Introduction to models of modified gravity

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Abstract

This is a sketchy written version of my lectures given at the workshop DARKMOD.

Contents

1 Introduction

So far, general relativity (GR) works extremely well:

- very precisely tested in the solar system, in binary pulsars; now, direct observations of gravitational waves.
- on cosmological scales, the ΛCDM model, with gravity described by general relativity supplemented by a cosmological constant, works rather well, despite a few tensions (which could simply be due to systematic effects).

So, why explore alternatives to GR ? The main motivations are often the following:

1. The tiny value of the cosmological constant required in the ΛCDM model is difficult to understand from a theoretical point of view. Instead, the observed acceleration could be the consequence of a modification of gravitation laws on cosmological scales.
2. Use modified gravity models as a way to parametrize deviations from GR and therefore to provide quantitative tests of GR.

How to modify gravity ?

1. Extra fields: scalar(s), vectors or tensors. Most models are based on a single scalar field.
2. Extra dimensions: for example braneworld scenarios (Randall-Sundrum, Dvali-Gabadadze-Poratti).
3. Breaking of diffeomorphism invariance: for example massive gravity, massive bimetric gravity, Lorentz-violating theories (Einstein-aether, Horava gravity).

What are the main requirements when trying to modify gravity ?

1. Internal consistency of the theory: in particular, no ghost, no gradient instability, no tachyonic instability with short time scale.
2. Compatibility with all present observational constraints: laboratory experiments, solar system tests, astrophysical & cosmological observations.

   This is potentially problematic for theories that involve extra fields, which produce new interactions that must be sufficiently suppressed on well explored scales. Hence the importance of a screening mechanism in the recently studied models.

These lectures present a limited number of models, focussing on the main models that have been investigated in the literature. Massive gravity will not be discussed, as a separate series of lectures is devoted to it. Many more details can be found in a number of recent reviews on modified gravity, e.g. [11],[19] and[2].

2 Traditional scalar-tensor theories

[For a review with the main relevant references, see e.g. [8].]

One can consider a large class of scalar-tensor theories of the form

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\left[F(\phi)R - Z(\phi)g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - U(\phi)\right] + S_m\left[\psi_m, g_{\mu\nu}\right] , \tag{1}$$

where S_m is the action for the matter fields, denoted by ψ_m. The above action can also be written, up to a redefinition of the scalar field, in the form

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\left[\chi R - \frac{\omega(\chi)}{\chi}g^{\mu\nu}\partial_\mu\chi\partial_\nu\chi - W(\chi)\right] + S_m\left[\psi_m, g_{\mu\nu}\right] . \tag{2}$$

A particular subset of theories are the well-known Jordan-Brans-Dicke theories, which can be written in the form

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\,\left[\phi\, R - \frac{\omega_{\rm BD}}{\phi}(\partial\phi)^2\right] + \int d^4x\sqrt{-g}\,\mathcal{L}_m[\psi_m; g_{\mu\nu}]\,, \tag{3}$$

where $\omega_{\rm BD}$ is constant.

2.1 Jordan versus Einstein frame

In the above actions, matter is *minimally coupled* to the metric $g_{\mu\nu}$, which is called the *Jordan frame* metric. An equivalent formulation of the theory is given in the *Einstein frame*, where the kinetic term for the metric is of the Einstein-Hilbert form. This can be done by a conformal transformation of the metric and a redefinition of the scalar field so that the action reads

$$S = \int d^4x\sqrt{-g_*}\left[\frac{M_P^2}{2}R_* - \frac{1}{2}g_*^{\mu\nu}\partial_\mu\phi_*\,\partial_\nu\phi_* - V(\phi_*)\right] + S_m\left[\psi_m, A^2(\phi_*)g^*_{\mu\nu}\right] , \tag{4}$$

written in terms of the Einstein frame metric $g^*_{\mu\nu}$. We have introduced the reduced Planck mass $M_P \equiv 1/\sqrt{8\pi G}$. It is also convenient to introduce the dimensionless scalar field

$$\varphi_* \equiv \frac{\phi_*}{M_P}\,. \tag{5}$$

The price to pay for recovering the standard Einstein-Hilbert term is that matter is no longer minimally coupled to the metric. There is instead an explicit coupling to the scalar field in the matter action. The Jordan and Einstein formulations of scalar-tensor theories are equivalent, but, depending on the specific question one is interested in, one or the other can be more convenient.

2.1.1 Conformal transformation

Let us compute explicitly the conformal transformation to go from the Einstein frame action (4) to the Jordan frame action (1). Let us first recall that for a metric g^*_{ab} conformally related to the metric g_{ab},

$$g^*_{ab} = \Omega^2 g_{ab}\,, \tag{6}$$

the Ricci scalar is given by

$$R_* = g_*^{ab} \tilde{R}^*_{ab} = \Omega^{-2} \left[R - 6 \frac{\nabla^a \nabla_a \Omega}{\Omega} \right] , \tag{7}$$

and we thus have

$$\int d^4x \sqrt{-g_*}\, R_* = \int d^4x \sqrt{-g} \left[\Omega^2 R - 6 \Omega \nabla^a \nabla_a \Omega \right] = \int d^4x \sqrt{-g} \left[\Omega^2 R + 6 \nabla^a \Omega \nabla_a \Omega \right] , \tag{8}$$

where the second equality follows from an integration by parts. By comparing (1) and (4) with the above relation, one finds that the conformal factor between the two metrics must be

$$\Omega^2 = A^{-2} = F(\phi) \,. \tag{9}$$

Consequently

$$6 \nabla^a \Omega \nabla_a \Omega = \frac{3}{2F} \left(\frac{dF}{d\phi} \right)^2 (\partial \phi)^2 , \tag{10}$$

and

$$\sqrt{-g_*} g_*^{\mu\nu} \partial_\mu \phi_* \partial_\nu \phi_* = \sqrt{-g}\, M_P^2 F(\phi) \left(\frac{d\varphi_*}{d\phi} \right)^2 g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi \,. \tag{11}$$

The identification of the two actions (1) and (4) thus yields the relations

$$\left(\frac{d\varphi_*}{d\phi} \right)^2 = \frac{3}{2F^2} \left(\frac{dF}{d\phi} \right)^2 + \frac{Z}{F} \tag{12}$$

and

$$V(\varphi_*) = \frac{U(\phi)}{2F^2(\phi)} \,. \tag{13}$$

2.1.2 Energy-momentum tensor (Einstein frame)

Let us now work in the Einstein frame. For convenience, we drop the symbol $*$ that indicated Einstein frame quantities and we denote the Jordan metric as $\tilde{g}_{\mu\nu}$, so that the Einstein frame action (4) now reads

$$S = \int d^4x \sqrt{-g} \left[\frac{M_P^2}{2} R - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi\, \partial_\nu \phi - V(\phi) \right] + S_m \left[\psi_m, \tilde{g}_{\mu\nu} = A^2(\phi) g_{\mu\nu} \right] . \tag{14}$$

The matter energy-momentum tensor defined in the Einstein frame is not conserved because of the direct coupling to the scalar field, and verifies the relation

$$\nabla_\mu T^\mu_\nu = \frac{A_\phi}{A} T\, \nabla_\nu \phi \,, \tag{15}$$

where $T_{\mu\nu}$ is the Einstein-frame energy-momentum tensor defined by

$$T^{\mu\nu} \equiv \frac{2}{\sqrt{-g}} \frac{\delta L_m}{\delta g_{\mu\nu}} , \qquad \left[S_m = \int d^4x\, L_m \right] , \tag{16}$$

and $T = g^{\mu\nu} T_{\mu\nu}$ its trace.

This result can be shown for example by starting from the conservation of the Jordan frame energy-momentum tensor

$$\tilde{\nabla}_\mu \tilde{T}^\mu_\nu = 0\,, \tag{17}$$

(where we use here a tilde to denote all Jordan frame quantities) and by reexpressing the covariant derivative associated with $\tilde{g}_{\mu\nu}$ in terms of the covariant derivative associated with $g_{\mu\nu}$. Note that all indices for Jordan frame tensors are lowered or raised by the Jordan metric $\tilde{g}_{\mu\nu} = A^2 g_{\mu\nu}$ and its inverse. The tensor $\tilde{T}^{\mu\nu}$ is defined as in (16), but with tilde quantities, so that

$$T^{\mu\nu} = A^6\,\tilde{T}^{\mu\nu}\,, \quad T^\mu_{\ \nu} = A^4\,\tilde{T}^\mu_{\ \nu}\,, \quad T_{\mu\nu} = A^2\,\tilde{T}_{\mu\nu}\,. \tag{18}$$

2.1.3 Equation of motion for the scalar field (Einstein frame)

The variation (with respect to ϕ) of the total action (14) gives

$$\delta S = \int d^4x\sqrt{-g}\left[\nabla^\mu\nabla_\mu\phi - \frac{dV}{d\phi}\right]\delta\phi + \delta S_m\,, \tag{19}$$

with

$$\delta S_m = \int d^4x\,\frac{\delta L_m}{\delta\tilde{g}_{\mu\nu}}\frac{\partial\tilde{g}_{\mu\nu}}{\partial\phi}\delta\phi = \int d^4x\,\frac{\delta L_m}{\delta g_{\mu\nu}}\left(2\frac{A_\phi}{A}\right)g_{\mu\nu}\delta\phi\,. \tag{20}$$

We thus get

$$\nabla^\mu\nabla_\mu\phi - \frac{dV}{d\phi} + \frac{A_\phi}{A}T = 0\,. \tag{21}$$

2.2 Gravitational equations (Jordan frame)

We now go back to the Jordan frame and compute the scalar and gravitational fields generated by a static spherical body.

2.2.1 Jordan-Brans-Dicke theories

For simplicity, we restrict our calculation to the Jordan-Brans-Dicke theories, but it is straightforward to extend it to the general case (1), briefly discussed in the next subsection.

Let us consider the perturbed metric

$$ds^2 = -(1+2\Phi)dt^2 + (1-2\Psi)\delta_{ij}dx^i dx^j\,, \tag{22}$$

due to the presence of a (static spherically symmetric) non-relativistic source $T^0_0 = -\rho$, which is treated perturbatively. The scalar field ϕ is also perturbed:

$$\phi = 1 + \delta\phi\,, \tag{23}$$

where we have taken the background value $\bar{\phi} = 1$ (which is always possible up to a rescaling of G in the action).

Then the linearized equations of motion for the metric yield

$$\nabla^2\Psi = 4\pi G\rho + \frac{1}{2}\nabla^2\delta\phi\,, \qquad \Psi - \Phi = \delta\phi\,, \tag{24}$$

while the equation of motion for the scalar field gives

$$(3+2\omega_{\rm BD})\nabla^2\delta\phi = -8\pi G\rho\,. \tag{25}$$

Combining these equations leads to

$$\nabla^2\Phi = 4\pi G\mu\rho\,, \qquad \Psi = \eta\Phi\,, \tag{26}$$

with

$$\mu = \frac{4+2\omega_{\rm BD}}{3+2\omega_{\rm BD}}\,, \qquad \eta = \frac{1+\omega_{\rm BD}}{2+\omega_{\rm BD}}\,. \tag{27}$$

When $\omega_{\rm BD}$ tends to ∞, one recovers GR.

The effective Newton's constant is given by $G_{\rm eff} = G\mu$ and η corresponds to the PPN parameter $\gamma_{\rm PPN}$. The most stringent constraint on $\omega_{\rm BD}$ comes from the Shapiro delay is [3]

$$\omega_{\rm BD} > 4\times 10^4\,. \tag{28}$$

2.2.2 General case

In the general case of theories (1), one can envisage two situations:

- If the scalar field has a potential, in constrast with the BD case, and if the effective mass is very large, then the scalar interaction is effectively suppressed on distances larger than the inverse mass, even if the scalar field is strongly coupled to matter.

- If, by contrast, the scalar field is very light, then its coupling to matter, characterized by the parameter

$$\xi \equiv \frac{d\ln A}{d\varphi}\,, \tag{29}$$

  must be very weak in order to satisfy the current observational constraints. Indeed the PPN parameter $\gamma_{\rm PPN}$ is given by

$$\gamma_{\rm PPN} - 1 = -\frac{4\xi^2}{1+2\xi^2} = -\frac{F_\phi^2}{ZF+2F_\phi^2}\,. \tag{30}$$

As we will see in the next section, one can use the presence of a potential for the scalar field to suppress the effective coupling of extended objects to the scalar field.

3 Chameleon models and screening mechanism

[See e.g. [12] and [4] for recent reviews]

Chameleon theories [15, 14] are based on a screening mechanism such that the scalar coupling of extended objects is effectively suppressed. This is due to an effective scalar field potential that depends on the matter density, resulting in a high effective scalar mass in dense environments.

Let us consider a scalar-tensor theory, described in the *Einstein frame* by the action

$$S = \int d^4x\sqrt{-g}\left[\frac{M_P^2}{2}R - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi - V(\phi)\right] + S_m\left[\psi_m, \tilde g_{\mu\nu} = A^2(\phi)g_{\mu\nu}\right]\,. \tag{31}$$

and some non-relativistic matter with energy density ρ (defined in the Einstein frame).

3.1 Effective potential

According to (21), the equation of motion for the scalar field reads

$$\nabla^\mu \nabla_\mu \phi = \frac{dV}{d\phi} + \frac{A_\phi}{A}\rho, \tag{32}$$

since $T = -\rho$ for non-relativistic matter. As we saw earlier, the Einstein frame energy-momentum tensor is not conserved and we will work instead with the density $\hat{\rho} \equiv \rho/A$ which is conserved in the Einstein frame.

The scalar equation of motion then reads

$$\nabla^\mu \nabla_\mu \phi = \frac{dV}{d\phi} + A_\phi \hat{\rho} = \frac{dV_{\text{eff}}}{d\phi}\,, \tag{33}$$

with

$$V_{\text{eff}} \simeq V(\phi) + \xi \frac{\phi}{M_p}\hat{\rho}\,, \tag{34}$$

given an expansion in $\phi/M_p \ll 1$.

For illustration, let us consider a potential of the form

$$V(\phi) = V_0 + \frac{\mu^{4+n}}{\phi^n} \tag{35}$$

Then V_{eff} has a minimum for

$$\phi_{\text{min}} = \left(\frac{n M_p \mu^{4+n}}{\xi \hat{\rho}}\right)^{1/(n+1)}\,, \tag{36}$$

and the effective (square) mass at this minimum is given by

$$m^2_{\text{eff}} = \left(\frac{d^2 V_{\text{eff}}}{d\phi^2}\right)_{\text{min}} = n(n+1)\frac{\mu^{4+n}}{\phi^{n+2}_{\text{min}}} = n(n+1)\mu^{4+n}\left(\frac{\xi\hat{\rho}}{n M_p \mu^{4+n}}\right)^{(n+2)/(n+1)}\,. \tag{37}$$

This effective mass increases with the mass density: $m^2_{\text{eff}} \sim \hat{\rho}^{(n+2)/(n+1)}$. Therefore, the scalar interaction can be suppressed in high density environments.

3.2 Scalar field profile

Let us now solve the static scalar equation for a spherical object of radius R with, for simplicity, constant density:

$$\Delta\varphi = \frac{1}{r^2}\frac{d}{dr}\left(r^2 \frac{d\varphi}{dr}\right) = 8\pi G \xi \rho + \frac{dV}{d\varphi}\,, \qquad \varphi \equiv \phi/M_p\,. \tag{38}$$

One can distinguish three domains:

- Region $0 < r < r_s$

  The scalar field is at the minimum of the effective potential (inside the object):

$$\varphi = \varphi_c \tag{39}$$

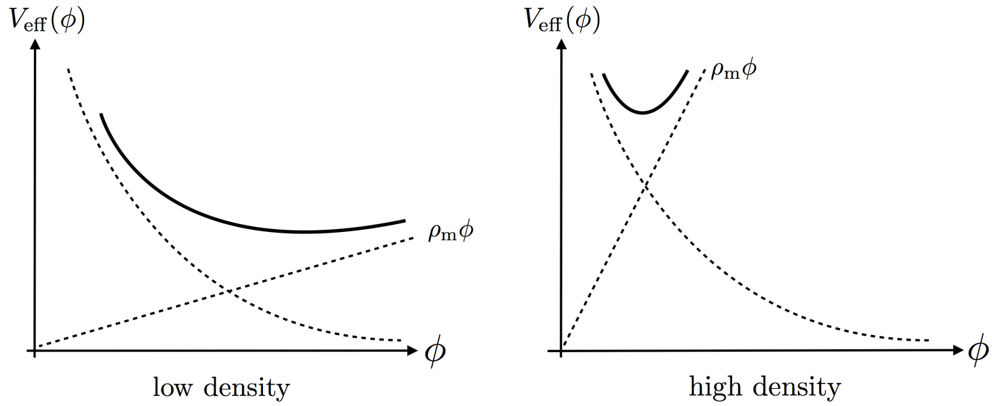

Figure 1: Effective potential at low and high densities [Plot taken from [13]].

- Region $r_s < r < R$

  The scalar field profile evolves, driven by the right hand side which is dominated by its first term:

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d\varphi}{dr}\right) \simeq 8\pi G\xi\rho\,. \tag{40}$$

  This is straightforward to integrate, giving

$$\varphi = \frac{4\pi G}{3}\xi\rho r^2 + \frac{A}{r} + B \tag{41}$$

  where A and B are two integration constants. These constants can be determined by matching φ and $d\varphi/dr$ at $r = r_s$, which yields

$$\varphi = \frac{4\pi G}{3}\xi\rho\left(r^2 + 2\frac{r_s^3}{r} - 3r_s^2\right) + \varphi_c \tag{42}$$

- Region $r > R$

  Outside the object, there is a new mimimum φ and new, much smaller, effective mass m_∞, associated with the (much lower) cosmological density. The scalar field profile is then of the Yukawa type:

$$\varphi = -\frac{C}{r}e^{-m_\infty(r-R)} + \varphi_\infty\,, \tag{43}$$

  where C is an integration constant. Matching $d\varphi/dr$ at $r = R$ yields

$$C = 8\pi G\,\xi\rho R^2\Delta R = 6\xi GM\frac{\Delta R}{R}\,, \tag{44}$$

  where we have assumed that $\Delta R \equiv R - r_s \ll R$. Moreover, matching φ at $r = R$ gives (keeping only up to first order terms in ΔR)

$$\varphi_c \simeq \varphi_\infty - 8\pi G\,\xi\rho R\Delta R\,. \tag{45}$$

From (45), one gets

$$\frac{\Delta R}{R} \simeq \frac{\varphi_\infty - \varphi_c}{8\pi G\,\xi\rho R^2} = \frac{\varphi_\infty - \varphi_c}{6\xi GM/R} = \frac{\varphi_\infty - \varphi_c}{6\xi|\Phi_\star|} \tag{46}$$

where we have used $M = (4/3)\pi\rho R^3$ and $\Phi_\star = -GM/R$. When the densities outside and inside are very different, $\varphi_c \ll \varphi_\infty$ and $\Delta R/R \simeq \varphi_\infty/(6\xi|\Phi_\star|)$. It is thus clear that the thin shell approximation that we have assumed, i.e. $\Delta R \ll R$, holds only if the condition

$$|\Phi_\star| \gg \frac{\varphi_\infty}{6\xi}\,. \tag{47}$$

is satisfied.

If the gravitational potential of the object is not sufficiently strong, then the screening is inefficient because the scalar field does not manage to reach the minimum of its potential in a large region inside the object.

3.3 Fifth force effects

The acceleration of a test particle is given by

$$\vec{a} = -\vec{\nabla}\Phi - \frac{d\ln A}{d\phi}\vec{\nabla}\phi\,, \tag{48}$$

where Φ is the Newtonian potential in the Einstein frame. This result can be derived as follows. A test particle follows geodesics in the Jordan frame, which implies in the nonrelativistic limit

$$\ddot{x}^i + \tilde{\Gamma}^i_{00} = 0\,. \tag{49}$$

This can be rewritten as

$$\ddot{x}^i + \Gamma^i_{00} = \Gamma^i_{00} - \tilde{\Gamma}^i_{00}\,, \tag{50}$$

which translates into

$$\ddot{x}^i + \partial^i\Phi = -\frac{A_\phi}{A}\partial^i\phi = -\xi\partial^i\varphi\,, \tag{51}$$

As a consequence, one can write

$$m_{\rm test}\ddot{x}^i = -m_{\rm test}\partial^i\Phi - q_{\rm test}\partial^i\varphi\,, \tag{52}$$

with

$$q_{\rm test} = \xi\, m_{\rm test}\,. \tag{53}$$

A body of mass M generates a gravitational field,

$$F_g = -\vec{\nabla}\Phi\,, \quad \Phi = -\frac{G_N M}{r}\,, \tag{54}$$

as well as a scalar field, which in the simple case where ξ and $V''(\phi) = m^2$ are constant, is given by[1]

$$F_\phi = -\xi\vec{\nabla}\varphi\,, \quad \varphi = -\frac{\xi M}{M_P^2}\frac{e^{-mr}}{4\pi r} = -\frac{2\xi G_N M}{r}e^{-mr}\,. \tag{55}$$

[1] In this particular case, the scalar field equation reads $\Delta\phi - m^2\phi = (\xi/M_P)\hat{\rho} = (\xi/M_P)M\delta^{(3)}(\vec{x})$, whose solution is

$$\phi = -\frac{\xi M}{M_P}\frac{e^{-mr}}{4\pi r}$$

.

The yields the ratio

$$\frac{F_\phi}{F_g} \simeq 2\xi^2 e^{-mr} \qquad r \lesssim m^{-1} \tag{56}$$

In the chameleon mechanism, we have found that, for an extended object where the thin shell approximation is valid, the scalar field outside the object is given by

$$\varphi = -\frac{2\xi\epsilon\, G_N M}{r} e^{-mr}\,, \tag{57}$$

with

$$\epsilon = 3\frac{\Delta R}{R} \ll 1\,, \tag{58}$$

in contrast with the result (55) where $\epsilon = 1$. This shows that the effective scalar charge of a screened object, $Q = \epsilon\,\xi M$, is suppressed with respect to that of an unscreened object.

4 $f(R)$ gravity

[See e.g. [28] and [6] for reviews]

One considers the Lagrangian

$$S = \frac{M_P^2}{2}\int d^4x\sqrt{-g}\,[R + f(R)] + S_{\rm matter}[\psi_m, g_{\mu\nu}]\,. \tag{59}$$

Variation with respect to the metric yields:

$$(1+f_R)R_{\mu\nu} - \frac{1}{2}(R+f)g_{\mu\nu} - [\nabla_\mu\nabla_\nu - g_{\mu\nu}\Box]f_R = M_P^{-2}T_{\mu\nu}\,. \tag{60}$$

The trace of this equation can be rewritten as an equation for the scalar f_R, also called *scalaron*,

$$\Box f_R = \frac{1}{3}\left(R + 2f - f_R R + M_P^{-2}\,T\right) \equiv \frac{dV_{\rm eff}(f_R)}{df_R}\,. \tag{61}$$

By taking the derivative of the right hand side, one obtains the effective mass

$$m^2_{\rm eff}(f_R) = \frac{1}{3}\left(\frac{1+f_R}{f_{RR}} - R\right)\,. \tag{62}$$

4.1 Constraints on the function f

- In high curvature regions, $f_R \sim 0$ and $|Rf_{RR}| \ll 1$, hence $m^2_{\rm eff} \simeq 1/(3f_{RR})$. The scalaron is not a tachyon if

$$f_{RR} > 0 \tag{63}$$

  in this regime.

- The graviton is not a ghost if we have everywhere

$$1 + f_R > 0. \tag{64}$$

- Solar system constraints must be satisfied, which requires $|f_R| \ll 1$ to evade fifth force constraints. Typically, we must have

$$|f_R| \ll 10^{-6}\,. \tag{65}$$

- One would like to recover standard GR in the early universe:

$$\frac{f(R)}{R} \to 0 \quad \text{and} \quad f_R \to 0 \quad \text{when} \quad R \to \infty\,. \tag{66}$$

Typical examples considered in the literature are the Hu-Sawicki model,

$$f(R) = -\frac{\alpha M^2}{1+(R/M^2)^{-\beta}}\,, \qquad \alpha, \beta > 0 \tag{67}$$

and the Starobinsky model

$$f(R) = \alpha M^2 \left[\left(1+\frac{R^2}{M^4}\right)^{-\beta/2} - 1\right]\,, \qquad \alpha, \beta > 0\,, \tag{68}$$

which take the same form in the relevant cosmological regime $R \gg M^2$:

$$f(R) \simeq \alpha M^2 \left[\left(\frac{R}{M^2}\right)^{-\beta} - 1\right] \tag{69}$$

In practice, in the high curvature regime, the function $F(R)$ is of the form

$$F(R) \simeq R - 2\Lambda + |f_{R0}|\,\frac{\bar{R}^{n+1}}{R^n}\,, \tag{70}$$

where $\bar{R}$ is the curvature today.

4.2 Equivalence with scalar-tensor theories

The $f(R)$ theories can be rewritten as scalar-tensor theories. Indeed, the action

$$S = \frac{M_p^2}{2}\int d^4x\sqrt{-g}\,\mathcal{F}(R), \tag{71}$$

is equivalent to the action

$$S = \frac{M_p^2}{2}\int d^4x\sqrt{-g}\,[\mathcal{F}(\sigma) + (R-\sigma)\mathcal{F}'(\sigma)]\,, \tag{72}$$

as can be checked by varying the two actions. Defining $\chi = \mathcal{F}'(\sigma)$ and $V = \mathcal{F}(\sigma) - \sigma\mathcal{F}'(\sigma)$, one gets

$$S = \frac{M_p^2}{2}\int d^4x\sqrt{-g}\,[\chi R - V(\chi)]\,, \tag{73}$$

where one recognises a JBD type theory with $\omega_{\rm BD} = 0$ but with a potential. These theories thus form a subclass of (1).

By resorting to a conformal transformation, as well as a field redefinition,

$$g^*_{\mu\nu} = e^{\sqrt{\frac{2}{3}}\frac{\phi}{M_P}} \,, \quad \phi = -\sqrt{\frac{3}{2}} M_P \ln \mathcal{F}'(\sigma) \,, \tag{74}$$

one gets the Einstein frame action

$$S = \int d^4x \sqrt{-g_*} \left[\frac{M_P^2}{2} R_* - \frac{1}{2} g_*^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right] + S_m \left[\psi_m, A^2(\phi) g^*_{\mu\nu} \right] \,, \tag{75}$$

with

$$A(\phi) = \exp\left(-\frac{1}{\sqrt{6}} \frac{\phi}{M_P} \right) \,. \tag{76}$$

The potential associated with (69) is given by

$$V(\phi) \approx \frac{\alpha}{2} M^2 M_P^2 \left[1 - (\beta + 1) \left(\sqrt{\frac{2}{3}} \frac{\phi}{\alpha \beta M_P} \right)^{\frac{\beta}{1+\beta}} \right] \,, \tag{77}$$

or, more simply, of the form

$$V = \Lambda - \mu^4 \left(\frac{\phi}{M_P} \right)^{\frac{n}{n+1}} \,. \tag{78}$$

5 Horndeski and Vainshtein mechanism

5.1 Horndeski theories

One can try to construct scalar-tensor theories with second order derivatives in their Lagrangian, i.e. of the form

$$S[\phi, g_{\mu\nu}] = \int d^4x \sqrt{-g} \, \mathcal{L}(\phi, \nabla_\mu \phi, \nabla_\mu \nabla_\nu \phi; g_{\mu\nu}) \equiv \int d^4x \, L \tag{79}$$

The corresponding Euler-Lagrange equation for the scalar field is given by

$$\nabla_\mu \nabla_\nu \left(\frac{\partial^2 L}{\partial \nabla_\mu \nabla_\nu \phi} \right) - \nabla_\mu \left(\frac{\partial L}{\partial \nabla_\mu \phi} \right) + \frac{\partial L}{\partial \phi} = 0 \,. \tag{80}$$

This leads in general to fourth-order equations of motion.

However, there exists a family of Lagrangians, discovered by Horndeski in 1974, such that the Euler-Lagrange equations for both the scalar field and the metric are second-order. These Horndeski theories are described by Lagrangians obtained by combining four Lagrangians that read, in modern notation,

$$L_2^H[G_2] \equiv G_2(\phi, X) \,, \tag{81}$$

$$L_3^H[G_3] \equiv G_3(\phi, X) \, \Box\phi \,, \tag{82}$$

$$L_4^H[G_4] \equiv G_4(\phi, X) \, {}^{(4)}\!R - 2 G_{4X}(\phi, X) \left[(\Box\phi)^2 - (\nabla^\mu \nabla^\nu \phi)(\nabla_\mu \nabla_\nu \phi) \right] \,, \tag{83}$$

$$L_5^H[G_5] \equiv G_5(\phi, X) \, {}^{(4)}\!G_{\mu\nu} \nabla^\mu \nabla^\nu \phi + \frac{1}{3} G_{5X}(\phi, X) \left[(\Box\phi)^3 - 3 \, \Box\phi \, (\nabla^\mu \nabla^\nu \phi)(\nabla_\mu \nabla_\nu \phi) + 2 \, (\nabla_\mu \nabla_\nu \phi)(\nabla^\sigma \nabla^\nu \phi)(\nabla_\sigma \nabla^\mu \phi) \right] \,. \tag{84}$$

where each of the Lagrangian depends on an abitrary function G_A of ϕ and $X \equiv \nabla_\mu\phi\nabla^\mu\phi$.

Horndeski's paper was completely forgotten until 2011. In the meantime, the so-called galileon theories had been introduced, motivated by considerations related to some braneworld models [25]. In fact, galileons turn out to be a particular case of Horndeski's theories when the metric is Minkowski and the functions G_A are given by

$$G_3 \propto X\,, \qquad G_4 \propto X^2\,, \qquad G_5 \propto X^2\,. \tag{85}$$

Galileon theories enjoy a special symmetry, dubbed galileon symmetry, characterized by the invariance under the transformation

$$\phi(x) \longrightarrow \phi(x) + c + b_\mu x^\mu\,. \tag{86}$$

By extending galileon theories and introducing a dynamical metric, Horndeski's theories were recovered [7, 18].

5.2 Vainshtein screening

5.2.1 Example: cubic galileon

Let us consider the example of the cubic galileon, described in the **Einstein frame**, by an action of the form

$$S = \int d^4x\sqrt{-g}\left[-k_2(\partial\phi)^2 - k_3\,(\partial\phi)^2\Box\phi\right] + S_m\left[\psi_m, A^2(\phi)g_{\mu\nu}\right] \equiv S = \int d^4x\sqrt{-g}L + S_m\,. \tag{87}$$

The equation of motion for the scalar field is given by

$$\nabla_\mu J^\mu = -\frac{A_\phi}{A}T, \tag{88}$$

with

$$J^\mu \equiv -\frac{1}{\sqrt{-g}}\frac{\delta S}{\delta\partial_\mu\phi} = \nabla_\nu\left(\frac{\partial L}{\partial\phi_{\mu\nu}}\right) - \frac{\partial L}{\partial\phi_\mu} = 2(k_2 + k_3\Box\phi)\nabla^\mu\phi - 2k_3(\nabla^\mu\nabla^\nu\phi)\phi_\nu\,. \tag{89}$$

The explicit equation of motion is given by

$$2k_2\Box\phi + 2k_3\left[(\Box\phi)^2 - \phi^{\mu\nu}\phi_{\mu\nu}\right] - 2k_3R_{\mu\nu}\phi^\mu\phi^\nu = -\alpha\frac{T}{M_P}\,. \tag{90}$$

In a Minkowski background, assuming spherical symmetry, one gets

$$\frac{1}{r^2}\partial_r\left(r^2J^r\right) = -\alpha\frac{T}{M_P} = \frac{\alpha}{M_P}\rho(r)\,, \qquad J^r = 2k_2\phi' + 4k_3\frac{\phi'^2}{r}\,. \tag{91}$$

For a point-like source of mass M, the integration yields

$$2k_2\phi' + 4k_3\frac{\phi'^2}{r} = \frac{\alpha M}{4\pi r^2 M_P}\,, \tag{92}$$

giving the solution

$$\phi' = -\frac{k_2 r}{4k_3}\left[1 \pm \sqrt{1 + \frac{k_3\alpha M}{k_2^2 M_P\pi r^3}}\right]\,. \tag{93}$$

Defining the Vainshtein radius[2]

$$r_V \equiv \left(\frac{k_3 \alpha M}{k_2^2 M_P \pi}\right)^{1/3}, \tag{94}$$

one can write the appropriate solution (with ϕ' going to zero at infinity):

$$\phi' = -\frac{k_2 r}{4k_3}\left[1 - \sqrt{1 + \frac{r_V^3}{r^3}}\right]. \tag{95}$$

One can identify two regimes:

1. the linear regime $r \gg r_V$:
$$\phi' \simeq \frac{k_2}{8k_3}\frac{r_V^3}{r^2} \tag{96}$$

2. the Vainshtein regime $r \ll r_V$
$$\phi' \simeq \frac{k_2}{4k_3}\frac{r_V^{3/2}}{r^{1/2}} \tag{97}$$

This implies

$$\frac{F_\phi}{F_g} \simeq \frac{\alpha^2}{k_2} \quad (r \gg r_V), \qquad \frac{F_\phi}{F_g} \simeq 2\frac{\alpha^2}{k_2}\left(\frac{r}{r_V}\right)^{3/2} \quad (r \ll r_V). \tag{98}$$

See [5] for a review on the laboratory tests of Vainshtein screening (in particular using Casimir force experiments).

5.2.2 More general case: Horndeski theories

One can also consider the Vainshtein mechanism at the level of linear perturbations in the context of Horndeski theories [16, 20].

6 DHOST theories

See [22] for a recent short review (with the relevant references).

6.1 Higher-order scalar-tensor theories

We now consider scalar-tensor theories whose action depends not only on ϕ and its gradient $\phi_\mu \equiv \nabla_\mu \phi$ as usual, but also on its second derivatives $\phi_{\mu\nu} \equiv \nabla_\mu \nabla_\nu \phi$. Restricting ourselves to the cubic order in $\phi_{\mu\nu}$ up to cubic order, we are interested by actions of the form

$$\begin{aligned} S[g,\phi] &= \int d^4x \sqrt{-g}\Big[f_0(X,\phi) + f_1(X,\phi)\Box\phi + f_2(X,\phi)\,R + C_{(2)}^{\mu\nu\rho\sigma}\,\phi_{\mu\nu}\,\phi_{\rho\sigma} + \\ &\qquad + f_3(X,\phi)\,G_{\mu\nu}\phi^{\mu\nu} + C_{(3)}^{\mu\nu\rho\sigma\alpha\beta}\,\phi_{\mu\nu}\,\phi_{\rho\sigma}\,\phi_{\alpha\beta}\Big], \end{aligned} \tag{99}$$

[2] In DGP, we have $r_c^2 \equiv \frac{2k_3}{\alpha M_P}$. To get $r_c \sim H_0^{-1}$, the Vainshtein radius is $r_V \sim 130$ pc.

where the functions f_i depend only on ϕ and $X \equiv \phi_\mu \phi^\mu$.

The tensors $C_{(2)}$ and $C_{(3)}$ being the most general tensors constructed with the metric $g_{\mu\nu}$ and the scalar field gradient ϕ_μ, it is easy to see that the terms quadratic in $\phi_{\mu\nu}$ can be rewritten as

$$C_{(2)}^{\mu\nu\rho\sigma}\,\phi_{\mu\nu}\,\phi_{\rho\sigma} = \sum_{A=1}^{5} a_A(X,\phi)\,L_A^{(2)}\,, \tag{100}$$

with

$$\begin{aligned} &L_1^{(2)} = \phi_{\mu\nu}\phi^{\mu\nu}\,, \qquad L_2^{(2)} = (\Box\phi)^2\,, \qquad L_3^{(2)} = (\Box\phi)\phi^\mu\phi_{\mu\nu}\phi^\nu\,,\\ &L_4^{(2)} = \phi^\mu\phi_{\mu\rho}\phi^{\rho\nu}\phi_\nu\,, \qquad L_5^{(2)} = (\phi^\mu\phi_{\mu\nu}\phi^\nu)^2\,, \end{aligned} \tag{101}$$

where the a_A are five arbitrary functions of X and ϕ. Similarly, the cubic terms can be written in terms of ten arbitrary functions b_A, as

$$C_{(3)}^{\mu\nu\rho\sigma\alpha\beta}\,\phi_{\mu\nu}\,\phi_{\rho\sigma}\,\phi_{\alpha\beta} = \sum_{A=1}^{10} b_A(X,\phi)\,L_A^{(3)}\,, \tag{102}$$

where

$$\begin{aligned} &L_1^{(3)} = (\Box\phi)^3\,, \quad L_2^{(3)} = (\Box\phi)\,\phi_{\mu\nu}\phi^{\mu\nu}\,, \quad L_3^{(3)} = \phi_{\mu\nu}\phi^{\nu\rho}\phi_\rho^\mu\,, \quad L_4^{(3)} = (\Box\phi)^2\,\phi_\mu\phi^{\mu\nu}\phi_\nu\,,\\ &L_5^{(3)} = \Box\phi\,\phi_\mu\phi^{\mu\nu}\phi_{\nu\rho}\phi^\rho\,, \quad L_6^{(3)} = \phi_{\mu\nu}\phi^{\mu\nu}\phi_\rho\phi^{\rho\sigma}\phi_\sigma\,, \quad L_7^{(3)} = \phi_\mu\phi^{\mu\nu}\phi_{\nu\rho}\phi^{\rho\sigma}\phi_\sigma\,,\\ &L_8^{(3)} = \phi_\mu\phi^{\mu\nu}\phi_{\nu\rho}\phi^\rho\,\phi_\sigma\phi^{\sigma\lambda}\phi_\lambda\,, \quad L_9^{(3)} = \Box\phi\,(\phi_\mu\phi^{\mu\nu}\phi_\nu)^2\,, \quad L_{10}^{(3)} = (\phi_\mu\phi^{\mu\nu}\phi_\nu)^3\,. \end{aligned} \tag{103}$$

6.2 Horndeski and Beyond Horndeski theories

The general action (99) includes in particular Horndeski theories. The quadratic part of the Horndeski action, is fully determined by the function $f_2 = G_4$, with the quadratic coefficient a_A given by

$$a_1 = -a_2 = 2f_{2,X}\,, \qquad a_3 = a_4 = a_5 = 0\,. \tag{104}$$

Similarly, the cubic part of Horndeski theories, depends only on the functions $f_3 = G_5$, while

$$3b_1 = -b_2 = \frac{3}{2}b_3 = f_{3,X}\,, \qquad b_A = 0\ (A = 4,\ldots,10)\,. \tag{105}$$

The so-called Beyond Horndeski (or GLPV) theories, introduced in [9], extend Horndeski theories by including two additional Lagrangians, each characterized by a single arbitrary function. The first of these Lagrangians, which can be written as $L_{(2)}^{\rm bH}[g_2]$, is quadratic and characterized by the coefficients

$$a_1 = -a_2 = Xg_2\,, \qquad a_3 = -a_4 = 2g_2\,, \qquad a_5 = 0\,. \tag{106}$$

The second new Lagrangian, which is cubic and will be denoted $L_{(3)}^{\rm bH}[g_3]$, depends on a single arbitrary function g_3 and its non vanishing coefficients b_A are given by

$$\frac{b_1}{X} = -\frac{b_2}{3X} = \frac{b_3}{2X} = -\frac{b_4}{3} = \frac{b_5}{6} = \frac{b_6}{3} = -\frac{b_7}{6} = g_3\,. \tag{107}$$

6.3 Degenerate Higher-order scalar-tensor (DHOST) theories

The crucial element that characterizes higher-order theories with a single scalar degree of freedom is the degeneracy of their Lagrangian, hence their name DHOST.

DHOST theories include seven subclasses of quadratic theories (four classes with $f_2 \neq 0$ and three classes with $f_2 = 0$) and nine subclasses of cubic theories (two with $f_3 \neq 0$ and seven with $f_3 = 0$). These quadratic and cubic subclasses can be combined to yield degenerate hybrid theories, involving both quadratic and cubic terms, but all combinations are not possible: only 25 combinations (out of 63) lead to degenerate theories, often with extra conditions on the functions a_A and b_A in the Lagrangian (see [1] for details and for the explicit form of the functions in each subclass).

A legitimate question about this classification is whether seemingly different DHOST theories could correspond the same theory in different guises, in other words whether some theories could be identified via field redefinitions [3]. Since the Lagrangian depends on a metric and on a scalar field, natural field redefinitions of the metric involve disformal transformations

$$\tilde{g}_{\mu\nu} = C(X,\phi) g_{\mu\nu} + D(X,\phi)\, \phi_\mu\, \phi_\nu\, . \tag{108}$$

Via this transformation, any action $\tilde{S}$ given as a functional of $\tilde{g}_{\mu\nu}$ and ϕ induces a new action S for $g_{\mu\nu}$ and ϕ, when one substitutes the above expression for $\tilde{g}_{\mu\nu}$ in $\tilde{S}$:

$$S[\phi, g_{\mu\nu}] \equiv \tilde{S}\left[\phi, \tilde{g}_{\mu\nu} = C\, g_{\mu\nu} + D\, \phi_\mu \phi_\nu\right]\, . \tag{109}$$

The actions S and $\tilde{S}$ are then said to be related by the disformal transformation (108).

Interestingly, there is a nice correspondence between the type of disformal transformations and the extent of the corresponding stable class of theories:

- Horndeski theories are stable under disformal transformations characterized by $C(\phi)$ and $D(\phi)$, i.e. conformal and disformal factors that depend only on ϕ, but not on X.
- Beyond Horndeski theories are stable under disformal transformations characterized by $C(\phi)$ and $D(\phi, X)$.
- Finally, DHOST theories are stable under the most general disformal transformations where C and D depend on both ϕ and X.

6.4 Beyond Horndeski: breaking of Vainshtein mechanism

In Beyond Horndeski theories, one finds a partial breaking of Vainshtein mechanism inside astrophysical bodies so that the equations of motion for the weak-field metric potentials defined by

$$ds^2 = -(1+2\Phi)dt^2 + (1-2\Psi)\,\delta_{ij} dx^i dx^j \tag{110}$$

are modified to [17, 21, 26, 27]

$$\frac{d\Phi}{dr} = \frac{G_N M(r)}{r^2} + \frac{\Upsilon_1 G_N}{4}\frac{d^2 M(r)}{dr^2} \tag{111}$$

$$\frac{d\Psi}{dr} = \frac{G_N M(r)}{r^2} - \frac{5\Upsilon_2 G_N}{4r}\frac{dM(r)}{dr}. \tag{112}$$

[3] The coupling to matter is ignored here. If, after a redefinition of the metric, two related theories are minimally coupled to matter, then they are physically distinct.

The dimensionless parameters Υ_i characterise deviations from GR of the beyond Horndeski type. They are directly related to the parameters appearing in the effective description of dark energy (see next section) that controls the linear cosmology of beyond Horndeski theories:

$$\Upsilon_1 = \frac{4\alpha_H^2}{c_T^2(1+\alpha_B) - \alpha_H - 1} \quad \text{and}$$
$$\Upsilon_2 = \frac{4\alpha_H(\alpha_H - \alpha_B)}{5(c_T^2(1+\alpha_B) - \alpha_H - 1)}. \tag{113}$$

7 Cosmology

7.1 Effective description of Dark Energy and Modified Gravity

In order to study the cosmology of DHOST theories, it is very convenient to resort to the unified formalism that has been developed for an effective description of Dark Energy and Modified Gravity (see e.g. [10] for a review).

This approach is based on a 3 + 1 decomposition of spacetime,

$$ds^2 = -N^2 dt^2 + h_{ij}(dx^i + N^i dt)(dx^j + N^j dt)\,, \tag{114}$$

in which the spatial slices coincide with uniform scalar field hypersurfaces. In this particular gauge, sometimes called unitary gauge, the action of DHOST theories is of the form

$$S = \int d^3x\, dt\, N\sqrt{h}\, L[N, K_{ij}, {}^3\!R_{ij}; t]\,, \tag{115}$$

where N is the lapse function which appears in the 3 + 1 form of the spacetime metric ; K_{ij} is the extrinsic curvature tensor and ${}^3\!R_{ij}$ the intrinsic curvature tensor.

The Friedmann equations associated with a spatially flat Friedmann-Lemaître-Robertson-Walker (FLRW) spacetime,

$$ds^2 = -\bar{N}^2(t)dt^2 + a^2(t)\delta_{ij}dx^i dx^j\,, \tag{116}$$

are then simply derived from the homogeneous action

$$S_{\rm homog} = \int dt\, N a^3 L[N = \bar{N}(t), K^i_j = \frac{\dot{a}}{\bar{N}a}\delta^i_j, {}^3\!R_{ij} = 0; t]\,. \tag{117}$$

To study the dynamics of linear perturbations, one needs to write down the action at quadratic order in perturbations. These perturbations are associated with the three basic ingredients of the action:

$$\delta N \equiv N - \bar{N}\,, \qquad \delta K^i_j = K^i_j - H\delta^i_j\,, \qquad \delta\, {}^3\!R^i_j = {}^3\!R^i_j\,, \tag{118}$$

where $H = \dot{a}/(\bar{N}a)$ is the Hubble parameter, and ${}^3\!R^i_j$ is already a perturbation since it vanishes in the background. The Lagrangian at quadratic order is then obtained via a Taylor expansion, which is formally written as

$$L(q_A) = \bar{L} + \frac{\partial L}{\partial q_A}\delta q^A + \frac{1}{2}\frac{\partial^2 L}{\partial q_A \partial q_B}\delta q^A \delta q^B + \dots\,. \tag{119}$$

where $q^A = \{N, K^i_j, {}^3\!R^i_j\}$.

All (quadratic and cubic) DHOST theories lead to a Lagrangian quadratic in linear perturbations of the form [23]

$$
\begin{aligned}
S_{\rm quad} \;\; = \;\; & \int d^3x\, dt\, a^3 \frac{M^2}{2}\Big\{\delta K_{ij}\delta K^{ij} - \left(1+\frac{2}{3}\alpha_{\rm L}\right)\delta K^2 + (1+\alpha_{\rm T})\left({}^3R\frac{\delta\sqrt{h}}{a^3} + \delta_2{}^3R\right) + H^2\alpha_{\rm K}\delta N^2 \\
& + 4H\alpha_{\rm B}\delta K\delta N + (1+\alpha_{\rm H})\,{}^3R\,\delta N + 4\beta_1\delta K\delta\dot N + \beta_2\delta\dot N^2 + \frac{\beta_3}{a^2}(\partial_i\delta N)^2\Big\}\,, \qquad (120)
\end{aligned}
$$

where $\delta_2{}^3R$ denotes the second order term in the perturbative expansion of ${}^3R$, where the parameters M, $\alpha_{\rm L}$, $\alpha_{\rm T}$, $\alpha_{\rm K}$, $\alpha_{\rm B}$, $\alpha_{\rm H}$, β_1, β_2 and β_3 are time-dependent functions[4]. Moreover, one finds that these parameters, for DHOST theories, are restricted to satisfy either one of the following sets of conditions:

$$
\mathcal{C}_{\rm I}: \qquad \alpha_{\rm L} = 0\,, \qquad \beta_2 = -6\beta_1^2\,, \qquad \beta_3 = -2\beta_1\left[2(1+\alpha_{\rm H}) + \beta_1(1+\alpha_{\rm T})\right]\,, \qquad (121)
$$

or

$$
\mathcal{C}_{\rm II}: \qquad \beta_1 = -(1+\alpha_{\rm L})\frac{1+\alpha_{\rm H}}{1+\alpha_{\rm T}}\,, \quad \beta_2 = -6(1+\alpha_{\rm L})\frac{(1+\alpha_{\rm H})^2}{(1+\alpha_{\rm T})^2}\,, \quad \beta_3 = 2\frac{(1+\alpha_{\rm H})^2}{1+\alpha_{\rm T}}\,. \qquad (122)
$$

The category $\mathcal{C}_{\rm I}$ contains the subclass of Horndeski theories and of those related to Horndeski via disformal transformations.

From the action (120), one can isolate the physical degrees of freedom, which reduce to one scalar and two tensor modes for DHOST. Their action is given by [23]

$$
S_{\rm quad,phys} = \int d^3x\, dt\, a^3 \left\{\frac{M^2}{2}\left[A\,\dot{\tilde\zeta}^2 - B\,\frac{(\partial\tilde\zeta)^2}{a^2}\right] + \frac{M^2}{8}\left[\dot\gamma_{ij}^2 - \frac{1+\alpha_{\rm T}}{a^2}(\partial_k\gamma_{ij})^2\right]\right\}\,, \qquad (123)
$$

where $\tilde\zeta \equiv \zeta - \beta_1\delta N$, ζ being the usual curvature perturbation of the spatial part of the metric and h_{ij} denotes the transverse-traceless perturbation of the metric. The explicit expressions for the coefficients A and B can be found in [23]. In particular, for models in the category $\mathcal{C}_{\rm II}$, one finds that $B = -(1+\alpha_{\rm T})$. Comparing with the tensor part, one sees that the coefficients of the gradient terms for the scalar and tensor modes have opposite signs and therefore, these modes cannot be stable simultaneously. This signals an instability for theories satisfying $\mathcal{C}_{\rm II}$.

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