CORRELATIONS IN QE(LIKE) NEUTRINO-NUCLEUS SCATTERING

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Outline

• Detailed microscopic cross sections calculations for QE(-like) scattering
• Influence of long-range correlations
• Influence of short-range correlations in 1- and 2-nucleon knockout processes
• Influence of seagull and pion-in-flight MEC contributions

• Scheme-dependent separation
• Double counting
Neutrino-hadron scattering

e.g.

Dip region: multinucleon mechanisms
Neutrino-nucleus interactions

\[ \hat{H}_w = \frac{G}{\sqrt{2}} \int d\vec{x} j_{\mu, \text{lepton}}(\vec{x}) j_{\mu, \text{hadron}}(\vec{x}) \]

Hadron current

\[ J^\mu = F_1(Q^2)\gamma^\mu + i \frac{\kappa}{2M_N} F_2(Q^2)\sigma^{\mu\nu} q_\nu + G_A(Q^2)\gamma^\mu \gamma_5 + \frac{1}{2M_N} G_F(Q^2) q^\mu \gamma_5 \]

Lepton tensor

\[ l_{\alpha\beta} \equiv \sum_{s,s'} [\bar{u}_l \gamma_\alpha (1 - \gamma_5) u_l]^\dagger [\bar{u}_\nu \gamma_\beta (1 - \gamma_5) u_\nu] \]
\[ J^\mu_N (\vec{x}) = J^\mu_{\text{convection}} (\vec{x}) + J^\mu_{\text{magnetization}} (\vec{x}) \]

with
\[ J^\mu_{c} (\vec{x}) = \frac{1}{2M_i} \sum_{i=1}^{A} G^{i,\alpha}_E \left[ \delta (\vec{x} - \vec{x}_i) \vec{v}_i - \vec{v}_i \delta (\vec{x} - \vec{x}_i) \right], \]

\[ J^\mu_{m} (\vec{x}) = \frac{1}{2M} \sum_{i=1}^{A} G^{i,\alpha}_M \vec{\nabla} \times \vec{\sigma}_i \delta (\vec{x} - \vec{x}_i), \]

\[ J^\mu_{A} (\vec{x}) = \sum_{i=1}^{A} \left[ G^{i,\alpha}_A \vec{\sigma}_i \delta (\vec{x} - \vec{x}_i) \right], \]

\[ J^0_{V,\alpha} (\vec{x}) = \rho^0_N (\vec{x}) = \sum_{i=1}^{A} G^{i,\alpha}_E \delta (\vec{x} - \vec{x}_i), \]

\[ J^0_{A,\alpha} (\vec{x}) = \rho^0_A (\vec{x}) = \frac{1}{2M_i} \sum_{i=1}^{A} G^{i,\alpha}_A \vec{\sigma}_i \cdot \left[ \delta (\vec{x} - \vec{x}_i) \vec{v}_i - \vec{v}_i \delta (\vec{x} - \vec{x}_i) \right], \]

\[ J^0_{P,\alpha} (\vec{x}) = \rho^0_P (\vec{x}) = \frac{m_P}{2M} \sum_{i=1}^{A} G^{i,\alpha}_P \vec{\nabla} \cdot \vec{\sigma}_i \delta (\vec{x} - \vec{x}_i) \]

for NC reactions
\[ G_{E}^{V,0} = \left( \frac{1}{2} - \sin^2 \theta_W \right) \tau_3 - \sin^2 \theta_W, \]
\[ G_{M}^{V,0} = \left( \frac{1}{2} - \sin^2 \theta_W \right) (\mu_p - \mu_n) \tau_3 - \sin^2 \theta_W (\mu_p + \mu_n), \]
\[ G_{A,0} = g_a \tau_3 = - \frac{1.262}{2} \tau_3 \]

for CC reactions
\[ G_{E}^{V,\pm} = \tau_\pm, \]
\[ G_{M}^{V,\pm} = (\mu_p - \mu_n) \tau_\pm, \]
\[ G_{A,\pm} = g_a \tau_\pm = - 1.262 \tau_\pm, \]

\[ G = (1 + Q^2/M^2)^{-2} \quad \text{Q}^2 \text{ dependence : dipole parametrization or BBBA07} \]
Inclusive QE 1-nucleon knockout cross sections

\[ \frac{d^2 \sigma}{d \Omega \, d \omega} = (2\pi)^4 \, k_f \varepsilon_f \, \sum_{s_f, s_i} \frac{1}{2J_i + 1} \, \sum_{M_f, M_i} \left| \langle f \, \left| \hat{H}_W \right| i \rangle \right|^2 \]

\[ \left( \frac{d^2 \sigma_{i \rightarrow f}}{d \Omega \, d \omega} \right)_{\text{NR}} = \frac{G^2 \varepsilon_f^2}{\pi} \, \frac{2 \cos^2 \left( \frac{\theta}{2} \right)}{2J_i + 1} \, \left[ \sum_{j=0}^{\infty} \sigma_{CL}^j + \sum_{j=1}^{\infty} \sigma_T^j \right] \]

\[ \sigma_{CL}^j = \left| \langle J_f, J_i \rangle \, \hat{M}_j (\kappa) + \frac{\omega}{|q|} \, \hat{L}_j (\kappa) \right|^2 \]

\[ \sigma_T^j = \left( -\frac{q_\mu^2}{2 |q|^2} + \tan^2 \left( \frac{\theta}{2} \right) \right) \left[ \left| \langle J_f \, \left| \hat{J}_{j,mag} (\kappa) \right| J_i \rangle \right|^2 + \left| \langle J_f \, \left| \hat{J}_{j,s} (\kappa) \right| J_i \rangle \right|^2 \right] \]

\[ + \tan \left( \frac{\theta}{2} \right) \sqrt{-\frac{q_\mu^2}{|q|^2} + \tan^2 \left( \frac{\theta}{2} \right)} \left[ 2 \Re \left( \langle J_f \, \left| \hat{J}_{j,mag} (\kappa) \right| J_i \rangle \, \langle J_f \, \left| \hat{J}_{j,s} (\kappa) \right| J_i \rangle^* \right) \right] \]
2-nucleon knockout cross sections

\[ \frac{d\sigma^X}{dE_f d\Omega_f dT_b d\Omega_b d\Omega_a} = \frac{p_a p_b E_a E_b}{(2\pi)^6} g_{rec}^{-1} \sigma^X \zeta \times \left[ v_{CC} W_{CC} + v_{CL} W_{CL} + v_{LL} W_{LL} + v_T W_T + v_{TT} W_{TT} + v_{TC} W_{TC} + v_{TL} W_{TL} + h(v_T W_T' + v_{TC} W_{TC'} + v_{TL} W_{TL'}) \right], \]

with:

\[ \mathcal{J}_\lambda = \langle \Phi_f^{(A-2)}(E^{exc}, J_R M_R); p_a m_{s_a}; p_b m_{s_b} | \hat{J}_\lambda(q) | \Phi_{gs} \rangle \]

\[ | \Phi^{2p2h} \rangle = | \Phi_f^{(A-2)}(E^{exc}, J_R M_R); p_a m_{s_a}; p_b m_{s_b} \rangle_{as} \]
Cross section calculations

Nuclear model

• Starting point: mean-field nucleus with Hartree-Fock single-particle wave functions
• Skyrme SkE2 force used to build the potential
• Pauli blocking
• Binding
• Long- and short-range correlations
• Meson Exchange Currents
- Nuclear radius $\approx 1.2A^{1/3}$ fm
- Nucleon is a diffuse system
  - Hard core (repulsion) $\approx 0.5$ fm
  - RMS charge radius from $(e,e') = 0.897(18)$ fm
- $0.07 \lesssim \text{NPF} \lesssim 0.42$
  - closest packing fraction of spheres $\approx 0.74$
  - packing fraction of Argon liquid $\approx 0.032$
  - packing fraction of Argon gas $\approx 3.75 \times 10^{-5}$
- The nuclear medium is a rather dense quantum liquid

C. Colle, PhD, UGent 2017

Packing fraction $\approx 0.012$
The Fermi gas model

Easiest microscopic independent particle model

\[ n = \frac{2}{(2\pi \hbar)^3} \int_0^{p_F} d^3p = 2 \frac{V}{(2\pi \hbar)^3} \left( \frac{4}{3} \pi p_F^3 \right) \]

\[ \Rightarrow p_F = \hbar \left( \frac{3n\pi}{V} \right)^{1/3} \sim 247 \text{ MeV/c} \]

\[ E_F = \frac{p_F^2}{2m} \sim 33 \text{ MeV} \]
The mean field model (or shell-model)

Iron-56 is the most abundant and the third most stable feature. It does not have \( Z \) or \( N \) equal to a magic number.

Note the oscillations of abundance depending upon whether \( Z \) and \( N \) are odd or even.

Energy first excited state

Nuclei with magic numbers of neutrons have neutron absorption cross-sections up to two orders of magnitude less than other nuclei with similar masses.
Solve Schrödinger (or Dirac) equation for nucleon in nuclear potential

Harmonic oscillator potential
→Work harder: add spin-orbit term

\[ \hat{H} \rightarrow \hat{H} + \xi(r) \vec{l} \cdot \vec{s} \]

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Not too approximate model for nuclei: already quite some correlations: the mean field model (or shell-model)

→Work harder: add spin-orbit term

J = l-1/2

J = l+1/2

![Diagram showing shell gaps, less bound than average, and more bound than average.](image)
Independent particle picture

\[ \rho(r) = \sum_{b<F} \varphi_b^*(r) \varphi_b(r) \]

Fig. 3.18. The nuclear density distribution for the least bound proton in $^{206}$Pb. The shell-model predicts the last (3s1/2) proton in $^{206}$Pb to have a sharp maximum at the centre, as shown at the left-hand side. On the right-hand side the nuclear charge density difference $\rho_c(206\text{Pb}) - \rho_c(205\text{TI}) = \varphi_{3s1/2}^2(r)$ is given [taken from (Frois 1983) and Doe 1983]
Woods-Saxon potential:

\[ V(r) = V_0 \frac{1}{1 + e^{\frac{r-R}{a}}} \]

Not too approximate model for nuclei: already quite some correlations: the mean field model (or shell-model).

Figure 1. Woods-Saxon potential for neutrons (a) and protons (b) along the Sn isotopic chain.
The Hartree-Fock mean field

• Nucleons are moving independent from each other in a mean field
• How do we obtain a reliable and consistent field?

\[ H = \sum_i T_i + \frac{1}{2} \sum_{i,j} V_{i,j} \]

\[ H = \sum_i (T_i + U(r_i)) + \left( \frac{1}{2} \sum_{i,j} V_{i,j} - \sum_i U(r_i) \right) \]

\[ H = \sum_i h_0(i) + H_{res} \]

Residual interaction
The Hartree-Fock recipe:

Nucleons fill up a number of orbitals and form a density that can be written in terms of the occupied states as:

$$\rho(r) = \sum_{b<F} \varphi_b^*(r)\varphi_b(r)$$

The potential at a position \( r' \), generated by the nucleon-nucleon two-body interaction \( V(r,r') \) is given by

$$U_H(r') = \sum_{b<F} \int \varphi_b^*(r)V(r,r')\varphi_b(r)dr$$

$$-\frac{\hbar^2}{2m} \Delta \varphi_i(r) + \sum_{b<f} \int \varphi_b^*(r')V(r,r')\varphi_b(r')dr' \cdot \varphi_i(r)$$

$$- \sum_{b<f} \int \varphi_b^*(r')V(r,r')\varphi_b(r)\varphi_i(r')dr' = \varepsilon_i \varphi_i(r)$$
Hartree-Fock recipe: Start with an initial guess for either the average field or the wave functions and use $V(r,r')$ to solve the coupled equations to obtain better values e.g.

\[-\frac{\hbar^2}{2m}\Delta \varphi_i(r) + U_H(r)\varphi_i(r) - \int U_F(r,r')\varphi_i(r')dr' = \varepsilon_i\varphi_i(r)\]

\[U_H(r') = \sum_{b<F} \int \varphi_b^*(r)V(r,r')\varphi_b(r)dr\]

\[U_F(r,r') = \sum_{b<F} \varphi_b^*(r')V(r,r')\varphi_b(r)\]
Wave function for the nucleus = Slater determinant

$$\Psi_{1,2,\ldots,A}(r_1, r_2, \ldots, r_A) = \frac{1}{\sqrt{A}} \begin{vmatrix} \varphi_1(r_1) & \varphi_2(r_2) & \cdots & \varphi_A(r_A) \\ \varphi_2(r_1) & \varphi_3(r_2) & \cdots & \varphi_1(r_A) \\ \vdots \\ \varphi_A(r_1) & \varphi_1(r_2) & \cdots & \varphi_A(r_A) \end{vmatrix}$$

$$E_0 = \sum_{i=1}^{A} \varepsilon_i$$

Antisymmetrization takes into account the Pauli principle
Long-range correlations are correlations over the whole size of the nucleus. They can redistribute the incoming energy transfer to the nucleus over all the nuclear constituents. They manifest themselves in collective excitations such as giant resonances.
Long-range correlations = probing collective effects at low energies

https://cyclotron.tamu.edu/research/nuclear-structure/
David Bohm & David Pines ‘52 – Condensed matter physics

Quantum mechanical interactions

The Hamiltonian corresponding to a system of individual electrons is re-expressed such that the long-range part of the Coulomb interactions between electrons is described in terms of collective fields.

This leads to the description of organized behavior of electrons brought along by long-range Coulomb interactions that couple together the motion of many electrons simultaneously = plasma oscillations.

Neglecting the coupling between plasma vibrations of different momenta.
Single particle propagator

\[ G = G^{(0)} + G^{(0)} V G \]
\[ G = G^{(0)} + G^{(0)} V G^{(0)} + G^{(0)} V G^{(0)} V G^{(0)} + \ldots \]

\[ \langle \alpha | \frac{1}{E - H + i\eta} | \beta \rangle = \langle \alpha | \frac{1}{E - H_0 + i\eta} | \beta \rangle \]
\[ + \sum_{\gamma, \delta} \langle \alpha | \frac{1}{E - H_0 + i\eta} | \gamma \rangle \langle \gamma | V | \delta \rangle \langle \delta | \frac{1}{E - H + i\eta} | \beta \rangle \]
Hamiltonian: $H = H_0 + U + V$

Auxiliary potential

2-body interaction

Dyson Equation:

$$G = G^{(0)} + G^{(0)} V G$$

$$G = G^{(0)} + G^{(0)} V G^{(0)} + G^{(0)} V G^{(0)} V G^{(0)} + \ldots$$

Self-energy
$\Sigma = \text{e.g.}$

- 2-body interaction $V$

- Axillary field $U$

...
Two-particle propagator

$G_{II}(\alpha, \beta, \gamma, \delta)$

Vertex function

...
\[ \Sigma = + + \text{Auxiliary field } U + \text{interaction } V + \Gamma \]

Higher order contributions/different topologies

Reformulation of the self-energy of a dressed particle
The Hartree-Fock mean field

\[ G^{HF}(\alpha, \beta; E) = G^{(0)}(\alpha, \beta; E) + \sum_{\gamma, \delta} G^{(0)}(\alpha, \gamma; E) \Sigma^{HF}(\gamma, \delta) G^{HF}(\delta, \beta; E') \]

Mean field already contains correlations!
Excited states: Particle-hole or polarization propagator

\[ \Pi(\alpha, \beta^{-1}; \gamma, \delta^{-1}; E) = \sum_{n \neq 0} \frac{\langle \Phi_0^N | a^\dagger_\beta a_\alpha | \Phi_n^N \rangle \langle \Phi_n^N | a^\dagger_\gamma a_\delta | \Phi_0^N \rangle}{E - (E_n^N - E_0^N) + i\eta} + \sum_{n \neq 0} \frac{\langle \Phi_0^N | a^\dagger_\gamma a_\delta | \Phi_n^N \rangle \langle \Phi_n^N | a^\dagger_\beta a_\alpha | \Phi_0^N \rangle}{E + (E_n^N - E_0^N) - i\eta} \]

\[ \Pi^{(0)}(\alpha, \beta^{-1}; \gamma, \delta^{-1}; E) = \int \frac{dE'}{2\pi} G^{(0)}(\alpha, \gamma; E + E') G^{(0)}(\delta^{-1}, \beta^{-1}; E') \]

\[ = \]

\[ \text{Diagram} \]
Excited states: Particle-hole or polarization propagator

\[ \Pi^{RPA}(\alpha, \beta^{-1}; \gamma, \delta^{-1}; E) = \Pi^{(0)}(\alpha, \beta^{-1}; \gamma, \delta^{-1}; E) \]

\[ + \sum_{\xi} \Pi^{(0)}(\alpha, \beta^{-1}; \zeta, \xi^{-1}; E) \langle \zeta \xi^{-1} | V_{ph} | \xi^{-1} \rangle \Pi^{RPA}(\varepsilon, \theta^{-1}; \gamma, \delta^{-1}; E) \]
Long-range correlations: Continuum RPA

- Green’s function approach
- Skyrme SkE2 residual interaction
- Self-consistent calculations

\[
|\Psi_{RPA}\rangle = \sum_c \left\{ X_{(\Psi,C)} |ph^{-1}\rangle - Y_{(\Psi,C)} |hp^{-1}\rangle \right\}
\]

\[
\Pi^{(RPA)}(x_1, x_2; \omega) = \Pi^{(0)}(x_1, x_2; \omega) + \frac{1}{\hbar} \int dx \int dx' \, \Pi^{(0)}(x, x; \omega) \tilde{V}(x, x') \Pi^{(RPA)}(x', x_2; \omega)
\]
Solving the RPA equations in coordinate space:

\[ |\Psi_C(E)\rangle = |p\ h^{-1}(E)\rangle + \int dx_1 \int dx_2 \ \tilde{V}(x_1, x_2) \]

\[ \sum_{c'} \mathcal{P} \int d\varepsilon_{p'} \left[ \psi_{h'}(x_1) \psi_{p'}^\dagger(x_1, \varepsilon_{p'}) \frac{E - \varepsilon_{p' h'}}{E + \varepsilon_{p' h'}} \right] \langle \Psi_0 | \hat{\psi}_2^\dagger \hat{\psi}_2 | \Psi_C(E) \rangle \]

What we really need is transition densities:

\[ \langle \Psi_0 | X_{\eta J} | \Psi_C(J; E) \rangle_{r_2} = - \langle h | X_{\eta J} | p(\varepsilon_{ph}) \rangle_r \]

\[ + \sum_{\mu, \nu} \int dr_1 \int dr_2 \ U_{\mu \nu}^J(r_1, r_2) \ \mathcal{R} \left( R_{\eta \mu; J}^{(0)}(r, r_1; E) \right) \langle \Psi_0 | X_{\nu J} | \Psi_C(J; E) \rangle_{r_2} \]

So in the end we have to solve a set of coupled equations, that after discretizing on a mesh in coordinate space, translates into a matrix inversion for the transition densities:

\[ \rho^{RPA}_C = - \frac{1}{1 - R U} \rho^{HF}_C \]
2-body interaction $V$?

Landau-Migdal interaction

$$V = c_0 \{ f_0(\rho) + f'(\rho) \tau_1 \tau_2 + g_0(\rho) \sigma_1 \sigma_2 + g'(\rho) \tilde{\sigma}_1 \tilde{\sigma}_2 \tilde{\tau}_1 \tilde{\tau}_2 \}$$

$$f(\rho(r)) = (1 - \rho(r)) f^{ext} + \rho(r) f^{int}$$

$$\rho(r) = \frac{1}{1 + e^{\frac{r-R}{a}}}$$
2-body interaction $V$ ?

Skyrme

$$
V(\vec{r}_1, \vec{r}_2) = t_0 \left( 1 + x_0 \hat{P}_\sigma \right) \delta(\vec{r}_1 - \vec{r}_2) \\
- \frac{1}{8} t_1 \left( (\vec{\nabla}_1 - \vec{\nabla}_2)^2 \delta(\vec{r}_1 - \vec{r}_2) + \delta(\vec{r}_1 - \vec{r}_2) (\vec{\nabla}_1 - \vec{\nabla}_2)^2 \right) \\
+ \frac{1}{4} t_2 (\vec{\nabla}_1 - \vec{\nabla}_2) \delta(\vec{r}_1 - \vec{r}_2) (\vec{\nabla}_1 - \vec{\nabla}_2) \\
+ i W_0 \left( \vec{\sigma}_1 + \vec{\sigma}_2 \right) \cdot (\vec{\nabla}_1 - \vec{\nabla}_2) \times \delta(\vec{r}_1 - \vec{r}_2) (\vec{\nabla}_1 - \vec{\nabla}_2) \\
+ \frac{1}{6} t_3 (1 - x_3) (1 + \hat{P}_\sigma) \rho \frac{(\vec{r}_1 + \vec{r}_2)}{2} \delta(\vec{r}_1 - \vec{r}_2) \\
+ \frac{e^2}{|\vec{r}_1 - \vec{r}_2|} + x_3 t_3 \delta(\vec{r}_1 - \vec{r}_2) \delta(\vec{r}_1 - \vec{r}_3) \\
- \frac{1}{24} t_4 \left\{ \left[ (\vec{\nabla}_1 - \vec{\nabla}_2)^2 + (\vec{\nabla}_2 - \vec{\nabla}_3)^2 + (\vec{\nabla}_3 - \vec{\nabla}_1)^2 \right] \right\} \\
\delta(\vec{r}_1 - \vec{r}_2) \delta(\vec{r}_1 - \vec{r}_3) + \delta(\vec{r}_1 - \vec{r}_2) \delta(\vec{r}_1 - \vec{r}_3) \\
\left\{ \left[ (\vec{\nabla}_1 - \vec{\nabla}_2)^2 + (\vec{\nabla}_2 - \vec{\nabla}_3)^2 + (\vec{\nabla}_3 - \vec{\nabla}_1)^2 \right] \right\} .
$$
Regularization of the residual interaction:

\[ V(Q^2) \rightarrow V(Q^2 = 0) \frac{1}{(1 + \frac{Q^2}{\Lambda^2})^2} \]
• Final state interactions:
  - taken into account through the calculations of the wave function of the outgoing nucleon in the (real) nuclear potential generated using the Skyrme force
  - influence of the spreading width of the particle states is implemented through a folding procedure

\[
R'(q, \omega') = \int_{-\infty}^{\infty} d\omega \ R(q, \omega) \ L(\omega, \omega'),
\]

\[
L(\omega, \omega') = \frac{1}{2\pi} \left[ \frac{\Gamma}{(\omega - \omega')^2 + (\Gamma/2)^2} \right].
\]
Relativistic corrections at higher energies (S. Jeschonnek and T. Donnelly, PRC57, 2438 (1998)):

- The outgoing nucleon obtains the correct relativistic momentum
  \[ p = \sqrt{T^2 + 2MT} \]
- Shifts the QE peak to the right relativistic position

Shift:
\[ \lambda \rightarrow \lambda(\lambda + 1) \quad \lambda = \frac{\omega}{2M_N} \]

Boost:
\[
\begin{align*}
R_{CC}^V(q, \omega) &\rightarrow \frac{q^2}{q^2 - \omega^2} R_{CC}^V(q, \omega), \\
R_{LL}^A(q, \omega) &\rightarrow \left(1 + \frac{q^2 - \omega^2}{4m^2}\right) R_{LL}^A(q, \omega), \\
R_T^V(q, \omega) &\rightarrow \frac{q^2 - \omega^2}{q^2} R_T^V(q, \omega), \\
R_T^A(q, \omega) &\rightarrow \left(1 + \frac{q^2 - \omega^2}{4m^2}\right) R_T^A(q, \omega), \\
R^N_{VA}(q, \omega) &\rightarrow \sqrt{\frac{q^2 - \omega^2}{q^2}} \sqrt{1 + \frac{q^2 - \omega^2}{4m^2}} R^N_{VA}(q, \omega).
\end{align*}
\]
Relativistic corrections at higher energies (S. Jeschonnek and T. Donnelly, PRC57, 2438 (1998)):

\[ \lambda \rightarrow \lambda (\lambda + 1) \quad \lambda = \frac{\omega}{2M_N} \]

The outgoing nucleon obtains the correct relativistic momentum

\[ p = \sqrt{T^2 + 2MT} \]

Shifts the QE peak to the right relativistic position

**Shift:**

**Boost:**

\[ R^V_{CC}(q, \omega) \rightarrow \frac{q^2}{q^2 - \omega^2} R^V_{CC}(q, \omega), \]

\[ R^A_{LL}(q, \omega) \rightarrow \left(1 + \frac{q^2 - \omega^2}{4m^2}\right) R^A_{LL}(q, \omega), \]

\[ R^V_T(q, \omega) \rightarrow \frac{q^2 - \omega^2}{q^2} R^V_T(q, \omega), \]

\[ R^A_T(q, \omega) \rightarrow \left(1 + \frac{q^2 - \omega^2}{4m^2}\right) R^A_T(q, \omega), \]

\[ R^{VA}_T(q, \omega) \rightarrow \sqrt{\frac{q^2 - \omega^2}{q^2}} \sqrt{1 + \frac{q^2 - \omega^2}{4m^2}} R^{VA}_T(q, \omega). \]
Coulomb correction for the outgoing lepton in charged-current interactions:

- Low energies: Fermi function
  \[ F(Z', E) = \frac{2\pi \eta}{1 - e^{-2\pi \eta}} \]
  \[ \eta \sim \mp Z' \alpha \]

- High energies: modified effective momentum approximation (J. Engel, PRC57,2004 (1998))
  \[ q_{\text{eff}} = q + 1.5 \left( \frac{Z' \alpha \hbar c}{R} \right) \]
  \[ \Psi_{\text{eff}}^{\text{eff}} = \zeta(Z', E, q) \Psi_i \]

\[ \zeta(Z', E, q) = \sqrt{\frac{q_{\text{eff}} E_{\text{eff}}}{qE}} \]

**Uncorrected**

**MEMA**
Fermi gas Hartree-Fock

\[ q = 95 \text{ [MeV/c]}, \quad Q^2 = 0.009 \text{ [(GeV/c)^2]} \]

\[ q = 121 \text{ [MeV/c]}, \quad Q^2 = 0.015 \text{ [(GeV/c)^2]} \]

\[ q = 508 \text{ [MeV/c]}, \quad Q^2 = 0.242 \text{ [(GeV/c)^2]} \]

\[ q = 675 \text{ [MeV/c]}, \quad Q^2 = 0.408 \text{ [(GeV/c)^2]} \]
CRPA : Comparison with electron scattering data

\[ ^{12}\text{C}(e, e') \]

\[
\frac{d^2 \sigma}{d\omega dQ} (\text{nb}/\text{MeV sr})
\]

- (a) \( q \approx 160 \text{[MeV/c]}, Q^2 \approx 0.026 ([\text{GeV/c}]^2) \)
- (b) \( q \approx 207 \text{[MeV/c]}, Q^2 \approx 0.042 ([\text{GeV/c}]^2) \)
- (c) \( q \approx 95 \text{[MeV/c]}, Q^2 \approx 0.009 ([\text{GeV/c}]^2) \)
- (d) \( q \approx 155 \text{[MeV/c]}, Q^2 \approx 0.024 ([\text{GeV/c}]^2) \)
- (e) \( q \approx 269 \text{[MeV/c]}, Q^2 \approx 0.071 ([\text{GeV/c}]^2) \)
- (f) \( q \approx 121 \text{[MeV/c]}, Q^2 \approx 0.015 ([\text{GeV/c}]^2) \)
- (g) \( q \approx 145 \text{[MeV/c]}, Q^2 \approx 0.021 ([\text{GeV/c}]^2) \)
- (h) \( q \approx 650 \text{[MeV/c]}, Q^2 \approx 0.381 ([\text{GeV/c}]^2) \)

\( E = 120 \text{ MeV}, \theta = 90^\circ \)
\( E = 120 \text{ MeV}, \theta = 145^\circ \)
\( E = 160 \text{ MeV}, \theta = 36^\circ \)
\( E = 161 \text{ MeV}, \theta = 50^\circ \)
\( E = 160 \text{ MeV}, \theta = 145^\circ \)
\( E = 200 \text{ MeV}, \theta = 36^\circ \)
\( E = 240 \text{ MeV}, \theta = 36^\circ \)
\( E = 440 \text{ MeV}, \theta = 145^\circ \)
$d^2 \sigma / d\omega d\Omega (\text{nb}/\text{MeV sr})$
$^{16}\text{O}(e, e')$

- Good overall agreement with e-scattering data


MiniBooNe $\nu_\mu$

- Satisfactory general agreement
- Good agreement for forward scattering
- Missing strength for low $T_\mu$, backward scattering
MiniBooNe $\bar{\nu}_\mu$

- Good general agreement
- Good agreement for forward scattering
- Missing strength for high $T_\mu$, backward scattering
- Better agreement with data than neutrino cross sections
neutrino vs anti-neutrinos

E=700 MeV

T2K $\nu_\mu$

- General agreement quite good
- Missing strength for low $p_\mu$
$^{40}\text{Ar}$
Forward scattering

\[ \frac{d^2 \sigma}{dp_\mu \, dc_{\mu}} \times \Phi_{T2K} \]

- \( E_{\nu_\mu} = 300, 350, 400, ..., 900, 950, 1000 \text{ MeV} \)
- \( \cos \theta_{\mu} = 0.97 \)
Short-range correlations

- SRC : short-range repulsive, tensor component of the nuclear force
- Individual nucleons receive large momenta compared to the Fermi momentum
- The short-range repulsive character of the nuclear force, which correlates with the Pauli exclusion principle, results in a large mean free path of the nucleons with respect to the size of the nucleus
- In an independent particle model nucleons move independently from each other in a mean field
- This approach fails to capture short-range features of nucleon-nucleon correlations

IPM single-particle orbitals are depleted by SRC and higher energy levels are populated
Two-body density:

\[ \rho^{[2]}(r_1, r_2) = \rho^{[1]}(r_1) \rho^{[1]}(r_2) g(r_{12}) \]

Independent particle model

Correlation function

\[ g(r_{12}) \]

Compare: \(^{12}\text{C} \ r \approx 2.86 \text{ fm} \]
Short-range correlations

\[ |\Psi\rangle = \frac{1}{\sqrt{N}} \hat{G} |\Phi\rangle \quad \text{with} \quad \hat{G} \approx \hat{S} \left( \prod_{i < j}^{A} \left[ 1 + \hat{l}(i, j) \right] \right) \]

\[ \hat{l}(i, j) = -g_c(r_{ij}) + f_{\sigma\tau}(r_{ij}) (\vec{\sigma}_i \cdot \vec{\sigma}_j) (\vec{\tau}_i \cdot \vec{\tau}_j) \]

\[ + f_{t\tau}(r_{ij}) \hat{S}_{ij} (\vec{\tau}_i \cdot \vec{\tau}_j), \]

\[ \hat{S}_{ij} = \frac{3}{r_{ij}^2} (\sigma_i \cdot r_{ij})(\sigma_j \cdot r_{ij}) - (\sigma_i \cdot \sigma_j) \]

\[ \langle \Psi_f | \hat{J}^{\text{nuc}}_{\mu} | \Psi_i \rangle = \frac{1}{\sqrt{N_i N_f}} \langle \Phi_f | \hat{J}^{\text{eff}}_{\mu} | \Phi_i \rangle \]

\[ \hat{J}^{\text{eff}}_{\mu} \approx \sum_{i=1}^{A} \hat{J}^{[1]}_{\mu}(i) + \sum_{i < j}^{A} \hat{J}^{[1],\text{in}}_{\mu}(i, j) + \left[ \sum_{i < j}^{A} \hat{J}^{[1],\text{in}}_{\mu}(i, j) \right] l(i, j) \]
Short-range correlations affect 1-nucleon and 2-nucleon knockout processes.

The SRC-prone nucleon pairs are predominantly in a back-to-back configuration with a small center-of-mass and high relative momentum.

Final-state interactions for the outgoing nucleons affect the experimental observations.

Data: CLAS $A(e,e',pN)$ data
- Strength residing in restricted part of phase space
- $p_b \approx p_b^{ave}$
- Quasi-deuteron kinematics

Figure 4.5: The $^{12}\text{C}(\nu_\mu, \mu^- N_a N_b)$ cross section ($N_a = p$, $N_b = p'$, n) at $\epsilon_{\nu_\mu} = 750$ MeV, $\epsilon_\mu = 550$ MeV, $\theta_\mu = 15^\circ$ and $T_p = 50$ MeV for in-plane kinematics. Left with SRCs, right with MECs, the bottom plot shows the $(\theta_a, \theta_b)$ regions with $P_{12} < 300$ MeV/c.
mass dependence $\sim A^{1.12}$: soft!
- Predominantly pn, s-pairs
- Universal over mass table
- Tensor force dominates at short distances

- Reduction of transverse response
- Enhancement of Coulomb-longitudinal

Vector and axial contributions have comparable strength
Tensor often dominates, but not for all kinematics
Vector and axial contributions have comparable strength
Tensor often dominates, but not for all kinematics
pn pairs dominate
SRC neutrinos 1p1h+2p2h

T. Van Cuyck et al
Meson-exchange currents

When an electroweak boson interacts with a pair of nucleons which are correlated through the exchange of a meson, this will cause the knockout of one or both of the particles from the nucleus. The boson was interacting with a current consisting of two nucleons, a two-body current, called a MEC.

Seagull

Pion in flight

Delta currents
Correlation currents

Already included in mean field models!

Contributions of heavier mesons:

\[ m_\pi \approx 135 \text{MeV}, \ m_\rho \approx 775 \text{MeV}, \ m_\omega \approx 782 \text{MeV} \]
III. MEC in 1p1h and 2p2h

Axial contributions:

\[ \tilde{\rho}_A^{[2],\text{axi}}(q) = \frac{i}{g_A} \left( \frac{f_{\pi NN}}{m_\pi} \right)^2 (I_V) \left( F_\pi(q_2^2) \Gamma_\pi^2(q_2^2) \frac{\sigma_2 \cdot q_2}{q_2^2 + m_\pi^2} - F_\rho(q_2^2) \Gamma_\rho^2(q_2^2) \frac{\sigma_1 \cdot q_1}{q_1^2 + m_\pi^2} \right) \]

Only seagull have axial counterpart
- timelike
- Partially constrained by PCAC
- Non-relativistic reduction not unambiguous

\[
\hat{\rho}_A^{[2],\text{axi}}(q) = \frac{i}{g_A} \left( \frac{f_{NN}}{m_\pi} \right)^2 (I_V) \left( F_\pi(q_2^2) \frac{\sigma_2 \cdot q_2}{q_2^2 + m_\pi^2} - F_\pi(q_1^2) \frac{\sigma_1 \cdot q_1}{q_1^2 + m_\pi^2} \right)
\]
Seagull and PIF in neutrino 1p1h

\[ W_{CC}(\text{GeV}^{-1}) \]

\[ W_{\tau}(\text{GeV}^{-1}) \]

\[ ^{12}\text{C}, q = 300 \text{ MeV}/c \]

\[ ^{12}\text{C}, q = 400 \text{ MeV}/c \]

\[ ^{12}\text{C}, q = 550 \text{ MeV}/c \]

\[ \omega (\text{MeV}) \]

\[ \omega (\text{MeV}) \]

\[ \omega (\text{MeV}) \]

Seagull and PIF in neutrino 2p2h
Summary

- Long- and short-range correlations in QE-like cross sections
- CRPA calculations provide extra strength for forward scattering arising from low-energy excitations
- This might affect CCQE neutrino cross sections as measured by MiniBooNe, T2K, ...
- SRC and MEC affect 1- and 2-nucleon knockout processes

- Scheme-dependent separation
- Double counting