Gravitational Waves in Doubly Coupled Bigravity

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Outline

Why and What Doubly Coupled Bigravity

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The Model

Birefringence
Graviton Propagation

Constraints

Cherenkov Radiation
Binary Pulsars

Astrophysical Sources

Gravitational Radiation

Bigravity

Two gravitons with both gravitons coupled to matter with a non-diagonal mass matrix

Why?

Ghost free massive gravity with one graviton massless and one massive on diagonalisation of the mass matrix Could have interesting predictions. Could give rise to signals in gravitational waves

The Model

The action consists of three terms. Here we are using the vielbein formulation

The Einstein Hilbert term

$$\boxed{S_G = \int d^4x \; e_1 \frac{R_1}{16\pi G_N} + \int d^4x \; e_2 \frac{R_2}{16\pi G_N}}$$

with vielbeins constrained to satisfy the symmetric condition

$$e^a_{1\mu} e^b_{2\nu} \eta_{ab} = e^a_{1\nu} e^b_{2\mu} \eta_{ab}$$

The potential term which gives rise to the graviton mass

$$\boxed{S_V = \Lambda^4 \sum_{ijkl} m^{ijkl} \int d^4x \; \epsilon_{abcd} \epsilon^{\mu\nu\rho\sigma} e^a_{i\mu} e^b_{j\nu} e^c_{k\rho} e^d_{l\sigma}}$$

Finally there's the matter term

$$\boxed{S_m(\psi_i, g_{\mu\nu})}.$$

with matter coupling to the Jordan metric, defined as

$$g_{\mu\nu} = \beta_1^2 g^1_{\mu\nu} + \beta_1\beta_2 Y_{\mu\nu} + \beta_2^2 g^2_{\mu\nu}$$

where the symmetric tensor is

$$Y_{\mu\nu} = \eta_{ab}(e^a_{1\mu} e^b_{2\nu} + e^a_{2\mu} e^b_{1\nu})$$

and $$\boxed{\Lambda^4 = m^2 M_{\rm Pl}^2}$$

m is related to the graviton mass, of O (H0)

choosing a cosmological background

$$\boxed{ds_1^2 = a_1^2(-d\eta^2 + dx^2)}$$

$$\boxed{ds_2^2 = a_2^2(-b^2 d\eta^2 + dx^2)}$$

The ratio of the two lapse functions, b, plays a crucial role in the modification of gravity in this class of models, as we will see later

Matter couples to the Jordan frame metric with scale factor

$$a_J = \beta_1 a_1 + \beta_2 a_2$$

The cosmological dynamics are governed
by the two Friedmann equations

$$3H_1^2 M_{\rm Pl}^2 = \beta_1 \frac{a_J^3}{a_1^3}\rho + 24\Lambda^4 m^{1jkl}\frac{a_j a_k a_l}{a_1^3}$$

$$\frac{3H_2^2 M_{\rm Pl}^2}{b^2} = \beta_2 \frac{a_J^3}{a_2^3}\rho + 24\Lambda^4 m^{2jkl}\frac{a_j a_k a_l}{a_2^3}$$

with H1 and H2 derived from the respective scale factors
and the m's arise from the potential term. There are two
branches. We consider the branch satisfying

$$\boxed{b = \frac{a_2 H_2}{a_1 H_1}}$$

The dynamics at both early and late times simplify. In both radiation and matter era when dark energy is negligible the ratio of scale factors converges to a constant

$$X \to X_m = \frac{\beta_2}{\beta_1}$$

When dark energy dominates, ie when the lambda terms dominate in both Friedmann equations then

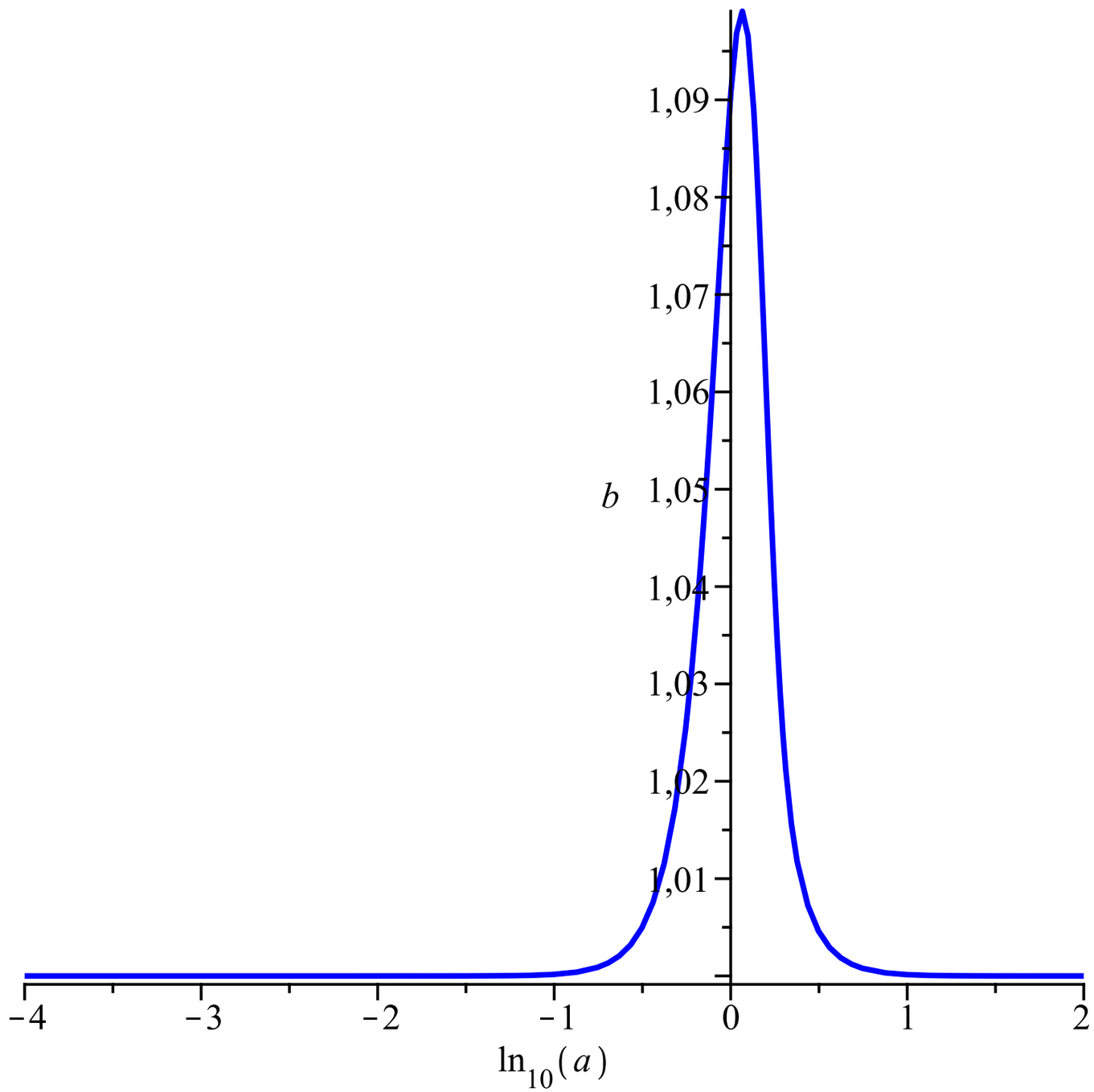
$$X \to X_d$$

$$\boxed{X_d = \frac{m^{2jkl} a_j a_k a_l}{m^{1jkl} a_j a_k a_l}}$$

For most of the evolution of the Universe we found $b = 1$

In between eras b deviates from 1 and X deviates from its asymptotic value. In particular in the current era $b \neq 1$

When b deviates from 1, as in the present era, we find non-standard gravitational wave propagation


1,09
1,08
1,07
1,06
b
1,05
1,04
1,03
1,02
1,01
−4
−3
−2
−1
0
1
2
ln10(a)

The potential term induces a mass term for the gravitons via a mass matrix

$$\boxed{M^2_{\alpha\beta}(a_\gamma) = -24m^2(b_\alpha b_\beta)^{1/2} m_{\alpha\beta}(a_\gamma)}$$

this is symmetric of order m^2

The mass matrix is non-diagonal and evolves with time.
This induces a mixing between the two gravitons which
could be observed as birefringence.

This leads to a coupling term in the evolution of the two gravitons

$$\frac{d^2\bar{h}_1}{d\eta^2} - \Delta\bar{h}_1 + (M^2_{11}(a_\gamma) - \frac{1}{a_1}\frac{d^2 a_1}{d\eta^2})\bar{h}_1 + M^2_{12}(a_\gamma)\bar{h}_2 = 0$$

$$\frac{d^2\bar{h}_2}{d\eta^2} - b^2\Delta\bar{h}_2 + (M^2_{22}(a_\gamma) - \frac{b^{1/2}}{a_2}\frac{d^2(a_2 b^{-1/2})}{d\eta^2})\bar{h_2} + M^2_{21}(a_\gamma)\bar{h}_1 = 0$$

Gravitational Waves

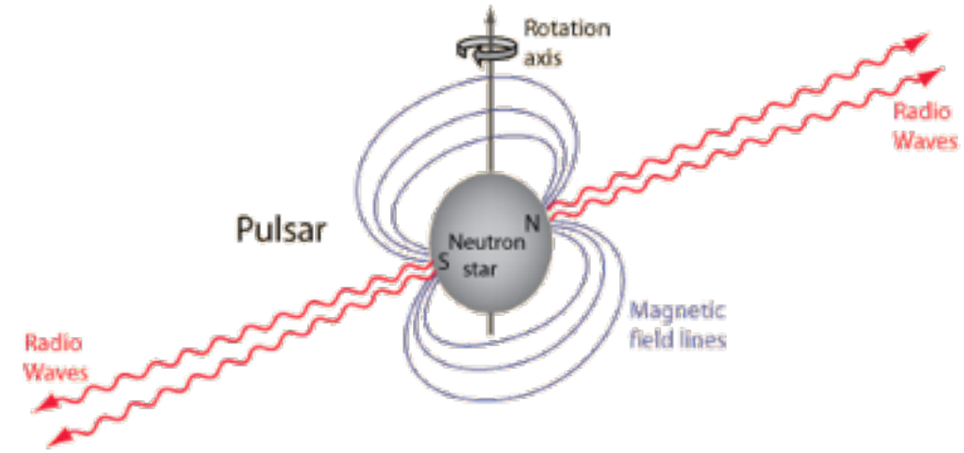
We can now study the emission of gravitational waves. Note that matter couples to the Jordan frame energy-momentum tensor and assuming we can neglect the cosmological evolution, we have

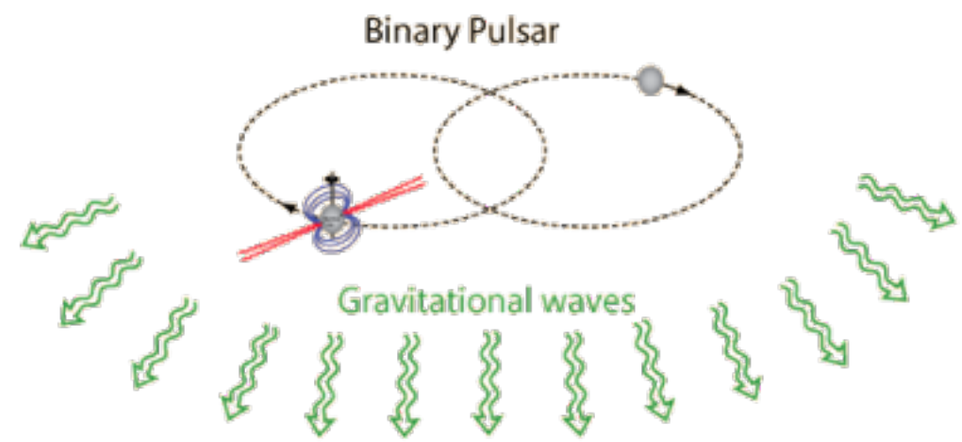
$$\frac{d^2\bar{h}^1_{ij}}{d\eta^2} - \Delta\bar{h}^1_{ij} + (M^2_{11}(a_\gamma) - \frac{1}{a_1}\frac{d^2 a_1}{d\eta^2})\bar{h}^1_{ij} + M^2_{12}(a_\gamma)\bar{h}^2_{ij} = \beta_1 \frac{\tilde{a}_J}{M_{\rm Pl}}\bar{T}_{ij}$$

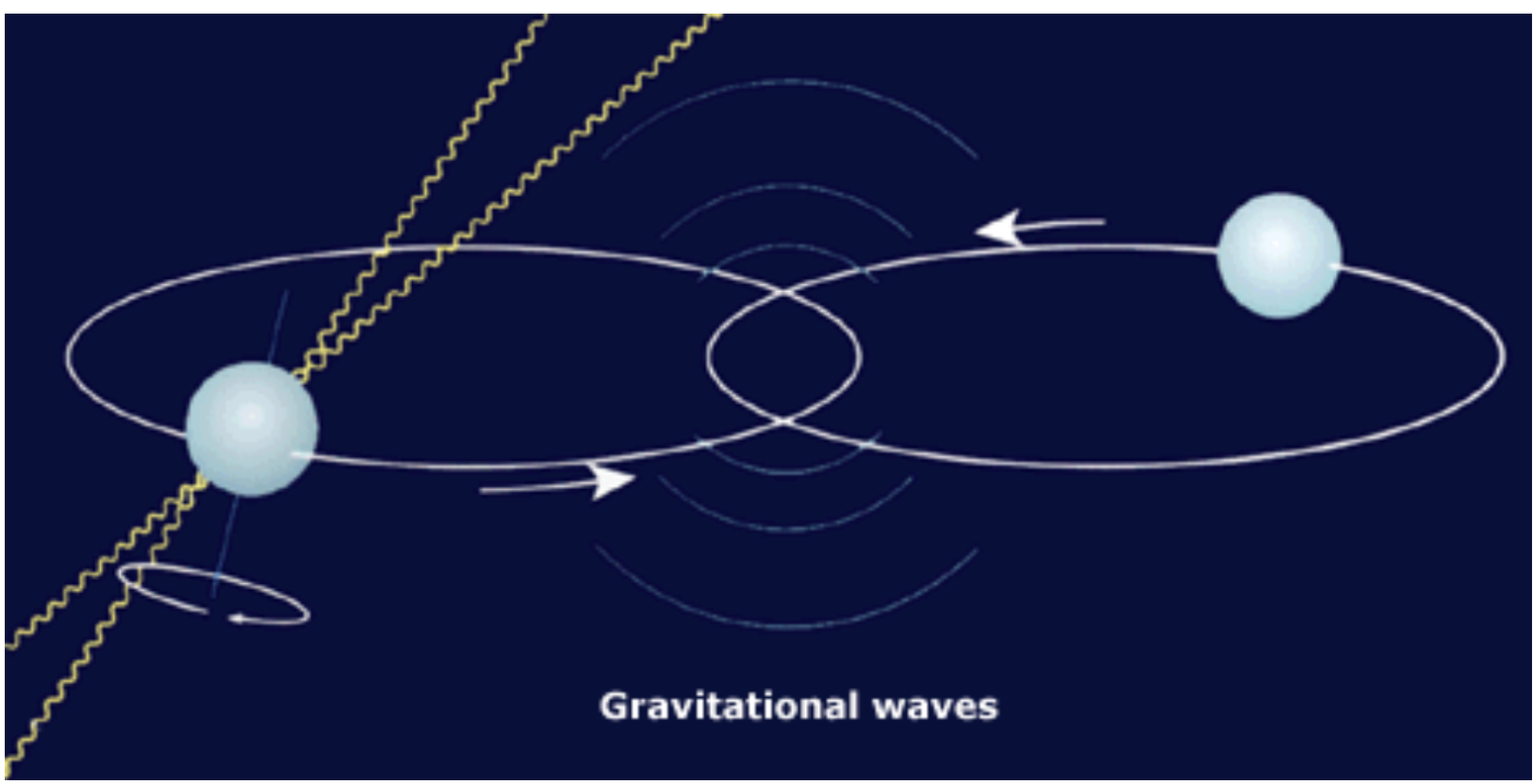
$$\frac{d^2\bar{h}^2_{ij}}{d\eta^2} - b^2\Delta\bar{h}^2_{ij} + ((M^2_{22}(a_\gamma) - \frac{b^{1/2}}{a_2}\frac{d^2(a_2 b^{-1/2})}{d\eta^2})\bar{h}^2_{ij} + M^2_{21}(a_\gamma)\bar{h}^1_{ij} = \beta_2 \frac{b^{1/2}\tilde{a}_J}{M_{\rm Pl}}\bar{T}_{ij}$$

We expect deviations from GR when b deviates from 1, ie today. We can analyse this in a different situations.

Binary Pulsars







We can compute the energy loss in binary pulsars in our bigravity model and compare to that in GR.

$$\frac{dE}{dt_J} = \left(\frac{\beta_1^2 + b\beta_2^2}{\beta_1^2 + \beta_2^2}\right)^2 \frac{a_J^4}{\tilde{a}_J^4} \frac{dE}{dt_{\rm GR}}$$

Note this only deviates from GR when $b \neq 1$

This is the case in the present Universe. Since the energy loss in binary pulsars in know we can use this to constrain bigravity

$$\boxed{0.995 < \left(\frac{\beta_1^2 + b\beta_2^2}{\beta_1^2 + \beta_2^2}\right)^2 \frac{a_J^4}{\tilde{a}_J^4} < 1}$$

which constrains b at $O(10^{-3})$

Now consider the propagation of gravitational waves. If the initial wave is due to a localised source we can approximate the waves as plane-waves

The propagation equations for the gravitons have eigenmodes

$$h_{\alpha\pm} = A_{\alpha\pm} e^{i(\omega_\pm t - i\vec{k}.\vec{x})}$$

with eigenfrequencies

$$\omega_+^2 \sim \vec{k}^2 + \tilde{M}_{11}^2 + \frac{\tilde{M}_{12}^4}{(1-b^2)\vec{k}^2 + \tilde{M}_{11}^2 - \tilde{M}_{22}^2}$$

$$\omega_-^2 \sim b^2\vec{k}^2 + \tilde{M}_{22}^2 - \frac{\tilde{M}_{12}^4}{(1-b^2)\vec{k}^2 + \tilde{M}_{11}^2 - \tilde{M}_{22}^2}$$

where the M's come from the mass matrix and are of order H0

This gives two eigenmodes

$$h_- = h_2 - Ch_1, \quad h_+ = h_1 + Ch_2$$

where

$$C = \frac{\tilde{M}_{12}^2}{(1-b^2)k^2 + \tilde{M}_{11}^2 - \tilde{M}_{22}^2}$$

with characteristic wave number

$$\bar{k}^2 = \frac{|\tilde{M}_{11}^2 - \tilde{M}_{22}^2|}{|1-b^2|}$$

which depends how small the deviation

$$\frac{|\omega_+ - \omega_-|}{k} \sim |b-1|$$

Computing the gravitational waves after emission we then find the Jordan frame gravitational wave to be

$$\bar{h}_J = a_J h_J = \beta_1 \bar{h}_1 + \beta_2 b^{1/2} \bar{h}_2$$

$$= \left(\frac{(\beta_1 + \beta_2 b^{1/2} C)^2}{1 + C^2} e^{i\omega_+ t} + \frac{(\beta_2 b^{1/2} - \beta_1 C)^2}{1 + C^2} e^{i\omega_- t} \right) e^{-i\vec{k}.\vec{x}} \frac{a_J^4}{\tilde{a}_J a_{\mathrm{GR}}^3} |_0 \frac{h_{\mathrm{GR}}}{\beta_1^2 + \beta_2^2}$$

this is the wave form of a far away source when the gravitational waves show birefringence

when $\omega_+ \sim \omega_-$

then $\omega = \dfrac{\omega_+ + \omega_-}{2}; \Delta\omega = \omega_- - \omega_+$

and if $\Delta\omega t \ll 1$

$$A_J = A\cos(\omega t - i\vec{k}.\vec{x} - \delta)$$

with a small time dependent phase shift

Graviton Speed

The wave propagates with energy

$$\omega = c_T k + \frac{\tilde{M}_{11}^2}{4k} + \frac{\tilde{M}_{22}^2}{4bk}$$

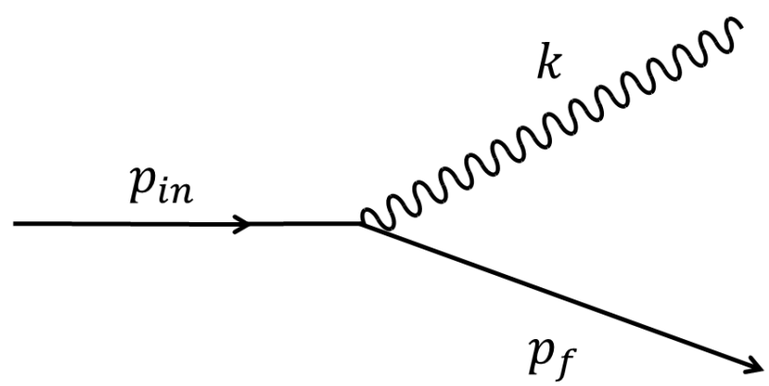
where

$$\boxed{c_T = \frac{1+b}{2}}$$

The effective speed is tightly constrained when b < 1 by Cerenkov radiation since cosmic rays can emit gravitons, depleting the number of cosmic rays observed.This gives

$$\boxed{(1-b) \lesssim 10^{-17}}$$

Cerenkov Radiation



Moore & Nelson '01;
Kimura et al '16

Graviton Decay

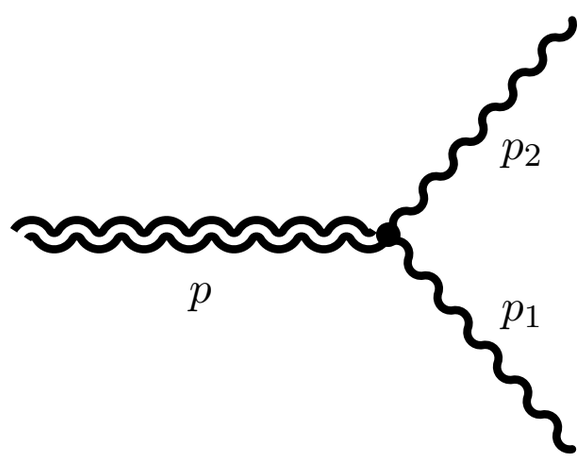


Figure 2: A graviton decaying into two photons

Brax, Burrage
& ACD '15

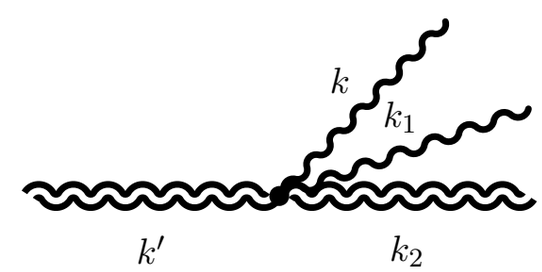


Figure 3: A graviton Cerenkov producing two photons

For b > 1 the strongest constraint comes from binary pulsars, but computing the graviton decay is good for the soul!

What are the prospects?

Consider gravitational waves at distance d from source and consider the square of the amplitude. We find the signal has a change in amplitude and time modulation and at time t=d

$$P_J(k) = \left(\frac{a_J^4}{\tilde{a}_J a_{\rm GR}^3}|_0\right)^2 \left(\frac{\beta_1^2 + b\beta_2^2}{\beta_1^2 + \beta_2^2}\right)^2 \left(1 - 4\frac{(\beta_1 + \beta_2 b^{1/2} C)^2 (\beta_2 b^{1/2} - \beta_1 C)^2}{(\beta_1^2 + b\beta_2^2)^2} \sin^2\left(\frac{(\omega_+ - \omega_-)d}{2}\right)\right) P_{\rm GR}(k)$$

with the spectrum modulated by a frequency dependent pre-factor

When one of the beta's vanishes we get

$$|\bar{h}_J|^2 = \left(\frac{a_J^4}{\tilde{a}_J a_{\rm GR}^3}|_0\right)^2 \left(1 - 4C^2 \sin^2\left(\frac{(\omega_+ - \omega_-)d}{2}\right)\right) |h_{\rm GR}|^2$$

Where C is constant for $k \lesssim \bar{k}$ and vanishes for large k

This retrieves the result for the singly coupled case, but note the term in sin^2 never vanishes

when $$|b-1| \lesssim \frac{1}{k_{\rm exp} d}$$

the modulation of the GR signal is relevant. It is possible that sources around d=100 Mpc and a sensitivity in nano Hertz range we might observe effects for $|b-1| \lesssim 10^{-7}$.

Conclusions

Emission and progagation of gravitational waves in doubly coupled bigravity

Deviations from GR governed by the parameter b, which only differs from 1 in the transient cosmological era between the matter era in the past and the future dark energy era

When $b \neq 1$ the speed of gravitational waves $c_T \neq 1$

but b is tightly constrained by cosmic rays and binary pulsar, so small deviation from unity. May still get possible modulations of the wave form in Jordan frame —ie when coupled to matter

Detection of gravitational wave and an electromagnetic counterpart would constrain b, or give rise to interesting detections

Multi-messenger signals are important