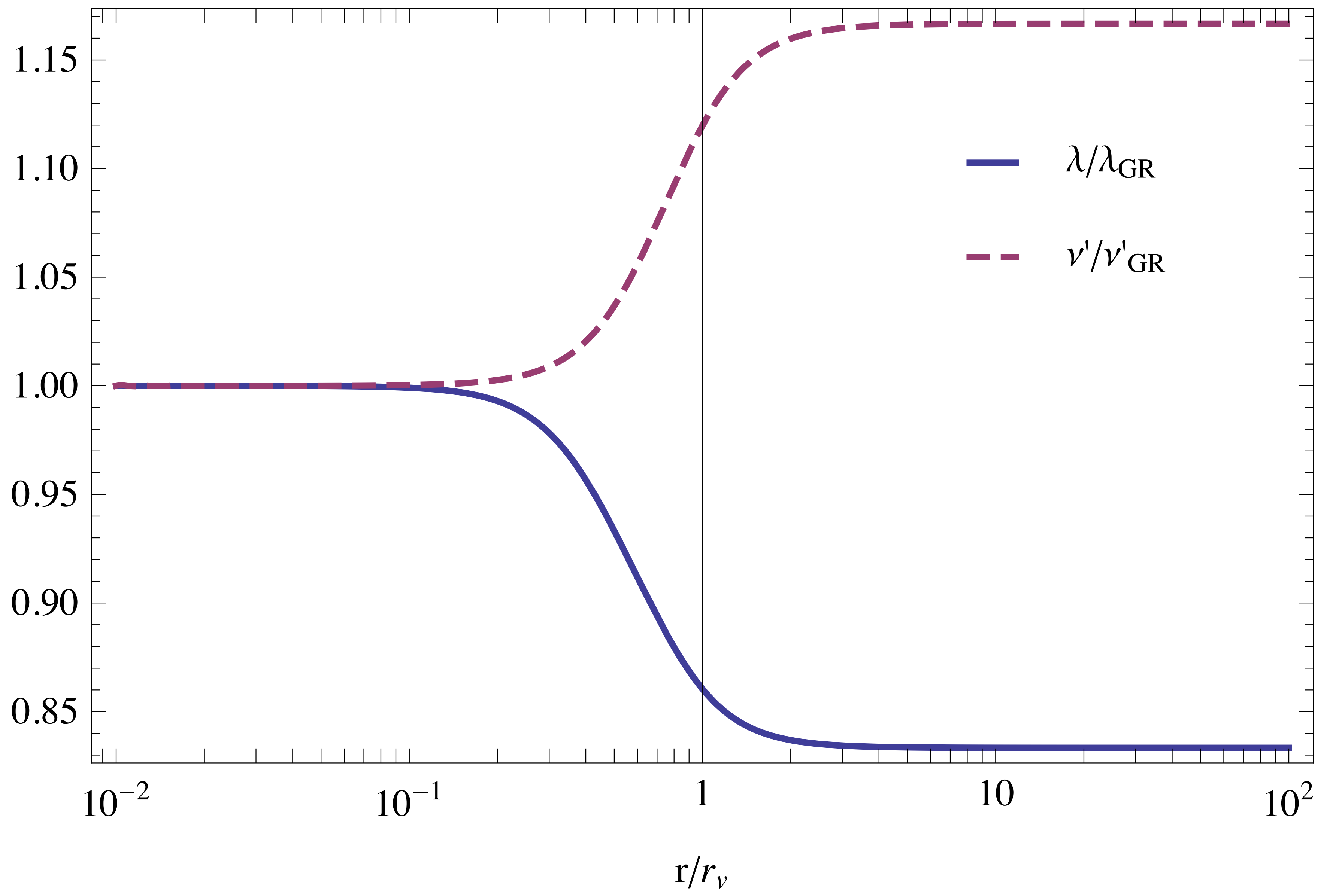
$$\mu^7 + ... = 0$$

Algebraic equation of 7th order on μ

All other metric functions depend on μ

Vainshtein mechanism in bi-gravity

recovery of GR



Non-Vainshtein recovery of GR

[EB unpublished]

Physical metric in the EF form,

$$ds_g^2 = -h(r)dv^2 + 2k(r)dvdr + r^2 d\Omega^2$$

Ansatz for the second metric

$$\frac{ds_f^2}{C^2} = -dv^2 + 2S\,dvdr + \left((r\mu)'^2 - S^2\right)dr^2 + (r\mu)^2 d\Omega^2$$

For C such that

$$\beta(C-1)^2 - 2\alpha(C-1) + 1 = 0$$

Recovery of GR up to a Lambda-term ~ m^2

BLACK HOLES

Black holes

Schwarzschild metric

Non-bidiagonal BHs [Salam & Strathdee'77] [Isham & Storey'78] [Koyama, Niz, Tasinato'11]+many others

Ansatz (bi-Eddington-Finkelstein form) [EB& Fabbri'13]

$$ds_g^2 = -\left(1 - \frac{r_g}{r}\right) dv^2 + 2dvdr + r^2 d\Omega^2,$$

$$ds_f^2 = C^2 \left[-\left(1 - \frac{r_f}{r}\right) dv^2 + 2dvdr + r^2 d\Omega^2\right]$$

Two choices:
$$\begin{cases} r_g = r_f & \text{bi-diagonal} \\ \beta(C-1)^2 - 2\alpha(C-1) + 1 = 0 & \text{non-bidiagonal} \end{cases}$$

For these choices the extra "mass" energy-momentum tensor reduces to effective cosmological constant

Charged Black holes

Electromagnetic field coupled to g [EB& Fabbri'13]

$$ds_g^2 = -\left(1 - \frac{r_g}{r} + \frac{r_Q^2}{r^2} - \frac{r^2}{l_g^2}\right) dv^2 + 2dvdr + r^2 d\Omega^2,$$

$$ds_f^2 = C^2 \left[-\left(1 - \frac{r_f}{r} - \frac{r^2}{l_f^2}\right) dv^2 + 2dvdr + r^2 d\Omega^2\right].$$

$$A_\mu = \left\{\frac{Q}{r}, 0, 0, 0\right\}$$

Rotating Black holes

Original Kerr metric

[EB& Fabbri'13]

$$ds_g^2 = -\left(1 - \frac{r_g r}{\rho^2}\right)\left(dv + a\sin^2\theta d\phi\right)^2$$
$$+ 2\left(dv + a\sin^2\theta d\phi\right)\left(dr + a\sin^2\theta d\phi\right) + \rho^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)$$
$$\rho^2 = r^2 + a^2\cos^2\theta$$

f is flat, but unusual form

$$ds_f^2 = C^2\left[-dv^2 + 2dvdr + 2a\sin^2\theta drd\phi + \rho^2 d\theta^2 + \left(r^2 + a^2\right)\sin^2\theta d\phi^2\right]$$

Obtained from $ds_M^2 = -dt^2 + dx^2 + dy^2 + dz^2$

by:

$$t = v - r,\ x + iy = (r - ia)e^{i\phi}\sin\theta,\ z = r\cos\theta$$
$$r \to Cr,\ v \to Cv,\ a \to Ca$$

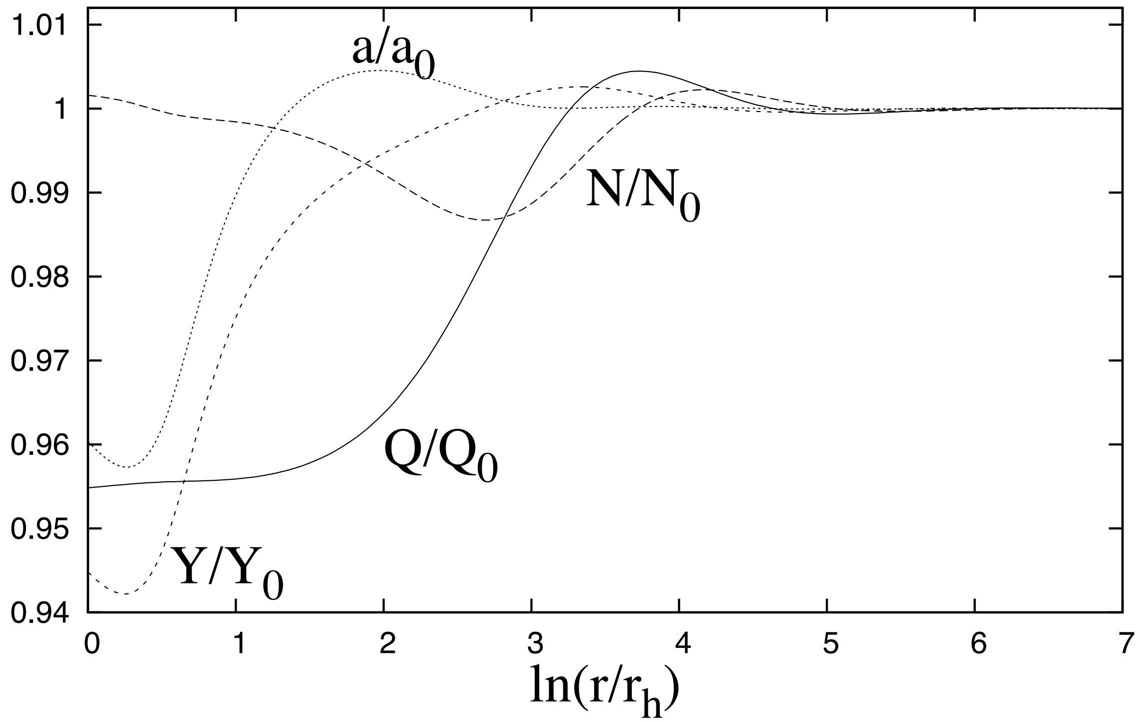
Hairy bi-diagonal black holes

Asymptotically AdS hairy solutions exist [Volkov'12]

$$
\begin{aligned}
g_{\mu\nu}dx^\mu dx^\nu &= -\,Q^2\,dt^2 + N^{-2}\,dr^2 + R^2 d\Omega^2\,,\\
f_{\mu\nu}dx^\mu dx^\nu &= -\,a^2\,dt^2 + b^2\,dr^2 + U^2 d\Omega^2\,,
\end{aligned}
$$

Numerical integration of a system of coupled ODEs

$$
\begin{cases}
N' = \mathcal{F}_1(r, N, Y, U, \mu, \kappa, \alpha_3, \alpha_4)\\
Y' = \mathcal{F}_2(r, N, Y, U, \mu, \kappa, \alpha_3, \alpha_4)\\
U' = \mathcal{F}_3(r, N, Y, U, \mu, \kappa, \alpha_3, \alpha_4).
\end{cases}
$$



Hairy bi-diagonal black holes

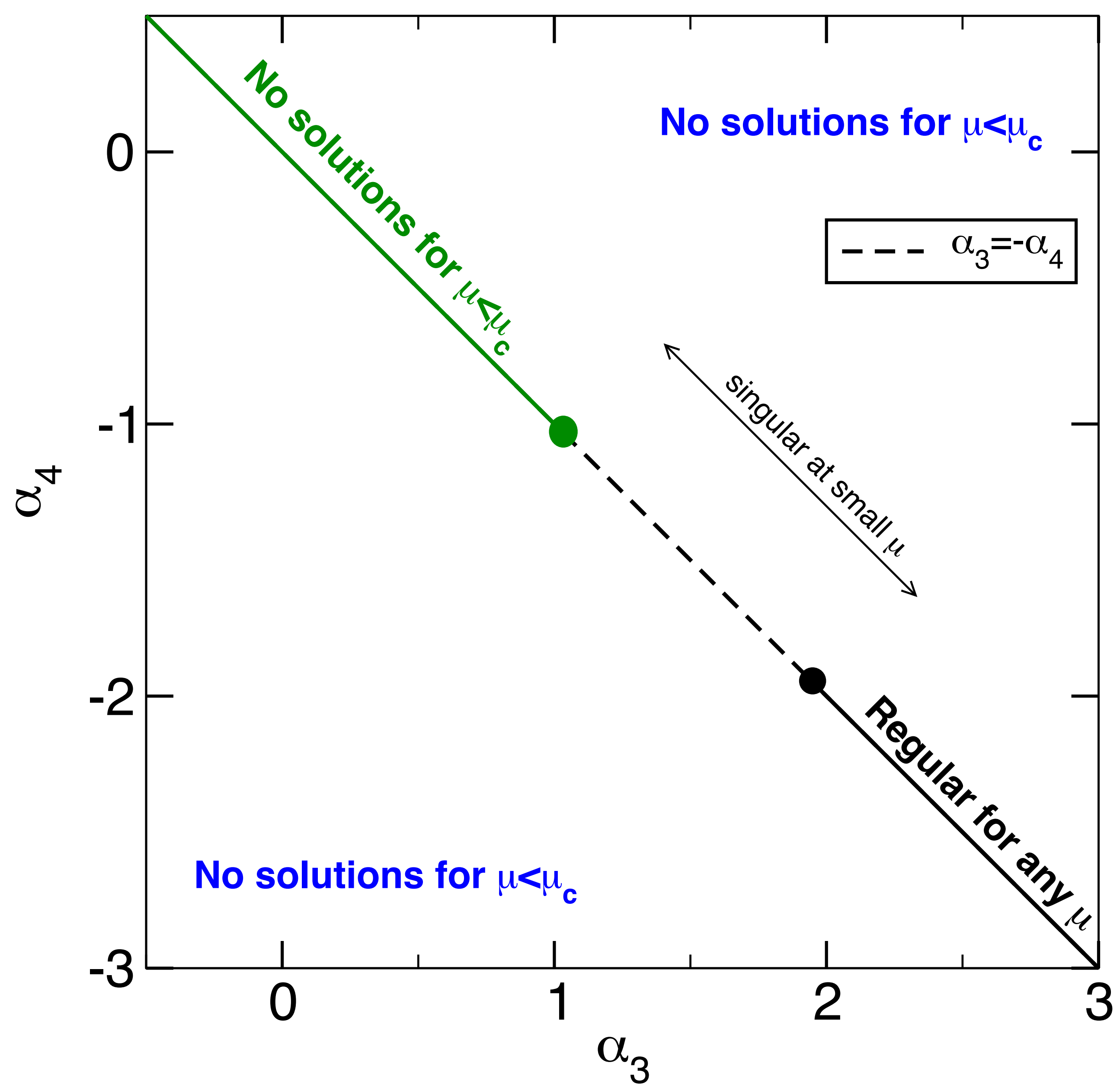
Asymptotically flat hairy solutions [Brito,Cardoso,Pani'13]

$$N = 1 - \frac{C_1}{2r} + \frac{C_2(1+r\mu)}{2r}e^{-r\mu},$$
$$Y = 1 - \frac{C_1}{2r} - \frac{C_2(1+r\mu)}{2r}e^{-r\mu},$$
$$U = r + \frac{C_2(1+r\mu+r^2\mu^2)}{\mu^2 r^2}e^{-r\mu},$$

Yukawa decay

For generic potential only for large BH mass.

$$r_s \sim 1/H$$



PERTURBATIONS of BLACK HOLES

Perturbations

spherically symmetric ansatz for perturbations

[EB & Fabbri'14]

Perturbations of both metrics

$$g_{\mu\nu} = g^{(0)}_{\mu\nu} + h^{(g)}_{\mu\nu}, \;\; f_{\mu\nu} = f^{(0)}_{\mu\nu} + h^{(f)}_{\mu\nu}$$

$$\delta G^{\mu}{}_{\nu} = m^2 \delta T^{\mu}{}_{\nu}, \quad \delta \mathcal{G}^{\mu}{}_{\nu} = \frac{m^2}{\kappa} \delta \left(\frac{\sqrt{-g}}{\sqrt{-f}} \mathcal{T}^{\mu}{}_{\nu} \right) \; .$$

$$\boldsymbol{h^{\mu\nu}_{(g)}} = e^{\Omega v} \begin{pmatrix} h^{vv}_{(g)}(r) & h^{vr}_{(g)}(r) & 0 & 0 \\ h^{vr}_{(g)}(r) & h^{rr}_{(g)}(r) & 0 & 0 \\ 0 & 0 & \frac{h^{\theta\theta}_{(g)}(r)}{r^2} & 0 \\ 0 & 0 & 0 & \frac{h^{\theta\theta}_{(g)}(r)}{r^2 \sin^2\theta} \end{pmatrix} \qquad (\Omega > 0)$$

$$\boldsymbol{h^{\mu\nu}_{(f)}} = \frac{e^{\Omega v}}{C^2} \begin{pmatrix} h^{vv}_{(f)}(r) & h^{vr}_{(f)}(r) & 0 & 0 \\ h^{vr}_{(f)}(r) & h^{rr}_{(f)}(r) & 0 & 0 \\ 0 & 0 & \frac{h^{\theta\theta}_{(f)}(r)}{r^2} & 0 \\ 0 & 0 & 0 & \frac{h^{\theta\theta}_{(f)}(r)}{r^2 \sin^2\theta} \end{pmatrix}$$

$h^{\mu\nu}_{(-)} \equiv h^{\mu\nu}_{(g)} - C^2 h^{\mu\nu}_{(f)}$ i.e. $h^{vv}_{(-)}(r) = h^{vv}_{(g)}(r) - h^{vv}_{(f)}(r)$

Spherical Perturbations

Regularity at horizons and infinity !

Perturbations for bidiagonal case

$$h^{(-)\mu}{}_{\nu} = h^{\mu}_{\nu} - \tilde{h}^{\mu}_{\nu}$$

$$h^{(+)\mu}{}_{\nu} = h^{\mu}_{\nu} + \kappa\tilde{h}^{\mu}_{\nu}$$

$$m' \equiv m\sqrt{1 + 1/\kappa}$$

$$\boxed{\mathcal{E}^{\alpha\beta}_{\mu\nu} h^{(+)}_{\alpha\beta} = 0}$$

$h^{(-)}_{\mu\nu}$ is massive

$h^{(+)}_{\mu\nu}$ is massless

$$\nabla^{\nu} h^{(-)}_{\mu\nu} = h^{(-)} = 0$$

$$\Box h^{(-)}_{\mu\nu} + 2R^{\sigma}{}_{\mu}{}^{\lambda}{}_{\nu} h^{(-)}_{\lambda_\sigma} = m'^2 h^{(-)}_{\mu\nu}$$

Bi-diagonal case

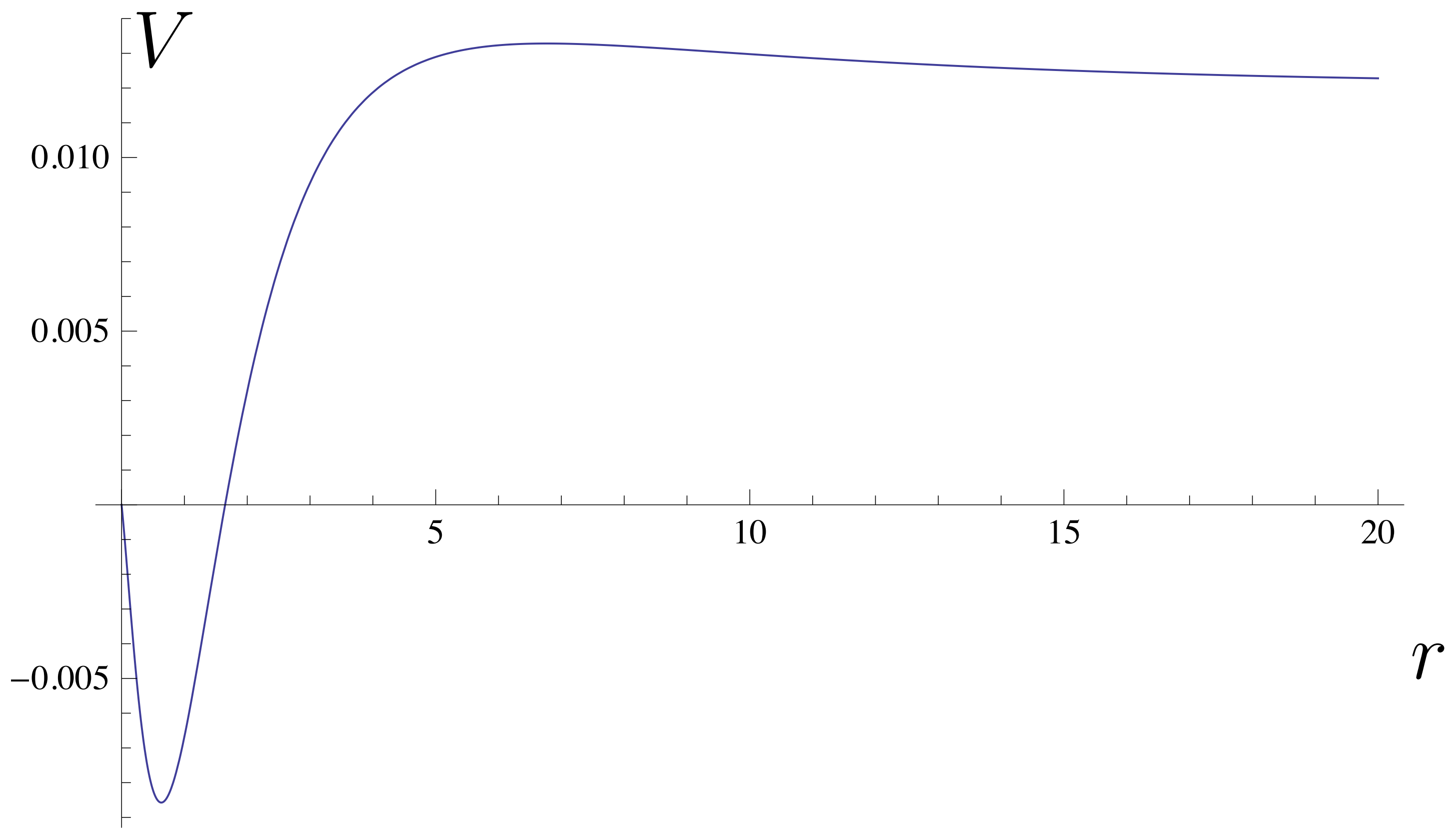
GL instability

A system of equations of second order plus 2 constraints on $H_{tt},\ H_{tr},\ H_{rr},\ K$

Playing with equations we can obtain a single equation on φ_0 (a combination of H_{tt}, H_{rr} and H_{tr})

$$\frac{d^2}{dr_*^2}\varphi_0 + \left[\omega^2 - V(r)\right]\varphi_0 = 0$$

Unstable ($\Omega > 0$) mode, satisfying boundary conditions?



$$V_0 = \left(1 - \frac{r_g}{r}\right)\left[\frac{2M}{r^3} + m'^2 + \frac{24M(M-r)m'^2 + 6r^3(r-4M)m'^4}{\left(2M + r^3 m'^2\right)^2}\right]$$

Bi-diagonal case: Instability

$$\boxed{0 < m' < \frac{\mathcal{O}(1)}{r_S}}$$

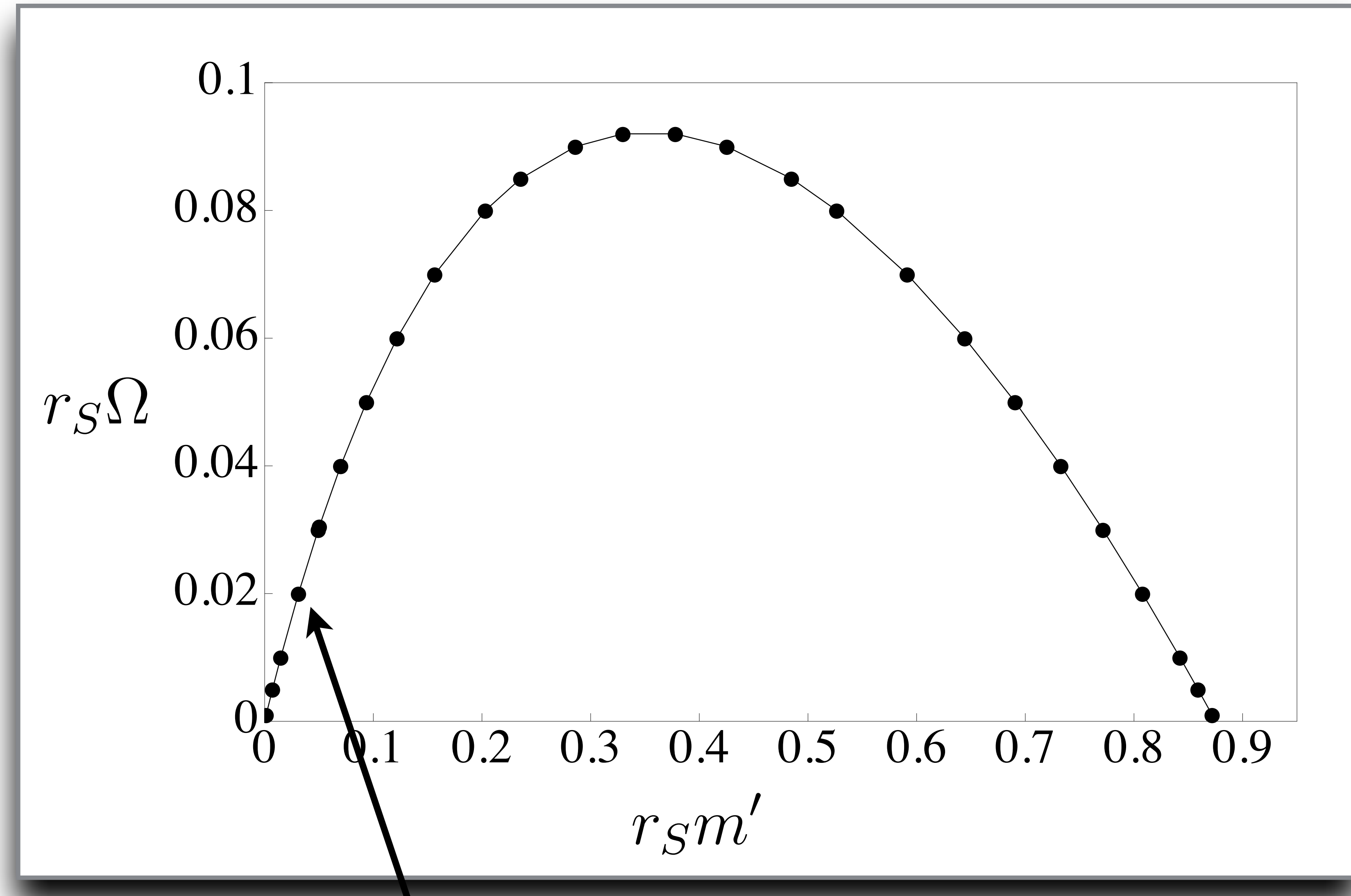
Instability

Confirmed independently by [Brito, Cardoso, Pani'13]

Instability of black holes

rate of instability

Rate of instability



for $m' \sim H \rightarrow \boxed{\tau \sim H^{-1}}$

Very slow instability !

Approximately linear dependance $\quad r_S \ll 1/m'$

$$\Omega = m'$$

Non-bidiagonal case

General solution for perturbations

[EB & Fabbri'14]

$$h^{\mu\nu}_{(g,f)} = h^{\mu\nu(g,f)}_{GR} + h^{\mu\nu(g,f)}_{(m)}$$

$$h^{\mu\nu}_{GR} = -\nabla^\mu \xi^\nu - \nabla^\nu \xi^\mu$$

Non-Bi-diagonal case

explicit solution for perturbations

$$h_{GR}^{\mu\nu(f)} = 0$$

$$h_{GR}^{\mu\nu(g)} = \boxed{e^{\Omega v}} \begin{pmatrix} 0 & \Omega c_1 & 0 & 0 \\ \Omega c_1 & c_0\left(\Omega - \frac{r_g}{2r^2}\right) & 0 & 0 \\ 0 & 0 & c_0 r^{-3} & 0 \\ 0 & 0 & 0 & c_0 \csc^2(\theta) r^{-3} \end{pmatrix}$$

$$h_{(m)}^{rr(g)} = \frac{\mathcal{A}(r_g - r_f)\boxed{e^{\Omega v}}}{4\Omega} m^2 h_{(-)}^{\theta\theta},$$

$$h_{(m)}^{rr(f)} = -\kappa^{-1} h_{(m)}^{rr(g)}.$$

Since at $r \to \infty$ $v = t + r$ the perturbations are not regular at infinity.

NO unstable modes

Non-bidiagonal solution is stable against radial perturbations

Non-Bi-diagonal case

Non-radial perturbations: QNM

[EB,Brito,Pani'15]

Decomposition of perturbations in axial and polar modes

The quasinormal modes are the same as those of a Schwarzschild BH in GR !

- QNM are vibration of a relativistic self-gravitating object.
- The boundary conditions are important.

For a mode to be QN, the perturbation must behave as ingoing wave near the horizon,

$$\sim e^{-i(\omega t+k_- r_*)}$$

and outgoing at the infinity,

$$\sim e^{i(k_+ r_* -\omega t)}$$

CONCLUSIONS

- It is possible to construct non-bidiagonal solutions in massive gravity, which are analogues of corresponding GR solutions (Schwarzschild, charged, rotating).
- There are hairy massive gravity black holes
- The non-bidiagonal black holes in massive gravity are stable
- The bi-diagonal spherically symmetric BHs are unstable due to the helicity-0 mode instability. The rate of instability is extremely small.
- Superradiant instability for rotating BHs in massive gravity.
- The fate of unstable BHs? The endpoint of gravitational collapse?
- Rotating hairy BHs?
- dS hairy black holes?
- Do perturbations around black holes contain ghosts?