Albert Einstein and the Modern Physics Revolution

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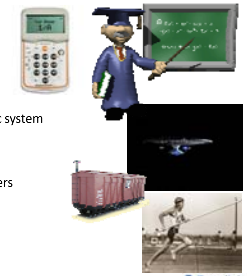
Saturday Morning Physics
Lecture number 2
October 7, 2017

Concepts for today

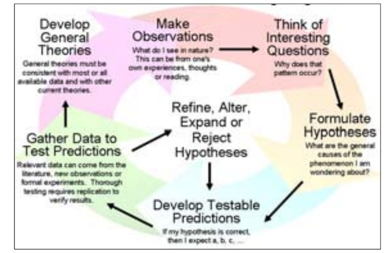
- Intuition
- Transformations
- **Special Relativity**
- Simultaneity
- If time allows
  - Mass equals energy
  - Spacetime

- Utilizing
  - Clickers
  - The metric system
  - Algebra
  - Star Trek
  - Trains
  - Pole vaulters



Prof. Dan Hooper
September 30, 2017

But what is "Science"?



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Are There Alternatives to the Scientific Method?

Reliance on tradition or authority

- This is something of a straw man opponent to science; few would argue that science conducted sufficiently fairly and carefully will often lead to conclusions that are likely to be false
- Many instances of reliance on tradition are actually a weak form of reliance on social science – if many people held position X in the past, then this provides a limited degree of empirical evidence that holding position X is likely to be helpful or advantageous

Reliance on pure reasoning (mathematics, philosophy)

- Many people think of mathematics as a part of science, but it is fundamentally not grounded in empiricism (a central part of the scientific method)
- Although philosophers of science hold a range of opinions on this issue, my view is that math helps to illuminate the relationships between ideas and can help to clarify our thinking, but does not itself tell us anything about our world

We are guided by our intuition

- Must be careful!
  - Ptolemeic view of the "universe" based on beliefs and on intuition
- Experience, observation and intuition are linked
- It can only takes you so far
- Intuition grows as we gain experience and make observations

What is Intuition?

(noun)

- The ability to understand something immediately, without the need for conscious reasoning.

"We shall allow our intuition to guide us"

- a thing that one knows or considers likely from instinctive feeling rather than conscious reasoning.
  "Your insights and intuitions as a native speaker are positively sought"

synonyms: instinct, hunch, feeling (in one's bones), inkling, (sneaking) suspicion, idea, sense, notion;

Modern Physics: Our Intuition is incomplete

Much is, at first, counter-intuitive

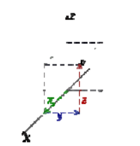
We must develop *new* intuition

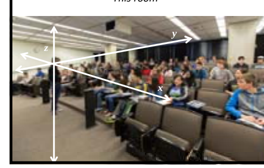
Terminology: A quantity is said to be

- **Invariant**
  - If different observers would obtain the same result from a measurement of this quantity.
    - *The mass of an object.*
- **Relative**
  - If different observers would obtain different result from a measurement of this quantity.
    - *The speed of an object.*

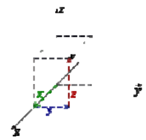
Terms: Frame of Reference

- A coordinate system and some way to fix stuff into that coordinate system.
  - *A football field*
  - *This room*





Terms: Inertial Frame of Reference

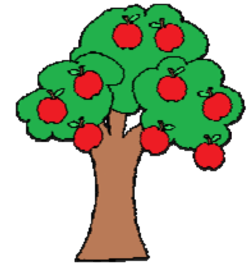


- A frame of reference in which the stuff in it has no forces acting on them.
  - The stuff is at rest or it moves at a constant velocity in a straight line.
  - *Examples:*
    - *The car that brought you here this morning, moving steadily*
    - *A train car moving steadily*
    - *The bridge of the Enterprise (maybe?)*

What are these symbols?

Physics	Greek letters
Velocity : v	Beta : β
Distance : d	Gamma : γ
Time : t	

Frame of reference

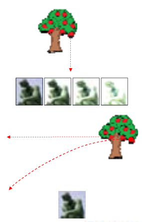


Another term: Coordinate Transformations

A regular transformation:

$$x = x_0 + vt$$
$$t = t_0$$

We call this the "Galilean Transformation"



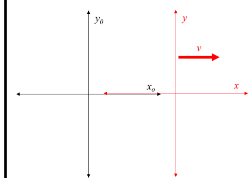
Galileo Galilei: 1564-1642

- Italian astronomer
- Physicist
- Engineer
- philosopher
- mathematician.

He has been called
- "father of observational astronomy",
- "father of modern physics",
- "father of the scientific method", and
- "father of science".

Galileo studied speed and velocity, gravity and free fall, **the principle of relativity**, inertia, projectile motion and also worked in applied science and technology, describing the properties of pendulums and "hydrostatic balances", inventing the thermoscope and various military compasses, and using the telescope for scientific observations of celestial objects.

Example of a coordinate transformation



$v = 5$ [m/s]
$t_0 = 10$ [sec]
$x_0 = 0$

What are x and t?

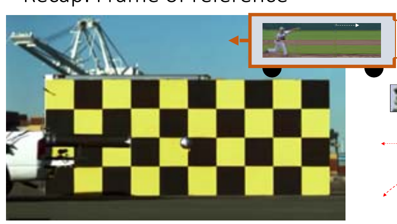
$$x = x_0 + vt$$
$$t = t_0$$

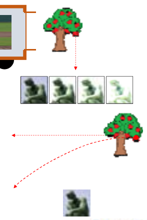
Coordinate Transformation Example: Shooting a ball from a moving train car

$v_{ball} = -v_{train} = 70$ MPH

What is the velocity of the ball when it emerges from the train?

Recap: Frame of reference



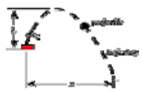


The Year

1905

Physics into 1905: Many successes!

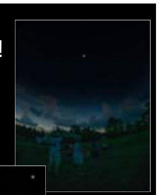
- Newton's laws of mechanics are known and are very accurate



- The laws of electricity and magnetism were discovered and have been determined to be very accurate

Physics into 1905: Many successes!

- Newton's laws of gravity are very accurate
  - Predicts orbits of the planets and their moons
  - Predicts appearance of comets
  - Predicts eclipses



But ..

There are a few loose ends to tidy up.

A big one is contained within
Maxwell's Equations of Electricity and Magnetism

Electricity and magnetism:

Maxwell's Equations

Maxwell's Equations, 1862

- James Clerk Maxwell, 1831-1879
- Form the foundation of classical electromagnetism, quantum field theory, classical optics, and electric circuits.

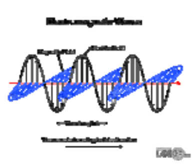
Maxwell's Equations *(for reference)*

Name	Differential equations	Meaning
Gauss's law	$\nabla \cdot \mathbf{E} = 4\pi\rho$	The electric flux leaving a volume is proportional to the charge inside.
Gauss's law for magnetism	$\nabla \cdot \mathbf{B} = 0$	There are no magnetic monopoles; the total magnetic flux through a closed surface is zero.
Maxwell–Faraday equation (Faraday's law of induction)	$\nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}$	The voltage induced in a closed loop is proportional to the rate of change of the magnetic flux that the loop encloses.
Ampère's circuital law (with Maxwell's addition)	$\nabla \times \mathbf{B} = \frac{1}{c}\left(4\pi\mathbf{J} + \frac{\partial \mathbf{E}}{\partial t}\right)$	The magnetic field integrated around a closed loop is proportional to the electric current plus displacement current (rate of change of electric field) that the loop encloses.

Note that the speed of light, c, is part of these equations

Maxwell's Equations: Powerful and accurate

- Electricity and magnetism are <u>the same thing</u>
- Predict the existence of light waves.
  - Light is a form of electromagnetism
  - Light travels with a velocity of 299,792,458 meters per second *("the speed of light", c)*
  - Wow! That's Fabulous!



But Maxwell's Equations have some strange features

The speed of light is the same for all observers

The equations are not invariant under a
Galilean coordinate transformation

The Speed of Light

- It seems like* speed is relative:

$$x' = x + vt$$
$$t' = t$$
$$v' = v_0 + v$$
$$c' \stackrel{?}{=} c + v\,??$$

- Maxwell's Equations say that the speed of light is invariant

** Intuition*

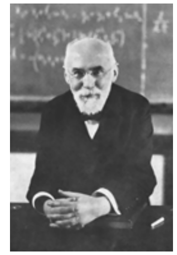
What *do* Maxwell's Equations say?

- Light travels at the same speed even if the observer is moving
- Velocities do not add up in the expected way

$$v' \neq v_0 + v$$

- How's that?
  - Mathematicians to the rescue

Hendrik Lorentz (1823-1928)



- Lorentz developed a *coordinate transformation* that kept Maxwell's Equations invariant
  - By doing deep math on these complicated equations – amazing!
- *Maxwell's Equations are invariant under the Lorentz transformation.*

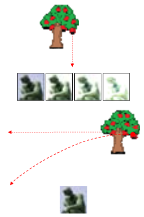
Coordinate Transformations

Galilean transformation:

$$x = x_0 + vt$$
$$t = t_0$$

Lorentz transformation:

$$x = \gamma(x_0 + vt)$$
$$t = \gamma\left(t_0 + \frac{vx}{c^2}\right)$$
$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$



Understanding of Lorentz Transformations in 1905

- Even at the speed of an orbiting GPS satellite, 14,000 KPH (3889 m/s), the gamma factor is almost irrelevant:

$$\gamma = 1/\sqrt{1-(v/c)^2}$$
$$= 1/\sqrt{1-(3889/299{,}792{,}458)^2}$$
$$= 1/\sqrt{1-0.00000000017}$$
$$\gamma = 1.000000000084$$

$$x = \gamma(x_0 + vt)$$
$$t = \gamma\left(t_0 + \frac{vx}{c^2}\right)$$
$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

* Intuition

Understanding of Lorentz Transformations in 1905

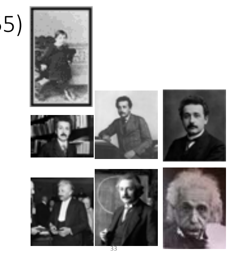
- However, for speeds close to the speed of light, Lorentz transformation predicted weird things.
  - This seemed* so radical that scientist were reluctant to accept it
  - For example, what happens when $v = c$?
  - Or worse: when $v > c$??
- Then there came along this patent clerk in Bern, Switzerland …

$$x = \gamma(x_0 + vt)$$
$$t = \gamma\left(t_0 + \frac{vx}{c^2}\right)$$
$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

* Intuition

Albert Einstein (1879 – 1955)

- At age 4 (1883)
- At 23 (1902): Patent clerk
- At 26 (1905): Published 4 papers that changed everything
- At 29 (1908): Professor at U Bern
- At 36 (1915): Published The General Theory of Relativity
- At 42 (1921): Nobel Prize
- At 54 (1933): Emigrated to America
- At 76 (1955): Completes *The Meaning of Relativity*, 5th edition

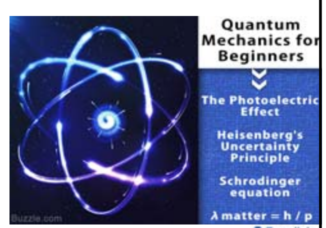


1905: Einstein's Extraordinary Year

- https://en.wikipedia.org/wiki/Annus_Mirabilis_papers
  - *Annus Mirabilis* = Extraordinary year
- Einstein published four spectacular papers, which derived explanations for:
  1. The Photoelectric Effect
  2. Brownian Motion
  3. Coordinate transformations in Maxwell's Equations (Special Relativity) ☆
  4. Mass-energy equivalence ☆

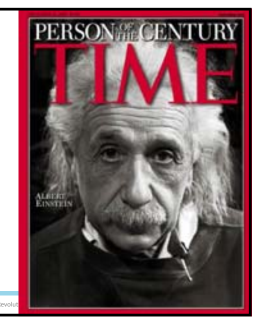
1905: Old Memes - Gone

- He introduced to humans:
  - Quantum Mechanics
  - Special Relativity
  - E=mc²



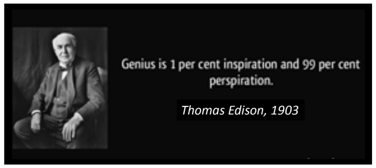
4 papers: More than a century of physics

- Each of these papers, individually, would have turned physics on its head
- All of them together from one person, in one year, is (to me) incomprehensible!



Genius

- Einstein was one of the hardest-working scientists of the 20th Century



Physics: Velocity (v), Distance (d) and Time (t)

Hint: *Use the units*

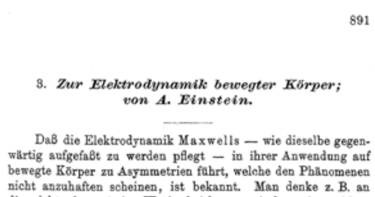
$v = d / t$	[meters per second] = [meters] ÷ [second]
$d = vt$	[m] = [m/s] × [s]
$t = d / v$	[s] = [m] ÷ [m/s]

And now …

Special Relativity

Special Relativity

"On the Electrodynamics of Moving Bodies"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3. *Zur Elektrodynamik bewegter Körper;*
von A. Einstein.

Daß die Elektrodynamik Maxwells — wie dieselbe gegenwärtig aufgefaßt zu werden pflegt — in ihrer Anwendung auf bewegte Körper zu Asymmetrien führt, welche den Phänomenen nicht anzuhaften scheinen, ist bekannt. Man denke z. B. an

Einstein stipulated

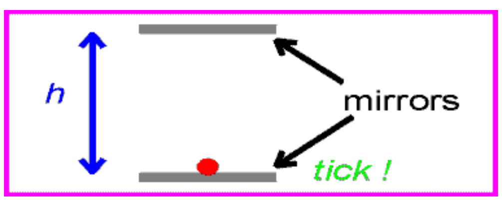
- The laws of nature are the same in all inertial frames of reference.
  - *Intuition*
- The speed of light in the vacuum is the same in all inertial frames of reference.
  - *Recognized as a (crazy?) feature of Maxwell's Equations*

What can we predict* from these two assumptions?

Let invent a new type of clock and see

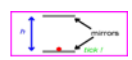
* Scientific method

The Light Clock



Time = distance / velocity

$$t = 2h / v$$
$$t = 2h / c$$
$$h = \tfrac{1}{2} tc$$

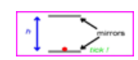


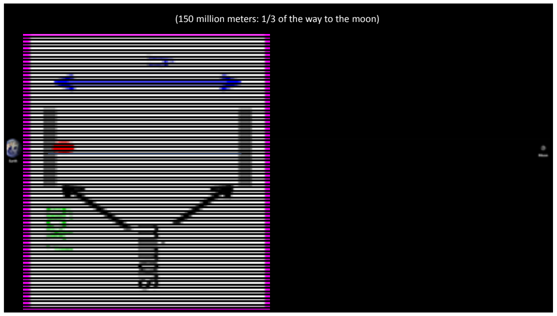
The clock ticks at a rate of $2h/c$ seconds

For a clock that ticks every t seconds, the mirrors are ½ tc apart

For the clock to tick at 1 second

$$h = \tfrac{1}{2} tc$$
$$h = \tfrac{1}{2} c$$
$$\rightarrow c = 299{,}792{,}458 \ [m/s]$$
$$h = 149{,}896{,}229 \ [m]$$





Remember, we assumed

The speed of light
is the same for all observers

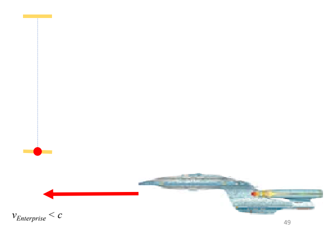
$$\beta \equiv v / c$$

"The Greek letter beta is defined to be v over c"

$$\gamma \equiv \frac{1}{\sqrt{1-\beta^2}}$$

"The Greek letter gamma is defined to be one over the square-root of one minus beta squared"

From the perspective of the Light Clock



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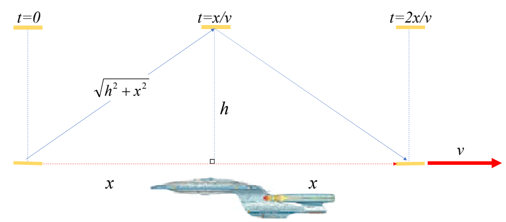
From the perspective of the Enterprise



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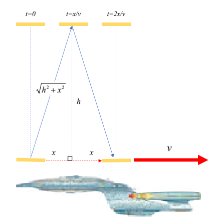
How long does it take for the initial "Tock"?



$$t = \frac{\sqrt{h^2+x^2}}{c}$$

$$\Rightarrow x = vt$$

$$t = \frac{\sqrt{h^2+(vt)^2}}{c}$$

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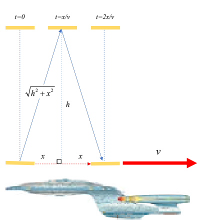
A little algebra …



$$t = \frac{\sqrt{h^2+(vt)^2}}{c}$$

$$tc = \sqrt{h^2+(vt)^2}$$

$$t^2c^2 = h^2+(vt)^2$$

$$t^2c^2 = h^2+v^2t^2$$

$$t^2c^2 - v^2t^2 = h^2$$

$$t^2\left(c^2-v^2\right) = h^2$$

$$h^2 = t^2\left(c^2-v^2\right) \ldots$$

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A little more algebra …

$$t = \frac{\sqrt{h^2+(vt)^2}}{c}$$

…

$$h^2 = t^2\left(c^2-v^2\right)$$

$$\beta \equiv \frac{v}{c}$$

$$h^2 = t^2c^2\left(1-\left(\frac{v}{c}\right)^2\right)$$

$$h^2 = t^2c^2\left(1-\beta^2\right)$$

$$h^2\frac{1}{c^2\left(1-\beta^2\right)} = t^2$$

$$t^2 = \frac{h^2}{c^2}\frac{1}{1-\beta^2}$$

$$t = \frac{h}{c}\Big/\sqrt{1-\beta^2}$$

$$t = t_0 / \sqrt{1-\beta^2}$$

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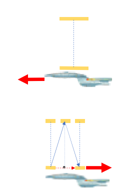
Lorentz Transformation of time!



The time for the light
to travel this path
is longer to the Enterprise observer
than it is
to the observer sitting on the clock!

The time is "**dilated**"

7 October 2017 McCrory: Einstein's 1905 Revolution Fermilab 54

Once again:

$$t = \frac{t_0}{\sqrt{1-\beta^2}}$$

$$t = \gamma \; t_0$$

Dilate

v.tr.
To make wider or larger; cause to expand.
v.intr.
1. To become wider or larger; expand.
2. To speak or write at great length on a subject ; expatiate.

In other words …

For the observer on the Enterprise and the observer on the mirrors to see the light to go between the mirrors the same way, …

The passage of time on the Enterprise has to be slower than the passage of time on the mirrors!

Recap: Time dilation

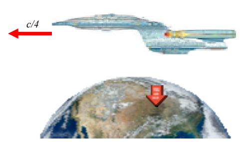
$$t = \gamma \; t_0$$

The time observed on the moving clock is longer than the observer's clock

$$t_0 = t / \gamma$$

The time on the observer's clock is shorter than the moving clock

How long is one second, as seen from the Earth, on the Enterprise for $v = 0.25c$? *



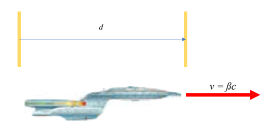
A. 0.9682
B. 3.1416
C. 0.7071
D. 1.0328
E. 1.1412

$$t_E = \gamma \; t_0$$

$$t_E = t_0 / \sqrt{1-\beta^2}$$

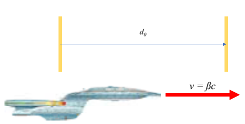
* ST:TNG Technical Manual: "High relativistic speeds are to be avoided unless absolutely necessary; impulse power should be limited to a maximum of ¼ lightspeed"

Light Clock also leads to length contraction



$$d = d_0 / \gamma$$

The time for the light to make one round trip as seen while sitting on the clock



t_0 : Time as seen on the clock
d_0 : Distance as seen on the clock
t : Time as seen on the Enterprise
d : Distance as seen on the Enterprise

$$t_0 = 2d_0 / c$$

$$t_1 = \frac{d}{c} - \frac{vt_1}{c}$$

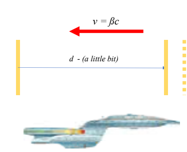
$$t_1\left(1+\frac{v}{c}\right) = \frac{d}{c}$$

$$t_1 = \frac{d/c}{1+v/c}$$

$$t_1 = \frac{d}{c(1+v/c)}$$

$$t_1 = \frac{d}{c+v}$$

The time for the light to go from left to right, as seen from NCC-1701D



$$t_2 = \frac{d}{c} + \frac{v}{c}t_2$$

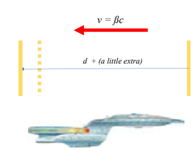
$$t_2\left(1-\frac{v}{c}\right) = \frac{d}{c}$$

$$t_2 = \frac{d/c}{1-v/c}$$

$$= \frac{d}{c(1-v/c)}$$

$$t_2 = \frac{d}{c-v}$$

The time for the light to go from right to left, as seen from NCC-1701D



The total time for the light to go from left to right, and back, as seen from NCC-1701D

$$t = t_1 + t_2$$

$$t = \frac{d}{c+v} + \frac{d}{c-v}$$

We'll use this:

$$\left(\frac{1}{a-b} + \frac{1}{a+b}\right) = \left(\frac{a+b}{(a-b)(a+b)} + \frac{a-b}{(a-b)(a+b)}\right) = \left(\frac{2a}{(a-b)(a+b)}\right) = \frac{2a}{a^2-b^2}$$

$$\begin{aligned} t &= \frac{d}{c-v} + \frac{d}{c+v} \\ &= d\left(\frac{2c}{c^2-v^2}\right) \\ &= \frac{2cd}{c^2\left(1-\frac{v^2}{c^2}\right)} \\ &= \frac{2d}{c}\left(\frac{1}{(1-\beta^2)}\right) \\ &= \frac{2d}{c}\gamma^2 \\ \frac{ct}{2\gamma^2} &= d \\ d &= \frac{ct}{2\gamma^2} \end{aligned}$$

$$\begin{aligned} d &= \frac{tc}{2\gamma^2} \\ &\Rightarrow t_0 = \gamma\ t \\ &\Rightarrow t = \frac{t_0}{\gamma} \\ &= \frac{t}{2\gamma}\frac{c}{\gamma} \\ &= \frac{t_0}{2}\frac{c}{\gamma} \\ d &= (t_0 c)/(2\gamma) \\ &\Rightarrow d_0 = t_0 c/2 \\ d &= \frac{d_0}{\gamma} \end{aligned}$$

Special Relativity Summary

$t = \gamma\ t_0$ Time Dilation

$d = \frac{1}{\gamma} d_0$ Length Contraction

With,

$\gamma = 1/\sqrt{1-\beta^2}$

$\beta = v/c$

A nice velocity

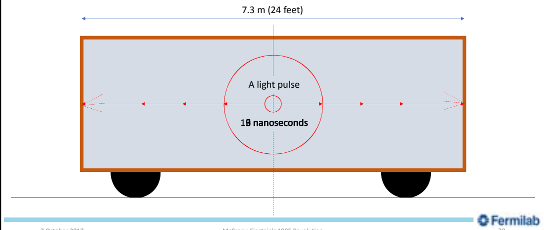
$$\begin{aligned} v &= 0.866\ c \\ \beta &= 0.866 \\ \gamma &= 1/\sqrt{1-(0.866)^2} \\ &= 1/\sqrt{1-.749956} \\ &\approx 1/\sqrt{1-0.75} \\ &= 1/0.5 \\ &= 2 \end{aligned}$$

Questions?

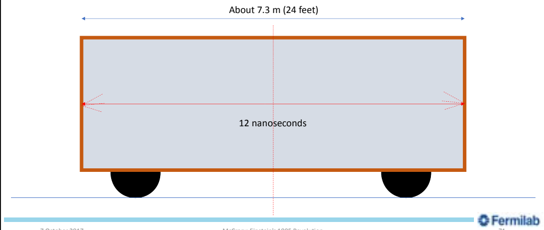
Simultaneity

- Noun form of "simultaneous"
- Two events happen simultaneously when you see/observe that they have occurred at the same time.
- The fact that the speed of light is the same to all observers means that our intuitive understanding of simultaneity needs to be extended.
  - Let's see why

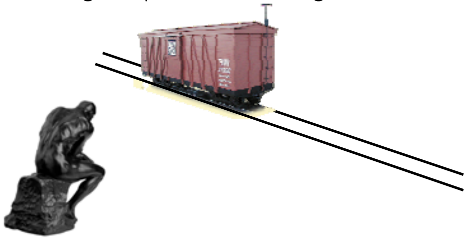
A thought experiment: Stationary train car



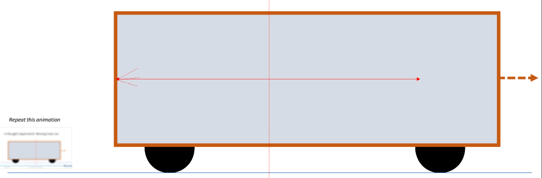
The light pulses hit the ends simultaneously



A thought experiment: Moving train car

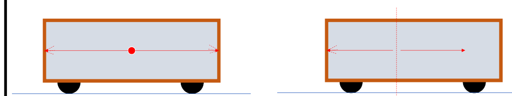


The light hits the back of the train car first



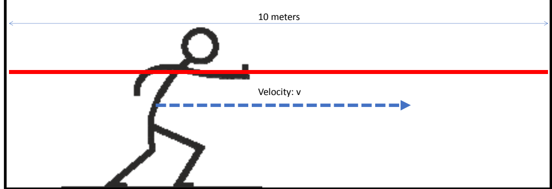
Simultaneity is different (!!!)

- Because the speed of light is the same for all observers …
- The observer **on the train** sees the light hit the ends of the train simultaneously
- The observer **on the ground** sees the light hit the ends of the train at different times.

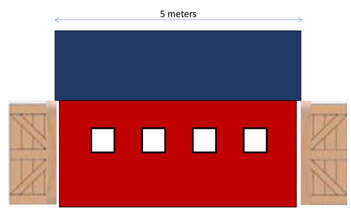


Scientists tried to disprove counter-intuitive aspects of Special Relativity

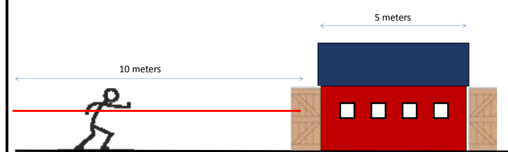
Physicist's view of a pole vaulter



Physicist's view of a barn

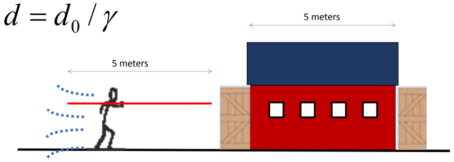


A pole vaulter and a barn

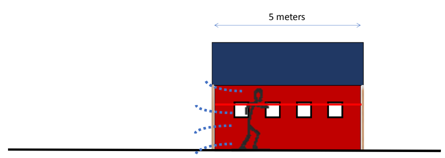


Let the speed of vaulter be 0.866c (γ = 2)

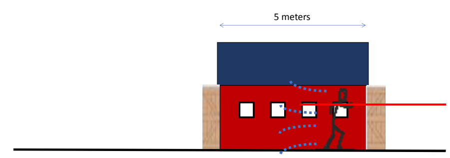
$$d = d_0 / \gamma$$



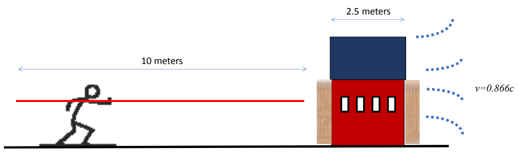
Barn, with doors shut, fully contains vaulter



And the vaulter keeps running, pole in tact



From the point of view of the vaulter



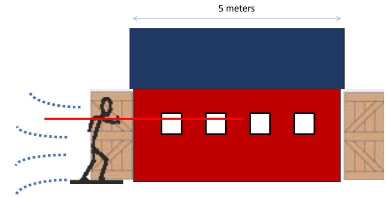
A Paradox?

Either the pole gets smashed by the doors or it does not!
It cannot be both whole and broken.

Resolving this is a little tricky.

Questions?

What causes the doors to close?



causality
noun

The relationship between cause and effect.

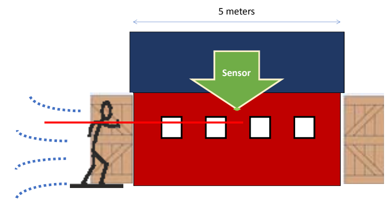
causal
adjective

Relating to or acting as a cause.

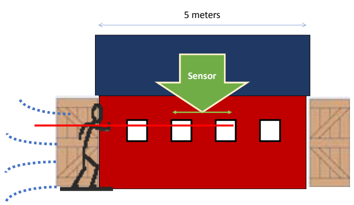
Causality can travel no faster than the speed of light

So, let's use a light-based signal to cause the doors to close

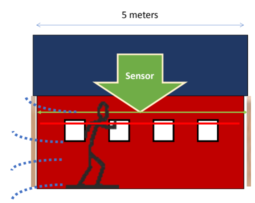
What causes the doors to close?



The signal propagates to the doors …

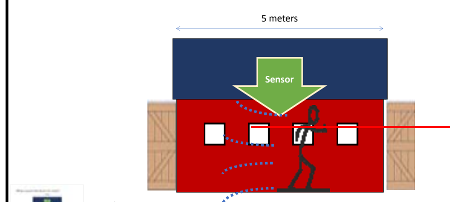


… And causes doors to close



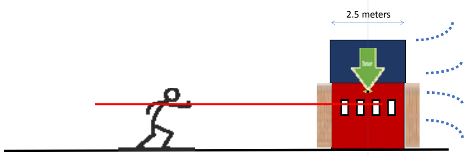
Just like the train car, previously!

And vaulter continues with an unbroken pole

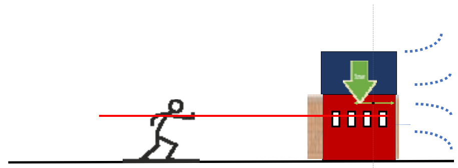


Questions?

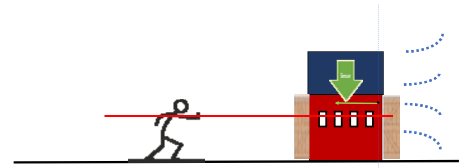
From the point of view of the vaulter



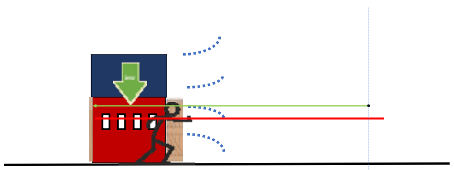
The front door closes (quickly)



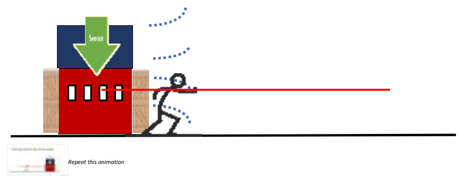
The front door opens



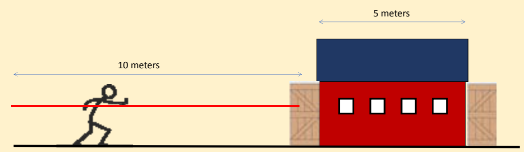
Back door closes when the signal reaches it



And vaulter continues with an unbroken pole



The paradox is resolved through careful application of cause-and-effect in Special Relativity



More on this apparent paradox …

- https://en.wikipedia.org/wiki/Ladder_paradox
- YouTube: "Relativity 5b - pole and barn paradox", by viascience.
  - https://www.youtube.com/watch?v=0TU1tKTOIj4

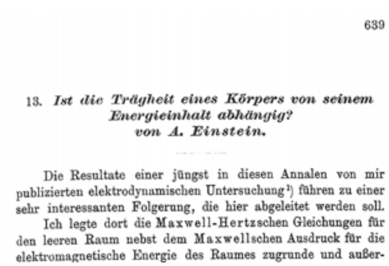
At first, this seemed counter-intuitive

We have improved our intuition

Equivalence of mass and energy

"Does the Inertia of a Body Depend Upon Its Energy Content?"

$E = mc^2$



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13. ***Ist die Trägheit eines Körpers von seinem Energieinhalt abhängig?***
von A. Einstein.

Die Resultate einer jüngst in diesen Annalen von mir publizierten elektrodynamischen Untersuchung[1]) führen zu einer sehr interessanten Folgerung, die hier abgeleitet werden soll. Ich legte dort die Maxwell-Hertzschen Gleichungen für den leeren Raum nebst dem Maxwellschen Ausdruck für die elektromagnetische Energie des Raumes zugrunde und außer-

Special Relativity: $E = mc^2$

- It is possible to derive this formula using algebra
  - Energy = force × distance
  - Force = the change in the momentum of an object
  - Lorentz transformations of time (that is, the velocity)
- But it is easier (and cleaner) to use calculus
  - http://www.emc2-explained.info/Emc2/Deriving.htm

$$E = mc^2$$

Derived using only math (mostly algebra) and the two conjectures:

1. The laws of nature are the same in all inertial frames of reference.
2. The speed of light in the vacuum is the same in all inertial frames of reference.

Mind: Blown!
Wonderful Theoretical Fun!

Energy and mass are the same thing

An aside: Our nomenclature at Fermilab

- Physicists at Fermilab set the speed of light equal to one

$$E = m$$

- This is *only* a change in units
- Emphasizing even more clearly that energy is the same thing as mass

What does this <u>mean</u>? $E = mc^2$

- What is the energy of 1 gram of matter?

$E = 0.001$ [kg]$\times(2.998 \times 10^8$ [m/s]$)^2$
$E = 1 \times 10^{-3} \times 8.988004 \times 10^{16}$ [kg m^2/sec^2]
$E = 8.988004 \times 10^{13}$ [Joules]
$E \approx 90$ Terajoules

→ Burning 691,538 gallons of gasoline
→ Exploding 42,964,554 pounds (21.5 kilo-tons (kT)) of TNT

First atomic bomb: Hiroshima (USA, fission)

- Hiroshima bomb: 63 Terajoules [6.3×10^{13} Joules] (15 kT)
- Equivalent to about 0.7 grams of matter converted to energy

Largest bomb: Tsar Bomba (USSR, fusion)

- Tsar Bomba: 240 Petajoules [2.4×10^{17} joules] (57 MT)
  - 3800 times larger than Hiroshima bomb
  - Converted about 2.7 kg of matter into energy

How do we know Special Relativity is accurate?

No Lorentz violations could be measured thus far, and exceptions in which positive results were reported have been refuted or lack further confirmations.

Everyday uses of Special Relativity

- Global Positioning System (GPS)
- Muon Decay

GPS is based on *very* accurate clocks

- GPS clock ticks must be known to an accuracy of **20-30 nanoseconds**
  - ~5 meters on the road.
- GPS satellites are constantly moving relative to Earthly observers
  - These clocks move a little more slowly
- Special Relativity predictions … next slide
- GPS also gets substantial corrections from General Relativity

http://www.astronomy.ohio-state.edu/~pogge/Ast162/Unit5/gps.html

GPS: How big an effect is satellite motion?

Speed of a GPS satellite is 14,000 KPH (3889 m/s)

$\beta_{satellite} = 3889 \,/\, 299{,}792{,}458$
$= 0.0000129633$
$\gamma_{satellite} = 1.00000000008402$

One day = 84600 seconds
$\gamma_{satellite} \times$ (one day) = 84600.0000071084 seconds
7.1 microseconds off

$\beta = v / c$

$\gamma = 1 / \sqrt{1-\beta^2}$

$t = t_0 \gamma$

This means that over a day, your position inaccuracy would grow from about 5 meters to about 300 meters.

An aside – GPS and General relativity

- The satellite is in micro-gravity
- The receiver is in Earth gravity
- General Relativity:
  - Earth clock runs 38 microseconds slower per day
  - 5.4X bigger effect than Special Relativity
  - About a mile of error per day

Muon Decay


THE
PHYSICAL REVIEW
A Journal of Experimental and Theoretical Physics Established by E. L. Nichols in 1893

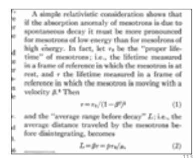
Variation of the Rate of Decay of Mesotrons with Momentum

INTRODUCTION

RECENT experiments on the variation of cosmic-ray intensity with altitude have shown that the rate of decrease of the mesotron

traveling in the atmosphere. No appreciable number of mesotrons, however, will disintegrate within a condensed absorber, even equivalent in mass to the whole thickness of the atmosphere,

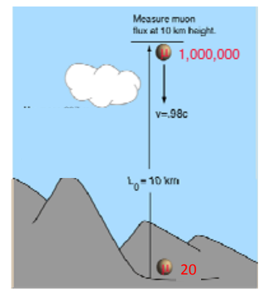
- 1941 experiment
- Now a common experiment in a graduate physics laboratory


A simple relativistic consideration shows that if the absorption anomaly of mesotrons is due to spontaneous decay it must be more pronounced for mesotrons of low energy than for mesotrons of high energy. In fact, let τ_0 be the "proper life-time" of mesotrons; i.e., the lifetime measured in a frame of reference in which the mesotron is at rest, and τ the lifetime measured in a frame of reference in which the mesotron is moving with a velocity β. Then

$$\tau = \tau_0/(1-\beta^2)^{\frac{1}{2}} \quad (1)$$

and the "average range before decay" L; i.e., the average distance traveled by the mesotrons before disintegrating, becomes

$$L = \beta\tau = p\tau_0/\mu, \quad (2)$$



No relativity

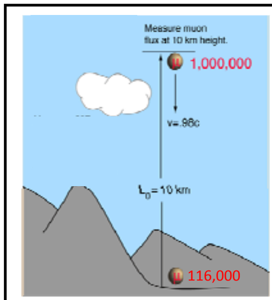
Distance: 10,000 meters
Time = distance / velocity

t = 1×10^4 [m]/((0.98) (3×10^8 [m/s]))

t = 34×10^{-6} [sec]

Muon lifetime = 2.2×10^{-6} [sec]
15.5X longer

Expect to see about 20 muons



With time dilation

Distance: 10,000 meters
Time = distance / velocity
$t_{Enterprise} = t_0 / \gamma$

t = (1×10^4 [m]/((0.98) (3×10^8 [m/s])))/ γ

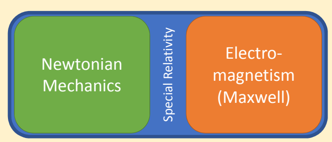
t = (34×10^{-6} [sec]) / γ

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{1-0.98^2}} = \frac{1}{\sqrt{0.0396}} = 5.025$$

Agrees with experiment

Special Relativity includes Newton and E&M

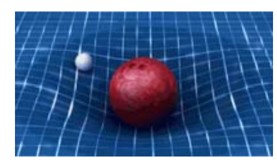
- Special Relativity extends Newtonian Mechanics to close-to-c speeds, but it does *not* contradict it
- It includes all of Newtonian Mechanics in its entirety.



Questions?

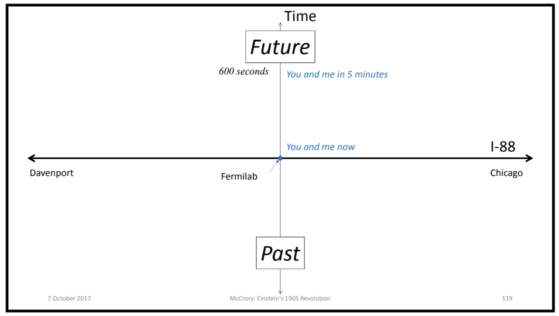
General Relativity …

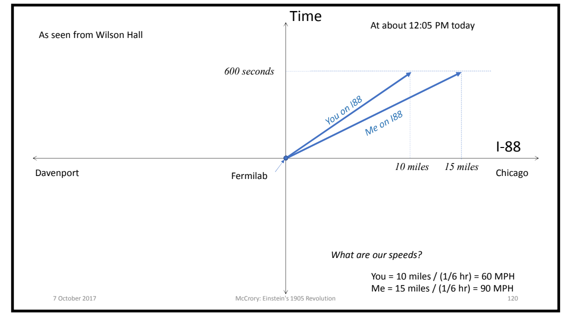
- Einstein's Relativity does not end here
- General relativity combines acceleration (non-inertial reference frames) with gravity
- Gravity is a result of the geometry of spacetime (!)

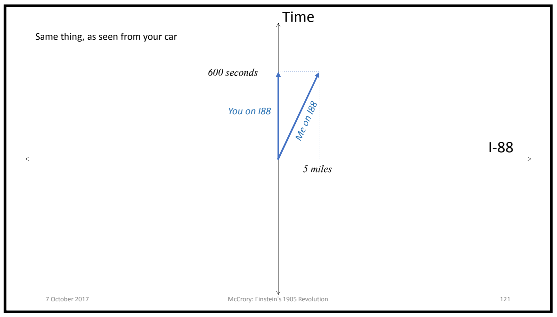


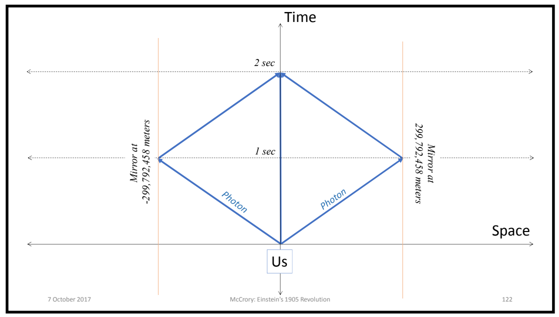
We are moving fast enough that you might think we have the time to go over General Relativity in some detail.
But, alas, we do not.

Spacetime

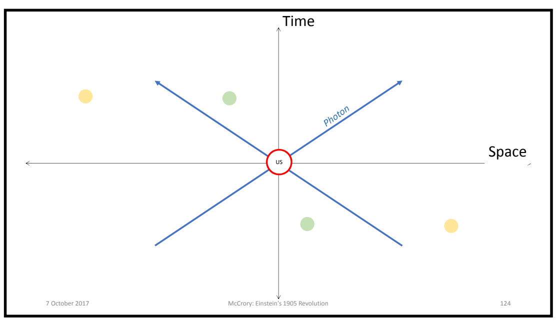


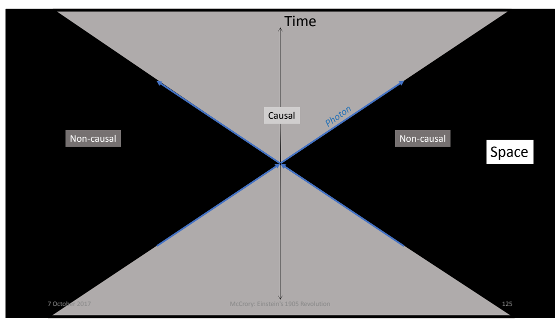






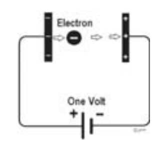
Causality:
The fastest a signal can travel is *c*





Units of mass and energy

- We use "Electron Volts" as an energy
  - 1.6×10^{-19} joules
- Since mass=energy, we also say that a particle has a mass in "electron volts"



$$m_e = 510999\ [eV]$$
$$= 511\ [keV]$$
$$m_p = 938272081[eV]$$
$$= 938\ [MeV]$$

Fermilab

7 October 2017 McCrory: Einstein's 1905 Revolution 126

Converting between energy and mass

- When two particles collide at high energy, the result has a lot of energy
- Often (usually) this energy becomes mass.

Intuition … *reprise*

- How do we know what we know?
  - Intuition
  - Deduction
  - Observation
- This talk:
  - Intuition can be incomplete
    - "Counter intuitive"
  - Deduction and observation can be amazing

In other words, Special Relativity is

A triumph of the Scientific Method

Special Relativity: Conclusions

- Time dilates
- Length contracts
- Simultaneity is relative
- Matter and energy are the same thing
- It agrees, impressively, with experiments
- Our old intuition must change
- Maxwell's Equations are correct
  - And they always have been
- Einstein was an insightful, hard working scientist

$$t = \gamma \ t_0$$

$$d = \frac{1}{\gamma} d_0$$

$$\gamma = 1/\sqrt{1-\beta^2}$$
$$\beta = v/c$$

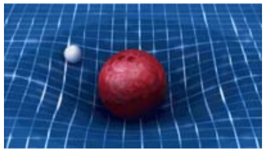
Conclusions

Weary, though fascinated!

- Einstein was one of the greatest minds ever
- 1905: Four papers changed humans' understanding of the physical world
- Special Relativity: Old intuition of space and time must be adjusted
- Algebra is cool

Relativity does not end here

- **General relativity**
  - Einstein's life's work
- Space is warped by mass
- Mass moves according to how space is warped
- Mind-blowing predictions:
  - Precession of the orbit of Mercury
  - Light is bent by a gravity field
  - Black holes
  - Expansion of the universe
  - Gravity waves



That's all, folks!

These slides:
1. https://smp.fnal.gov
2. "Session I: Fall 2017"

Feedback

http://tinyurl.com/SMPFeedback-Fall17

The other 1905 Einstein papers were also profound

Brownian Motion

Brownian Motion

"Investigations on the theory of Brownian Movement"

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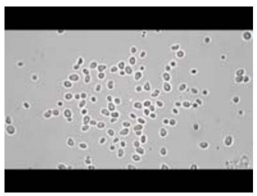
5. **Über die von der molekularkinetischen Theorie der Wärme geforderte Bewegung von in ruhenden Flüssigkeiten suspendierten Teilchen;**
von A. Einstein.

In dieser Arbeit soll gezeigt werden, daß nach der molekularkinetischen Theorie der Wärme in Flüssigkeiten suspendierte Körper von mikroskopisch sichtbarer Größe infolge der Mole-

Brownian Motion

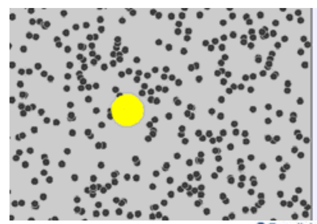
Botanist Robert Brown, 1773-1858

- A tiny, visible object (suspended in water) will move about randomly.
- Could the pollen grains be alive?
- Nope. Shown to happen with lab-created inanimate stuff



Brownian Motion: Einstein

- Einstein solved the problem, assuming "molecules" are what is pushing the object around.
- His calculations revealed the size of these molecules
  - Avagadro's Number



Photoelectric Effect

"On a Heuristic Point of View about the Creation and Conversion of Light"

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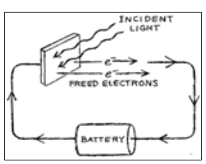
6. **Über einen die Erzeugung und Verwandlung des Lichtes betreffenden heuristischen Gesichtspunkt;**
von A. Einstein.

Zwischen den theoretischen Vorstellungen, welche sich die Physiker über die Gase und andere ponderable Körper gebildet haben, und der Maxwellschen Theorie der elektromagnetischen Prozesse im sogenannten leeren Raume besteht

The Photoelectric Effect

- Covered last week by Professor Hooper
- Emission of electrons when light shines onto a material … sometimes
- This phenomenon was discovered by Hertz and Hallwachs in 1887.



Interpretation based on Maxwell's Equations

- Light is a wave
- Brighter light → More energetic electrons
- Brighter light → More prompt emission of electrons
  - Dim light: electrons would be released slowly and at low energy

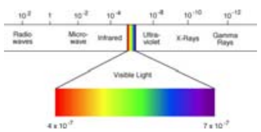
Observations

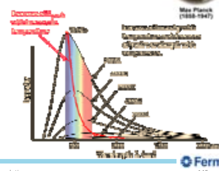
- In 1902, Lenard observed that the energy of individual emitted electrons increased with the *color* of the light, but not the intensity.
- Furthermore, when the color was blue enough, electrons would be emitted instantly

Max Planck

Max Planck (1858-1947)

- Studied the radiation emitted from a glowing body
  - "Black Body Radiation"
  - (Radiation: energy from light)



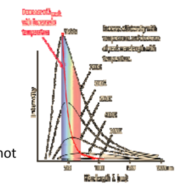


How to fit this frequency curve?

- Assume that electromagnetic energy could be emitted only in quantized form
- He introduced a new constant, h:

$$E = h\nu$$

  - h is Planck's constant, 6.62607×10⁻³⁴ [Joules seconds]
  - ν ("nu") is the frequency of the radiation.
- Scientists appreciated the novelty and correctness (mathematically) of this assumption, but they did not (particularly) like what this meant!



Max Planck and Einstein

- Einstein showed that Planck's assertion, that light is made up of particles, would resolve the photoelectric effect
- Light comes in quanta, and they behave like particles in the Photoelectric effect
- These particles (now called "photons") hit electrons in the atoms of the metal and can only kick them out if the energy of the photos is high enough

Wave/particle duality

- The photoelectric effect helped to propel the then-emerging concept of wave–particle duality in the nature of light.
- Light simultaneously possesses the characteristics of both waves and particles, each being manifested according to the circumstances.
- It was soon realized that particles with mass also have a wavelength

Full equation for total energy

$$E = \sqrt{p^2c^2 + m_0^2c^4}$$

- In other words …
- Anything with energy has momentum
- A photon has momentum

$$\begin{aligned}
E &= mc^2 \\
E^2 &= m^2c^4 \\
&= \gamma\ m_0^2c^4 \\
&= \frac{m_0^2c^4}{1-\frac{v^2}{c^2}} \\
m^2c^4 &= \frac{m_0^2c^4}{1-\frac{v^2}{c^2}} \\
m^2c^4\left(1-\frac{v^2}{c^2}\right) &= m_0^2c^4 \\
m^2c^4 - m^2v^2c^2 &= m_0^2c^4 \\
&\rightarrow p = mv \\
m^2c^4 - p^2c^2 &= m_0^2c^4 \\
m^2c^4 &= p^2c^2 + m_0^2c^4 \\
E &= \sqrt{p^2c^2 + m_0^2c^4}
\end{aligned}$$

Return

Energy and mass are the same thing

An aside: No new physics in 20th Century?

- One school of thought:
  - Classical mechanics could cope with highly complex problems involving macroscopic situations
  - Thermodynamics and kinetic theory were well established
  - Geometrical and physical optics could be understood in terms of electromagnetic waves
  - Conservation laws for energy and momentum (and mass) were widely accepted
  - It was generally accepted that all the important laws of physics had been discovered
  - Research would be concerned with clearing up minor problems and particularly with improvements of method and measurement.
- However, A. A. Michelson's in his 1899 lectures *Light Waves and Their Uses*:
  - What would be the use of extreme refinement in the science of measurement?
  - […] all future discovery must lie [here].
  - The more important fundamental laws and facts of physical science have all been discovered, and these are now so firmly established that the possibility of their ever being supplanted in consequence of new discoveries is exceedingly remote.
  - Nevertheless, it has been found that there are apparent exceptions to most of these laws, and this is particularly true when the observations are pushed to a limit, i. e., whenever the circumstances of experiment are such that extreme cases can be examined. Such examination almost surely leads, not to the overthrow of the law, but to the discovery of other facts and laws whose action produces the apparent exceptions.

Some perspective on *Annus Miralilis*

- Worked as an examiner at the Patent Office in Bern, Switzerland
  - Limited access to scientific reference materials,
  - Co-worker: "[Einstein] could not have found a better sounding board for his ideas in all of Europe".
- Einstein tackles some of the era's most important physics questions
- These papers solved the primary scientific problems of the era:
  - Lord Kelvin lecture titled "Nineteenth-Century Clouds over the Dynamical Theory of Heat and Light"
    - Michelson–Morley experiment
    - Black body radiation.
  - Coordinate transformation of Maxwell's Equations.