# some theory tools for neutrino interactions with nucleons

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*thanks Andreas and Pilar!*

# Overview

- topic 0: why

- topic 1: amplitude analysis and z expansion

- topic 2: muon capture and nucleon axial radius

- topic 3: radiative corrections and SCET

# topic 0. why

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why bother with neutrino interactions? Isn't this too hard/ too different/ somebody else's problem?

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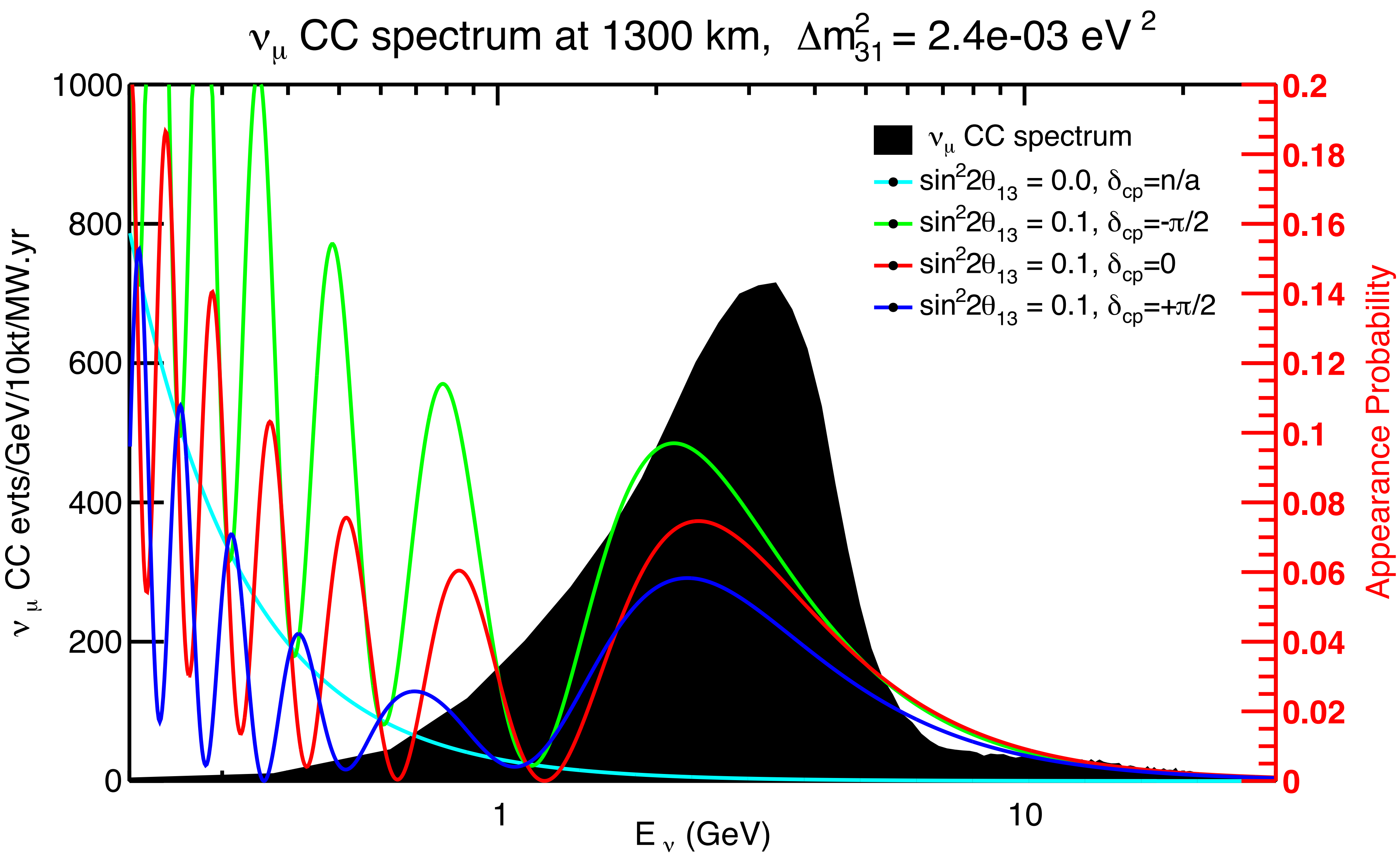
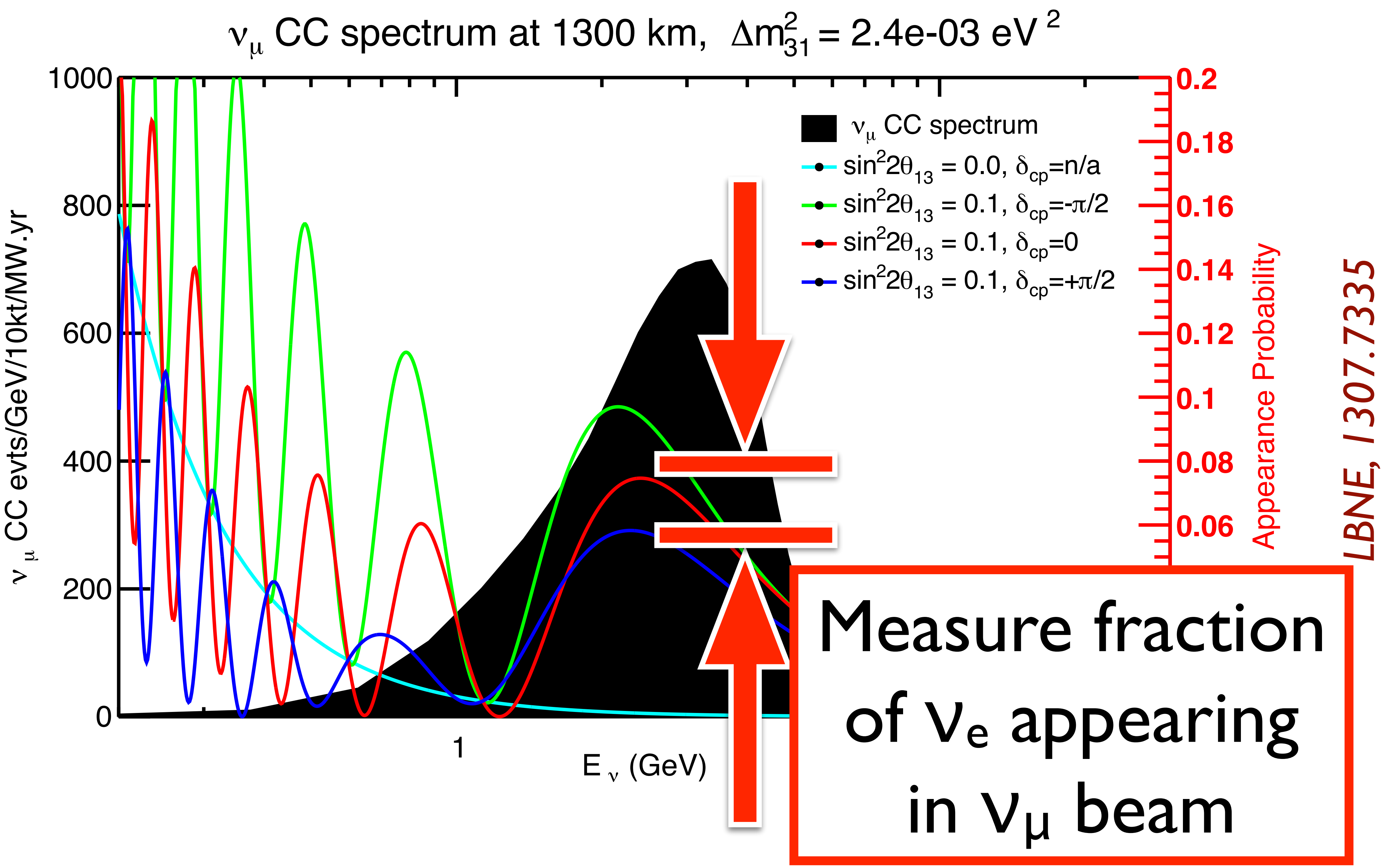
**"The good news is that it's not my problem"**

long baseline neutrino oscillation experiment is **simple** in **conception**:




*LBNE, 1307.7335*

long baseline neutrino oscillation experiment is **simple** in **conception**:



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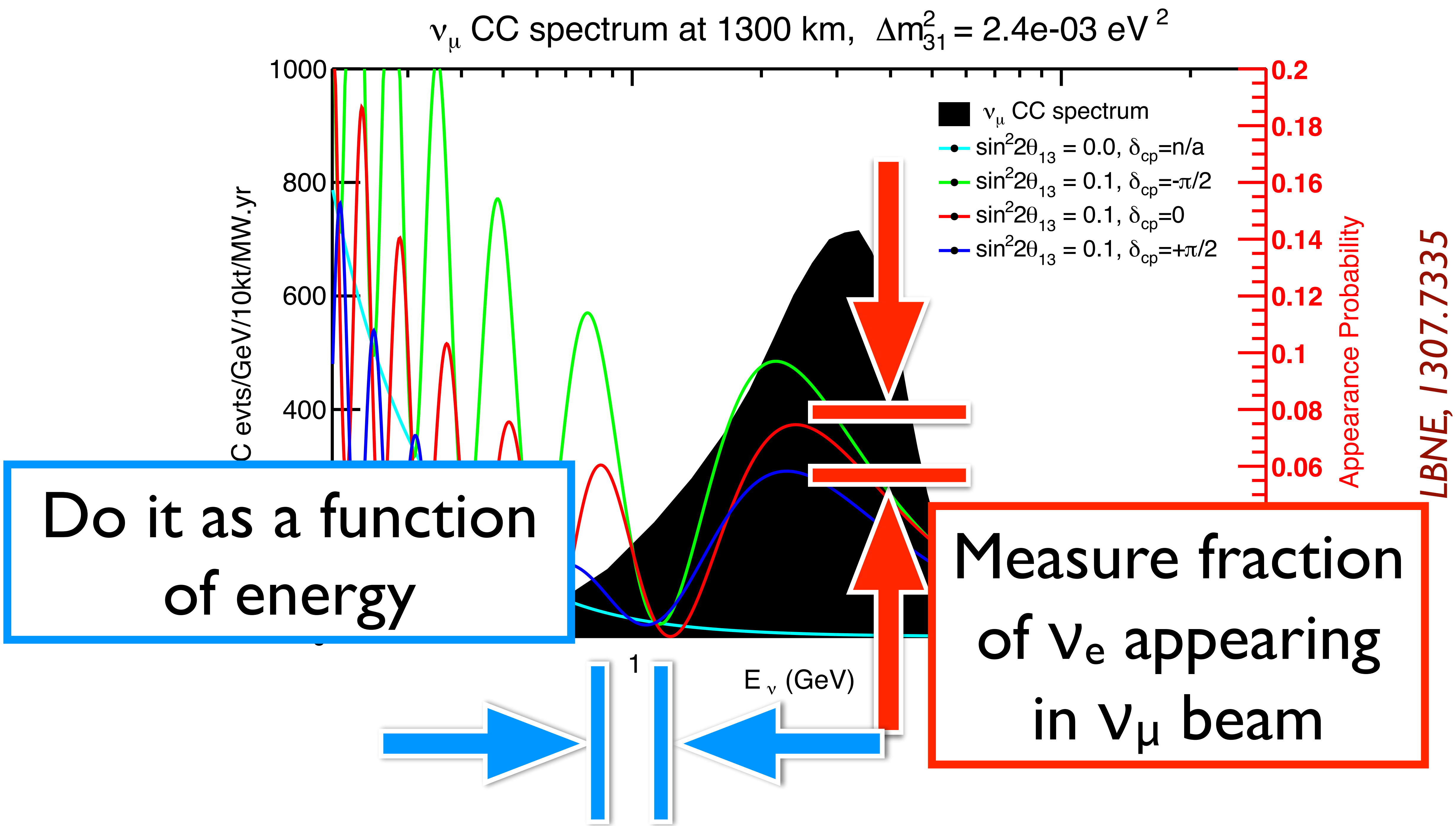

long baseline neutrino oscillation experiment is **difficult** in **practice**:

simple picture is complicated by

- $\nu_e$ versus $\nu_\mu$ cross section differences

need theory for $\sigma_{\nu e}/\sigma_{\nu\mu}$, at ~% precision of measurement

and also

- intrinsic $\nu_e$ component of beam
- degeneracy of uncertainty in detector response and neutrino interaction cross sections
- imperfect energy reconstruction

aided by near detector but

- beam divergence and oscillation (near flux$\neq$far flux)

need theory for $\sigma_{\nu\mu}$, at a precision depending on the experimental capabilities current paradigm:

constrain neutrino interactions by

- determining nucleon level amplitudes
- parameterizing/measuring/calculating nuclear modifications

folk paradigms:

constrain neutrino interactions by

- starting at the quark level
- computing nuclear response

“perfect theory”

constrain neutrino interactions by

- starting directly at the nuclear level
- parameterizing and measuring every cross section

“perfect expt.”

in any paradigm:

near detector has access to primarily $\nu_\mu$ neutrinos

$\nu e$ appearance signal is directly impacted by $\nu_\mu/\nu_e$ cross section differences

- kinematics
- 2nd class currents (G parity violation)
- radiative corrections (QED and EW)
- signal definition

having talked the talk, do some walking:

- $\nu_\mu/\nu_e$ in the time reversal process ($\mu\ p \to \nu\ n$)
- nucleon input uncertainty (e-p, $\nu\ d \to \nu\ n$)
- radiative corrections at GeV (e-p)

nuclear corrections: see talks of W. Van Order, S. Pastore, A. Ankowski, N. Jachowicz, A. Lovato. experiment: S. Bolognesi; lots of references: NUSTEC white paper 1706.03621

## Notes:

beyond neutrino oscillations related applications relying on quantitative nucleon structure:

- fundamental constants (probable 7 sigma shift in Rydberg)
- sigma terms and WIMP-DM direct detection
- $g_A$ and BBN
- …

QED is “easy”. But QED + nucleon structure is “hard”

entering a precision realm where percent level corrections to nucleon structure need to be calculated, not just estimated

## topic 1. amplitude analysis and z expansion

first, e-p elastic scattering

second, $\nu$-n CC scattering

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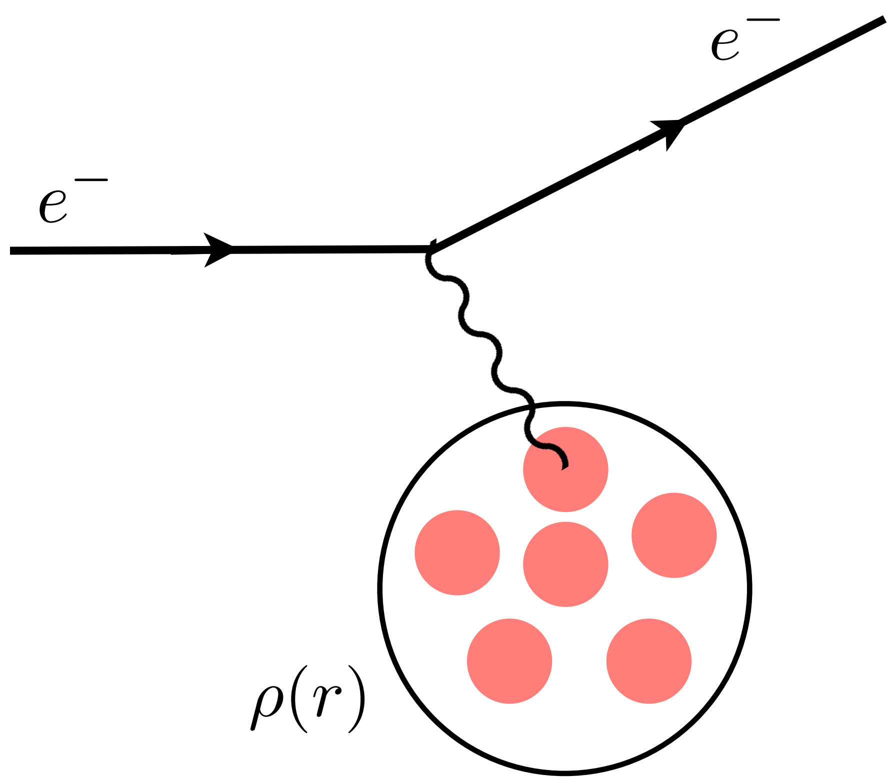
# recall scattering from extended classical charge distribution:

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{pointlike}} |F(q^2)|^2$$

$$F(q^2) = \int d^3r\, e^{i\boldsymbol{q}\cdot\boldsymbol{r}} \rho(\boldsymbol{r})$$

$$= \int d^3r \left[1 + i\boldsymbol{q}\cdot\boldsymbol{r} - \frac{1}{2}(\boldsymbol{q}\cdot\boldsymbol{r})^2 + \ldots\right] \rho(r)$$

$$= 1 - \frac{1}{6}\langle r^2 \rangle \boldsymbol{q}^2 + \ldots$$




for the relativistic, QM, case, *define* radius as slope of form factor

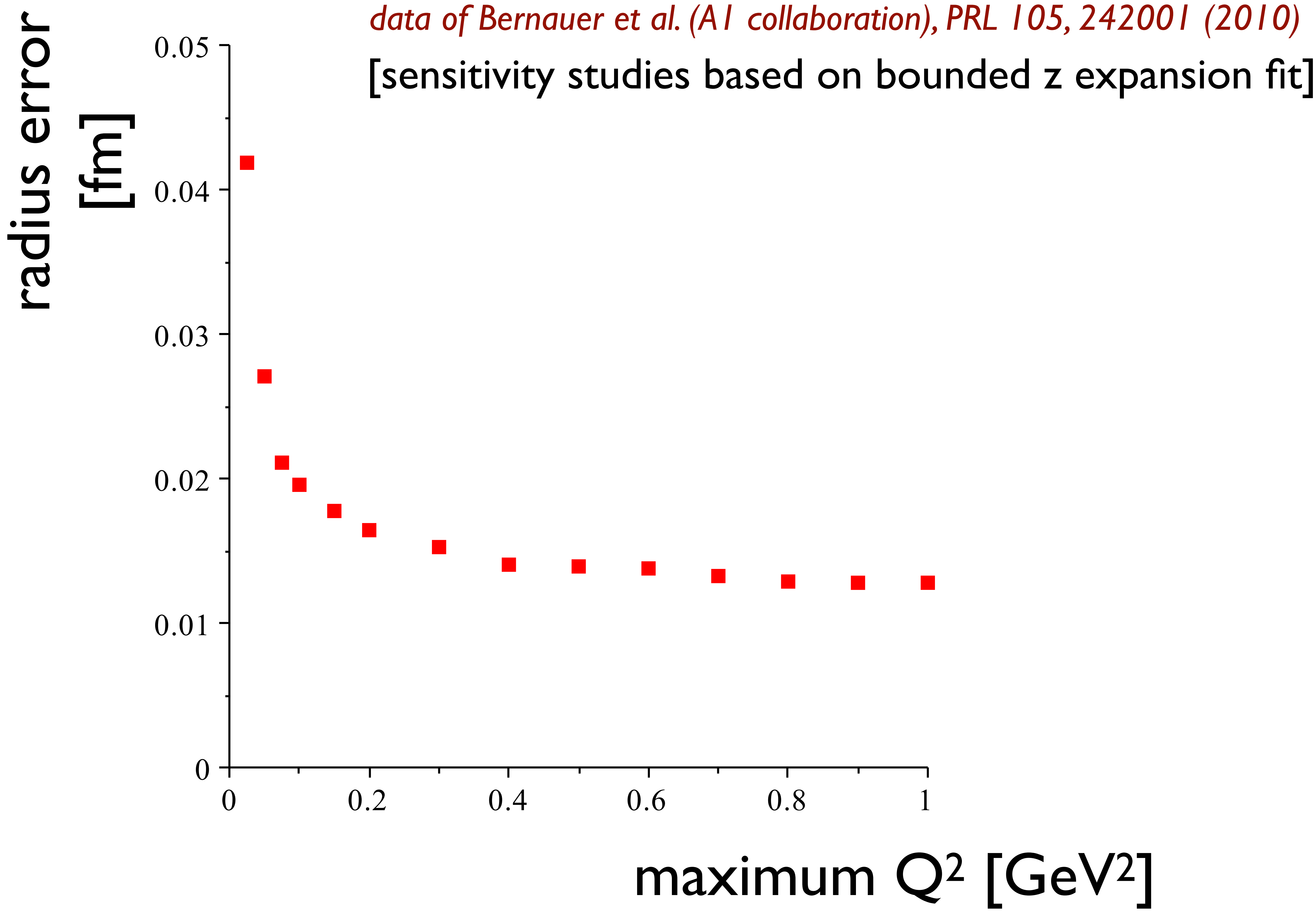
$$\boxed{r_E^2 \equiv 6 \frac{d}{dq^2} G_E(q^2)\Big|_{q^2=0}}$$

*(up to radiative corrections)*

$$\langle J^\mu \rangle = \gamma^\mu F_1 + \frac{i}{2m_p}\sigma^{\mu\nu} q_\nu F_2$$

$$G_E = F_1 + \frac{q^2}{4m_p^2} F_2 \qquad G_M = F_1 + F_2$$

## Radius extraction requires data over a $Q^2$ range where a simple Taylor expansion of the form factor is invalid



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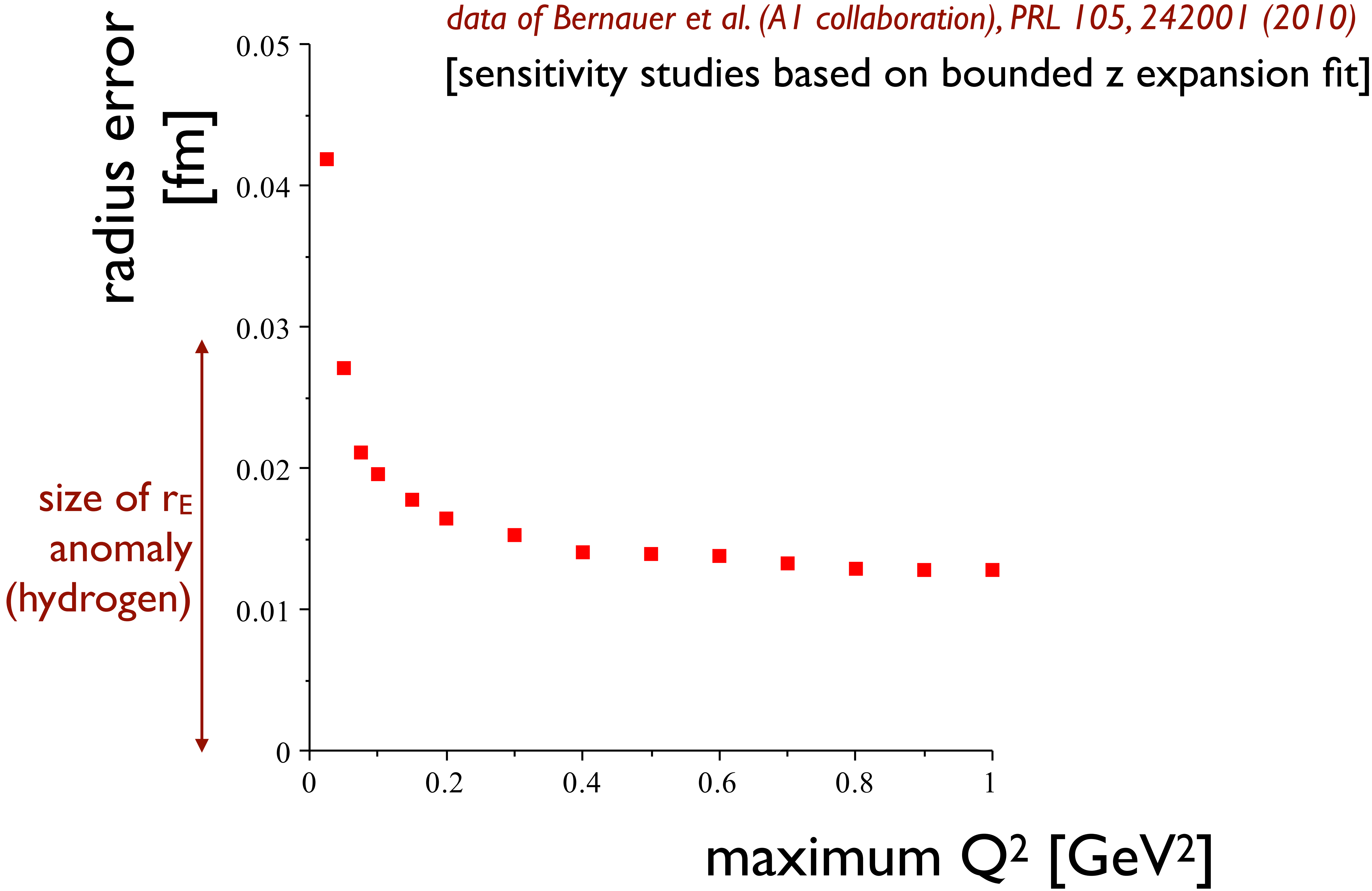

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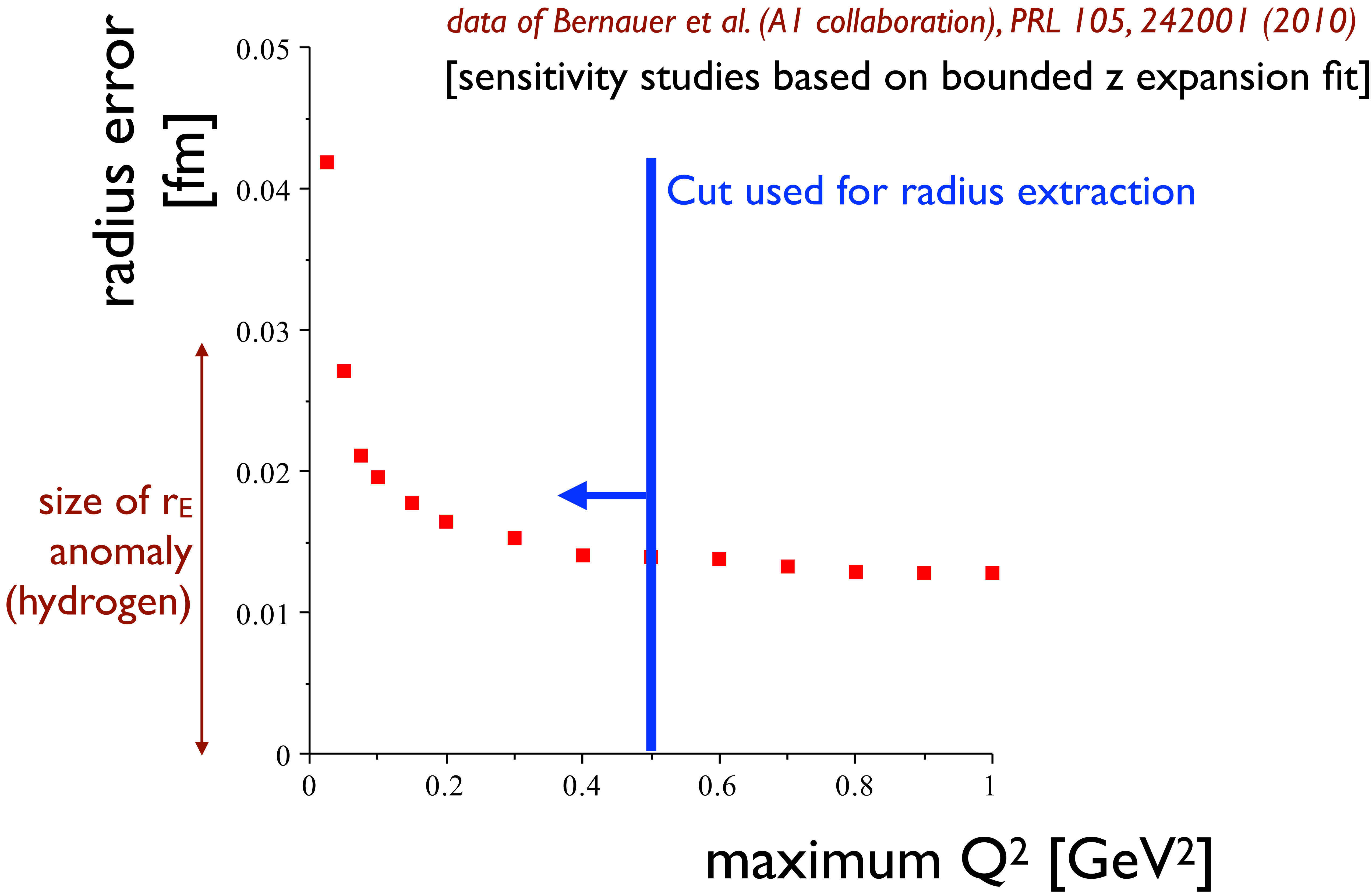

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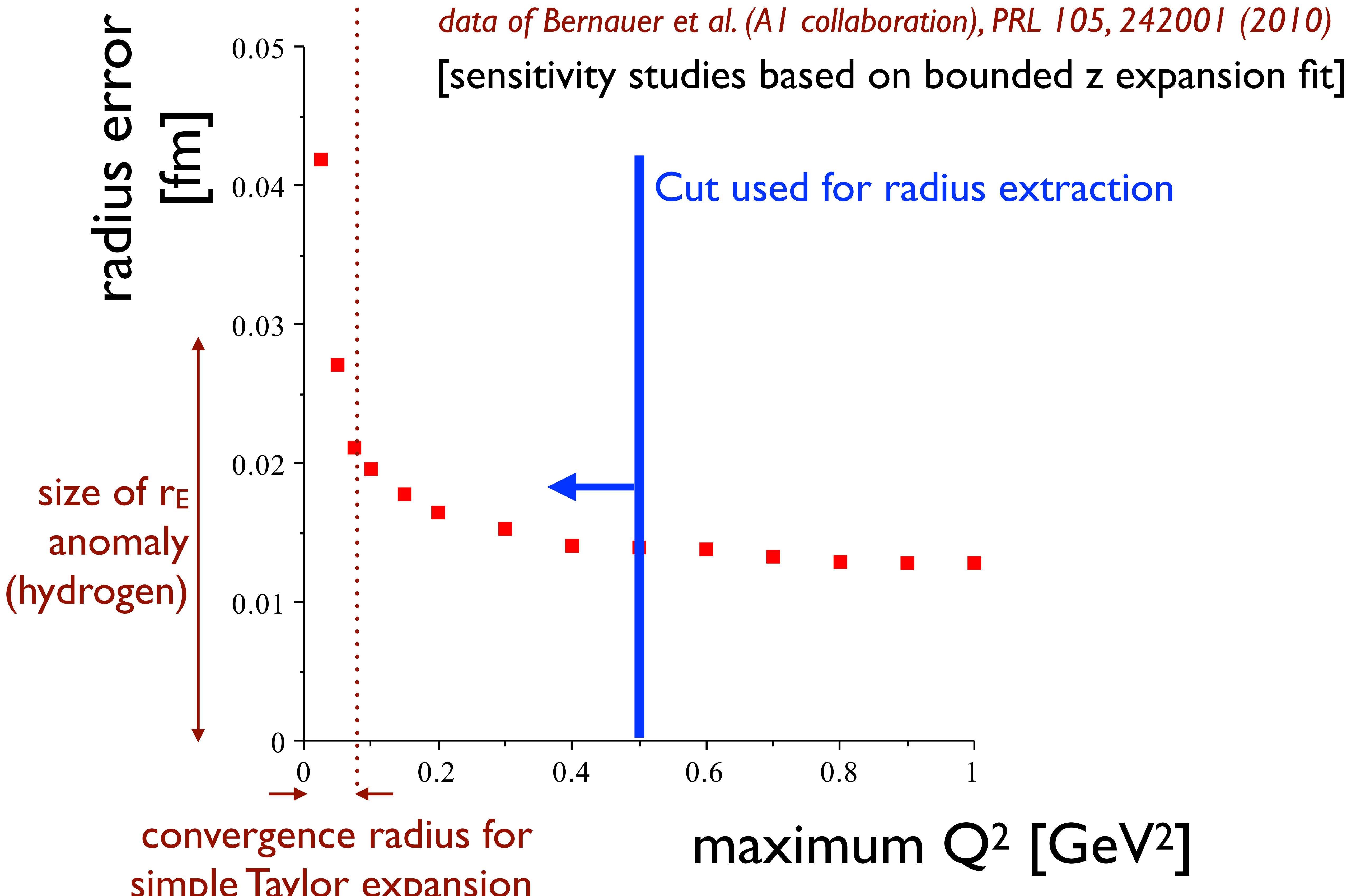
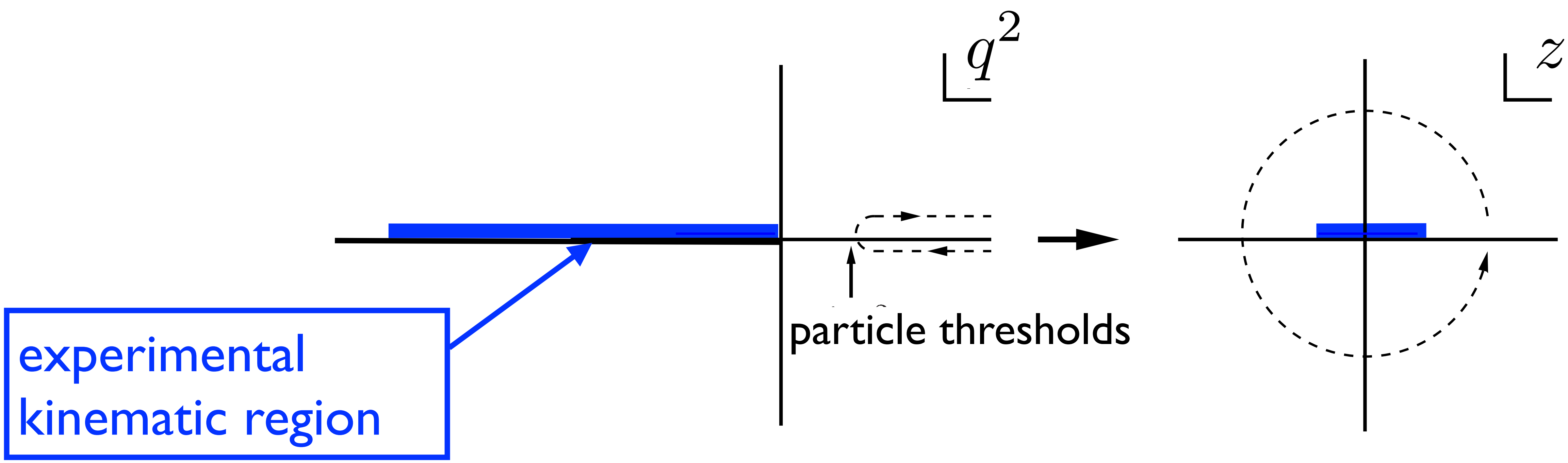


# That’s ok: underlying QCD tells us that Taylor expansion of form factor in appropriate variable is convergent

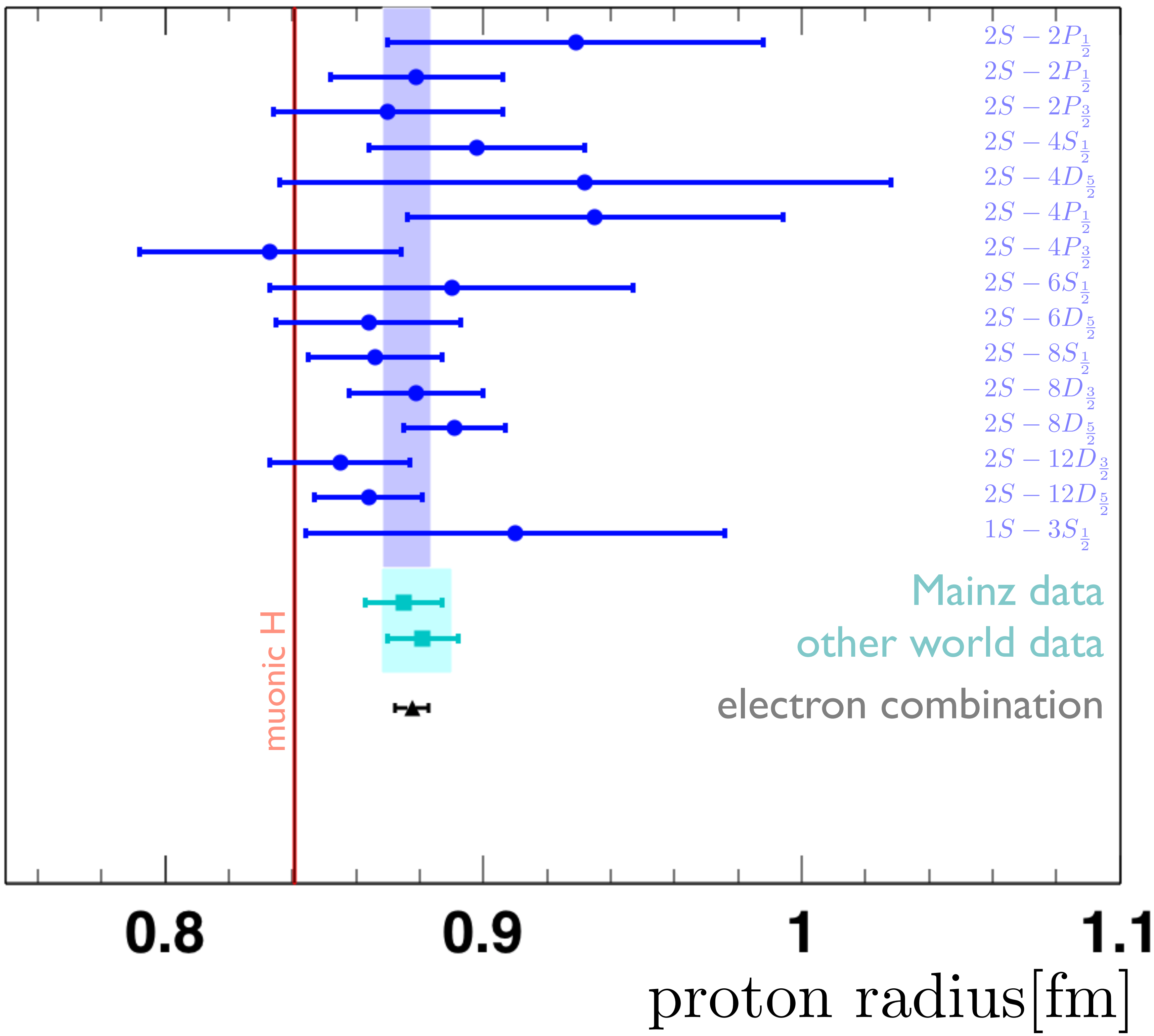



$$z(q^2, t_{\rm cut}, t_0) = \frac{\sqrt{t_{\rm cut} - q^2} - \sqrt{t_{\rm cut} - t_0}}{\sqrt{t_{\rm cut} - q^2} + \sqrt{t_{\rm cut} - t_0}}$$

$$F(q^2) = \sum_k a_k [z(q^2)]^k$$

coefficients in rapidly convergent expansion encode nonperturbative QCD

# Reanalysis of scattering data reveals strong influence of shape assumptions




**Errors larger, but discrepancy remains**

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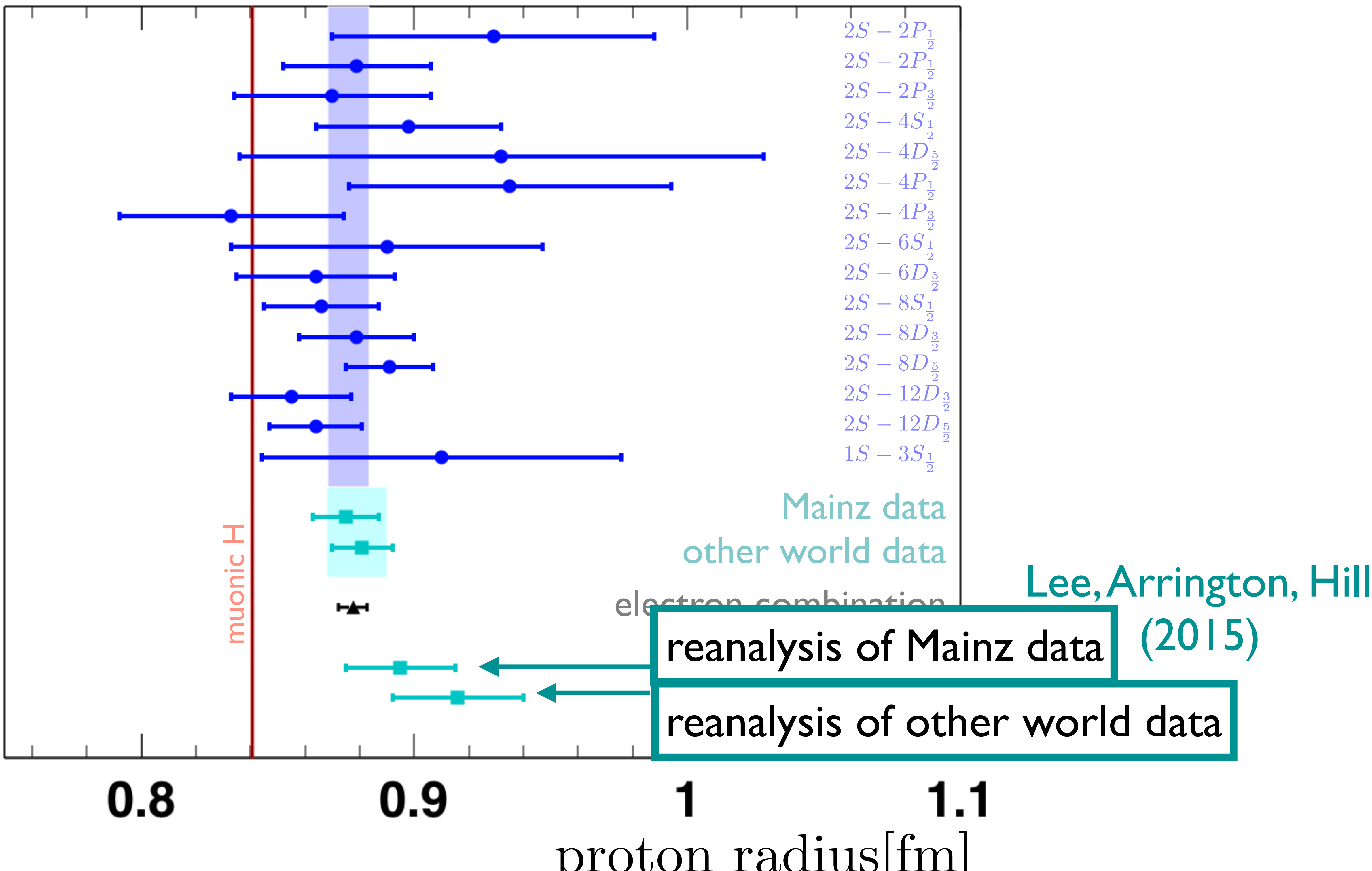
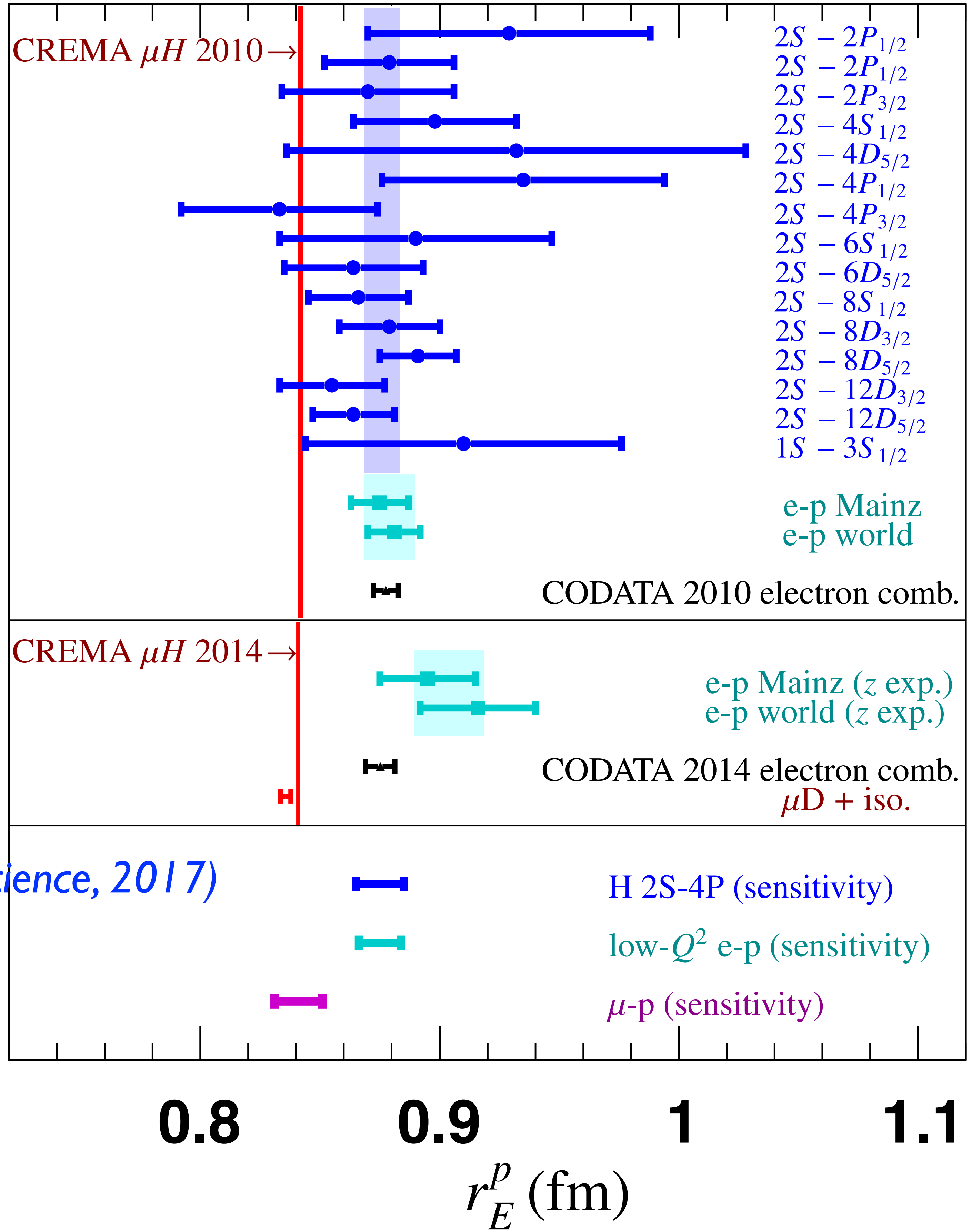



**Errors larger, but discrepancy remains**




*update: Beyer et al. (Science, 2017)*

topic 1. amplitude analysis and z expansion

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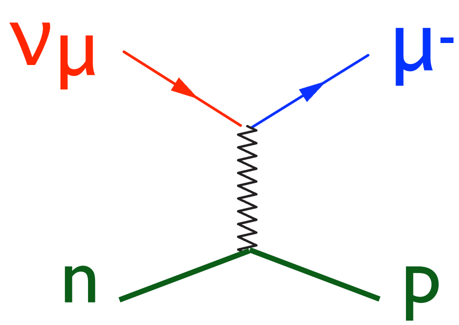
## Start with the basic process




$$\sigma(\nu n \to \mu p) = |\cdots F_A(q^2) \cdots|^2$$

poorly known axial-vector form factor

A common ansatz for $F_A$ has been employed for the last ~40 years:

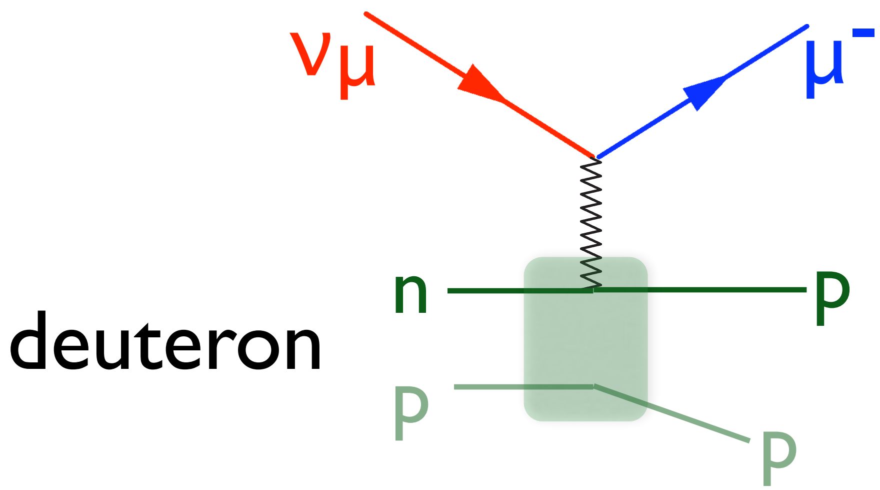
$$F_A^{\text{dipole}}(q^2) = F_A(0)\left(1 - \frac{q^2}{m_A^2}\right)^{-2}$$

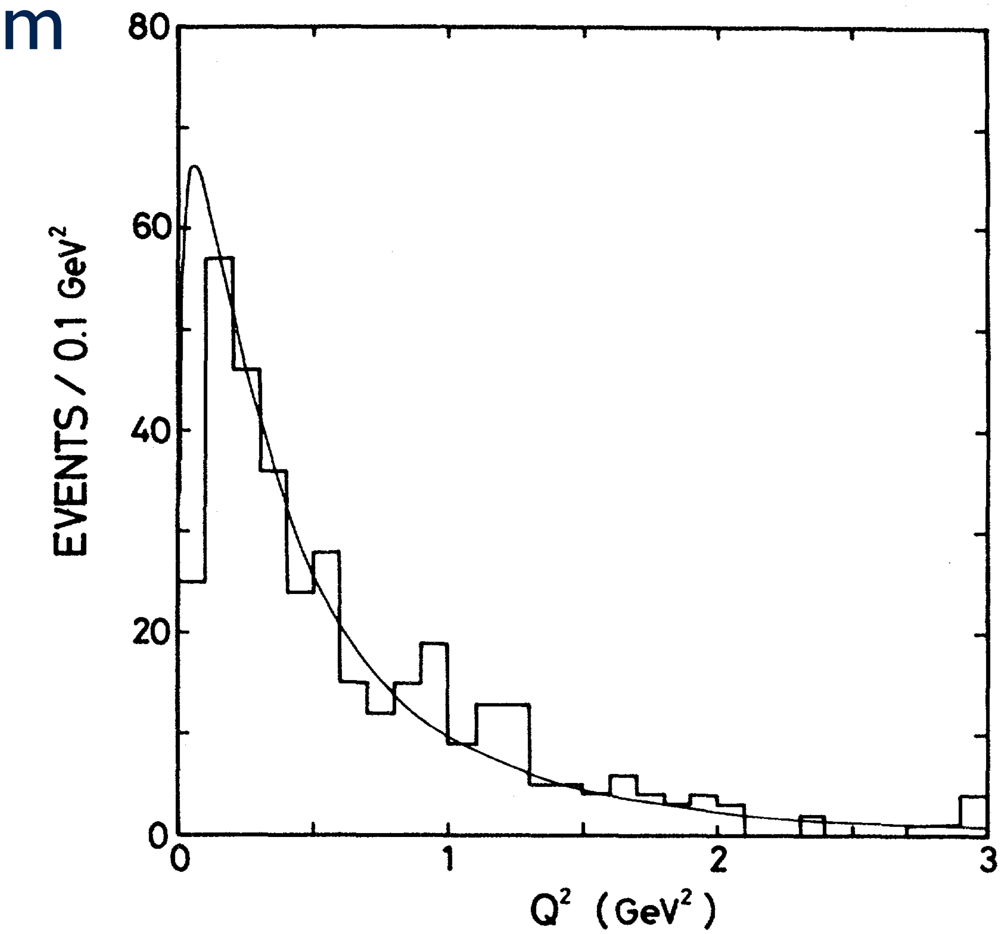
Inconsistent with QCD.

Typically quoted uncertainties are (too) small (e.g. compared to proton charge form factor!)

$$\frac{1}{F_A(0)} \frac{dF_A}{dq^2}\bigg|_{q^2=0} \equiv \frac{1}{6} r_A^2 \qquad r_A = 0.674(9)\ \text{fm}$$

# Best source of almost-free neutrons: deuterium




*Fermilab 15-foot deuterium bubble chamber, PRD 28, 436 (1983)*

## Deuterium bubble chamber data

- small(-ish) nuclear effects
- small(-ish) experimental uncertainties
- small statistics, ~3000 events in world data

*also:*

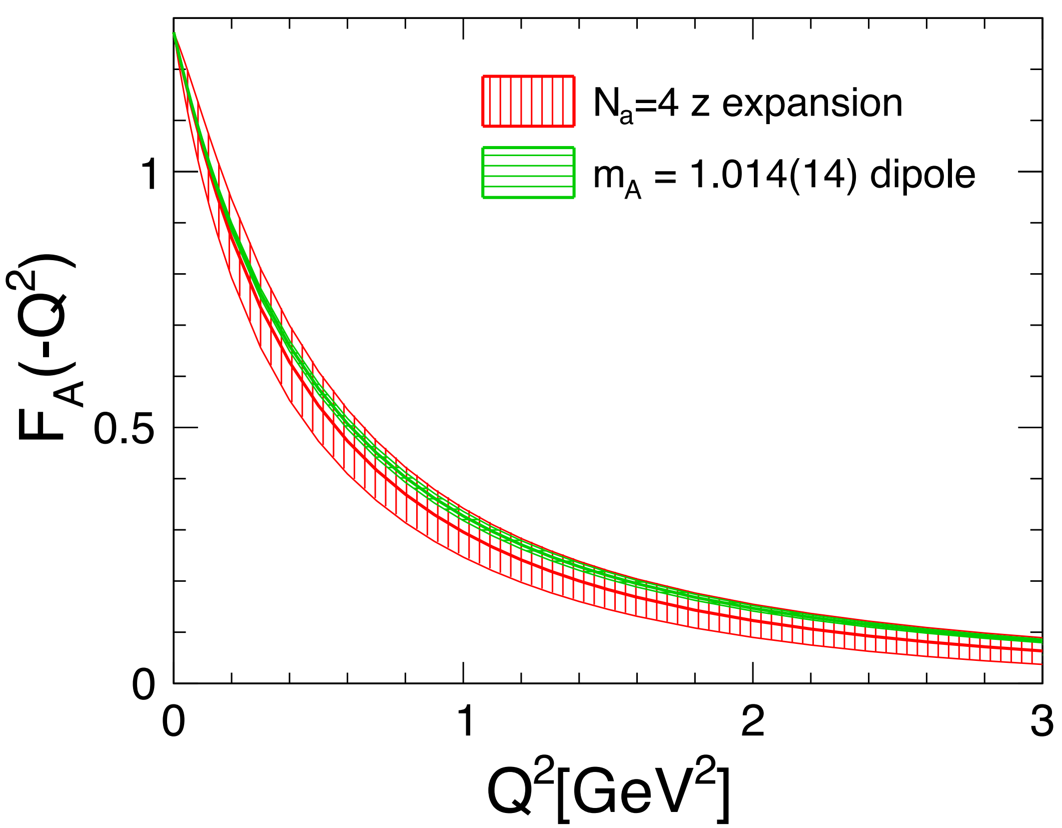
*ANL 12-foot deuterium bubble chamber, PRD 26, 537 (1982)*

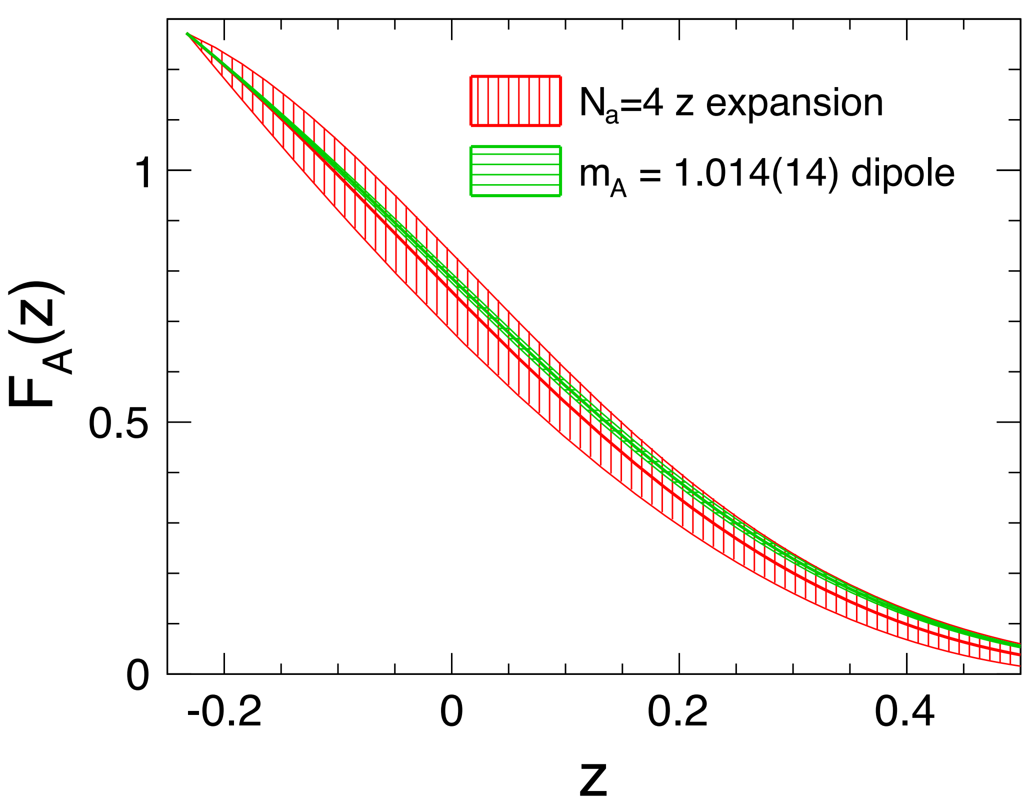
*BNL 7-foot deuterium bubble chamber, PRD23, 2499 (1981)*

- $F_A$ with complete error budget:

$$[a_1, a_2, a_3, a_4] = [2.30(13), -0.6(1.0), -3.8(2.5), 2.3(2.7)]$$

$$C_{ij} = \begin{pmatrix} 1 & 0.350 & -0.678 & 0.611 \\ 0.350 & 1 & -0.898 & 0.367 \\ -0.678 & -0.898 & 1 & -0.685 \\ 0.611 & 0.367 & -0.685 & 1 \end{pmatrix}$$



## Derived observables: 1) axial radius

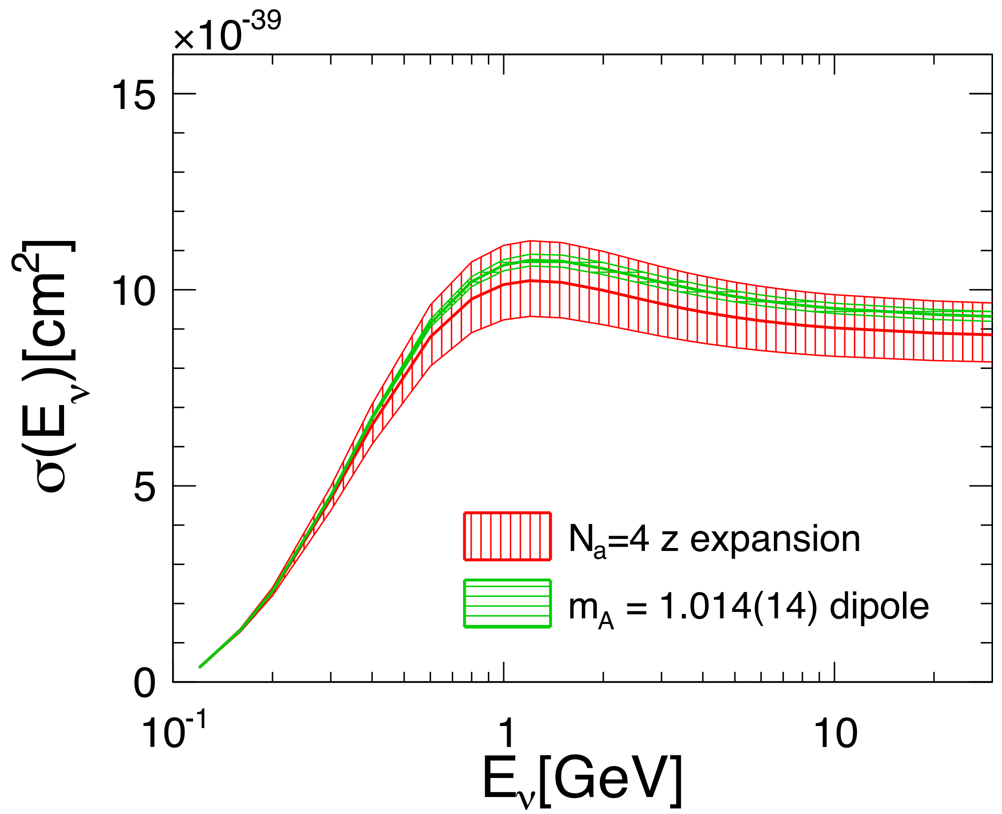
$$\frac{1}{F_A(0)} \frac{dF_A}{dq^2}\bigg|_{q^2=0} \equiv \frac{1}{6} r_A^2$$

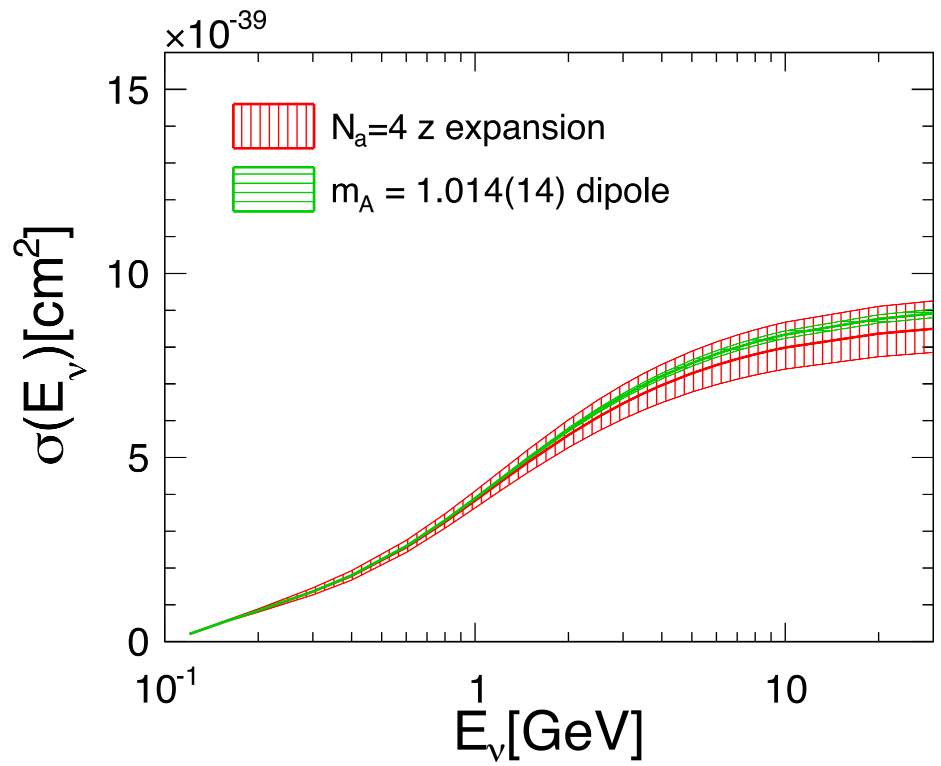
$$r_A^2 = 0.46(22)\,\mathrm{fm}^2$$

- order of magnitude larger uncertainty compared to historical dipole fits

- impacts comparison to other data, e.g. pion electroproduction, muon capture

# Derived observables: 2) neutrino-nucleon quasi elastic cross sections

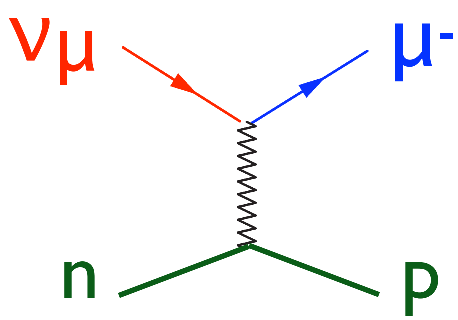


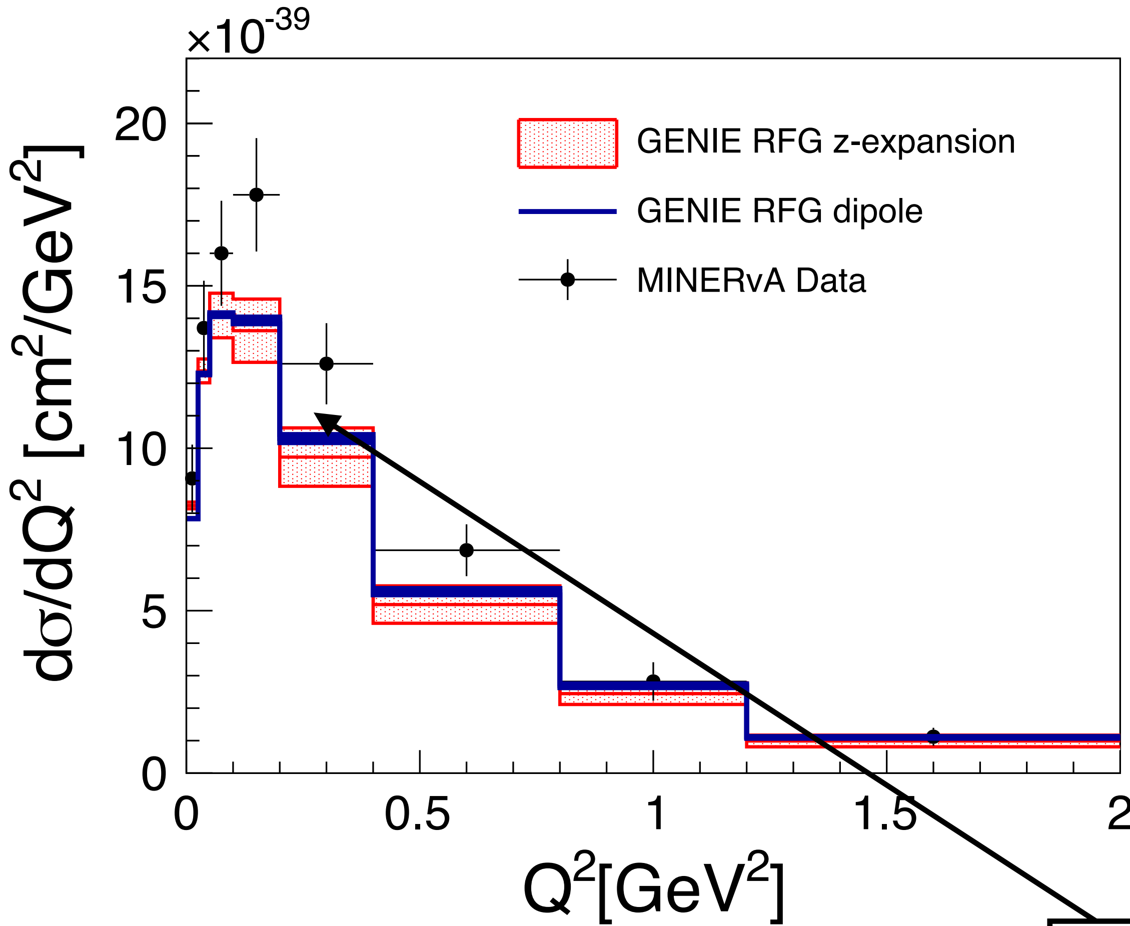




$$\sigma_{\nu n \to \mu p}(E_\nu = 1\,\mathrm{GeV}) = 10.1(0.9) \times 10^{-39}\,\mathrm{cm}^2$$

$$\sigma_{\nu n \to \mu p}(E_\nu = 3\,\mathrm{GeV}) = 9.6(0.9) \times 10^{-39}\,\mathrm{cm}^2$$

# discriminating nuclear models



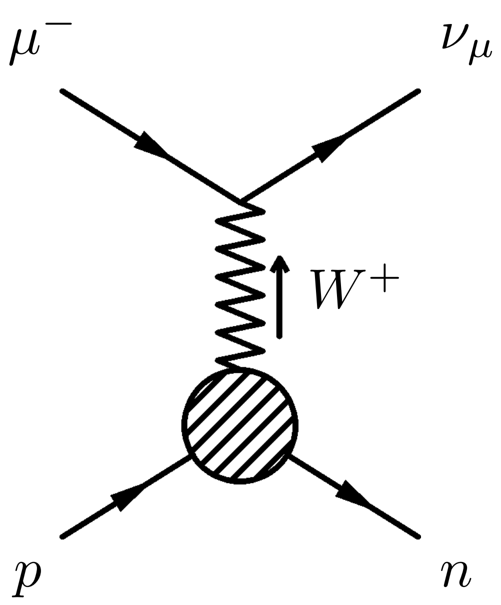




$$\sigma(\nu n \to \mu p) = |\cdots F_A(q^2) \cdots|^2$$

poorly known axial form factor

want to extract nuclear and flux effects from this comparison: but large nucleon level form factor uncertainty

## topic 2. muon capture



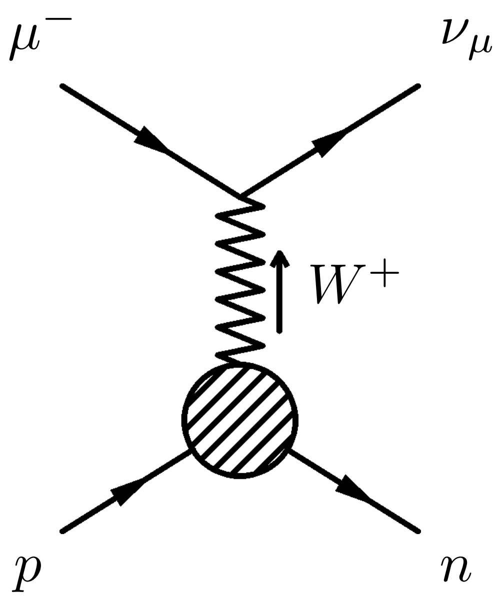

muon capture from ground state of muonic hydrogen:

- probes axial nucleon structure: FP, FA

- already competitive determination of rA

- potential for significant improvement

*from RJH, Kammel, Marciano, Sirlin 1708.08462*




$$\mathcal{L} = \mathcal{L}_{\mathrm{SM}}$$

$$\Big\downarrow \text{ perturbative matching}$$

$$\mathcal{L} = -\frac{G_F V_{ud}}{\sqrt{2}} \bar{\nu}_\mu \gamma^\mu (1-\gamma_5)\mu \, \bar{d}\gamma_\mu (1-\gamma_5) u + \text{H.c.} + \ldots$$

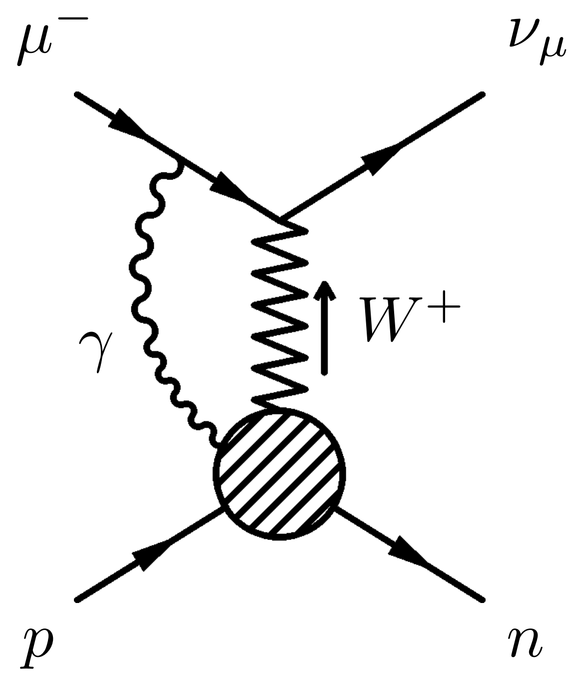
$$\Big\downarrow \text{ nonperturbative matching}$$

$$H = \frac{p^2}{2m_r} - \frac{\alpha}{r} + \delta V_{\mathrm{VP}} - i\frac{G_F^2 |V_{ud}|^2}{2} \left[c_0 + c_1 (\boldsymbol{s}_\mu + \boldsymbol{s}_p)^2\right] \delta^3(\boldsymbol{r})$$

$$\Lambda = \underbrace{G_F^2 |V_{ud}|^2}_{\text{weak}} \times \underbrace{[c_0 + c_1 F(F+1)]}_{\text{hadronic}} \times \underbrace{|\psi_{1\mathrm{S}}(0)|^2}_{\text{atomic}} + \ldots$$

factorization: weak hadronic atomic

$$c_0 = \frac{E_\nu^2}{2\pi M^2}(M - m_n)^2 \left[ \frac{2M - m_n}{M - m_n} F_1(q_0^2) + \frac{2M + m_n}{M - m_n} F_A(q_0^2) - \frac{m_\mu}{2m_N} F_P(q_0^2) \right.$$
$$\left. + (2M + 2m_n - 3m_\mu) \frac{F_2(q_0^2)}{4m_N} \right]^2$$




expansion in small quantities:

$$\epsilon \sim \alpha \sim \frac{m_\mu^2}{m_\rho^2} \sim \frac{m_u - m_d}{m_\rho} \lesssim 10^{-2}$$

- axial radius enters at first order in epsilon, so need all other first order corrections (to ~10%, for a 10% measurement of rA2)

- will see that other corrections are at first-and-a-half order; need to ensure against numerical enhancements (need these to ~100%)

momentum expansion:

sensitivity to momentum dependence in the capture process

$$q_0^2 \equiv m_\mu^2 - 2m_\mu E_\nu = -0.8768\, m_\mu^2 \quad \sim \epsilon$$

in our power counting, rA2 competes with gP, and other well-determined quantities (g≡normalization, r2≡slope)

$$1 + \Big[g_1,\ g_A\Big] + \sqrt{\epsilon}\Big[g_2\Big] + \epsilon\Big[r_1^2,\ r_A^2,\ g_P\Big] + \ldots$$

gA: neutron lifetime ✔

g1,g2,r12: e-p, e-n scattering + H, muH (see below) ✔ ✔

$$F_P(q_0^2) = \frac{2m_N g_{\pi NN} f_\pi}{m_\pi^2 - q_0^2} - \frac{1}{3}\, g_A\, m_N^2\, r_A^2 + \ldots$$

gpiNN: pion-nucleon scattering, and NN scattering ✔

## α expansion:

$$\mathrm{RC} = \underbrace{\mathrm{RC(electroweak)}}_{\text{matching, running in 4 Fermi theory}} + \overbrace{\mathrm{RC(finite\ size)}}^{\text{hadronic matrix element}} + \underbrace{\mathrm{RC(electron\ VP)}}_{\text{computed within QM}}$$

$$\mathrm{RC(electroweak)} = \frac{\alpha}{2\pi}\left[\overbrace{4\log\frac{m_Z}{m_p}}^{\text{large log}} \underbrace{-\,0.595 + 2C}_{\text{finite terms (estimate with OPE)}} + \overbrace{g(m_\mu, \beta_\mu = 0)}^{\text{Sirlin g function (IR subtraction)}}\right] + \cdots = +0.0237(10)$$ ✔

$$\mathrm{RC(finite\ size)} = -0.005(1)$$

(should be done better: computed in large nucleus ansatz rE>>rA) ✔

$$\mathrm{RC(electron\ VP)} = +0.0040(2).$$ ✔

isospin violation:

vector form factors: CC from isovector NC

deviations in F1(0): second order in IV (definition of CVC) ✔

deviations in F1(q2): first order in IV plus first order in q2 ✔

deviations in F2(0): first order in IV plus 0.5 order in kinematic prefactor (numerical estimate: 3.2e-4 << %) ✔

2nd class currents:

$$\langle n|(V^\mu - A^\mu)|p\rangle = \bar{u}_n\Big[F_1(q^2)\gamma^\mu + \frac{iF_2(q^2)}{2m_N}\sigma^{\mu\nu}q_\nu - F_A(q^2)\gamma^\mu\gamma^5 - \frac{F_P(q^2)}{m_N}q^\mu\gamma^5 + \frac{F_S(q^2)}{m_N}q^\mu - \frac{iF_T(q^2)}{2m_N}\sigma^{\mu\nu}q_\nu\gamma^5\Big]u_p + \dots ,$$

contribution of FS,FT: first order in IV plus 0.5 order in kinematic prefactor ✔

results:

$$\bar{g}_P{}^{\rm MuCap}\big|_{r_A^2=0.46(22)\,\rm fm^2} = 8.19\;(48)_{\rm exp}\;(69)_{\bar{g}_A}\;(6)_{\rm RC} = 8.19(84)$$

$$\bar{g}_P{}^{\rm theory} = 8.25(25)$$

$$g_{\pi NN}^{\rm MuCap} = 13.04\;(72)_{\rm exp}\;(8)_{g_A}\;(67)_{r_A^2}\;(10)_{\rm RC} = 13.04(99)$$

$$g_{\pi NN}^{\rm external} = 13.12(10)$$

turning the tables, take QCD for granted and extract rA2:

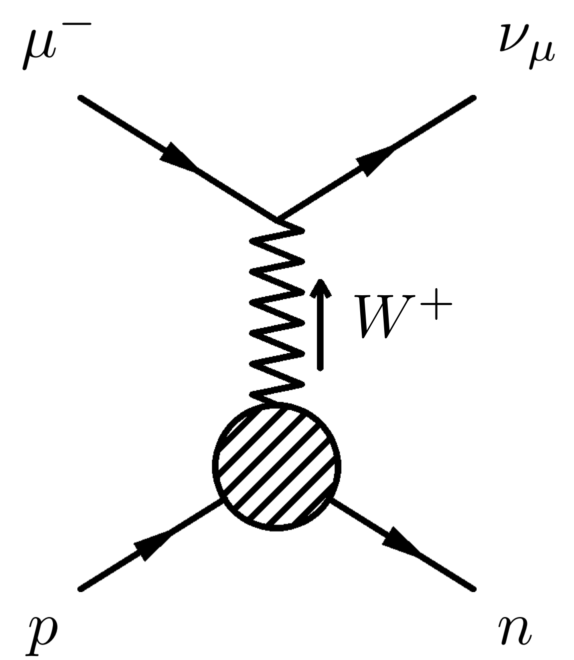
$$r_A^2({\rm MuCap}) = 0.43\;(24)_{\rm exp}\;(3)_{g_A}\;(3)_{g_{\pi NN}}\;(3)_{\rm RC} = 0.43(24)\;{\rm fm}^2$$

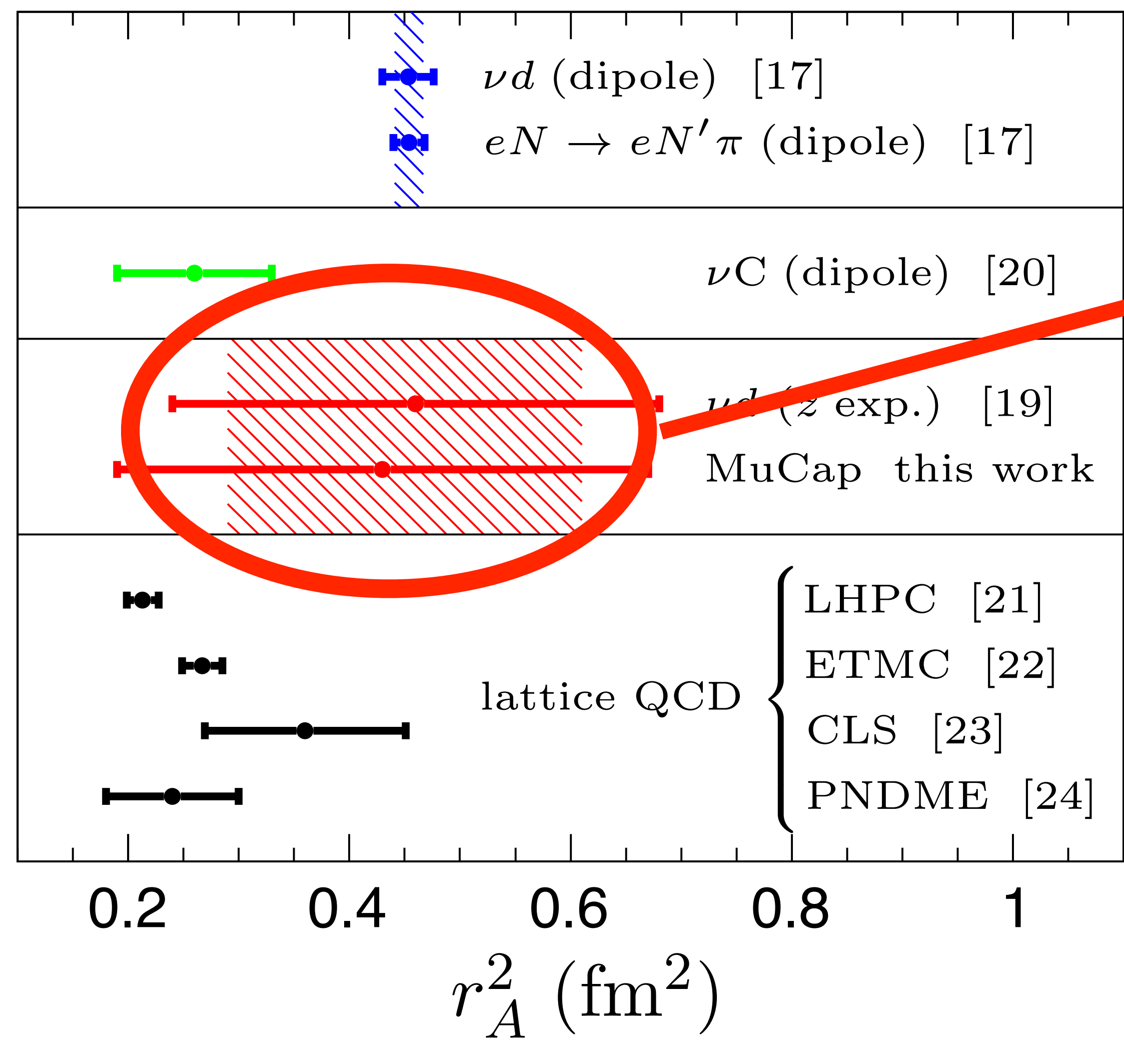
competitive with other methods with existing data, and potential for improvement

$$\delta r_A^2(\text{future exp.}) = (0.08)_{\rm exp}\;(0.03)_{g_A}\;(0.03)_{g_{\pi NN}}\;(0.03)_{\rm RC} = 0.10\;{\rm fm}^2$$

factor 3 improvement

# muon capture constraints



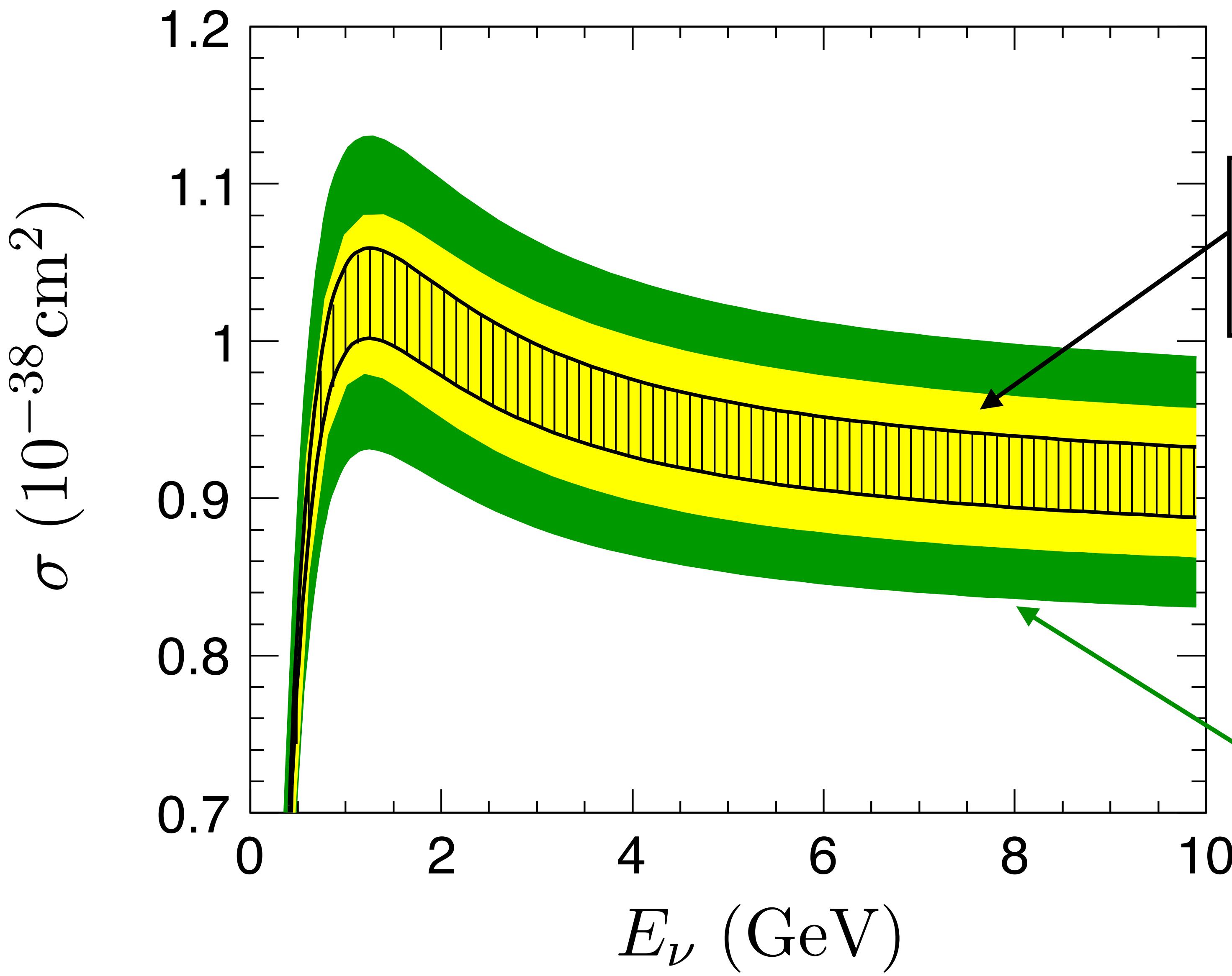

complete error budgets

*RJH, Kammel, Marciano, Sirlin 1708.08462*

*lattice average: see also Yao, Alvarez-Ruso, Vicente-Vacas 1708.08776 [ rA2=0.26(4) ]*

- potential factor ~3 improvement from next generation muon capture experiment

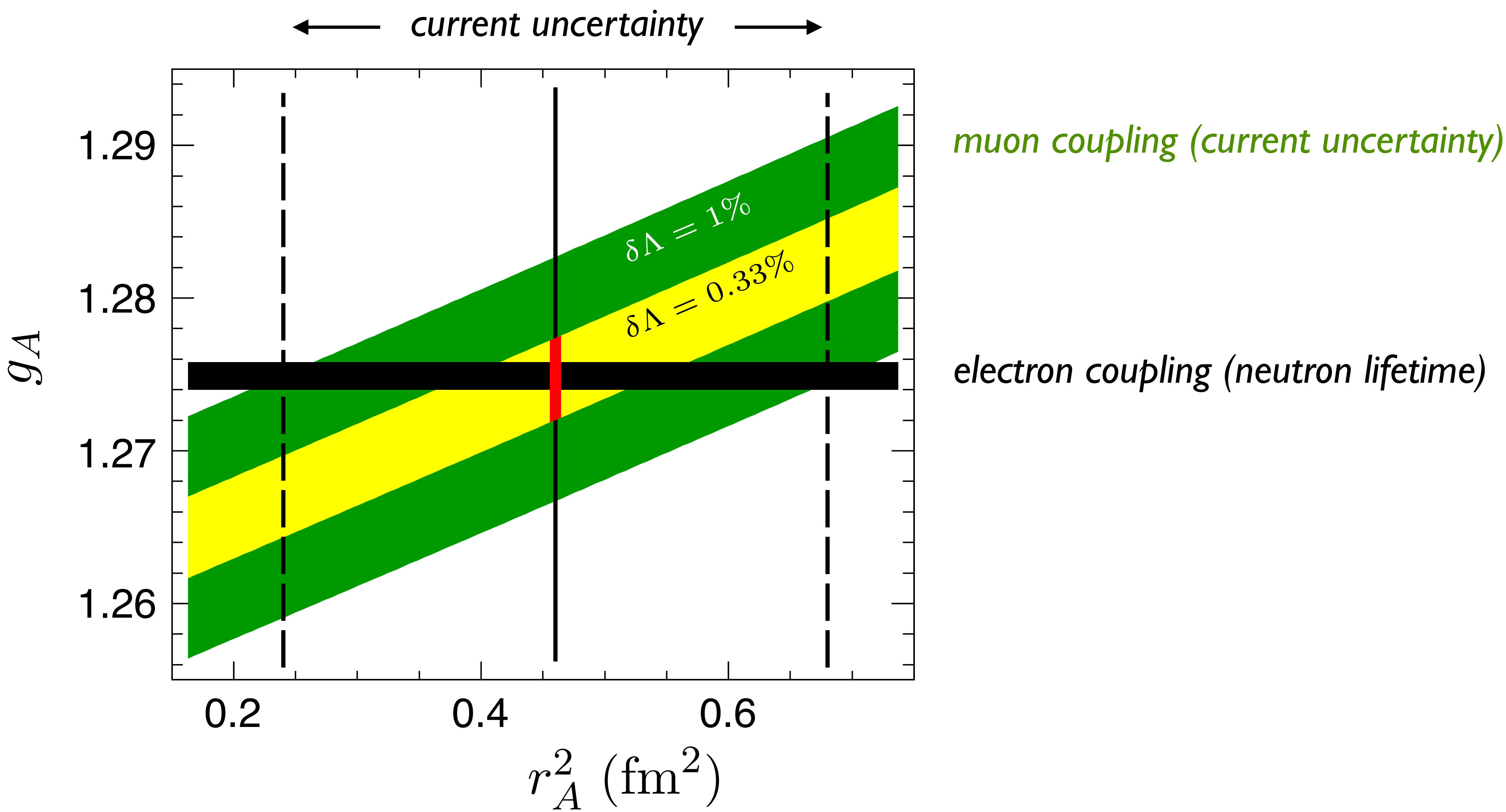
# implications for quasielastic neutrino cross sections



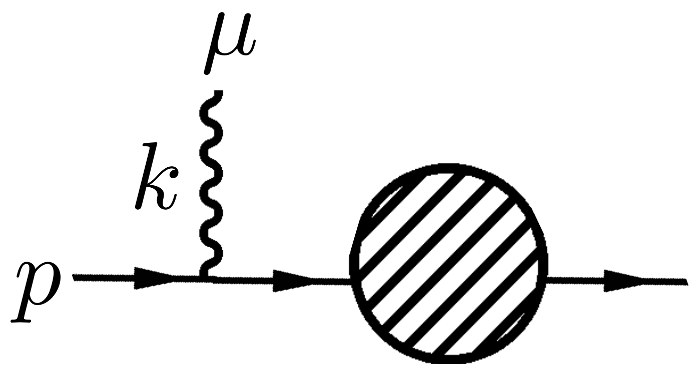

with radius constraint: ( hatched: external radius error $\delta r_A^2$=20% )

existing error (no external radius constraint)

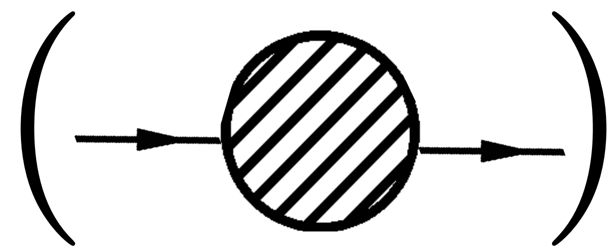
# test of electron-muon universality



# topic 3. radiative corrections and SCET




$$= e\,\frac{p^{\mu}}{p\cdot k}$$



$$+\ldots$$

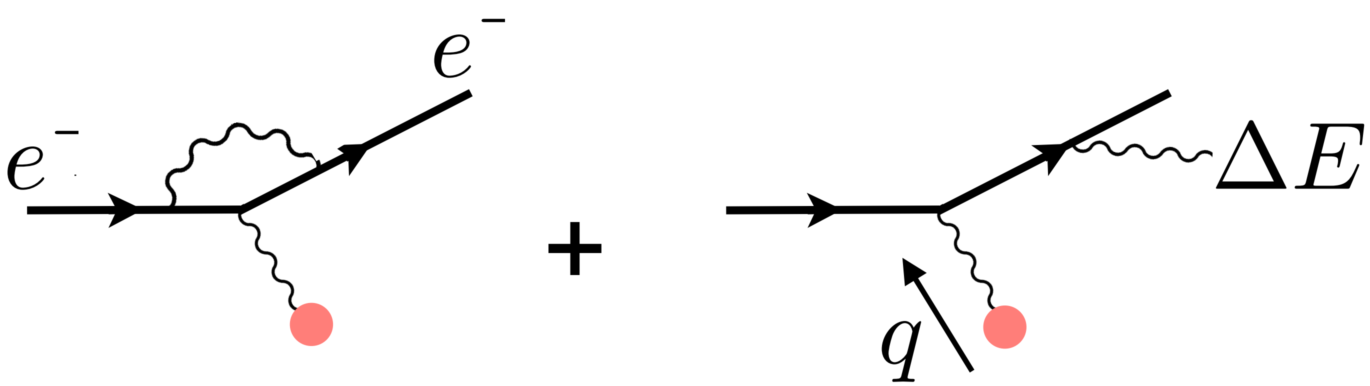
- eikonal coupling
- factorization of soft region
- proof by induction *Yennie, Frautschi, Suura (1961)*

$\Rightarrow$ exponentiation of IR divergences, cancellation between real and virtual

But exponentiation of IR divergences does not imply exponentiation of the entire first order correction

## Large logarithms spoil QED perturbation theory when $-q^2=Q^2\sim\mathrm{GeV}^2$

$$|F(q^2)|^2 \to |F(q^2)|^2 \left( 1 - \underbrace{\frac{\alpha}{\pi} \log \frac{Q^2}{m_e^2} \log \frac{E^2}{(\Delta E)^2}}_{\approx 0.5} + \dots \right)$$




## Experimental ansatz sums exponentiates 1st order:

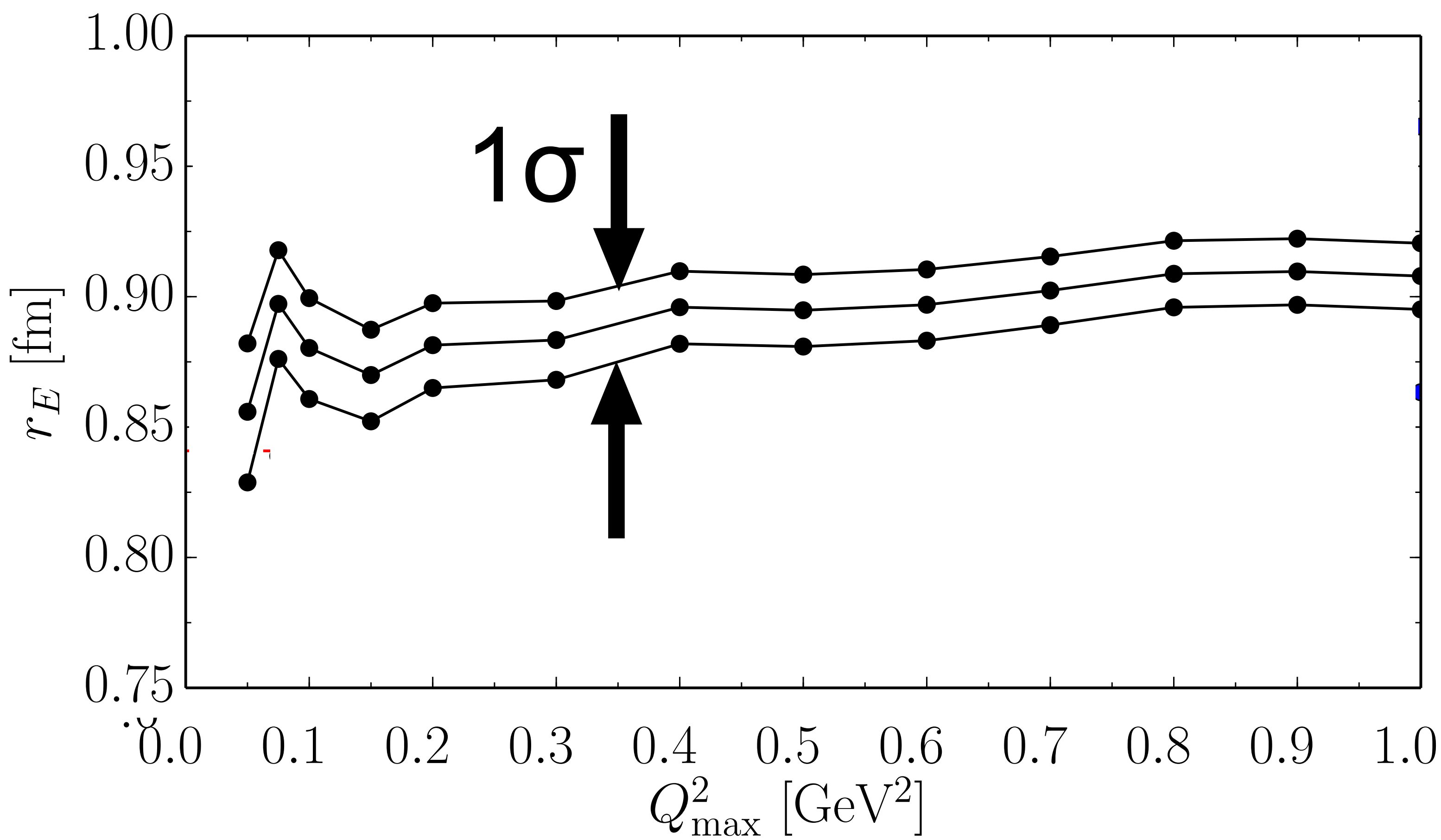
$$|F(q^2)|^2 \left( 1 - \frac{\alpha}{\pi} \log \frac{Q^2}{m_e^2} \log \frac{E^2}{(\Delta E)^2} + \dots \right) \to |F(q^2)|^2 \exp \left[ -\frac{\alpha}{\pi} \log \frac{Q^2}{m_e^2} \log \frac{E^2}{(\Delta E)^2} \right]$$

## Captures leading logarithms when

$$Q \sim E\,, \quad \Delta E \sim m_e$$

## As consistency check, error budget should contain the difference from resumming:

$$\log^2 \frac{Q^2}{m_e^2} \qquad \text{vs.} \quad \log \frac{Q^2}{m_e^2} \log \frac{E^2}{(\Delta E)^2}$$

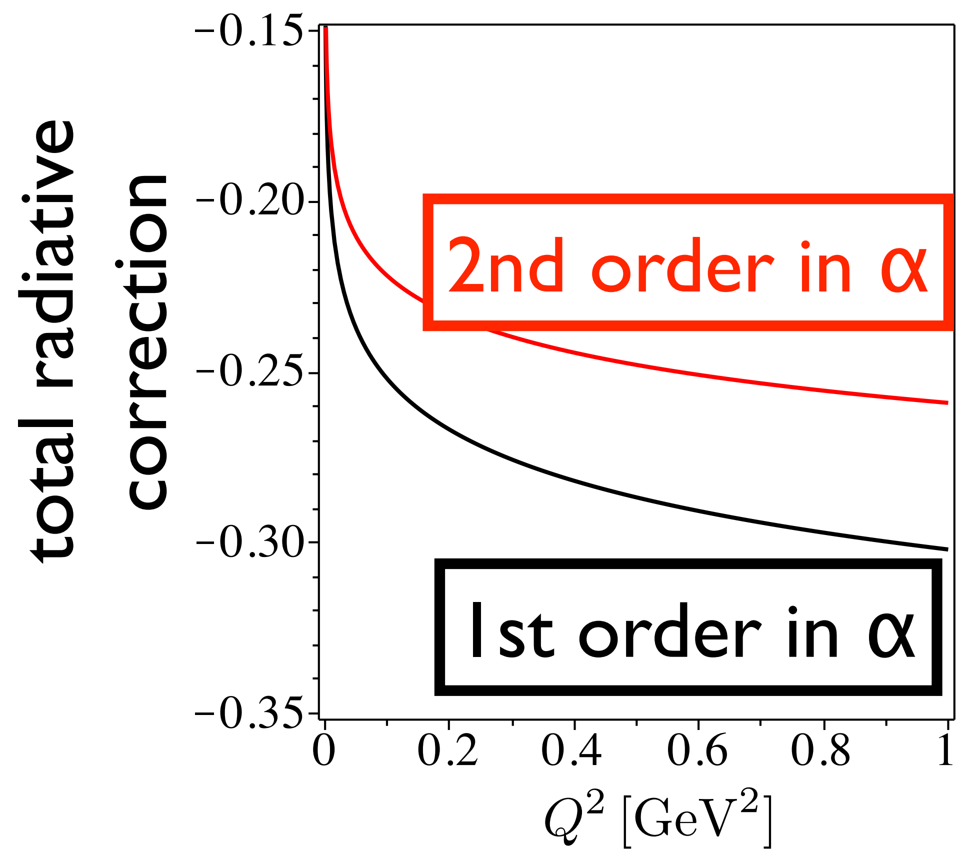



*electron energy:*

$E = 1\,\mathrm{GeV}$

*electron energy loss cut:*

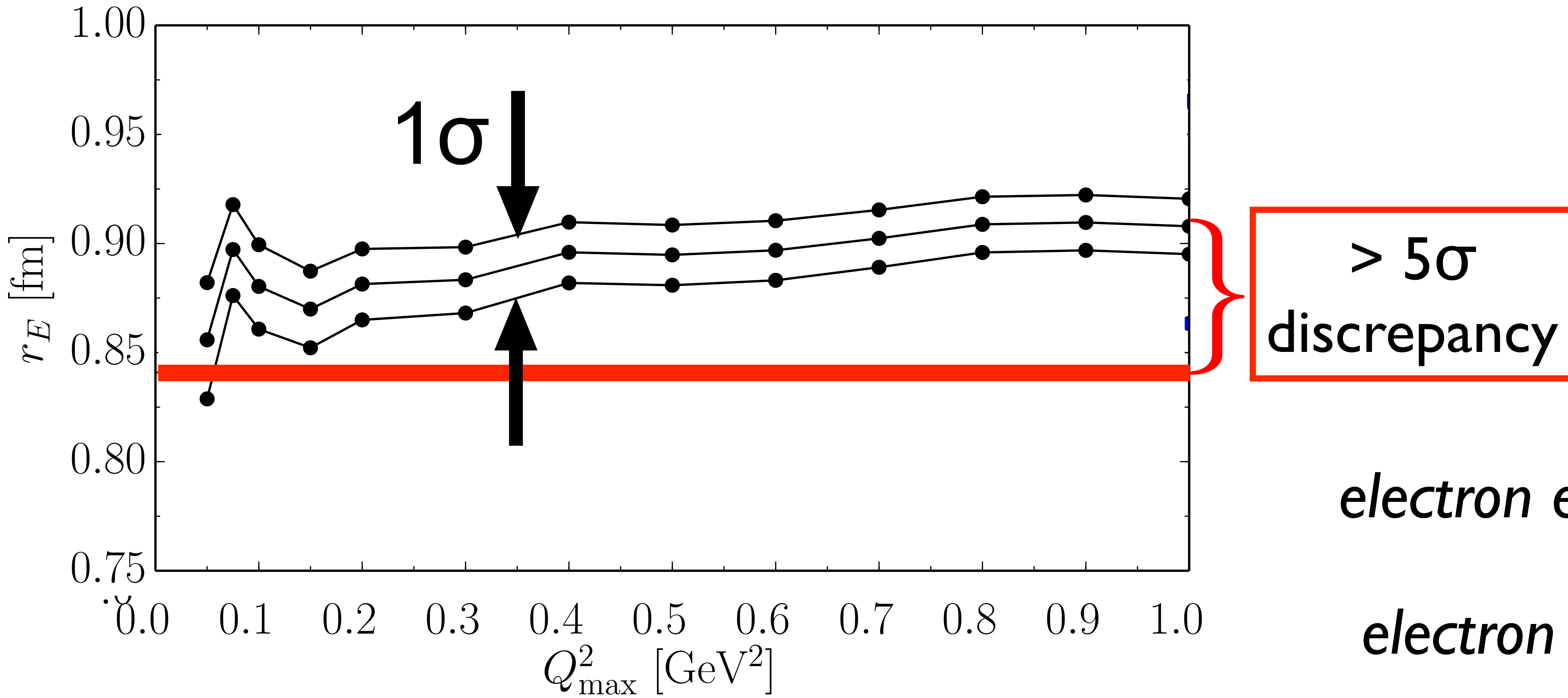
$\Delta E = 5\,\mathrm{MeV}$




- quoted systematics in A1 electron-proton scattering data are 0.2-0.5 %

- leading order radiative corrections ~30%

- need to systematically account for subleading logarithms, recoil, nuclear charge and structure

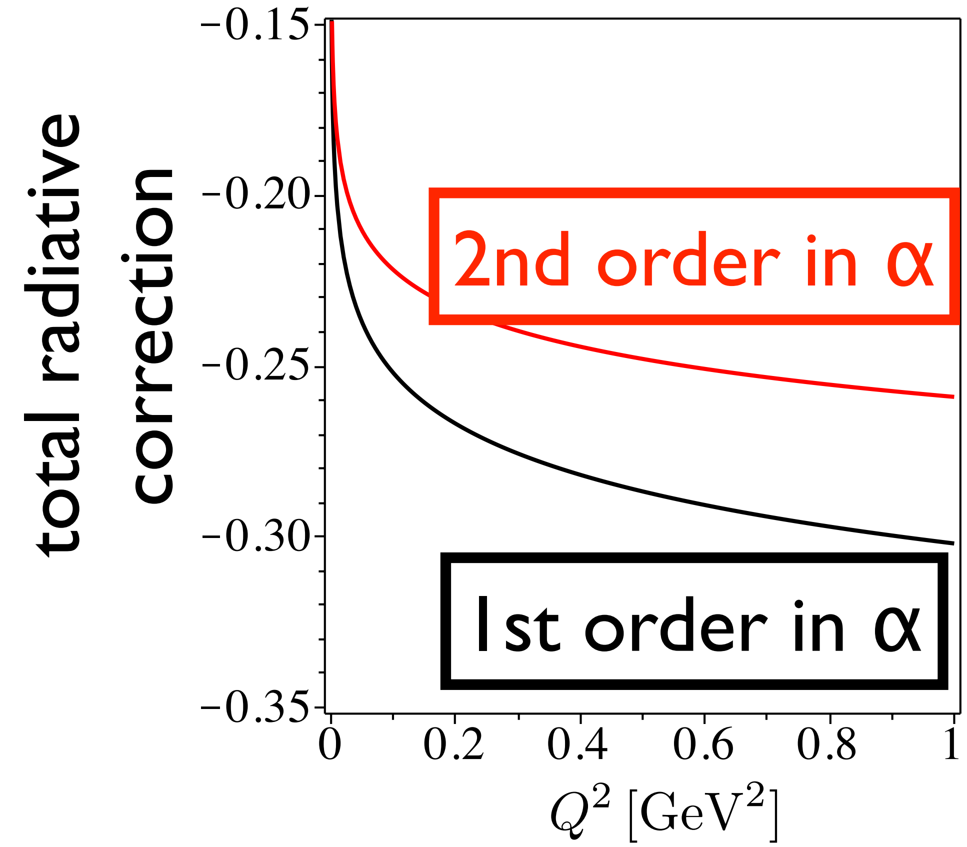



*electron energy:*

$E = 1\,\mathrm{GeV}$

*electron energy loss cut:*

$\Delta E = 5\,\mathrm{MeV}$

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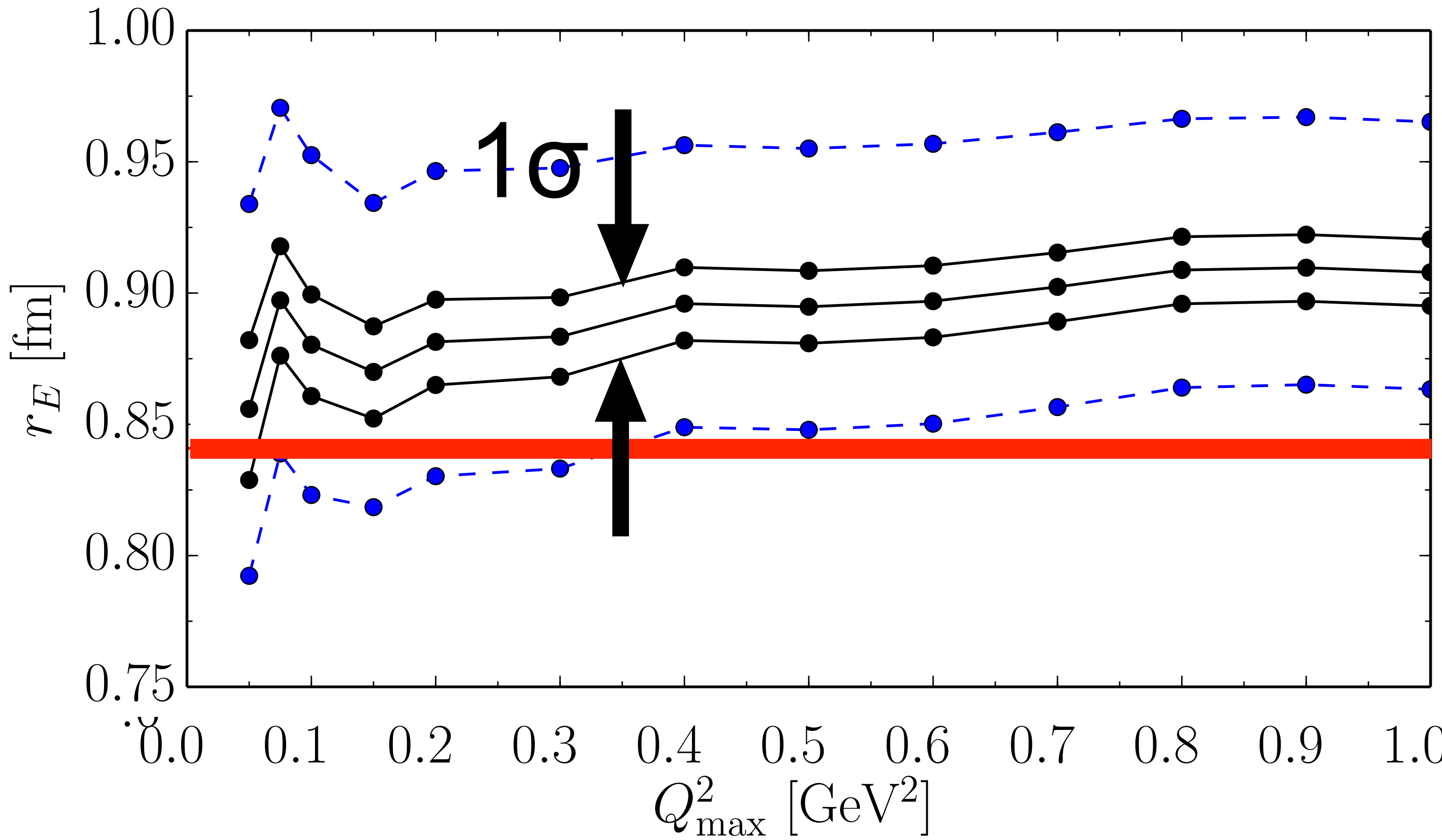


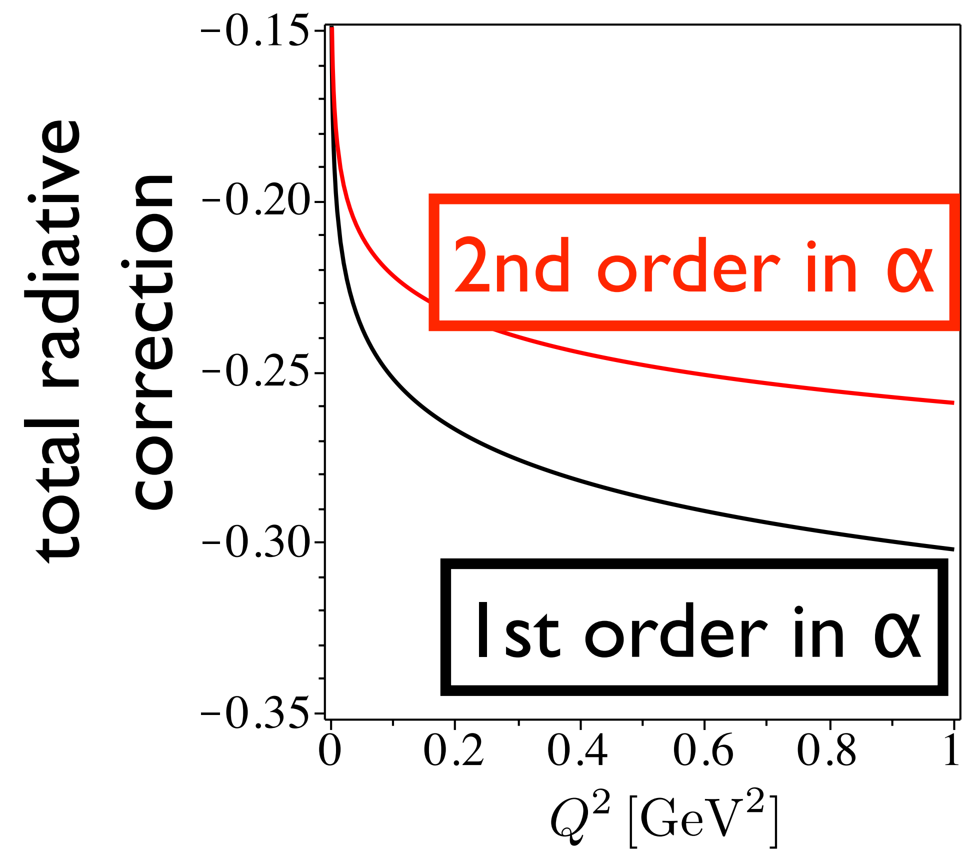

potentially large uncertainty from radiative corrections

*electron energy:*

$E = 1\,\mathrm{GeV}$

*electron energy loss cut:*

$\Delta E = 5\,\mathrm{MeV}$

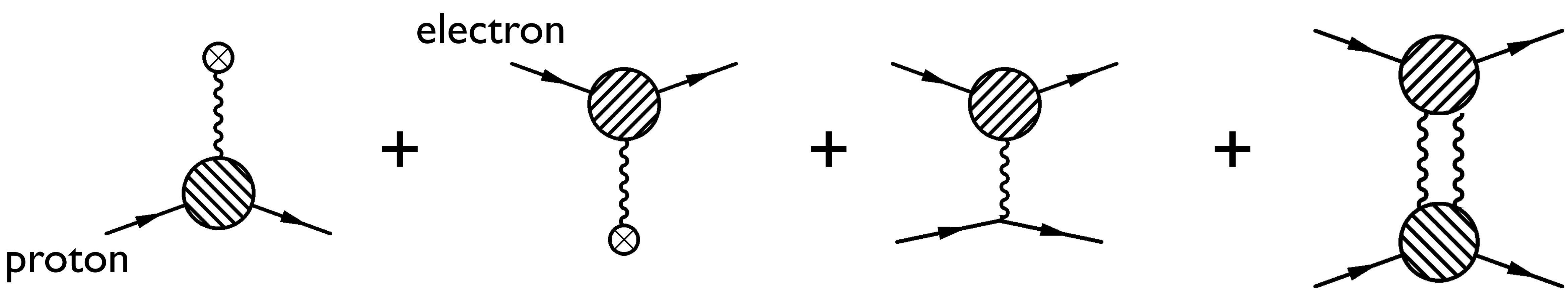



- quoted systematics in A1 electron-proton scattering data are 0.2-0.5 %

- leading order radiative corrections ~30%

- need to systematically account for subleading logarithms, recoil, nuclear charge and structure

# treat the problem in stages:



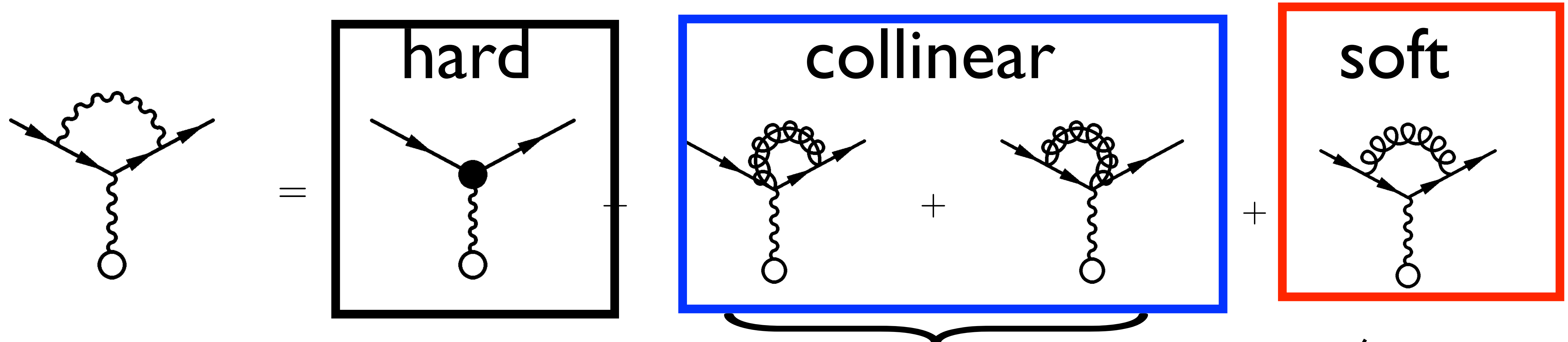

static source, nonrel. limit

static source, rel. limit

with recoil corrections

with nuclear charge corrections (two photon exchange)

- factorization




hadron structure
(Born form factors, ...)

$$d\sigma \propto \boxed{H\left(\frac{Q^2}{\mu^2}\right)}\ \boxed{J\left(\frac{m^2}{\mu^2}\right) R\left(\frac{m^2}{\mu^2},\frac{p\cdot p'}{m^2}\right)}\ \boxed{S\left(\frac{\Delta E}{\mu},\frac{p\cdot p'}{m^2},\frac{E}{m},\frac{E'}{m}\right)}$$

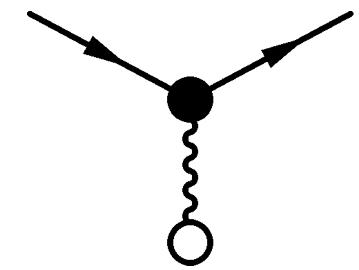
[ remainder function starting
at 2-loop (collinear anomaly/rapidity logs) ]

*Becher, Melnikov (2007)*
*Chiu, Golf, Kelley, Manohar (2007)*
*Becher, Neubert (2010)*
*Chiu, Jain, Neill, Rothstein (2012)*
...

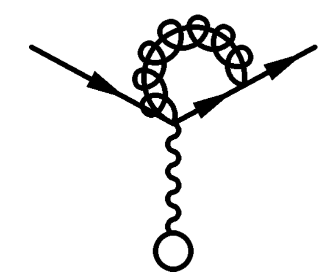
- physical electron mass regulates collinear divergences
- R given by ratio of Wilson loop matrix elements in m≠0/m=0

Sudakov form factor at one loop:

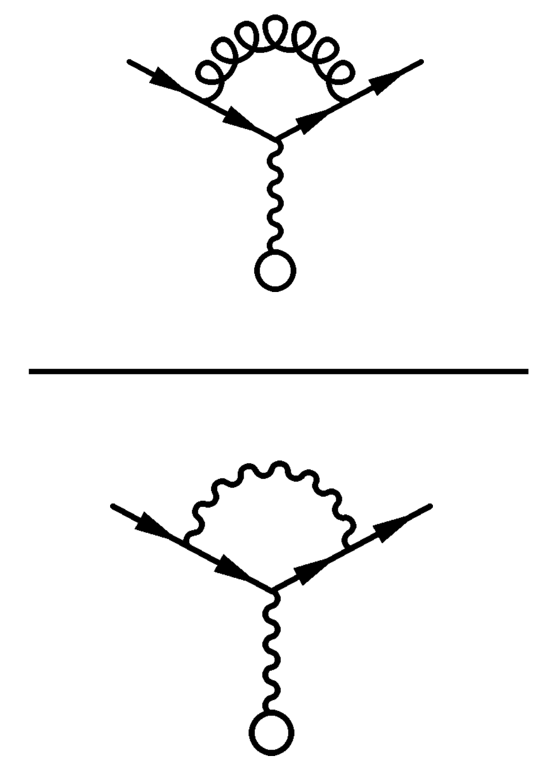
**Hard** $$F_H(\mu) = 1 + \frac{\alpha}{4\pi}\left[-\log^2\frac{Q^2}{\mu^2} + 3\log\frac{Q^2}{\mu^2} - 8 + \frac{\pi^2}{6}\right]$$



**Collinear** $$F_J(\mu) = 1 + \frac{\alpha}{4\pi}\left[\log^2\frac{m^2}{\mu^2} - \log\frac{m^2}{\mu^2} + 4 + \frac{\pi^2}{6}\right]$$



**Soft** $$F_S(\mu) = 1 + \frac{\alpha}{4\pi}\left[2\log\frac{\lambda^2}{\mu^2}\left(\log\frac{Q^2}{m^2} - 1\right)\right]$$



Large logarithms regardless of choice for μ

$F_S$: exponentiates (evaluate at any scale)

$F_J$: evaluate at μ~m

$F_H$: evaluate at μ~M~Q

*(two-loop matching, real+virtual see 1605.02613)*

$$F = F_H F_J F_S$$

## Two photon exchange

- Nuclear charge corrections introduce new spin structures (helicity counting: 3 amplitudes at leading power in $m_e/Q$)

$$F_H(\mu)\gamma^\mu \otimes \gamma_\mu \to \sum_{i=1}^{3} c_i(\mu)\,\Gamma_i^{(e)} \otimes \Gamma_i^{(p)}$$

- In principle, can use e+ and e- data to separately determine 1-photon exchange and 2-photon exchange contributions to $c_i$

- However, with available data, measure combination of 1-photon and 2-photon contributions.

- Regardless of treatment of hard coefficients, remaining radiative corrections are universal

$$d\sigma = H(M) \times \underbrace{\frac{H(\mu)}{H(M)} \times J(\mu) \times S(\mu)}$$

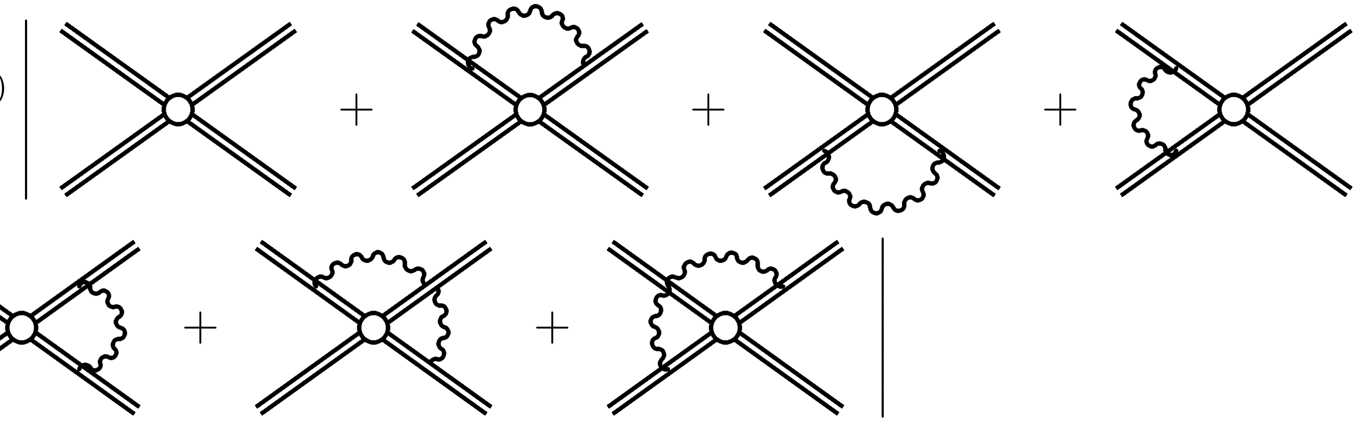
want to extract this

correct data by this factor

- J: refers to collinear region, same as before

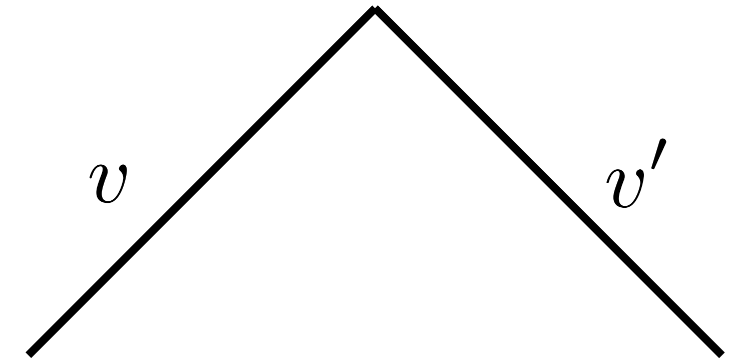
- S: include nuclear charge for general soft function (computed through 2-loop order)

$$\sqrt{S(\mu, \Delta E = 0)} = Z_h^{(e)} Z_h^{(p)}$$



- H(μ)/H(M): must now account for large logs in this factor

- resummation




governed by Wilson loops with cusps:

$$\bar{h} iv\cdot Dh \to \bar{h}^{(0)} S_v^\dagger iv\cdot D S_v h^{(0)} = \bar{h}^{(0)} iv\cdot\partial h^{(0)}\,, \quad S_v(x) = P\exp\left[i\int_{-\infty}^0 ds v\cdot A_s(x+sv)\right]$$

renormalization of hard function of interest:

$$\frac{d\log H}{d\log\mu} = 2\left[\gamma_{\rm cusp}(\bar\alpha)\log\frac{Q^2}{\mu^2} + \gamma_{\rm cusp}(v\cdot v',\bar\alpha) + 2\gamma_{\rm cusp}(\bar\alpha)\log\frac{v\cdot p'}{-v\cdot p - i0} + \gamma(\bar\alpha)\right].$$

universal functions

$$\text{electron}: \; p^\mu$$
$$\text{proton}: \; Mv^\mu$$

solution, summing large logarithms:

$$\log\frac{H(\mu_L)}{H(\mu_H)} = -\frac{\alpha}{2\pi}\log^2\frac{\mu_H^2}{\mu_L^2} + \dots$$

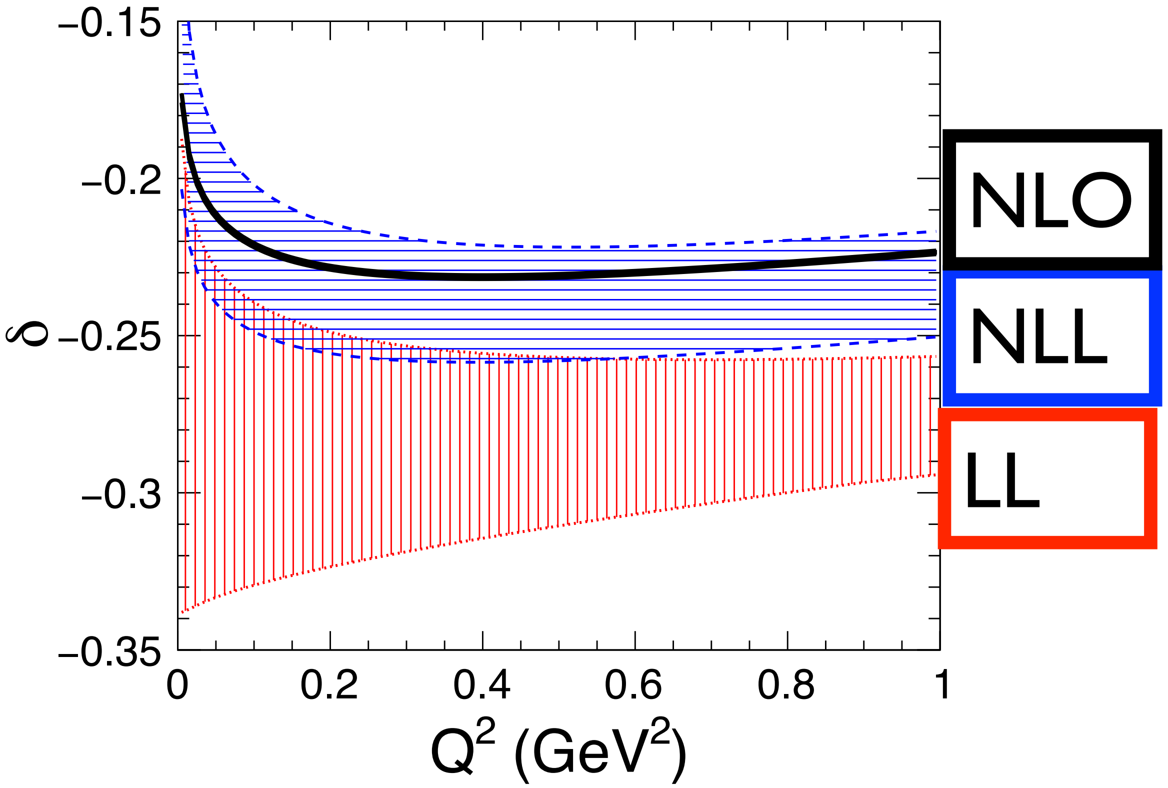
$$d\sigma = H(M) \times \underbrace{\frac{H(\mu)}{H(M)} \times J(\mu) \times S(\mu)}_{\text{total radiative correction}}$$

numerically: $\alpha L^2 = \alpha \log^2 \frac{Q^2}{m^2} \sim 1 \quad \Rightarrow \quad \alpha L \sim \alpha^{\frac{1}{2}}$ , etc.

*electron energy:* $E = 1\,\mathrm{GeV}$

*electron energy loss cut:* $\Delta E = 5\,\mathrm{MeV}$

total radiative correction



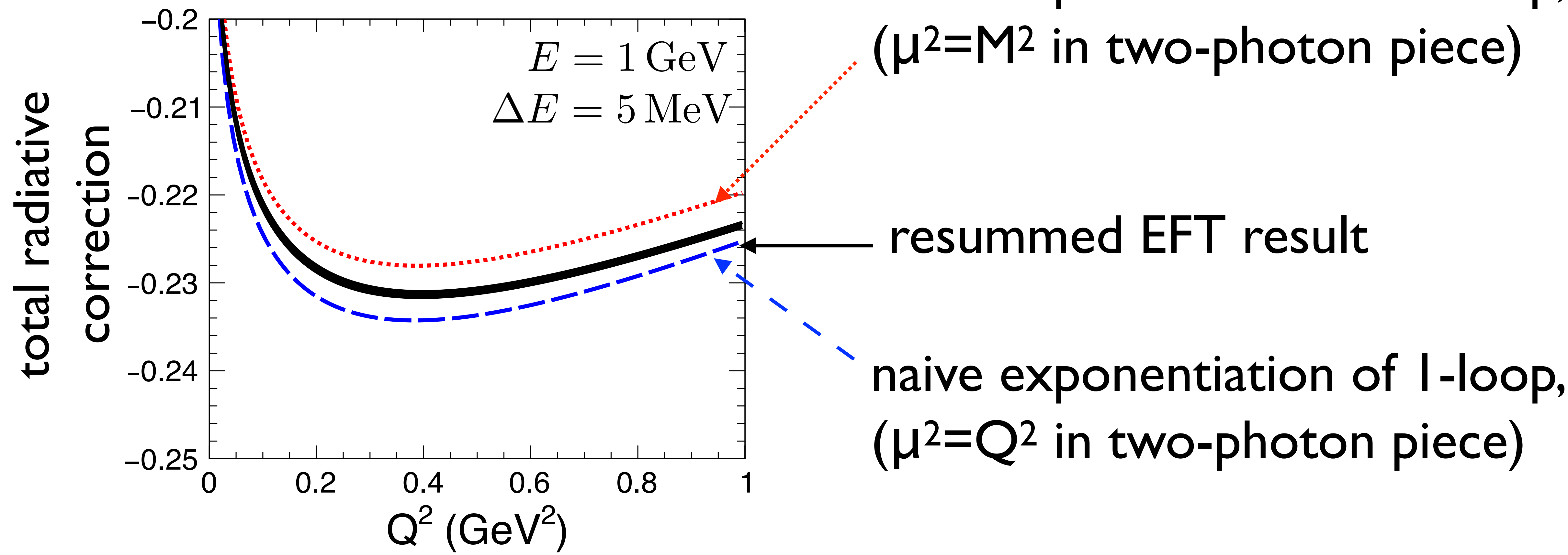

correct through:

NLO: $\mathcal{O}(\alpha)$

NLL: $\mathcal{O}(\alpha^{\frac{1}{2}})$

LL: $\mathcal{O}(1)$

Comparison to previous implementations of radiative corrections, e.g. in A1 analysis of electron-proton scattering data




- discrepancies at 0.5-1% compared to currently applied radiative correction models (cf. 0.2-0.5% systematic error budget of A1 experiment)

- conflicting implicit scheme choices for 1PE and 2PE

- complete analysis: account for floating normalizations, correlated shape variations when fitting together with backgrounds

EFT analysis clarifies several issues involving conflicting and/or implicit conventions and scheme choices

**1) Scheme choice and definition of radius and “Born” form factors**

**2) Scheme dependence of two-photon exchange**

**3) Limitations of naive exponentiation**

## 1) Scheme choice and definition of radius and "Born" form factors

$$\langle J^\mu \rangle = \bar{u}_{v'} \left[ \tilde{F}_1 \gamma^\mu + \tilde{F}_2 \frac{i}{2} \sigma^{\mu\nu} (v'_\nu - v_\nu) \right] u_v$$

Massive particle form factor (e.g. for proton):

$$\tilde{F}_i = F_H F_S$$

$$F_H(q^2, \mu = M) \equiv F_i(q^2)^{\text{Born}} \equiv \tilde{F}_i(q^2) F_S^{-1}(w, \mu = M)$$

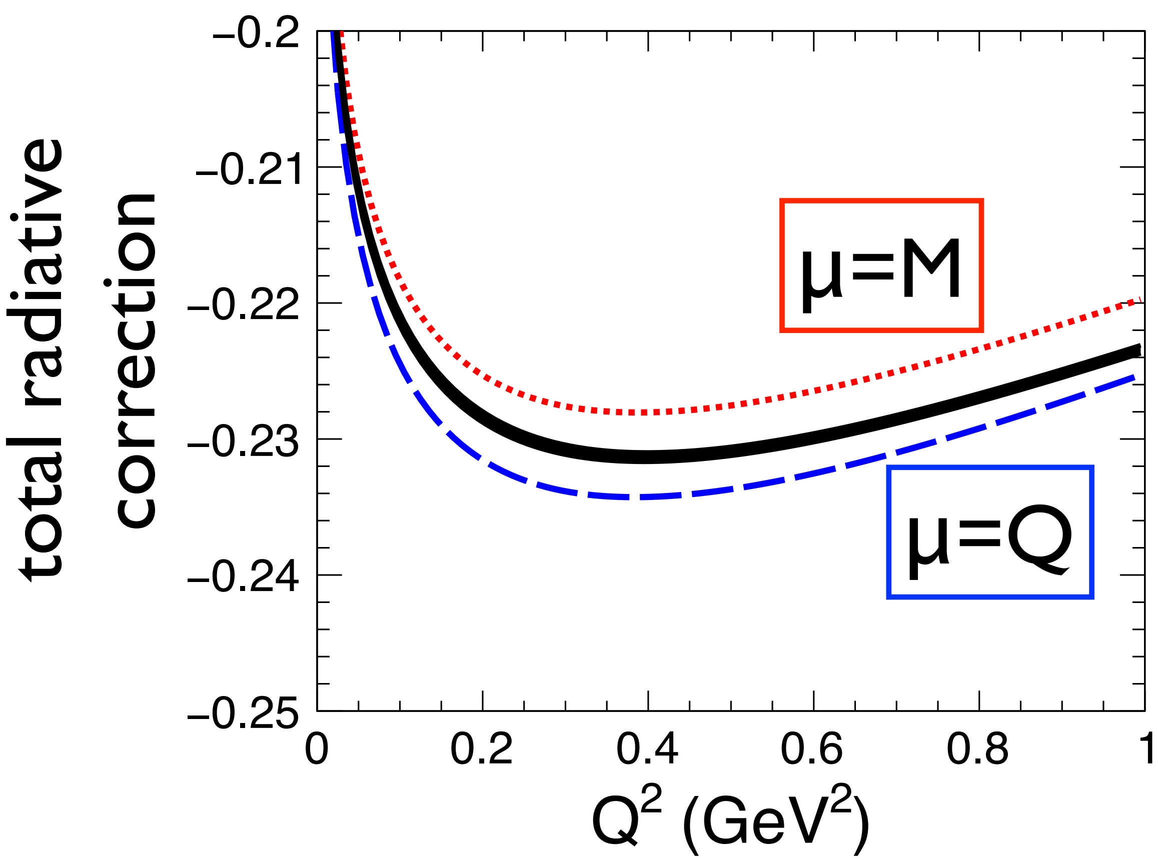
hard coefficient

soft function

Multiple conventions in the literature. Different "Born" form factors, different radii (differences typically below current precision)

## 2) Scheme dependence of two-photon exchange

As for form factors, define hadronic functions in the general 2→2 scattering process as the hard component in the factorization formula at factorization scale μ=M




Prevailing conventions have used conflicting μ=M for 1 photon exchange, μ=Q for 2 photon exchange

A scale-variation estimate of uncertainty in the 2 photon exchange subtraction

## 3) Limitations of naive exponentiation

- Renormalization analysis for subleading logs :

$$\log \frac{H(\mu_L)}{H(\mu_H)} = -\frac{\alpha}{2\pi} \log^2 \frac{\mu_H^2}{\mu_L^2} + \dots$$

⇒ New terms at order $\alpha^2 L^3$, $\alpha^2 L^2$, $\alpha^3 L^4$, …

- *Total* versus *individual* real photon energy below ΔE :

$$S^{(2)} = \frac{1}{2!}[S^{(1)}]^2 - \frac{16\pi^2}{3}(L-1)^2 .$$

$$S = \sum_n \left(\frac{\alpha}{4\pi}\right)^n S^{(n)}$$

⇒ New terms at order $\alpha^2 L^2$

complete analysis: account for floating normalizations, correlated shape variations when fitting together with backgrounds.

a difficult archeological problem. PRP from e-p appears to require something more (expt. syst.: ? / theory systematic: hard TPE) summary

# Summary

- topic 0: critical theory input needed for $\nu_e/\nu_\mu$ cross section differences and $\nu$ amplitudes at the nucleon level

- topic 1: amplitude analysis and z expansion: need to do better for elementary amplitudes

- topic 2: muon capture: template for general $\nu_e/\nu_\mu$ analysis and world's best (in a tie) $r_A$ determination

- topic 3: radiative corrections and SCET: template for exclusive $\nu_e/\nu_\mu$ analysis and cautionary tale for % level