Albert Einstein and the Modern Physics Revolution

Dr. Elliott S. McCrory
Saturday Morning Physics
Lecture number 2
October 7, 2017

Concepts for today

• Intuition
• Transformations
• Special Relativity
• Simultaneity
• If time allows
  • Mass equals energy
  • Spacetime
• Utilizing:
  • Clickers
  • The metric system
  • Algebra
  • Star Trek
  • Trans
  • Pole vaulters

But what is “Science”? [Slide]

Are There Alternatives to the Scientific Method? [Slide]

Reliance on tradition or authority:
• This is something of a straw man argument against science: who would argue that science conducted sufficiently fairly and carefully will often lead to conclusions that are likely to be false?
• Many instances of reliance on tradition are actually a weak form of reliance on social science — if many people hold position X in the past, then this provides a limited degree of empirical evidence that holding position X is likely to be helpful or advantageous.

Reliance on pure reasoning (mathematics, philosophy):
• Many people think of mathematics as a part of science, but it is fundamentally not grounded in empiricism (as a central part of the scientific method).
• Although philosophers of science hold a range of opinions on this issue, my view is that math helps to illuminate the relationships between ideas and can help to clarify our thinking, but does not itself tell us anything about our world.

We are guided by our intuition

• Must be careful!
  • Holistic view of the "universe" based on beliefs and on intuition
  • Experience, observation and intuition are linked
  • It can only takes you so far
  • Intuition grows as we gain experience and make observations

What is Intuition?

[Definition: The ability to understand something immediately, without the need for conscious reasoning.]

“We shall allow our intuition to guide us”

• A thing that one knows or considers likely from instinctive feeling rather than conscious reasoning.

synonyms: instinct, hunch, feeling (in one’s bones); inkling, (sneaking) suspicion, idea, sense, notion;

Modern Physics: Our Intuition is incomplete

Much is, at first, counter-intuitive

We must develop new intuition

Terminology: A quantity is said to be

• Invariant
  • If different observers would obtain the same result from a measurement of this quantity.
    • The mass of an object.

• Relative
  • If different observers would obtain different result from a measurement of this quantity.
    • The speed of an object.

Terms: Frame of Reference

• A coordinate system and some way to fix stuff into that coordinate system.
  • A football field
  • This room
Terms: Inertial Frame of Reference

- A frame of reference in which the stuff in it has no forces acting on them.
- The stuff is at rest or it moves at a constant velocity in a straight line.
- Examples:
  - The car that brought you here this morning, moving steadily
  - A train car moving steadily

What are these symbols?

Physics
- Velocity: \( v \)
- Distance: \( d \)
- Time: \( t \)

Greek letters
- Beta: \( \beta \)
- Gamma: \( \gamma \)

Another term: Coordinate Transformations

A regular transformation:

\[
X = X_0 + vt,
\]

\[
t = t_0.
\]

We call this the "Galilean Transformation"

Example of a coordinate transformation

\[
\begin{align*}
X &= X_0 + Vt \\
Y &= Y_0 + Vt \\
\end{align*}
\]

Example:

Shooting a ball from a moving train car

\( \gamma_{\text{ball}} = -\gamma_{\text{train}} = -70 \text{ MPH} \)

Recap: Frame of reference

1905
Physics into 1905: Many successes!

• Newton's laws of mechanics are known and are very accurate
• The laws of electricity and magnetism were discovered and have been determined to be very accurate

But ..

There are a few loose ends to tidy up.

A big one is contained within Maxwell's Equations of Electricity and Magnetism

Electricity and magnetism: Maxwell’s Equations

Maxwell’s Equations, 1862

• James Clerk Maxwell, 1831-1879
• Form the foundation of classical electromagnetism, quantum field theory, classical optics, and electric circuits.

Maxwell’s Equations (for reference)

<table>
<thead>
<tr>
<th>Name</th>
<th>Differential equations</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gauss’s law</td>
<td>( \nabla \cdot \mathbf{E} = \rho )</td>
<td>The electric flux through a volume is proportional to the charge inside.</td>
</tr>
<tr>
<td>Gauss’s law for magnetism</td>
<td>( \nabla \cdot \mathbf{B} = 0 )</td>
<td>There are no magnetic monopoles: the total magnetic flux through a closed surface is zero.</td>
</tr>
<tr>
<td>Faraday-Maxwell equation</td>
<td>( \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} )</td>
<td>The change in the magnetic flux through a closed loop is proportional to the rate of change of the electric field inside the loop.</td>
</tr>
<tr>
<td>Ampère’s law (with Maxwell’s addition)</td>
<td>( \nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} )</td>
<td>The magnetic field generated around a closed loop is proportional to the electric current plus displacement current (rate of change of electric field) inside the loop.</td>
</tr>
</tbody>
</table>

Note that the speed of light, \( c \), is part of these equations

Maxwell’s Equations: Powerful and accurate

• Electricity and magnetism are the same thing
• Predict the existence of light waves.
  • Light is a form of electromagnetism
  • Light travels with a velocity of 309,792,458 meters per second ("the speed of light", \( c \))
  • Wow! That’s Fabulous!

But Maxwell’s Equations have some strange features

The speed of light is the same for all observers

The equations are not invariant under a Galilean coordinate transformation

The Speed of Light

• It seems like* speed is relative: \( x' = x + vt \)
  \[ f' = f \]
  \[ \lambda' = \lambda + \lambda \]
  \[ c^2 = c + \lambda' ?? \]

• Maxwell’s Equations say that the speed of light is invariant

* Intuition
What do Maxwell’s Equations say?

- Light travels at the same speed even if the observer is moving
- Velocities do not add up in the expected way
  \[ v' \neq v_x + v_y \]
- How’s that?
  - Mathematicians to the rescue

Hendrik Lorentz (1823-1928)

- Lorentz developed a coordinate transformation that kept Maxwell’s Equations invariant
  - By doing deep math on these complicated equations – amazing!
- Maxwell’s Equations are invariant under the Lorentz transformation.

Hendrik Lorentz (1823-1928)

Coordinate Transformations

Galilean transformation: Lorentz transformation:

\[
\begin{align*}
    x &= x_0 + vt \\
    t &= t_0
\end{align*}
\]

\[
\begin{align*}
    x &= \gamma (x_0 + vt) \\
    t &= \gamma \left( t_0 + \frac{vx}{c^2} \right)
\end{align*}
\]

\[
\gamma = \frac{1}{\sqrt{1-v^2/c^2}}
\]

Understanding of Lorentz Transformations in 1905

- Even at the speed of an orbiting GPS satellite, 14,000 KPH (3889 m/s), the gamma factor is almost irrelevanta: 
  \[
  \gamma = 1 + \frac{v^2}{c^2} \approx 1
  \]
- However, for speeds close to the speed of light, Lorentz transformation predicted weird things:
  - This seemed so radical that scientists were reluctant to accept it.
  - For example, what happens when \( v = c \)?
    1. Even worse: when \( v > c \)?
    2. Then there came along this patent clerk in Bern, Switzerland ...

1905: Einstein’s Extraordinary Year

- https://en.wikipedia.org/wiki/Annus_Mirabilis_papers
  - Annus Mirabilis = Extraordinary year
- Einstein published four spectacular papers, which derived explanations for:
  1. The Photoelectric Effect
  2. Brownian Motion
  3. Coordinate transformations in Maxwell’s Equations (Special Relativity)
  4. Mass-energy equivalence \( E=mc^2 \)

1905: Old Memes - Gone

- He introduced to humans:
  - Quantum Mechanics
  - Special Relativity
  - E=mc²

4 papers: More than a century of physics

- Each of these papers, individually, would have turned physics on its head
- All of them together from one person, in one year, is (to me) incomprehensible!
Genius

Einstein was one of the hardest-working scientists of the 20th Century.

Special Relativity

Physics: Velocity (v), Distance (d) and Time (t)

Hint:

\[ v = \frac{d}{t} \quad \text{(meters per second)} \]
\[ d = vt \quad \text{(meters)} \]
\[ t = \frac{d}{v} \quad \text{(seconds)} \]

Einstein stipulated

• The laws of nature are the same in all inertial frames of reference.
• Intuition
• The speed of light in the vacuum is the same in all inertial frames of reference.
• Recognized as a (crazy?) feature of Maxwell's Equations

What can we predict* from these two assumptions?

Let invent a new type of clock and see

The Light Clock

The clock ticks at a rate of \( \frac{h}{c} \) seconds

For the clock to tick at 1 second

\[ h = \frac{1}{2} tc \]
\[ h = \frac{1}{2} c \]
\[ c = 299,792,458 \quad \text{[m/s]} \]
\[ h = 149,896,229 \quad \text{[m]} \]
Remember, we assumed

The speed of light is the same for all observers

\[ \beta \equiv \frac{v}{c} \]

"The Greek letter beta is defined to be \( v \) over \( c \)"

\[ \gamma \equiv \frac{1}{\sqrt{1 - \beta^2}} \]

"The Greek letter gamma is defined to be one over the square-root of one minus beta squared"

From the perspective of the Light Clock

From the perspective of the Enterprise

How long does it take for the initial "Tock"?

\[ t = \frac{\sqrt{h^2 + x^2}}{c} \]

\[ \Rightarrow x = vt \]

\[ t = \frac{\sqrt{h^2 + (vt)^2}}{c} \]

A little algebra ...

\[ t = \frac{\sqrt{h^2 + (vt)^2}}{c} \]

\[ t = \sqrt{c^2 (1 - \beta^2)} = \frac{1}{c} \]  

\[ t = \frac{h^2}{c^2 (1 - \beta^2)} \]

\[ t = \frac{h^2}{c^2 (1 - \beta^2)} \]

A little more algebra ...

\[ t = \frac{\sqrt{h^2 + (vt)^2}}{c} \]

\[ t = \frac{h^2}{c^2 (1 - \beta^2)} \]

Lorentz Transformation of time!

The time for the light to travel this path is longer to the Enterprise observer than it is to the observer sitting on the clock!

The time is "dilated"
Once again:
\[ t = \frac{t_0}{\sqrt{1 - \beta^2}} \]
\[ t = \gamma t_0 \]

Dilate
\( \nu \text{tr.} \)
To make wider or larger; cause to expand.
\( \nu \text{intr.} \)
1. To become wider or larger; expand.
2. To speak or write at great length on a subject; expatiate.

In other words...
For the observer on the Enterprise and the observer on the mirrors to see the light to go between the mirrors the same way, ...

The passage of time on the Enterprise has to be slower than the passage of time on the mirrors!

**Recap: Time dilation**

\[ t = \gamma t_0 \]

The time observed on the moving clock is longer than the observer’s clock.

\[ t_0 = t / \gamma \]

The time on the observer’s clock is shorter than the moving clock.

**How long is one second, as seen from the Earth, on the Enterprise for \( v = 0.25c \)?**

- A. 0.9682
- B. 3.1416
- C. 0.7071
- D. 1.0328
- E. 1.1412

**Light Clock also leads to length contraction**

\[ d = \frac{d_0}{\gamma} \]

The time for the light to make one round trip as seen while sitting on the clock

\[ t_0 = \frac{2d_0}{c} \]

The time for the light to go from left to right, as seen from NCC-1701D

\[ t_1 = \frac{d}{c} \left( \frac{1 + \frac{v}{c}}{1 - \frac{v}{c}} \right) \]

The time for the light to go from right to left, as seen from NCC-1701D

\[ t_2 = \frac{d}{c} \left( \frac{1 - \frac{v}{c}}{1 + \frac{v}{c}} \right) \]
The total time for the light to go from left to right, and back, as seen from NCC-1701D

\[ t = t_1 + t_2 \]

\[ t = \frac{d}{c+v} + \frac{d}{c-v} \]

We'll use this:

\[
\left( \frac{1}{a-h} + \frac{1}{a+b} \right) = \frac{a+b}{(a-b)(a+b)} - \frac{a-b}{(a-b)(a+h)} = \frac{2a}{(a-b)(a+h)} = \frac{2a}{a^2-b^2}
\]

Special Relativity Summary

- Time Dilation: \[ t = \gamma t_0 \]
- Length Contraction: \[ d = \frac{1}{\gamma} d_0 \]

A nice velocity \[ v = 0.866 \ c \]

\[ \beta = \frac{v}{c} = 0.866 \]

\[ \gamma = 1/\sqrt{1-(0.866)^2} = 1/\sqrt{1-0.749956} = 1/\sqrt{0.75} = 0.5 \]

\[ \gamma = 2 \]

Simultaneity

- Noun form of "simultaneous"
- Two events happen simultaneously when you see/observe that they have occurred at the same time.
- The fact that the speed of light is the same to all observers means that our intuitive understanding of simultaneity needs to be extended.

Let's see why.

A thought experiment: Stationary train car

A light pulse hits the ends simultaneously

A thought experiment: Moving train car

The light pulses hit the ends simultaneously
The light hits the back of the train car first

Simultaneity is different (!!!)
- Because the speed of light is the same for all observers...
- The observer on the train sees the light hit the ends of the train simultaneously
- The observer on the ground sees the light hit the ends of the train at different times.

Scientists tried to disprove counter-intuitive aspects of Special Relativity

Physicist’s view of a pole vaulter

Physicist’s view of a barn

A pole vaulter and a barn

Let the speed of vaulter be $0.866c$ ($\gamma = 2$)

Barn, with doors shut, fully contains vaulter

And the vaulter keeps running, pole in tact
From the point of view of the vaulter

30 meters

2.5 meters

v = 0.866c

A Paradox?

Either the pole gets smashed by the doors or it does not!
It cannot be both whole and broken.
Resolving this is a little tricky.

What causes the doors to close?

causality

definition noun
The relationship between cause and effect.

causal adjective
Relating to or acting as a cause.

Causality can travel no faster than the speed of light

So, let’s use a light-based signal to cause the doors to close

What causes the doors to close?

The signal propagates to the doors ...

... And causes doors to close

And vaulter continues with an unbroken pole
From the point of view of the vaulter

The front door closes (quickly)

The front door opens

Back door closes when the signal reaches it

And vaulter continues with an unbroken pole

The paradox is resolved through careful application of cause-and-effect in Special Relativity

More on this apparent paradox ...

- https://en.wikipedia.org/wiki/Ladder_paradox
- YouTube: "Relativity 5b - pole and barn paradox", by viascience.
- https://www.youtube.com/watch?v=UTI1yK7Q44

At first, this seemed counter-intuitive

We have improved our intuition

Equivalence of mass and energy
Special Relativity: \( E = mc^2 \)

- It is possible to derive this formula using algebra
  - Energy = force \times distance
  - Force = the change in the momentum of an object
  - Lorentz transformations of time (that is, the velocity)
- But it is easier (and cleaner) to use calculus

Energy \( E = mc^2 \)

Derived using only math (mostly algebra) and the two conjectures:
1. The laws of nature are the same in all inertial frames of reference.
2. The speed of light in the vacuum is the same in all inertial frames of reference.

An aside: Our nomenclature at Fermilab

- Physicists at Fermilab set the speed of light equal to one
  \( E = m \)
- This is only a change in units
- Emphasizing even more clearly that energy is the same thing as mass

What does this mean? \( E = mc^2 \)

- What is the energy of 1 gram of matter?
  \[ E = 0.001 \text{ kg} \times (2.998 \times 10^8 \text{ m/s})^2 \]
  \[ E = 1 \times 10^5 \times 8.988004 \times 10^{16} \text{ [Joules]} \]
  \[ E = 8.988004 \times 10^{13} \text{ [Joules]} \]
  \[ E = 90 \text{ Terajoules} \]
  - Burning 691,538 gallons of gasoline
  - Exploding 42,964,554 pounds (21.5 kilo-tons (kT)) of TNT

First atomic bomb: Hiroshima (USA, fission)

- Hiroshima bomb: 63 Terajoules \( [6.3 \times 10^{13} \text{ Joules}] \) \( [15 \text{ kT}] \)
- Equivalent to about 0.7 grams of matter converted to energy

Largest bomb: Tsar Bomba (USSR, fusion)

- Tsar Bomba: 240 Petajoules \( [2.4 \times 10^{17} \text{ Joules}] \) \( [57 \text{ MT}] \)
  - 3800 times larger than Hiroshima bomb
  - Converted about 2.7 kg of matter into energy

How do we know Special Relativity is accurate?

- No Lorentz violations could be measured thus far, and exceptions in which positive results were reported have been refuted or lack further confirmations.
Everyday uses of Special Relativity
• Global Positioning System (GPS)
• Muon Decay

GPS is based on very accurate clocks
• GPS clock ticks must be known to an accuracy of 20-30 nanoseconds
• ~5 meters on the road.
• GPS satellites are constantly moving relative to Earthly observers
• These clocks move a little more slowly
• Special Relativity predictions ... next slide
• GPS also gets substantial corrections from General Relativity

GPS: How big an effect is satellite motion?
Speed of a GPS satellite is 14,000 KPH (3889 m/s)
\[ \beta = \frac{v}{c} \]
\[ \gamma = \frac{1}{\sqrt{1 - \beta^2}} \]
\[ t = \gamma t' \]

An aside – GPS and General relativity
• The satellite is in micro-gravity
• The receiver is in Earth gravity
• General Relativity
  • Earth clock runs 38 microsec slower per day
  • 5X bigger effect than Special Relativity
  • About a mile of error per day

Muon Decay
• 1941 experiment
• Now a common experiment in a graduate physics laboratory

Special Relativity includes Newton and E&M
• Special Relativity extends Newtonian Mechanics to close-to-c speeds, but it does not contradict it
• It includes all of Newtonian Mechanics in its entirety.
Spacetime

Causality: The fastest a signal can travel is \( c \)

Units of mass and energy

- We use “Electron Volts” as an energy
- \( 1.6 \times 10^{-19} \text{ joules} \)
- Since mass=energy, we also say that a particle has a mass in “electron volts”

\[
\begin{align*}
m_e &= 510999 \text{ [eV]} \\
&= 511 \text{ [keV]} \\
m_p &= 938272081 \text{ [eV]} \\
&= 938 \text{ [MeV]} \\
\end{align*}
\]
Converting between energy and mass
• When two particles collide at high energy, the result has a lot of energy
• Often (usually) this energy becomes mass.

Intuition ... reprise
• How do we know what we know?
  • Intuition
  • Deduction
  • Observation
• This talk:
  • Intuition can be incomplete
  • "Counter intuitive"
  • Deduction and observation can be amazing

In other words, Special Relativity is
A triumph of the Scientific Method

Special Relativity: Conclusions
• Time dilates
• Length contracts
• Simultaneity is relative
• Matter and energy are the same thing
• It agrees, impressively, with experiments
• Our old intuition must change
• Maxwell’s Equations are correct
  • And they always have been
• Einstein was an insightful, hard working scientist

Conclusions

Relativity does not end here
• General relativity
  • Einstein’s life’s work
  • Space is warped by mass
  • Mass moves according to how space is warped
  • Mind-blowing predictions:
    • Precession of the orbit of Mercury
    • Light is bent by a gravity field
    • Black holes
    • Expansion of the universe
    • Gravity waves

That’s all, folks!

Feedback
http://tinyurl.com/SMPFeedback-Fall17
The other 1905 Einstein papers were also profound.

Brownian Motion

- A tiny, visible object (suspended in water) will move about randomly.
- Could the pollen grains be alive?
- Nope. Shown to happen with lab-created inanimate stuff.

Brownian Motion: Einstein

- Einstein solved the problem, assuming "molecules" are what is pushing the object around.
- His calculations revealed the size of these molecules:
  - Avogadro’s Number

The Photoelectric Effect

- Covered last week by Professor Hooper.
- Emission of electrons when light shines onto a material... sometimes.
- This phenomenon was discovered by Hertz and Hallewachs in 1887.

Interpretation based on Maxwell’s Equations

- Light is a wave
- Brighter light → More energetic electrons
- Brighter light → More prompt emission of electrons
  - Dim light: electrons would be released slowly and at low energy.

Observations

- In 1902, Lenard observed that the energy of individual emitted electrons increased with the color of the light, but not the intensity.
- Furthermore, when the color was blue enough, electrons would be emitted instantly.
**Max Planck**

- Studied the radiation emitted from a glowing body
  - "Black Body Radiation"
  - (Radiation: energy from light)

**How to fit this frequency curve?**

- Assume that electromagnetic energy could be emitted only in quantized form
- He introduced a new constant, \( h \):
  - \( h \) is Planck’s constant.
  - \( 6.62607 \times 10^{-34} \) joule seconds
  - \( \nu \) is the frequency of the radiation.
  - Scientists appreciated the novelty and correctness (mathematically) of this assumption, but they did not (particularly) like what this meant!

**Max Planck and Einstein**

- Einstein showed that Planck’s assertion, that light is made up of particles, would resolve the photoelectric effect
- Light comes in quanta, and they behave like particles in the Photoelectric effect
- These particles (now called "photons") hit electrons in the atoms of the metal and can only kick them out if the energy of the photos is high enough

**Wave/particle duality**

- The photoelectric effect helped to propel the then-emerging concept of wave–particle duality in the nature of light.
- Light simultaneously possesses the characteristics of both waves and particles, each being manifested according to the circumstances.
- It was soon realized that particles with mass also have a wavelength

**Full equation for total energy**

\[
E = \sqrt{p^2 c^2 + m^2 c^4} - mc^2
\]

- In other words...
- Anything with energy has momentum
- A photon has momentum

**An aside: No new physics in 20th Century?**

- One school of thought:
  - Classical mechanics could cope with highly complex problems involving macroscopic situations
  - Thermodynamics and kinetic theory were well established
  - Every branch of classical physics was already built well into the framework of mathematical analyses
  - It was generally accepted that all the important laws of physics had been discovered
  - Thermodynamics, for example, had been extended to cover problems and particularly with improvements of statistical methods

- However, A. A. Michelson’s in his 1889 lectures on light interferometry (Foucault):

- What conclusion can be drawn from interferometry in the science of measurement?
- What can be done with the development of modern means of measurement?
- The modern development of physics based on the concept of mechanics is a development of great scientific importance
- The work of physicists, including Michelson and others, is a triumph of the basic principles of physics
- Michelson's work focused on the development of tools and techniques for interferometry
- The work of early 20th century physicists in developing quantum mechanics was a response to the limitations of classical mechanics and the need for a new theoretical framework

**Some perspective on Annus Mirabilis**

- Worked as an examiner at the Patent Office in Bern, Switzerland
- Limited access to scientific reference materials,
- Co-worker: "[Einstein] could not have found a better sounding board for his ideas in all of Europe"
- Einstein tackles some of the era’s most important physics questions
- These papers solved the primary scientific problems of the era:
  - Lord Kelvin lecture titled "Nineteenth-Century Clouds over the Dynamical Theory of Heat and Light"
  - Michelson-Morley experiment
  - Black body radiation
  - Coordinate transformation of Maxwell's Equations.