FACULTY OF SCIENCES

DEPARTMENT OF PHYSICS AND ASTRONOMY

CORRELATIONS IN QE(LIKE) NEUTRINO-NUCLEUS SCATTERING

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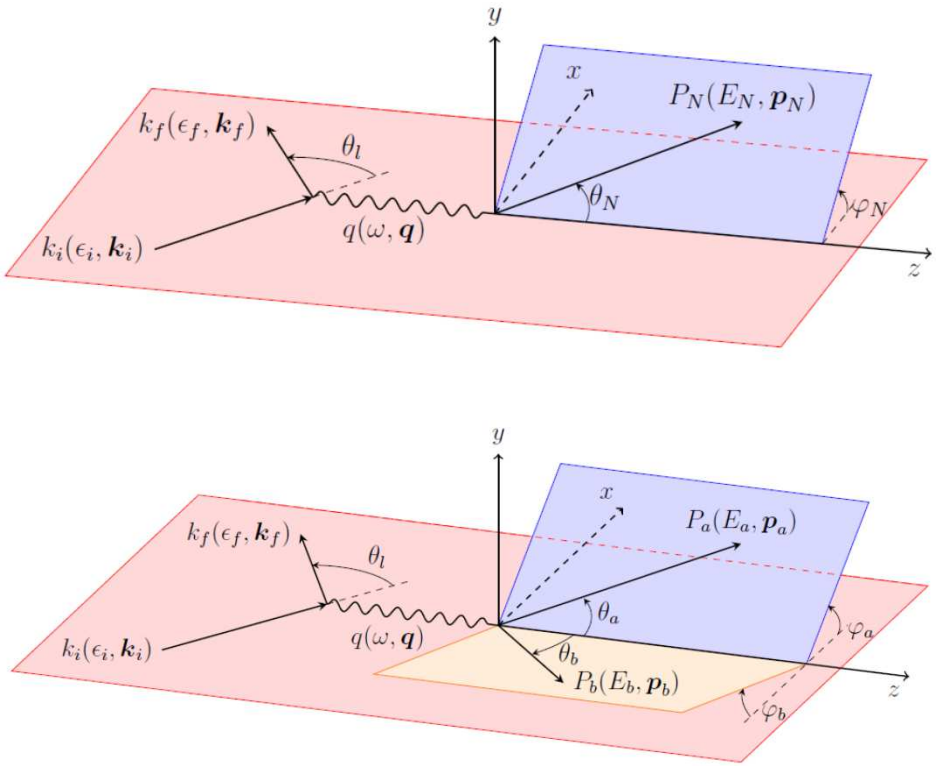
Outline

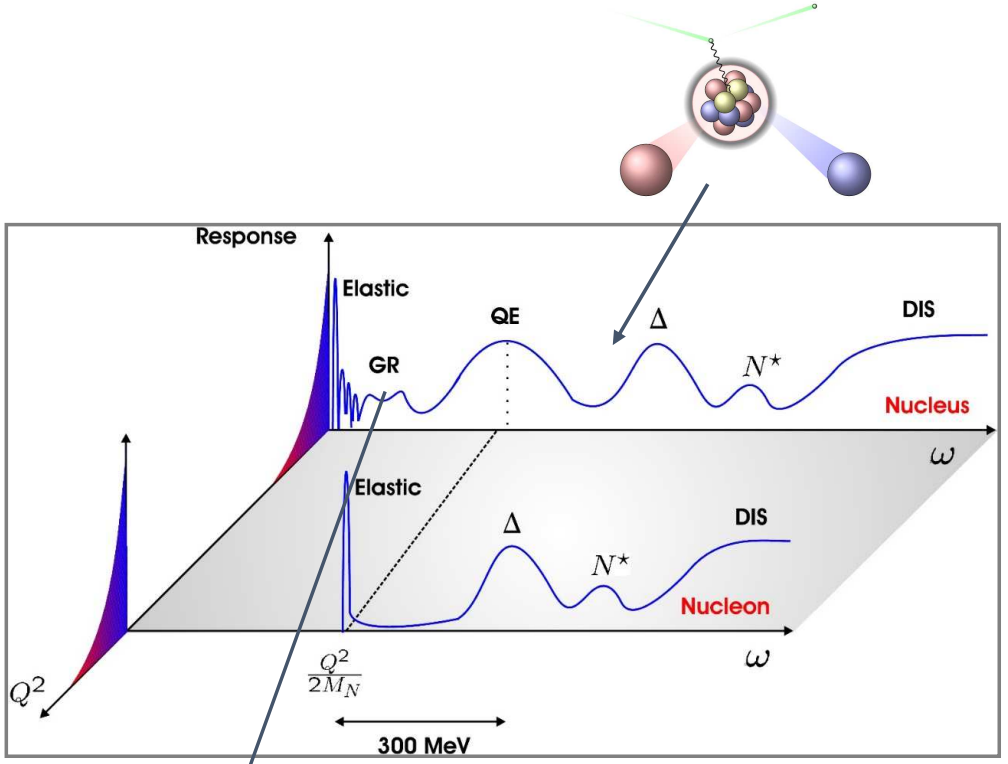
- Detailed microscopic cross sections calculations for QE(-like) scattering
- influence of long-range correlations
- influence of short-range correlations in 1- and 2-nucleon knockout processes
- Influence of seagull and pion-in-flight MEC contributions

- Scheme-dependent separation
- Double counting

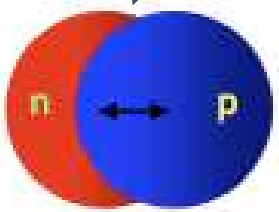
Neutrino-hadron scattering



Dip region : multinucleon mechanisms

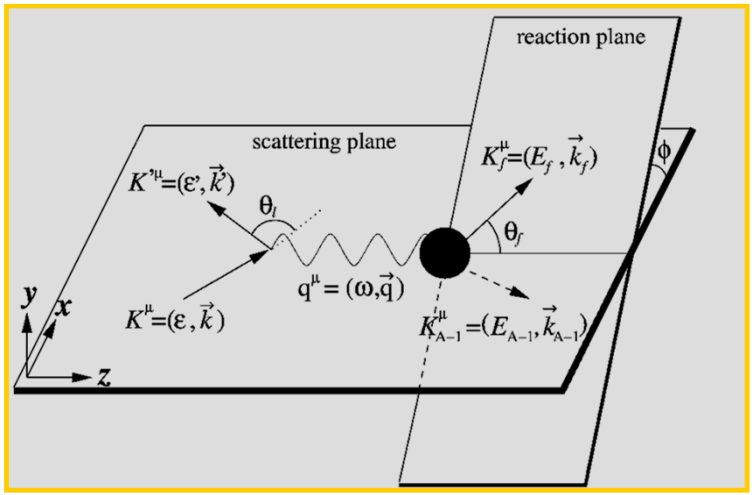


e.g.



Neutrino-nucleus interactions

$$\hat{H}_W = \frac{G}{\sqrt{2}} \int d\vec{x}\, \hat{J}_{\mu,lepton}(\vec{x})\, \hat{J}^{\mu,hadron}(\vec{x})$$



Hadron current

$$J^{\mu} = F_1(Q^2)\gamma^{\mu} + i\frac{\kappa}{2M_N}F_2(Q^2)\sigma^{\mu\nu}q_{\nu} + G_A(Q^2)\gamma^{\mu}\gamma_5 + \frac{1}{2M_N}G_P(Q^2)q^{\mu}\gamma_5$$

Lepton tensor

$$l_{\alpha\beta} \equiv \overline{\sum_{s,s'}} [\overline{u}_l \gamma_{\alpha}(1-\gamma_5)u_l]^{\dagger}[\overline{u}_{\nu}\gamma_{\beta}(1-\gamma_5)u_{\nu}]$$

$$\vec{J}_V^{\alpha}(\vec{x}) = \vec{J}^{\alpha}_{convection}(\vec{x}) + \vec{J}^{\alpha}_{magnetization}(\vec{x})$$

with
$$\vec{J}_c^{\alpha}(\vec{x}) = \frac{1}{2Mi}\sum_{i=1}^{A} G_E^{i,\alpha}\left[\delta(\vec{x}-\vec{x}_i)\overrightarrow{\nabla}_i - \overleftarrow{\nabla}_i\,\delta(\vec{x}-\vec{x}_i)\right],$$

$$\vec{J}_m^{\alpha}(\vec{x}) = \frac{1}{2M}\sum_{i=1}^{A} G_M^{i,\alpha}\overrightarrow{\nabla}\times\vec{\sigma}_i\,\delta(\vec{x}-\vec{x}_i),$$

$$\vec{J}_A^{\alpha}(\vec{x}) = \sum_{i=1}^{A} G_A^{i,\alpha}\,\vec{\sigma}_i\,\delta(\vec{x}-\vec{x}_i),$$

$$J_V^{0,\alpha}(\vec{x}) = \rho_V^{\alpha}(\vec{x}) = \sum_{i=1}^{A} G_E^{i,\alpha}\,\delta(\vec{x}-\vec{x}_i),$$

$$J_A^{0,\alpha}(\vec{x}) = \rho_A^{\alpha}(\vec{x}) = \frac{1}{2Mi}\sum_{i=1}^{A} G_A^{i,\alpha}\,\vec{\sigma}_i\cdot\left[\delta(\vec{x}-\vec{x}_i)\overrightarrow{\nabla}_i - \overleftarrow{\nabla}_i\,\delta(\vec{x}-\vec{x}_i)\right]$$

$$J_P^{0,\alpha}(\vec{x}) = \rho_P^{\alpha}(\vec{x}) = \frac{m_\mu}{2M}\sum_{i=1}^{A} G_P^{i,\alpha}\,\overrightarrow{\nabla}\cdot\vec{\sigma}_i\,\delta(\vec{x}-\vec{x}_i)$$

for NC reactions

$$G_E^{V,o} = \left(\frac{1}{2} - \sin^2\theta_W\right)\tau_3 - \sin^2\theta_W,$$

$$G_M^{V,o} = \left(\frac{1}{2} - \sin^2\theta_W\right)(\mu_p-\mu_n)\,\tau_3 - \sin^2\theta_W\,(\mu_p+\mu_n)$$

$$G^{A,0} = g_a\,\frac{\tau_3}{2} = -\frac{1.262}{2}\,\tau_3$$

for CC reactions

$$G_E^{V,\pm} = \tau_\pm$$

$$G_M^{V,\pm} = (\mu_p-\mu_n)\,\tau_\pm$$

$$G^{A,\pm} = g_a\,\tau_\pm = -1.262\,\tau_\pm$$

$G = (1+Q^2/M^2)^{-2}$ Q^2 dependence : dipole parametrization or BBBA07 :

Inclusive QE 1-nucleon knockout cross sections

$$\frac{d^2\sigma}{d\Omega\, d\omega} = (2\pi)^4\, k_f \varepsilon_f \sum_{s_f,s_i} \frac{1}{2J_i+1} \sum_{M_f,M_i} \left|\left\langle f \left| \widehat{H}_W \right| i \right\rangle\right|^2$$

$$\left(\frac{d^2\sigma_{i\to f}}{d\Omega d\omega}\right)_{\frac{\nu}{\nu}} = \frac{G^2 \varepsilon_f^2}{\pi} \frac{2\cos^2\left(\frac{\theta}{2}\right)}{2J_i+1} \left[\sum_{J=0}^{\infty} \sigma_{CL}^J + \sum_{J=1}^{\infty} \sigma_T^J\right]$$

$$\sigma_{CL}^J = \left|\left\langle J_f \left\| \widehat{\mathcal{M}}_J(\kappa) + \frac{\omega}{|\vec{q}|} \widehat{\mathcal{L}}_J(\kappa) \right\| J_i \right\rangle\right|^2$$

$$\sigma_T^J = \left(-\frac{q_\mu^2}{2|\vec{q}|^2} + \tan^2\left(\frac{\theta}{2}\right)\right) \left[\left|\left\langle J_f \left\| \widehat{\mathcal{T}}_J^{mag}(\kappa) \right\| J_i \right\rangle\right|^2 + \left|\left\langle J_f \left\| \widehat{\mathcal{T}}_J^{el}(\kappa) \right\| J_i \right\rangle\right|^2\right]$$

$$\mp \tan\left(\frac{\theta}{2}\right) \sqrt{-\frac{q_\mu^2}{|\vec{q}|^2} + \tan^2\left(\frac{\theta}{2}\right)} \left[2\Re\left(\left\langle J_f \left\| \widehat{\mathcal{T}}_J^{mag}(\kappa) \right\| J_i \right\rangle \left\langle J_f \left\| \widehat{\mathcal{T}}_J^{el}(\kappa) \right\| J_i \right\rangle^*\right)\right]$$

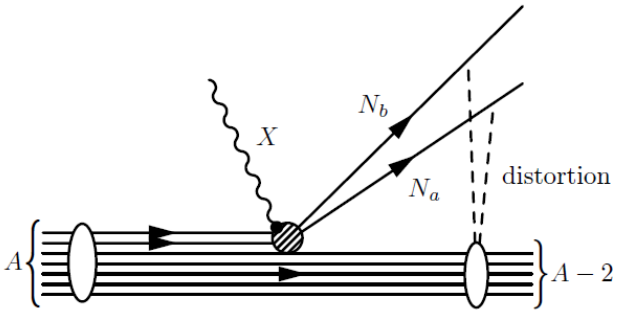
2-nucleon knockout cross sections

2-nucleon knockout :

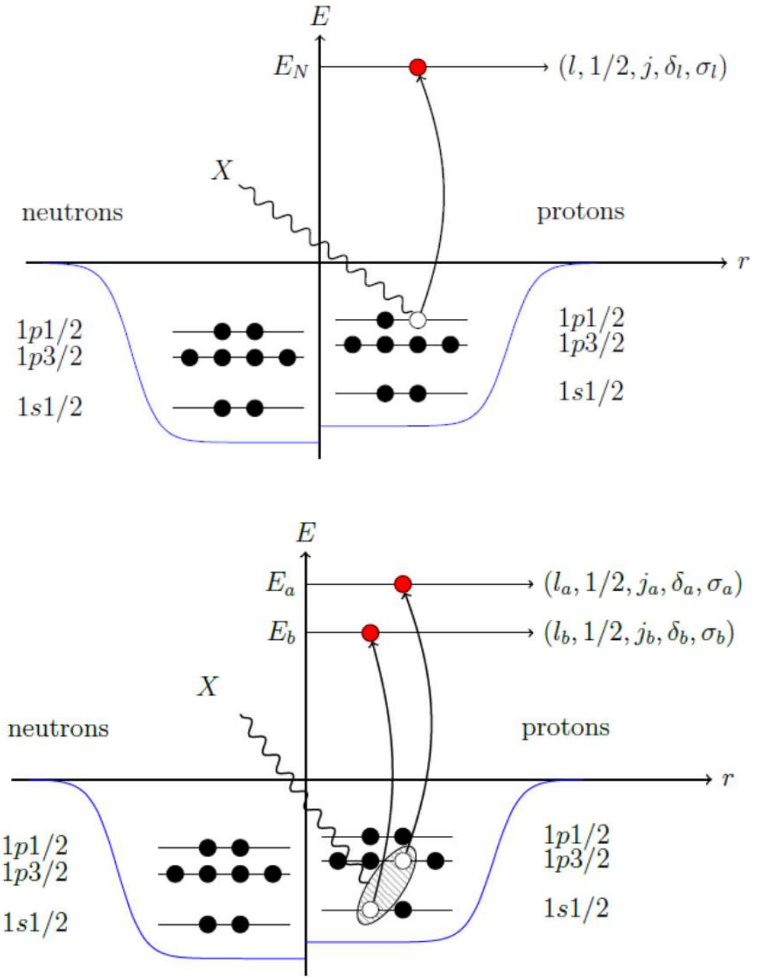
$$\frac{\mathrm{d}\sigma^X}{\mathrm{d}E_f \mathrm{d}\Omega_f \mathrm{d}T_b \mathrm{d}\Omega_b \mathrm{d}\Omega_a} = \frac{p_a p_b E_a E_b}{(2\pi)^6} g_{rec}^{-1} \sigma^X \zeta$$
$$\times \big[v_{CC} W_{CC} + v_{CL} W_{CL} + v_{LL} W_{LL} + v_T W_T + v_{TT} W_{TT} + v_{TC} W_{TC}$$
$$+ v_{TL} W_{TL} + h(v_{T'} W_{T'} + v_{TC'} W_{TC'} + v_{TL'} W_{TL'}) \big],$$

with :

$$\mathcal{J}_\lambda = \langle\, \Phi_f^{(A-2)}(E^{exc}, J_R M_R);\, \boldsymbol{p}_a m_{s_a};\, \boldsymbol{p}_b m_{s_b} \,|\, \widehat{J}_\lambda(\boldsymbol{q}) \,|\, \Phi_{gs} \,\rangle$$



$$|\, \Phi^{2p2h} \,\rangle = |\, \Phi_f^{(A-2)}(E^{exc}, J_R M_R);\, \boldsymbol{p}_a m_{s_a};\, \boldsymbol{p}_b m_{s_b} \,\rangle_{as}$$



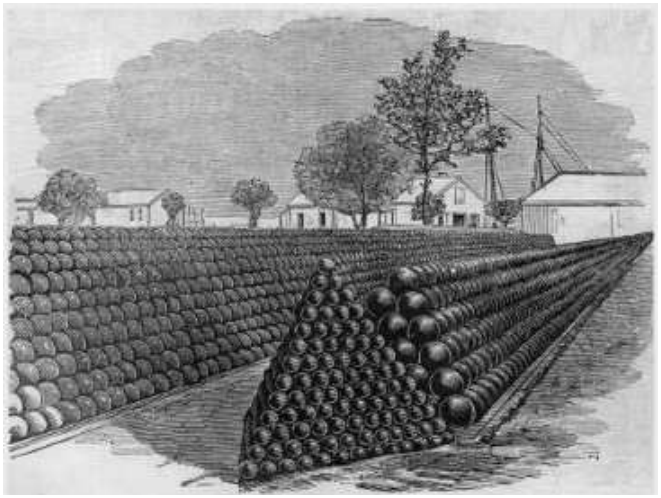
Cross section calculations

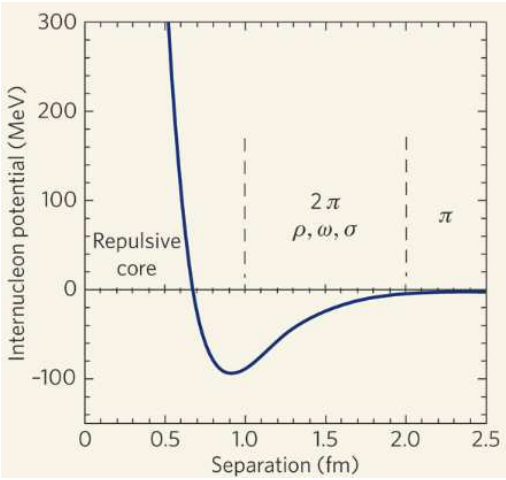
Nuclear model

- Starting point : mean-field nucleus with **Hartree-Fock** single-particle wave functions
- Skyrme SkE2 force used to build the potential
- Pauli blocking
- Binding
- **Long-** and **short-range correlations**
- **Meson Exchange Currents**

A model for the nucleus …

- Nuclear radius $\approx 1.2A^{\frac{1}{3}}$ fm
- Nucleon is a diffuse system
  - Hard core (repulsion) ≈ 0.5 fm
  - RMS charge radius from (e,e') $= 0.897(18)$ fm
- $0.07 \lesssim \text{NPF} \lesssim 0.42$
  - closest packing fraction of spheres ≈ 0.74
  - packing fraction of Argon liquid ≈ 0.032
  - packing fraction of Argon gas $\approx 3.75 \cdot 10^{-5}$
- The nuclear medium is a rather dense quantum liquid

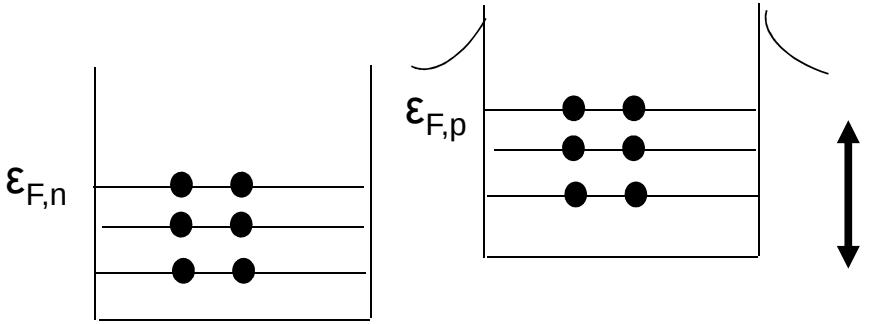




C. Colle, PhD, UGent 2017

The Fermi gas model

Easiest microscopic independent particle model



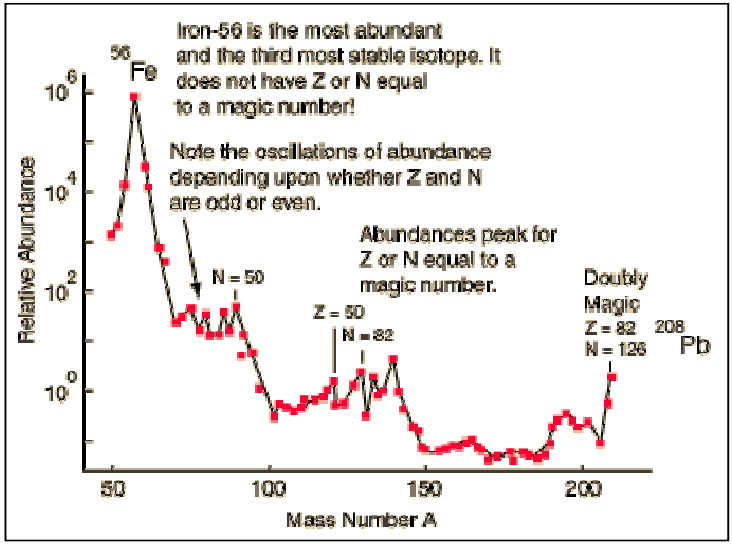
Depth of the potential ~ 41 MeV

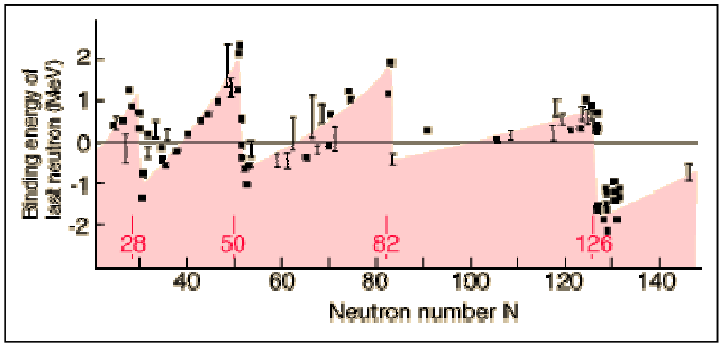
$$n = 2\frac{V}{(2\pi\hbar)^3}\int_0^{p_F} d^3p = 2\frac{V}{(2\pi\hbar)^3}\left(\frac{4}{3}\pi p_F^3\right)$$

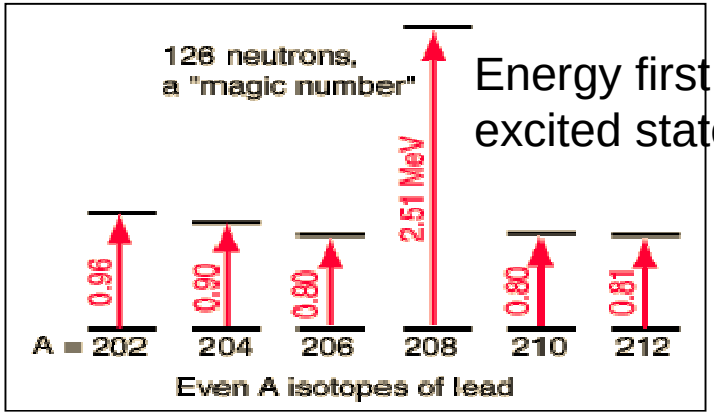
$$\Rightarrow \quad p_F = \hbar\left(\frac{3n\pi}{V}\right)^{1/3} \sim 247\,\mathrm{MeV/c}$$

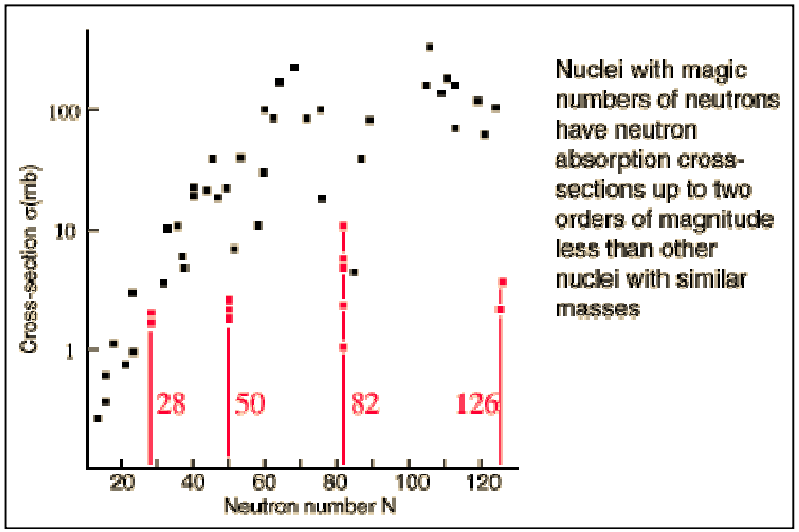
$$E_F = \frac{p_F^2}{2m} \sim 33\,\mathrm{MeV}$$

The mean field model (or shell-model)









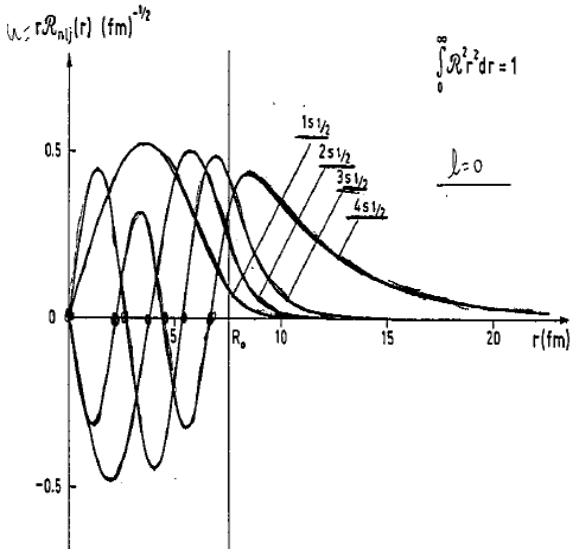
The mean field model (or shell-model)

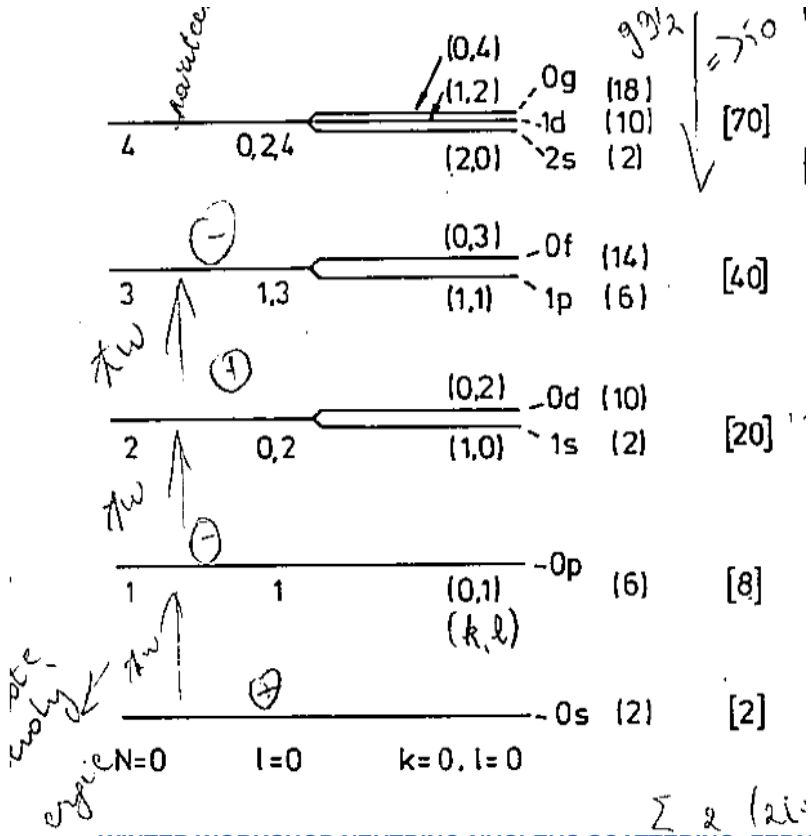
→ Solve Schrödinger (or Dirac) equation for nucleon in nuclear potential

$$\varphi(\vec{r}) = u(r)Y_m^l(\Omega)$$

spherical

Harmonic oscillator potential

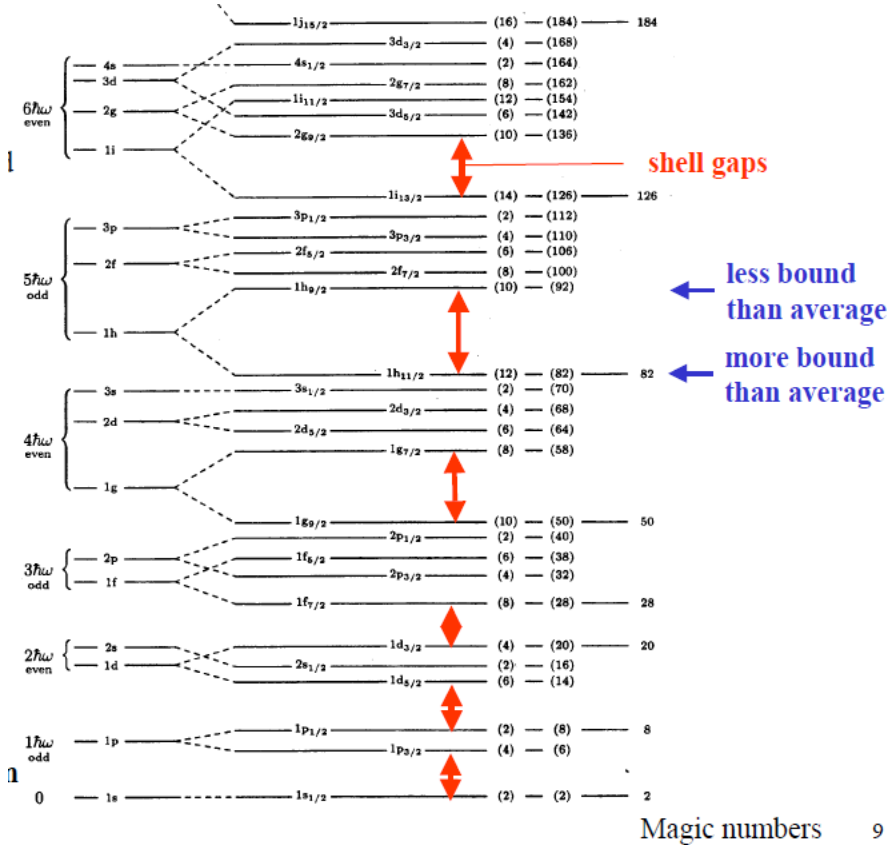


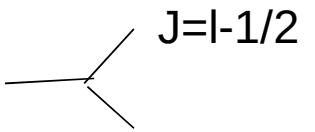


Not too approximate model for nuclei : already quite some correlations : the mean field model (or shell-model)

→Work harder : add spin-orbit term

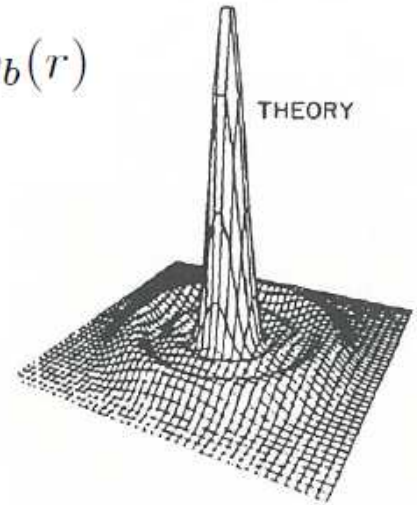
$$\hat{H} \rightarrow \hat{H} + \xi(r)\bar{l}\cdot\bar{s}$$





Independent particle picture ???

$$\rho(r) = \sum_{b<F} \varphi_b^*(r)\varphi_b(r)$$



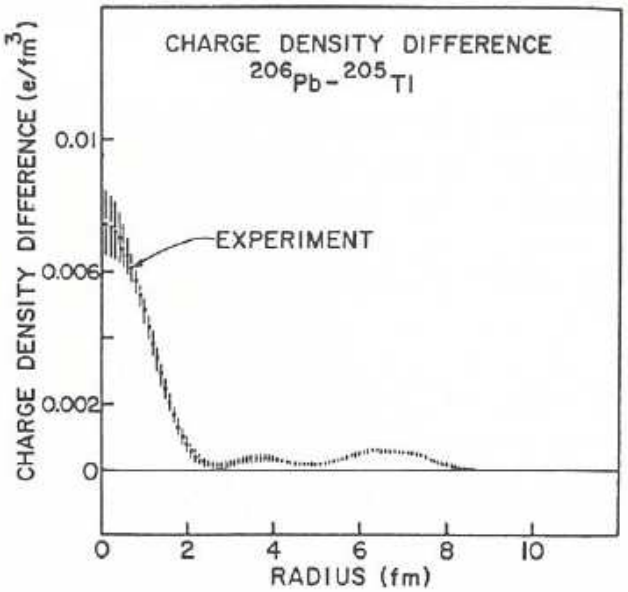


Fig. 3.18. The nuclear density distribution for the least bound proton in ^{206}Pb. The shell-model predicts the last $(3s_{1/2})$ proton in ^{206}Pb to have a sharp maximum at the centre, as shown at the left-hand side. On the right-hand side the nuclear charge density difference $\varrho_c\,(^{206}\mathrm{Pb}) - \varrho_c\,(^{205}\mathrm{Tl}) = \varphi^2_{3s_{1/2}}(r)$ is given [taken from (Frois 1983) and Doe 1983)]

Not too approximate model for nuclei : already quite some correlations : the mean field model (or shell-model)

Woods-Saxon potential :

$$V(r) = V_0 \frac{1}{1 + e^{\left(\frac{r-R}{a}\right)}}$$

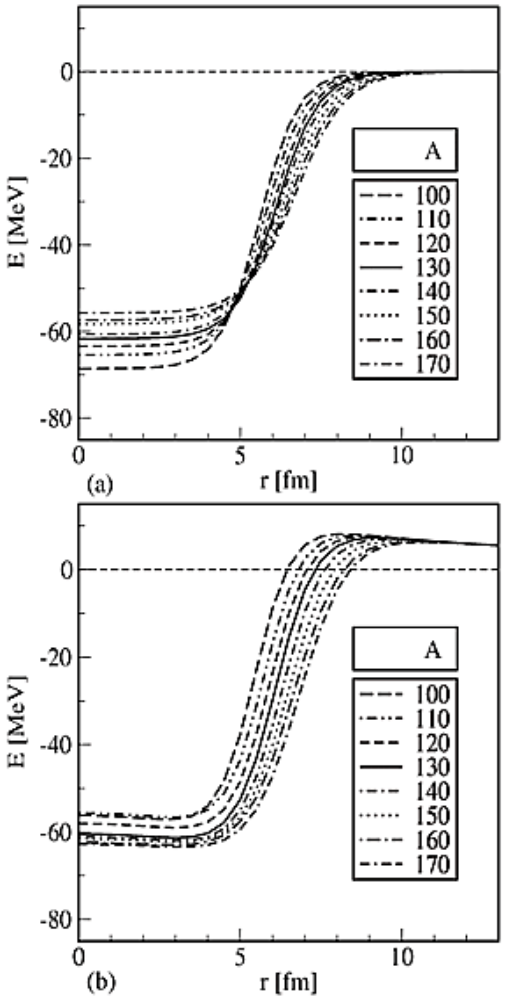


Figure 1. Woods-Saxon potential for neutrons (a) and protons (b) along the Sn isotopic chain.

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The Hartree-Fock mean field

- Nucleons are moving independent from each other in a mean field
- How do we obtain a reliable and consistent field ?

$$H = \sum_i T_i + \frac{1}{2}\sum_{i,j} V_{i,j}$$

$$H = \sum_i \left(T_i + U(r_i)\right) + \left(\frac{1}{2}\sum_{i,j} V_{i,j} - \sum_i U(r_i)\right)$$

$$H = \sum_i h_0(i) + H_{res}$$

Residual interaction

The Hartree-Fock recipe :

Nucleons fill up a number of orbitals and form a density that can be written in terms of the occupied states as :

$$\rho(r) = \sum_{b<F} \varphi_b^*(r)\varphi_b(r)$$

The potential at a position r', generated by the nucleon-nucleon two-body interaction V(r,r') is given by

$$U_H(r') = \sum_{b<F} \int \varphi_b^*(r) V(r,r') \varphi_b(r) dr$$

$$- \frac{\hbar^2}{2m}\Delta\varphi_i(r) + \sum_{b<f} \int \varphi_b^*(r') V(r,r') \varphi_b(r') dr' \cdot \varphi_i(r)$$

$$- \sum_{b<f} \int \varphi_b^*(r') V(r,r') \varphi_b(r) \varphi_i(r') dr' = \varepsilon_i \varphi_i(r)$$

$$-\frac{\hbar^2}{2m}\Delta\varphi_i(r) + U_H(r)\varphi_i(r) - \int U_F(r,r')\varphi_i(r')dr' = \varepsilon_i\varphi_i(r)$$

$$U_H(r') = \sum_{b<F}\int \varphi_b^*(r)V(r,r')\varphi_b(r)dr$$

$$U_F(r,r') = \sum_{b<F} \varphi_b^*(r')V(r,r')\varphi_b(r)$$

Hartree-Fock recipe :Start with an initial guess for either the average field or the wave functions and use V(r,r') to solve the coupled equations to obtain better values

e.g.

$$\begin{array}{ccccc} U^{(0)}_{H(F)}(r) & & U^{(1)}_{H(F)}(r) & & U^{(2)}_{H(F)}(r) \\ \downarrow & \nearrow & \downarrow & \nearrow & \downarrow \\ \varphi_i^{(0)}(r) & & \varphi_i^{(1)}(r) & & \varphi_i^{(2)}(r) \\ \varepsilon_i^{(0)} & & \varepsilon_i^{(1)} & & \varepsilon_i^{(2)} \end{array}$$

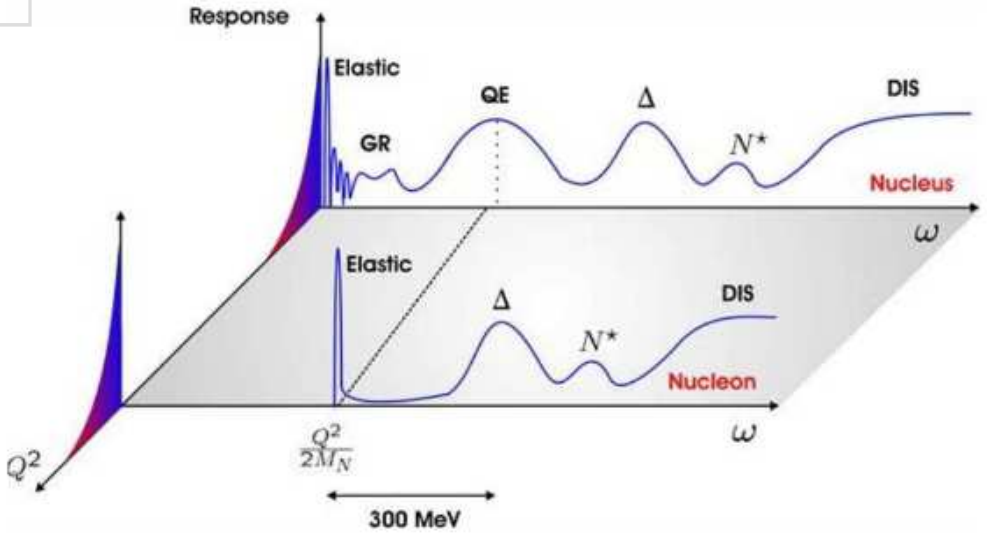
Wave function for the nucleus = Slater determinant

$$\Psi_{1,2,\ldots,A}(r_1, r_2, \ldots, r_A) = \frac{1}{\sqrt{A}} \begin{vmatrix} \varphi_1(r_1) & \varphi_2(r_2) & \cdots & \varphi_A(r_A) \\ \varphi_2(r_1) & \varphi_3(r_2) & \cdots & \varphi_1(r_A) \\ \vdots & & & \end{vmatrix}$$

$$E_0 = \sum_{i=1}^{A} \varepsilon_i$$

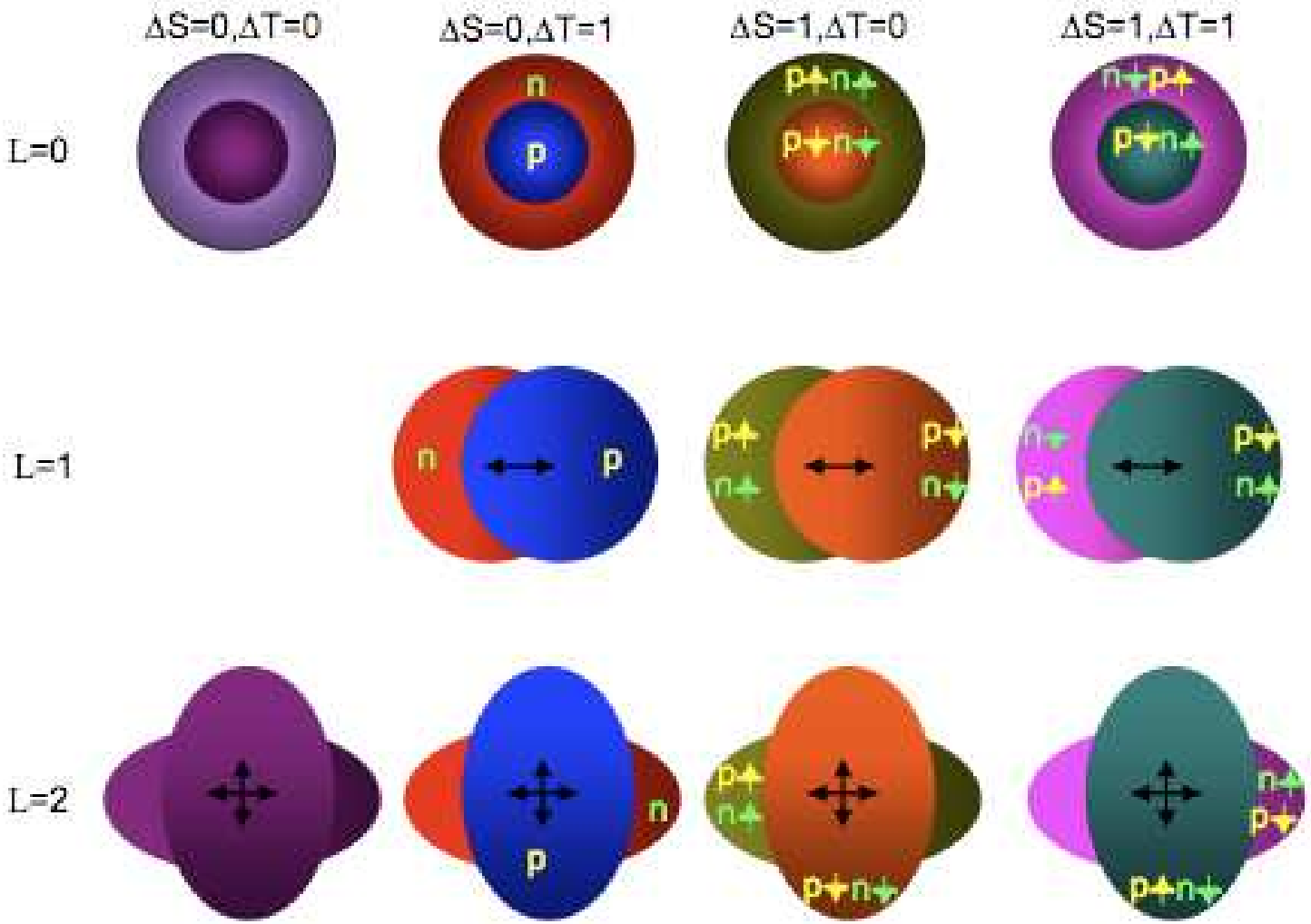
Antisymmetrization takes into account the Pauli principle

Long-range correlations : Continuum Random Phase Approximation



- Long-range correlations are correlations over the whole size of the nucleus
- They can redistribute the incoming energy transfer to the nucleus over all the nuclear constituents.
- They manifest themselves in collective excitations such as giant resonances

Long-range correlations = probing collective effects at low energies



https://cyclotron.tamu.edu/research/nuclear-structure/

RPA … What's in a name

David Bohm & David Pines '52 – Condensed matter physics

Quantum mechanical interactions interaction between electrons

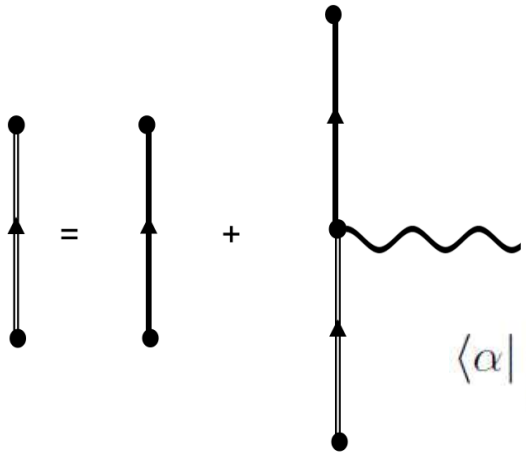
The Hamiltonian corresponding to a system of individual electrons is re-expressed such that the long-range part of the Coulomb interactions between electrons is described in terms of collective fields

This leads to the description of organized behavior of electrons brought along by long-range Coulomb interactions that couple together the motion of many electrons simultaneously = plasma oscillations

Neglecting the coupling between plasma vibrations of different momenta

Single particle propagator

$$G = G^{(0)} + G^{(0)} V G$$
$$G = G^{(0)} + G^{(0)} V G^{(0)} + G^{(0)} V G^{(0)} V G^{(0)} + \ldots$$



$$\langle \alpha | \frac{1}{E - H + i\eta} | \beta \rangle = \langle \alpha | \frac{1}{E - H_0 + i\eta} | \beta \rangle + \sum_{\gamma,\delta} \langle \alpha | \frac{1}{E - H_0 + i\eta} | \gamma \rangle \langle \gamma | V | \delta \rangle \langle \delta | \frac{1}{E - H + i\eta} | \beta \rangle$$

Hamiltonian : $H=H_0+U+V$

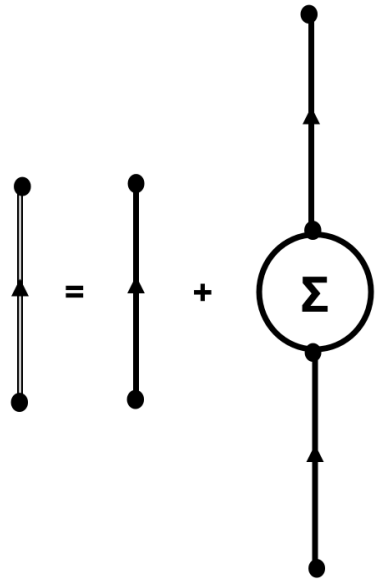
Auxiliary potential

2-body interaction

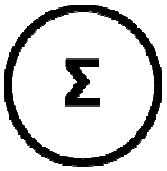
$$G = G^{(0)} + G^{(0)}VG$$

$$G = G^{(0)} + G^{(0)}VG^{(0)} + G^{(0)}VG^{(0)}VG^{(0)} + \dots$$

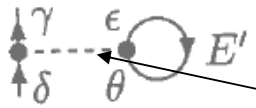
Dyson Equation :



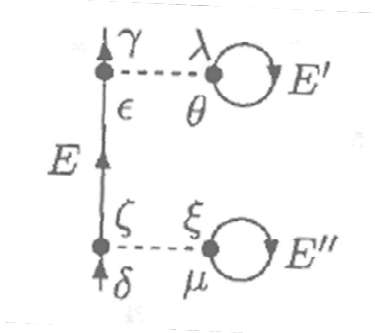
Self-energy

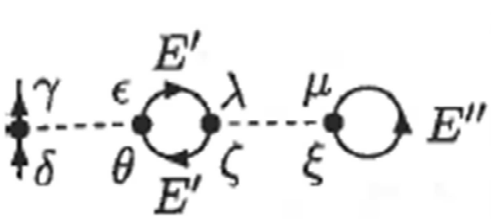


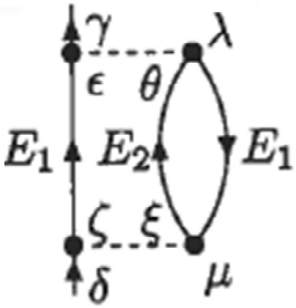
= e.g.



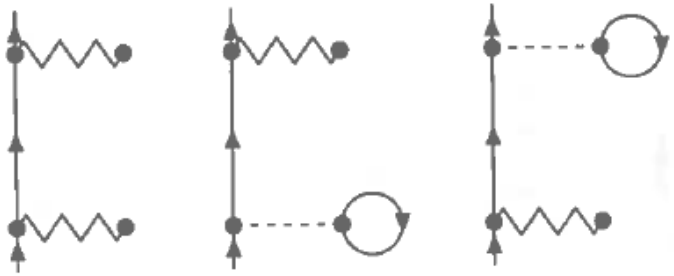
2-body interaction V







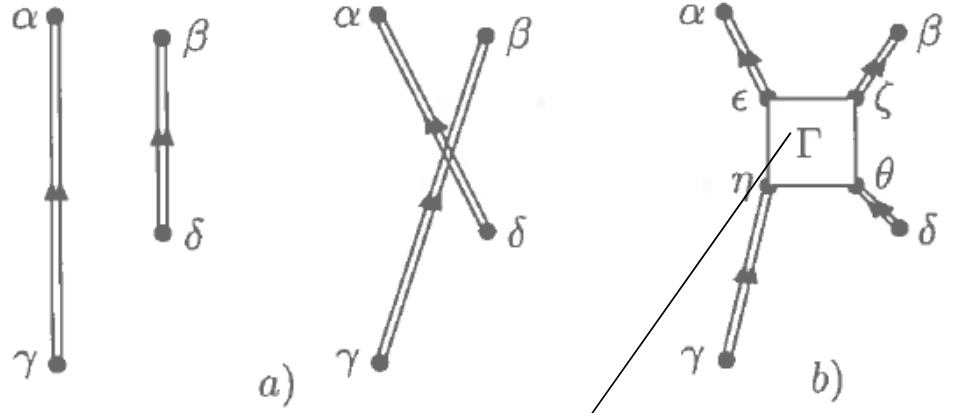
Axiliary field U



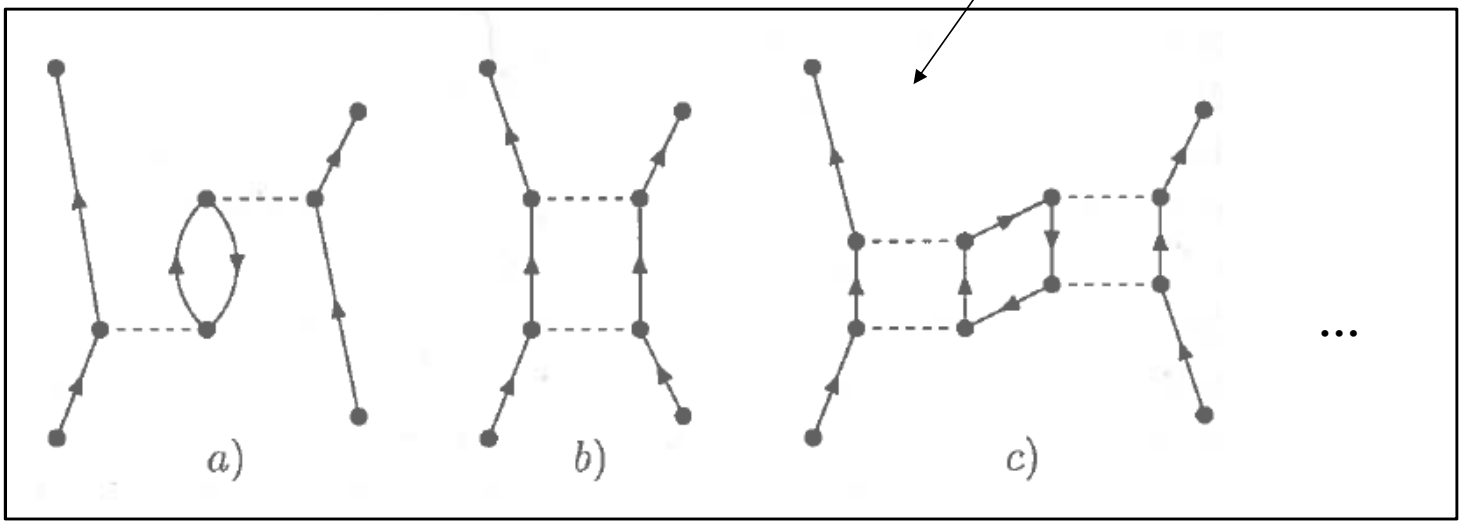
...

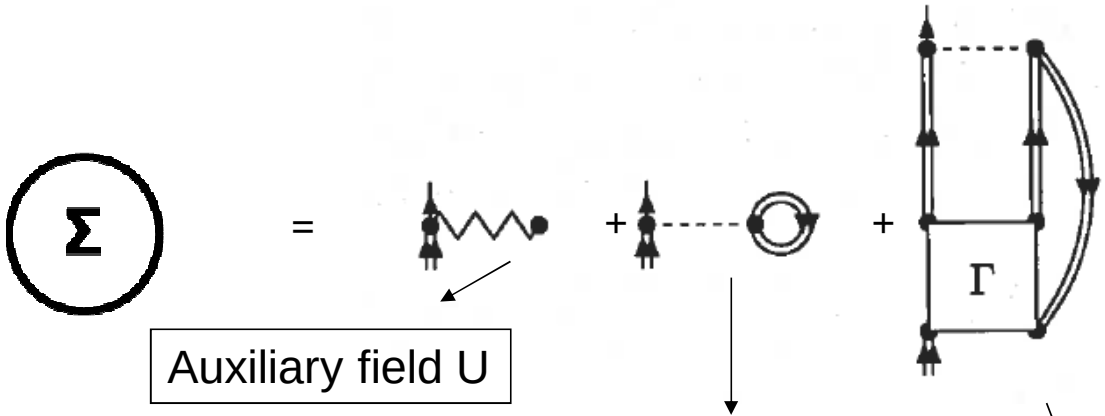
Two-particle propagator

$G_{II}(\alpha, \beta, \gamma, \delta)$



Vertex function



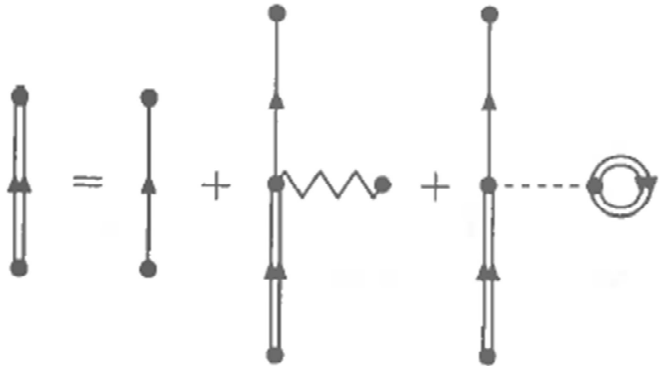


Higher order contributions/different topologies

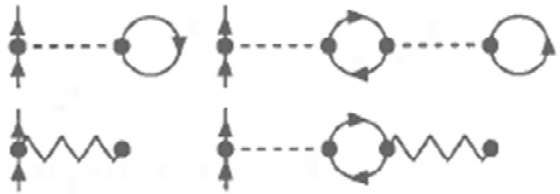
⇨ Reformulation of the self-energy of a dressed particle

The Hartree-Fock mean field

$$G^{HF}(\alpha,\beta;E) = G^{(0)}(\alpha,\beta;E) + \sum_{\gamma,\delta} G^{(0)}(\alpha,\gamma;E)\Sigma^{HF}(\gamma,\delta)G^{HF}(\delta,\beta;E)$$



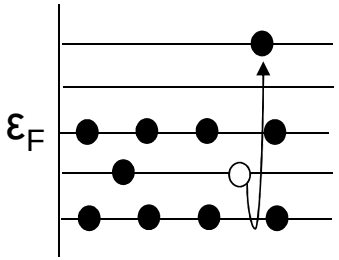
$$\Sigma^{HF}(\gamma,\delta;E) = -\langle\gamma|\,U\,|\delta\rangle - i\int\frac{dE'}{2\pi}\sum_{\mu\nu}\langle\gamma\mu|\,V\,|\delta\nu\rangle\,G^{HF}(\nu\mu;E')$$



Mean field already contains correlations !

Excited states : Particle-hole or polarization propagator

$$\Pi(\alpha,\beta^{-1};\gamma,\delta^{-1};E) = \sum_{n\neq 0}\frac{\left\langle\Phi_0^N\right|a_\beta^\dagger a_\alpha\left|\Phi_n^N\right\rangle\left\langle\Phi_n^N\right|a_\gamma^\dagger a_\delta\left|\Phi_0^N\right\rangle}{E-(E_n^N-E_0^N)+i\eta} - \sum_{n\neq 0}\frac{\left\langle\Phi_0^N\right|a_\gamma^\dagger a_\delta\left|\Phi_n^N\right\rangle\left\langle\Phi_n^N\right|a_\beta^\dagger a_\alpha\left|\Phi_0^N\right\rangle}{E+(E_n^N-E_0^N)-i\eta}$$



$$\Pi^{(0)}(\alpha,\beta^{-1};\gamma,\delta^{-1};E) = \int\frac{dE'}{2\pi}G^{(0)}(\alpha,\gamma;E+E')G^{(0)}(\delta^{-1},\beta^{-1};E')$$

=

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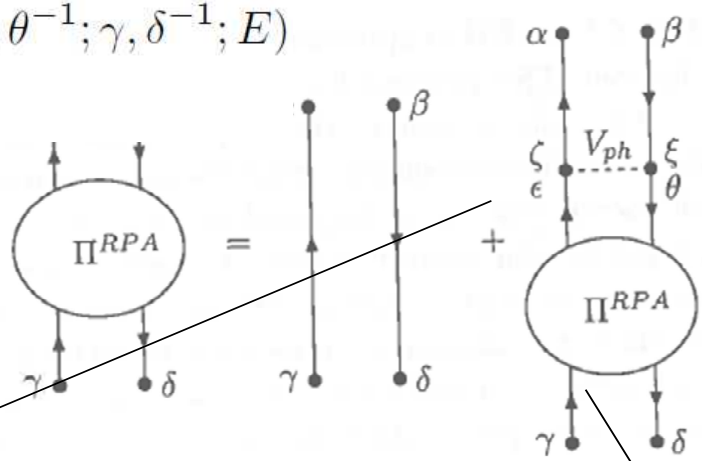
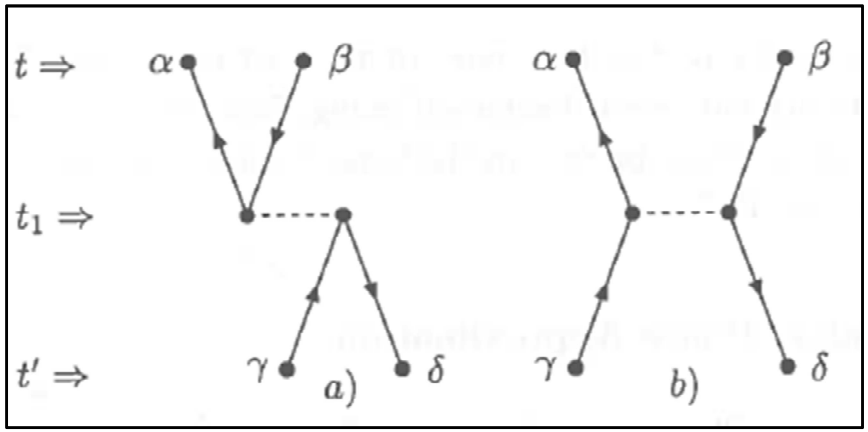
Excited states : Particle-hole or polarization propagator

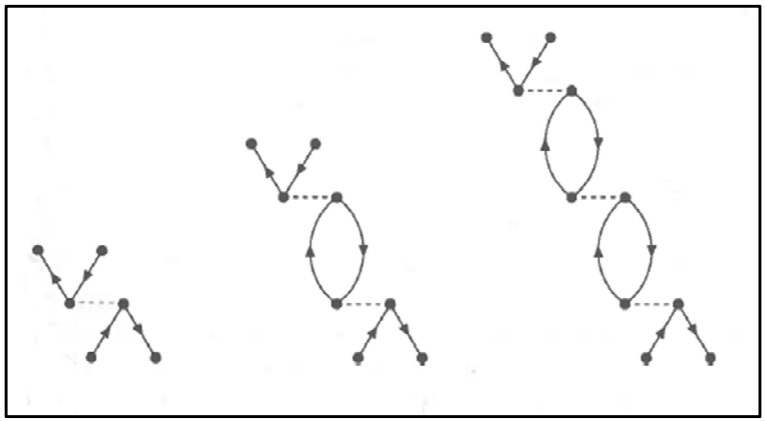
$$\Pi^{RPA}(\alpha,\beta^{-1};\gamma,\delta^{-1};E) = \Pi^{(0)}(\alpha,\beta^{-1};\gamma,\delta^{-1};E)$$
$$+\sum_{\varepsilon\theta\zeta\xi}\Pi^{(0)}(\alpha,\beta^{-1};\zeta,\xi^{-1};E)\left\langle\zeta\xi^{-1}\right|V_{ph}\left|\varepsilon\theta^{-1}\right\rangle\Pi^{RPA}(\varepsilon,\theta^{-1};\gamma,\delta^{-1};E)$$



First order



Higher order
RPA diagrams



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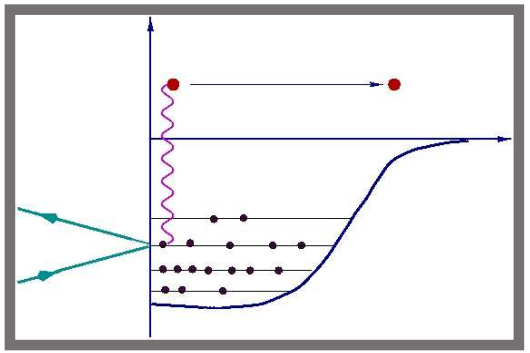
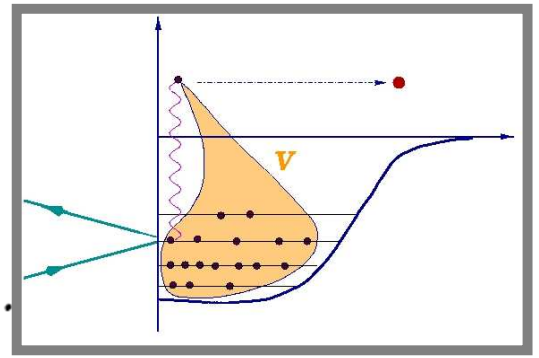
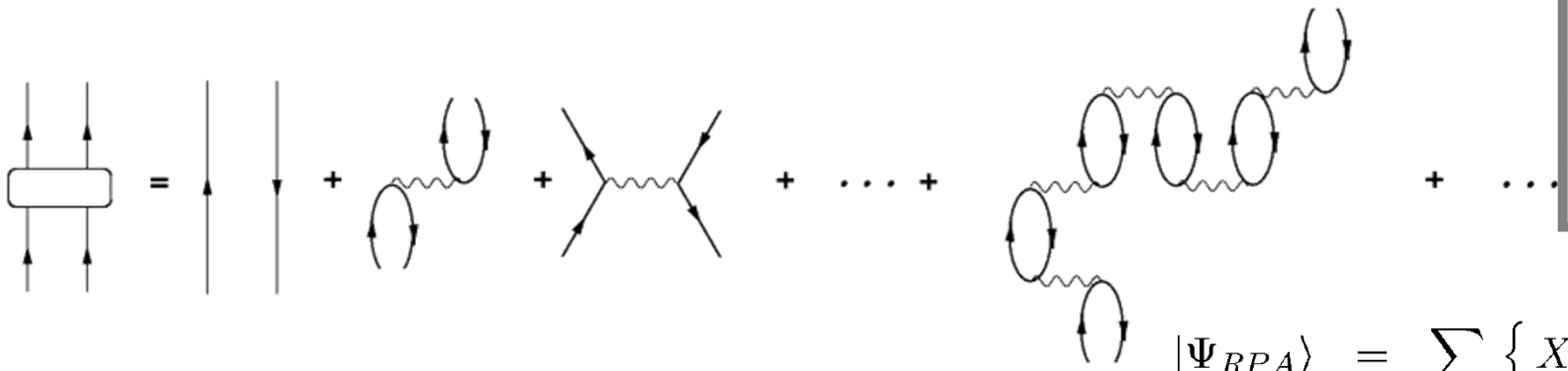
Long-range correlations : Continuum RPA

- Green's function approach
- Skyrme SkE2 residual interaction
- self-consistent calculations







$$|\Psi_{RPA}\rangle = \sum_c \left\{ X_{(\Psi,C)} \left| ph^{-1} \right\rangle - Y_{(\Psi,C)} \left| hp^{-1} \right\rangle \right\}$$

$$\Pi^{(RPA)}(x_1, x_2; \omega) = \Pi^{(0)}(x_1, x_2; \omega) + \frac{1}{\hbar} \int dx \int dx' \; \Pi^{(0)}(x_1, x; \omega) \, \tilde{V}(x, x') \, \Pi^{(RPA)}(x', x_2; \omega)$$

Solving the RPA equations in coordinate space :

$$|\Psi_C(E)\rangle = \left|ph^{-1}(E)\right\rangle + \int dx_1 \int dx_2 \; \widetilde{V}(x_1,x_2) \sum_{c'} \mathcal{P} \int d\varepsilon_{p'} \left[\frac{\psi_{h'}(x_1)\psi^{\dagger}_{p'}(x_1,\varepsilon_{p'})}{E-\varepsilon_{p'h'}} \left|p'h'^{-1}(\varepsilon_{p'h'})\right\rangle - \frac{\psi^{\dagger}_{h'}(x_1)\psi_{p'}(x_1,\varepsilon_{p'})}{E+\varepsilon_{p'h'}} \left|h'p'^{-1}(-\varepsilon_{p'h'})\right\rangle \right] \left\langle \Psi_0 \left| \hat{\psi}^{\dagger}(x_2)\hat{\psi}(x_2) \right| \Psi_C(E) \right\rangle$$

What we really need is transition densities :

$$\langle \Psi_0 || X_{\eta J} || \Psi_C(J;E) \rangle_r = - \langle h || X_{\eta J} || p(\varepsilon_{ph}) \rangle_r + \sum_{\mu,\nu} \int dr_1 \int dr_2 \; U^J_{\mu\nu}(r_1,r_2) \; \mathcal{R}\left(R^{(0)}_{\eta\mu;J}(r,r_1;E)\right) \; \langle \Psi_0 || X_{\nu J} || \Psi_C(J;E) \rangle_{r_2}$$

So in the end we have to solve a set of coupled equations, that after discretizing on a mesh in coordinate space, translates into a matrix inversion for the transition densities:

$$\rho_C^{RPA} = -\frac{1}{1-R\,U}\;\rho_C^{HF}$$

2-body interaction V ?

Landau-Migdal interaction

$$V = c_0\{f_0(\rho) + f_0'(\rho)\vec{\tau}_1\vec{\tau}_2 + g_0(\rho)\vec{\sigma}_1\vec{\sigma}_2 + g_0'(\rho)\vec{\sigma}_1\vec{\sigma}_2\vec{\tau}_1\vec{\tau}_2\}$$

$$f(\rho(r)) = (1-\rho(r))\ f^{ext} + \rho(r)\ f^{int}$$

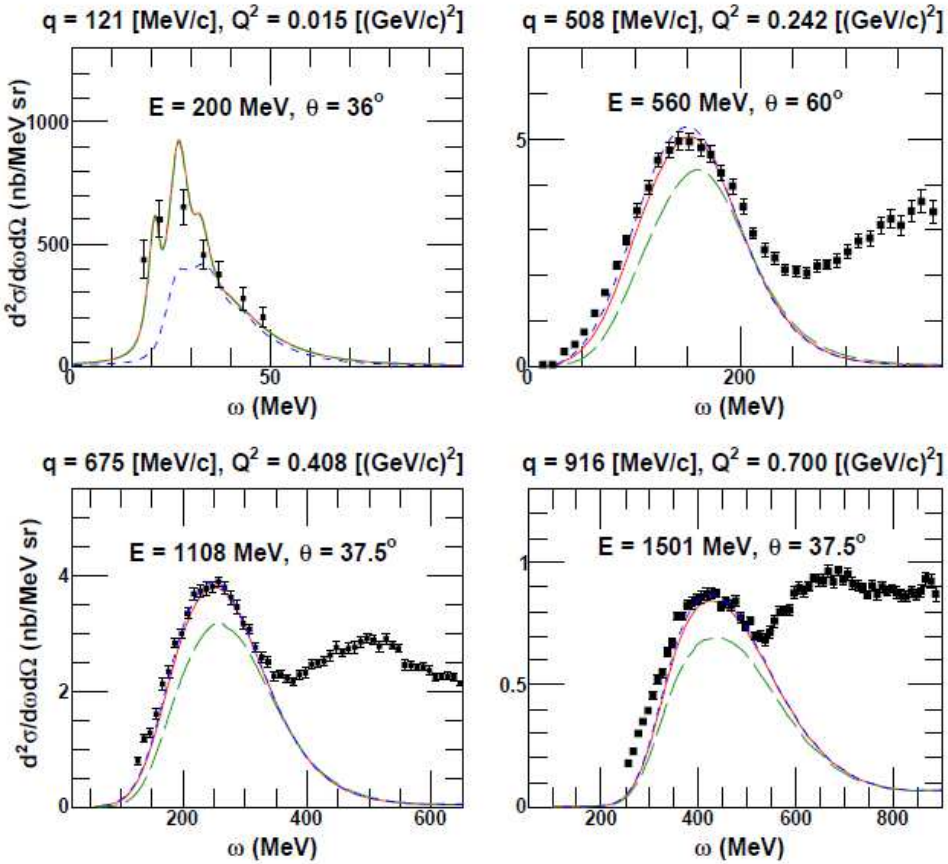
$$\rho(r) = \frac{1}{1+e^{\frac{r-R}{a}}}$$

2-body interaction V ?

Skyrme

$$
\begin{aligned}
V(\vec{r}_1,\vec{r}_2) \;=\;& t_0\,(1+x_0\hat{P}_\sigma)\,\delta(\vec{r}_1-\vec{r}_2) \\
& -\frac{1}{8}t_1\left[(\overleftarrow{\nabla}_1-\overleftarrow{\nabla}_2)^2\,\delta(\vec{r}_1-\vec{r}_2)+\delta(\vec{r}_1-\vec{r}_2)\,(\overrightarrow{\nabla}_1-\overrightarrow{\nabla}_2)^2\right] \\
& +\frac{1}{4}t_2(\overleftarrow{\nabla}_1-\overleftarrow{\nabla}_2)\,\delta(\vec{r}_1-\vec{r}_2)\,(\overrightarrow{\nabla}_1-\overrightarrow{\nabla}_2) \\
& +iW_0\,(\vec{\sigma}_1+\vec{\sigma}_2)\cdot(\overleftarrow{\nabla}_1-\overleftarrow{\nabla}_2)\times\delta(\vec{r}_1-\vec{r}_2)\,(\overrightarrow{\nabla}_1-\overrightarrow{\nabla}_2) \\
& +\frac{1}{6}t_3\,(1-x_3)\,(1+\hat{P}_\sigma)\,\rho\,\frac{(\vec{r}_1+\vec{r}_2)}{2}\,\delta(\vec{r}_1-\vec{r}_2) \\
& +\frac{e^2}{|\vec{r}_1-\vec{r}_2|}+x_3t_3\,\delta(\vec{r}_1-\vec{r}_2)\,\delta(\vec{r}_1-\vec{r}_3) \\
& -\frac{1}{24}t_4\left\{\left[(\overleftarrow{\nabla}_1-\overleftarrow{\nabla}_2)^2+(\overleftarrow{\nabla}_2-\overleftarrow{\nabla}_3)^2+(\overleftarrow{\nabla}_3-\overleftarrow{\nabla}_1)^2\right]\right\} \\
& \qquad \delta(\vec{r}_1-\vec{r}_2)\,\delta(\vec{r}_1-\vec{r}_3)+\delta(\vec{r}_1-\vec{r}_2)\,\delta(\vec{r}_1-\vec{r}_3) \\
& \qquad \left\{\left[(\overrightarrow{\nabla}_1-\overrightarrow{\nabla}_2)^2+(\overrightarrow{\nabla}_2-\overrightarrow{\nabla}_3)^2+(\overrightarrow{\nabla}_3-\overleftarrow{\nabla}_1)^2\right]\right\}.
\end{aligned}
$$

•Regularization of the residual interaction :



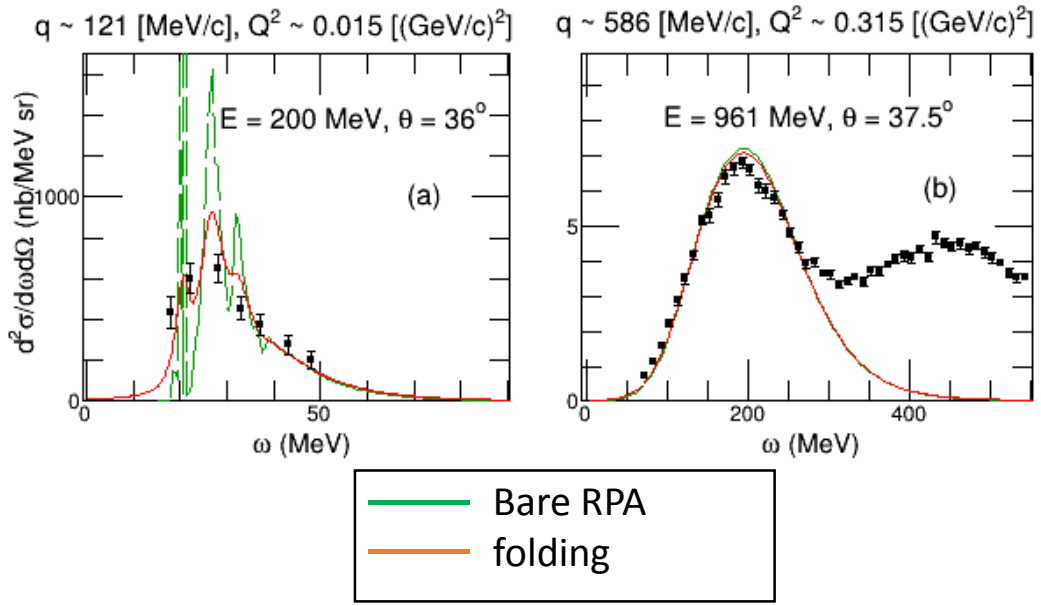
$$V(Q^2) \rightarrow V(Q^2 = 0)\ \frac{1}{(1 + \frac{Q^2}{\Lambda^2})^2}$$

Uncorrected
dipole

- Final state interactions :

-taken into account through the calculations of the wave function of the outgoing nucleon in the (real) nuclear potential generated using the Skyrme force

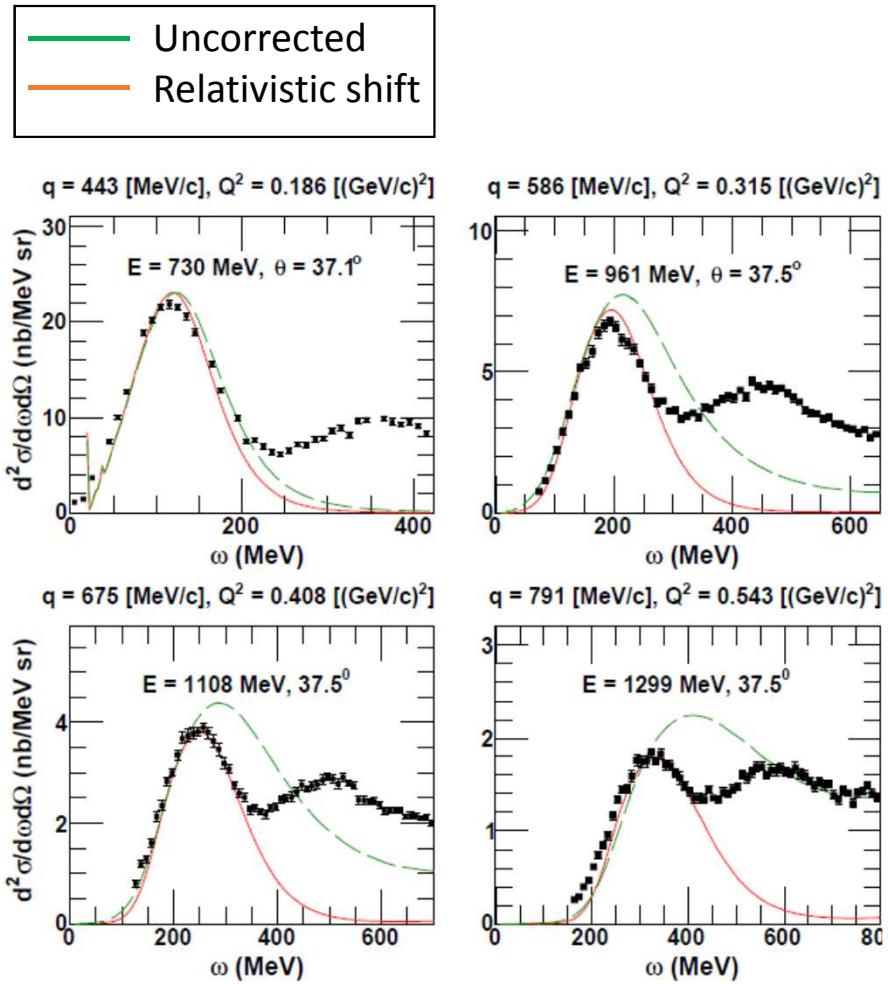
-influence of the spreading width of the particle states is implemented through a folding procedure



$$R'(q,\omega') = \int_{-\infty}^{\infty} d\omega \; R(q,\omega) \; L(\omega,\omega'),$$

$$L(\omega,\omega') = \frac{1}{2\pi}\left[\frac{\Gamma}{(\omega-\omega')^2+(\Gamma/2)^2}\right].$$

•Relativistic corrections at higher energies (S. Jeschonnek and T. Donnelly, PRC57, 2438 (1998)):



Shift :

$$\lambda \to \lambda(\lambda+1) \qquad \lambda = \omega/2M_N$$

- The outgoing nucleon obtains the correct relativistic momentum

$$p = \sqrt{T^2 + 2MT}$$

- Shifts the QE peak to the right relativistic position

Boost :

$$R^{\mathrm{V}}_{\mathrm{CC}}(q,\omega) \to \frac{q^2}{q^2-\omega^2} R^{\mathrm{V}}_{\mathrm{CC}}(q,\omega)\,,$$

$$R^{\mathrm{A}}_{\mathrm{LL}}(q,\omega) \to \left(1+\frac{q^2-\omega^2}{4m^2}\right) R^{\mathrm{A}}_{\mathrm{LL}}(q,\omega)\,,$$

$$R^{\mathrm{V}}_{\mathrm{T}}(q,\omega) \to \frac{q^2-\omega^2}{q^2} R^{\mathrm{V}}_{\mathrm{T}}(q,\omega)\,,$$

$$R^{\mathrm{A}}_{\mathrm{T}}(q,\omega) \to \left(1+\frac{q^2-\omega^2}{4m^2}\right) R^{\mathrm{A}}_{\mathrm{T}}(q,\omega)\,,$$

$$R^{\mathrm{VA}}_{\mathrm{T}'}(q,\omega) \to \sqrt{\frac{q^2-\omega^2}{q^2}}\sqrt{1+\frac{q^2-\omega^2}{4m^2}}\, R^{\mathrm{VA}}_{\mathrm{T}'}(q,\omega)\,.$$

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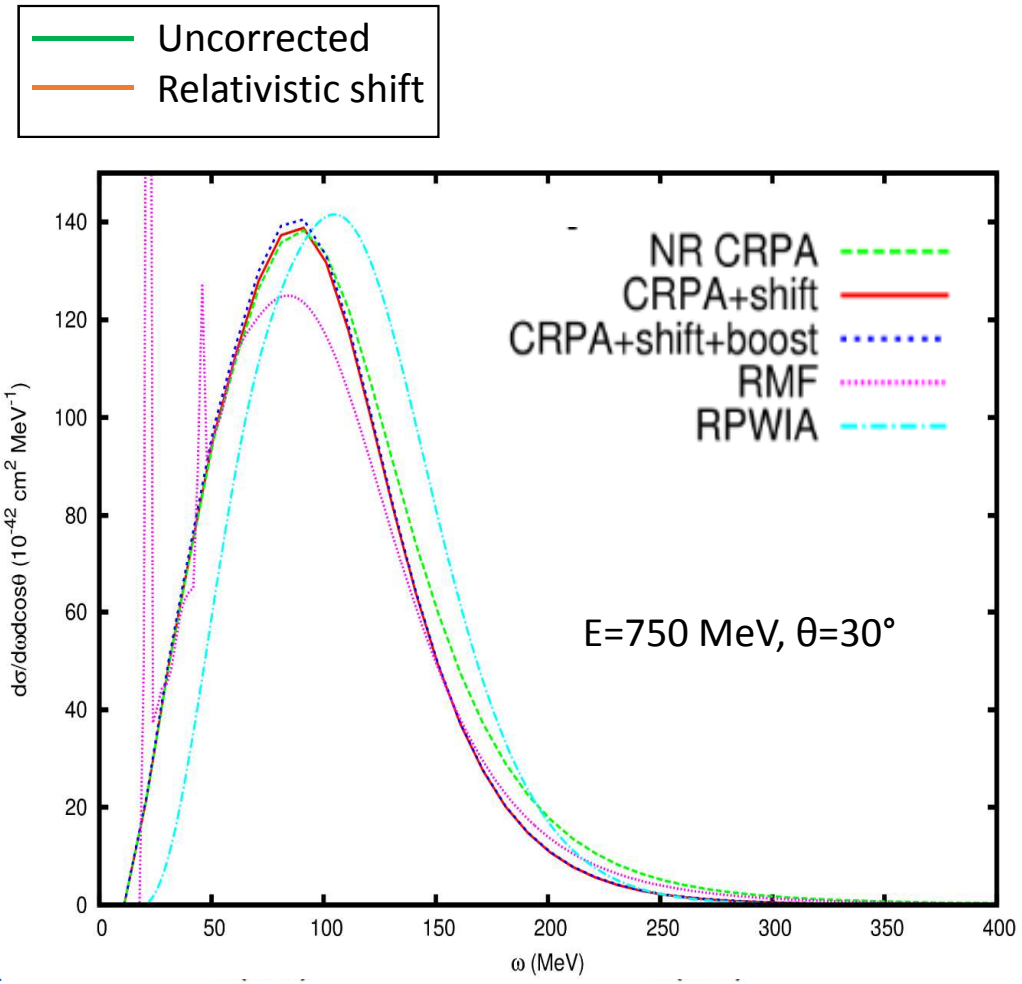

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$$R^{\mathrm{V}}_{\mathrm{T}}(q,\omega) \to \frac{q^2-\omega^2}{q^2} R^{\mathrm{V}}_{\mathrm{T}}(q,\omega)\,,$$

$$R^{\mathrm{A}}_{\mathrm{T}}(q,\omega) \to \left(1+\frac{q^2-\omega^2}{4m^2}\right) R^{\mathrm{A}}_{\mathrm{T}}(q,\omega)\,,$$

$$R^{\mathrm{VA}}_{\mathrm{T}'}(q,\omega) \to \sqrt{\frac{q^2-\omega^2}{q^2}}\sqrt{1+\frac{q^2-\omega^2}{4m^2}}\, R^{\mathrm{VA}}_{\mathrm{T}'}(q,\omega)\,.$$

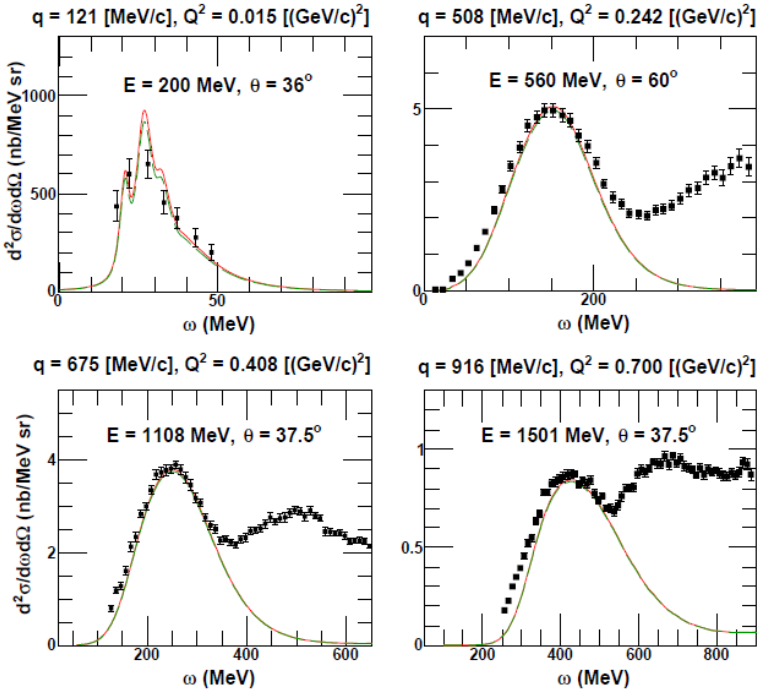
•Coulomb correction for the outgoing lepton in charged-current interactions :

✓ Low energies : Fermi function

$$F(Z',E) = \frac{2\pi\eta}{1-e^{-2\pi\eta}} \qquad \eta \sim \mp Z'\alpha$$

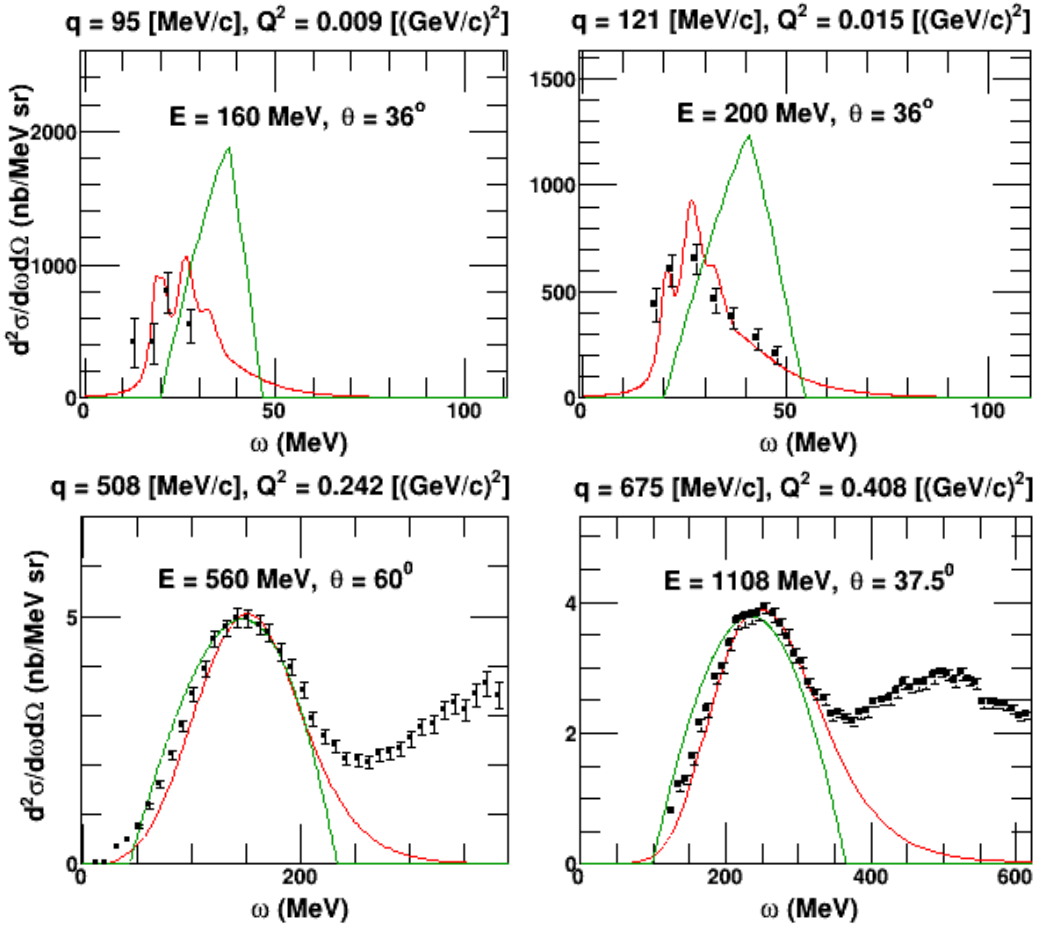
✓ High energies : modified effective momentum approximation (J. Engel, PRC57,2004 (1998))

$$q_{eff} = q + 1.5\left(\frac{Z'\alpha\hbar c}{R}\right), \qquad \Psi_l^{eff} = \zeta(Z',E,q)\,\Psi_l\,,$$



$$\zeta(Z',E,q) = \sqrt{\frac{q_{eff}E_{eff}}{qE}}$$

Uncorrected
MEMA

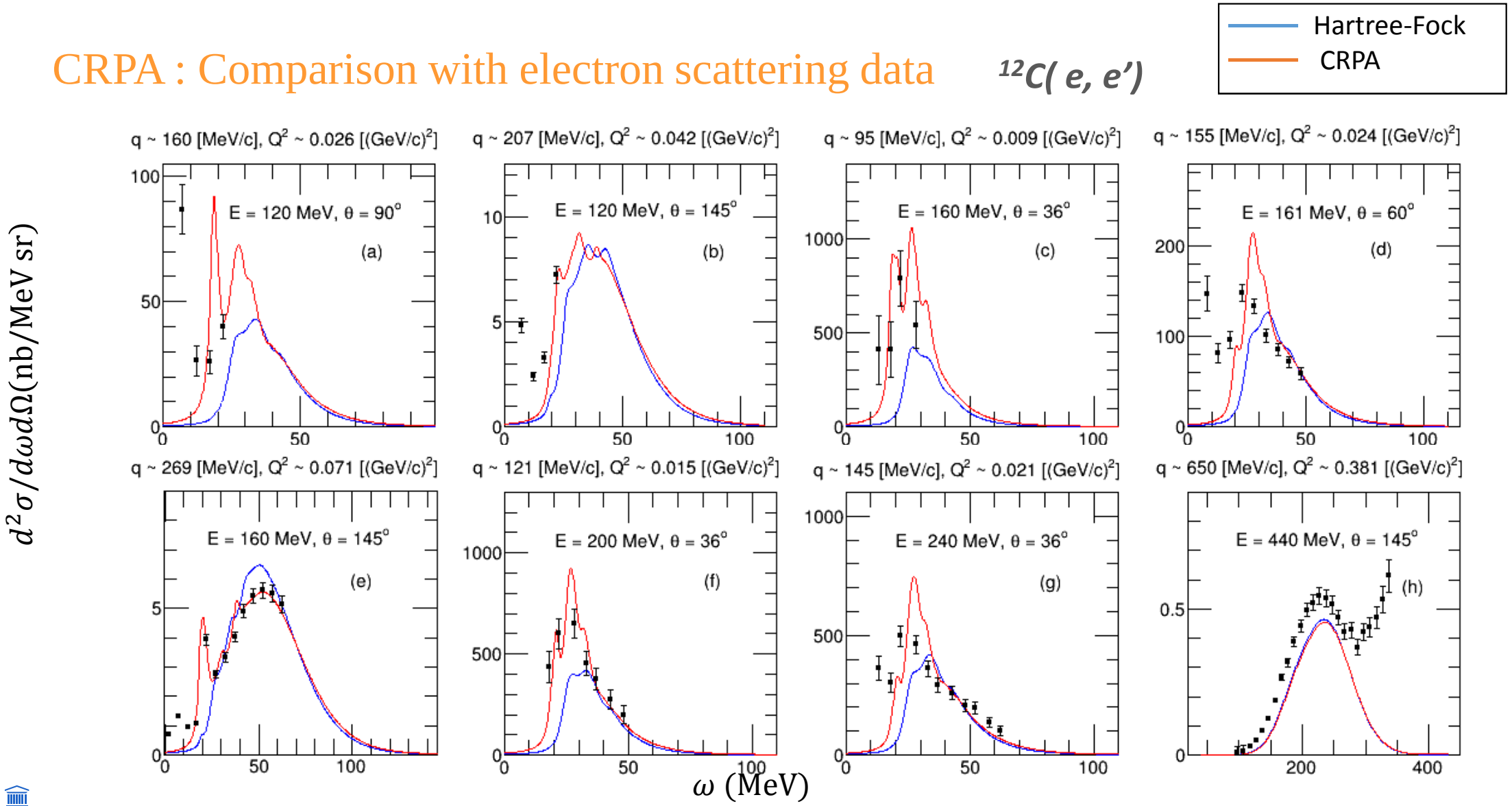


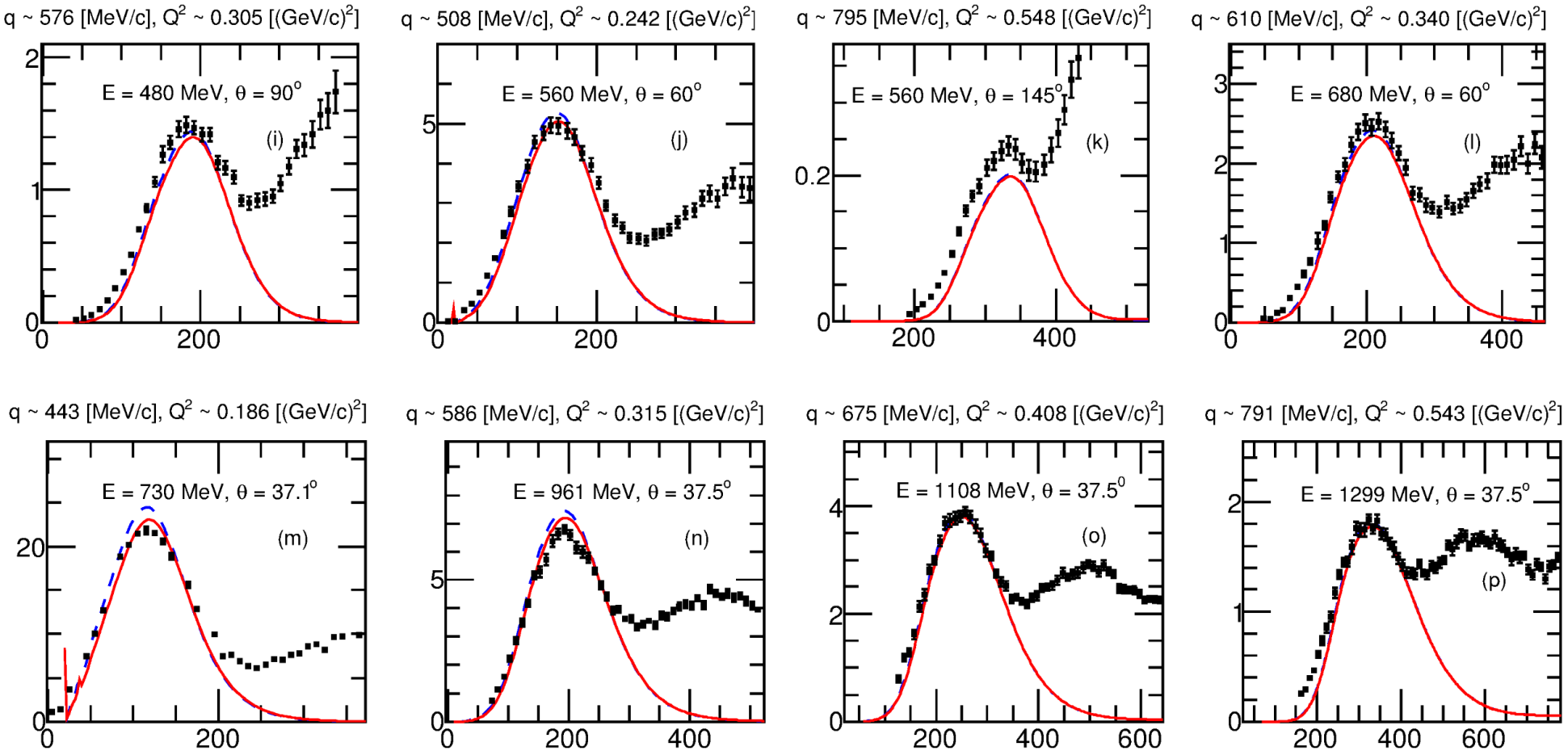
Fermi gas

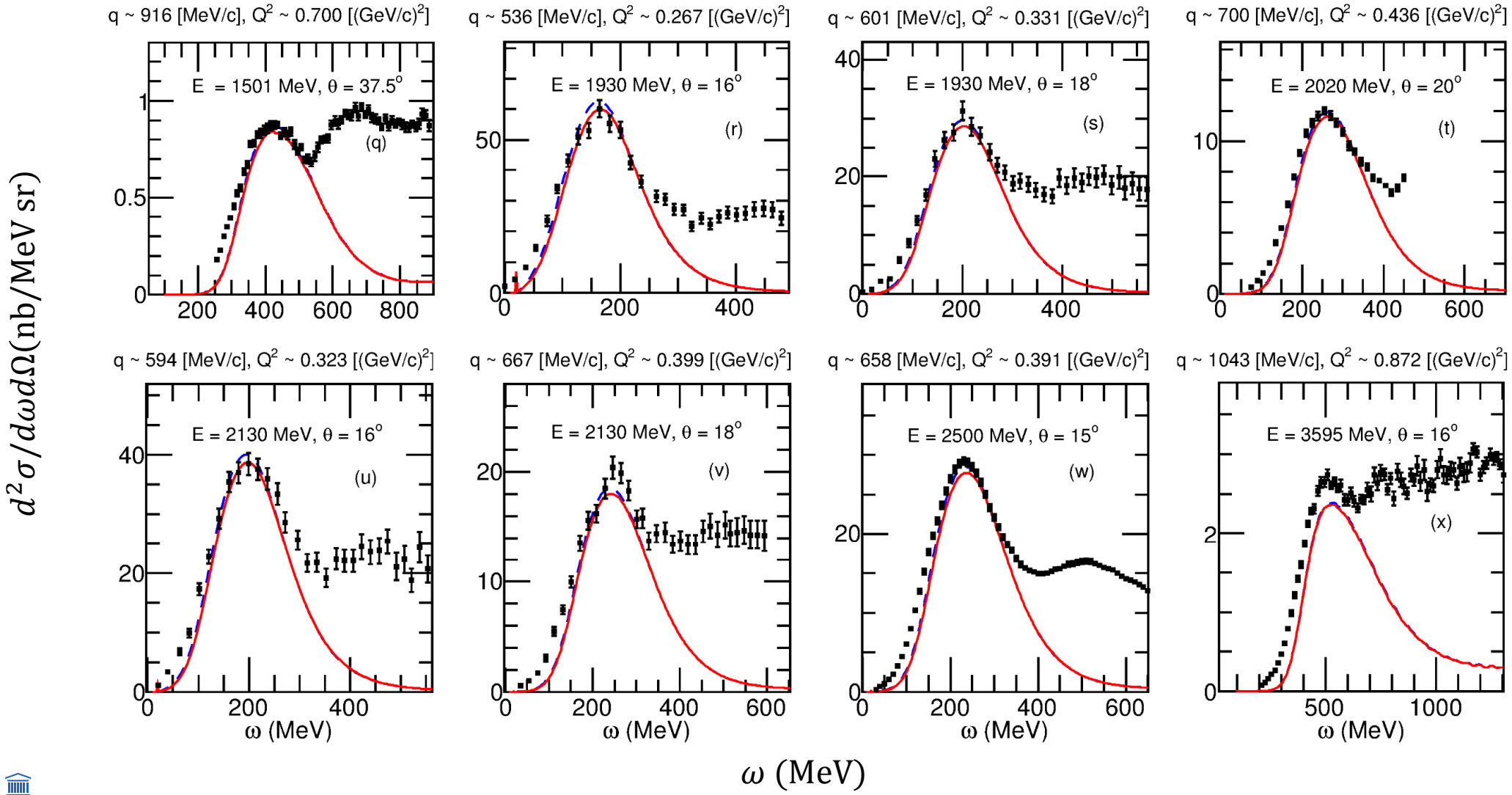
Hartree-Fock

CRPA : Comparison with electron scattering data

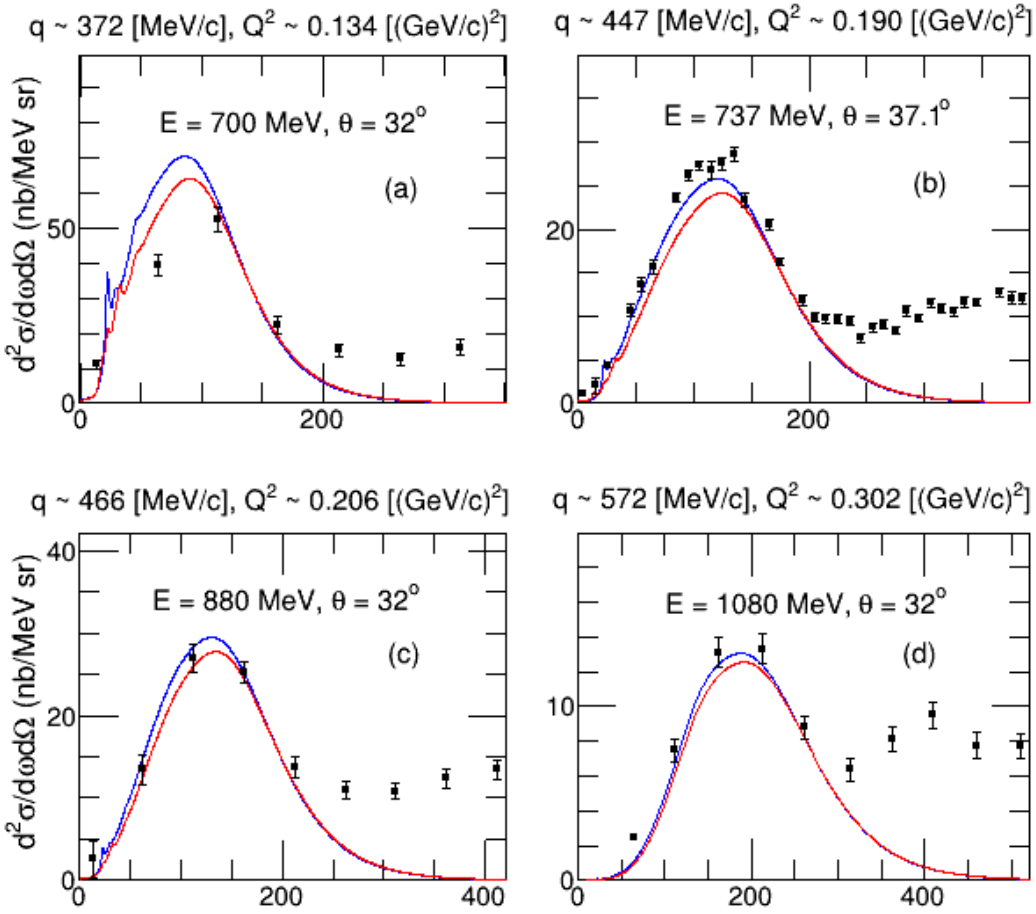
$^{12}C(e, e')$




q ~ 576 [MeV/c], Q² ~ 0.305 [(GeV/c)²]
E = 480 MeV, θ = 90°
(i)
q ~ 508 [MeV/c], Q² ~ 0.242 [(GeV/c)²]
E = 560 MeV, θ = 60°
(j)
q ~ 795 [MeV/c], Q² ~ 0.548 [(GeV/c)²]
E = 560 MeV, θ = 145°
(k)
q ~ 610 [MeV/c], Q² ~ 0.340 [(GeV/c)²]
E = 680 MeV, θ = 60°
(l)
q ~ 443 [MeV/c], Q² ~ 0.186 [(GeV/c)²]
E = 730 MeV, θ = 37.1°
(m)
q ~ 586 [MeV/c], Q² ~ 0.315 [(GeV/c)²]
E = 961 MeV, θ = 37.5°
(n)
q ~ 675 [MeV/c], Q² ~ 0.408 [(GeV/c)²]
E = 1108 MeV, θ = 37.5°
(o)
q ~ 791 [MeV/c], Q² ~ 0.543 [(GeV/c)²]
E = 1299 MeV, θ = 37.5°
(p)
$d^2\sigma/d\omega d\Omega$(nb/MeV sr)
ω (MeV)


q ~ 916 [MeV/c], Q² ~ 0.700 [(GeV/c)²]
E = 1501 MeV, θ = 37.5°
(q)
q ~ 536 [MeV/c], Q² ~ 0.267 [(GeV/c)²]
E = 1930 MeV, θ = 16°
(r)
q ~ 601 [MeV/c], Q² ~ 0.331 [(GeV/c)²]
E = 1930 MeV, θ = 18°
(s)
q ~ 700 [MeV/c], Q² ~ 0.436 [(GeV/c)²]
E = 2020 MeV, θ = 20°
(t)
q ~ 594 [MeV/c], Q² ~ 0.323 [(GeV/c)²]
E = 2130 MeV, θ = 16°
(u)
q ~ 667 [MeV/c], Q² ~ 0.399 [(GeV/c)²]
E = 2130 MeV, θ = 18°
(v)
q ~ 658 [MeV/c], Q² ~ 0.391 [(GeV/c)²]
E = 2500 MeV, θ = 15°
(w)
q ~ 1043 [MeV/c], Q² ~ 0.872 [(GeV/c)²]
E = 3595 MeV, θ = 16°
(x)
ω (MeV)
$d^2\sigma/d\omega d\Omega$(nb/MeV sr)
ω (MeV)

$^{16}O(e, e')$



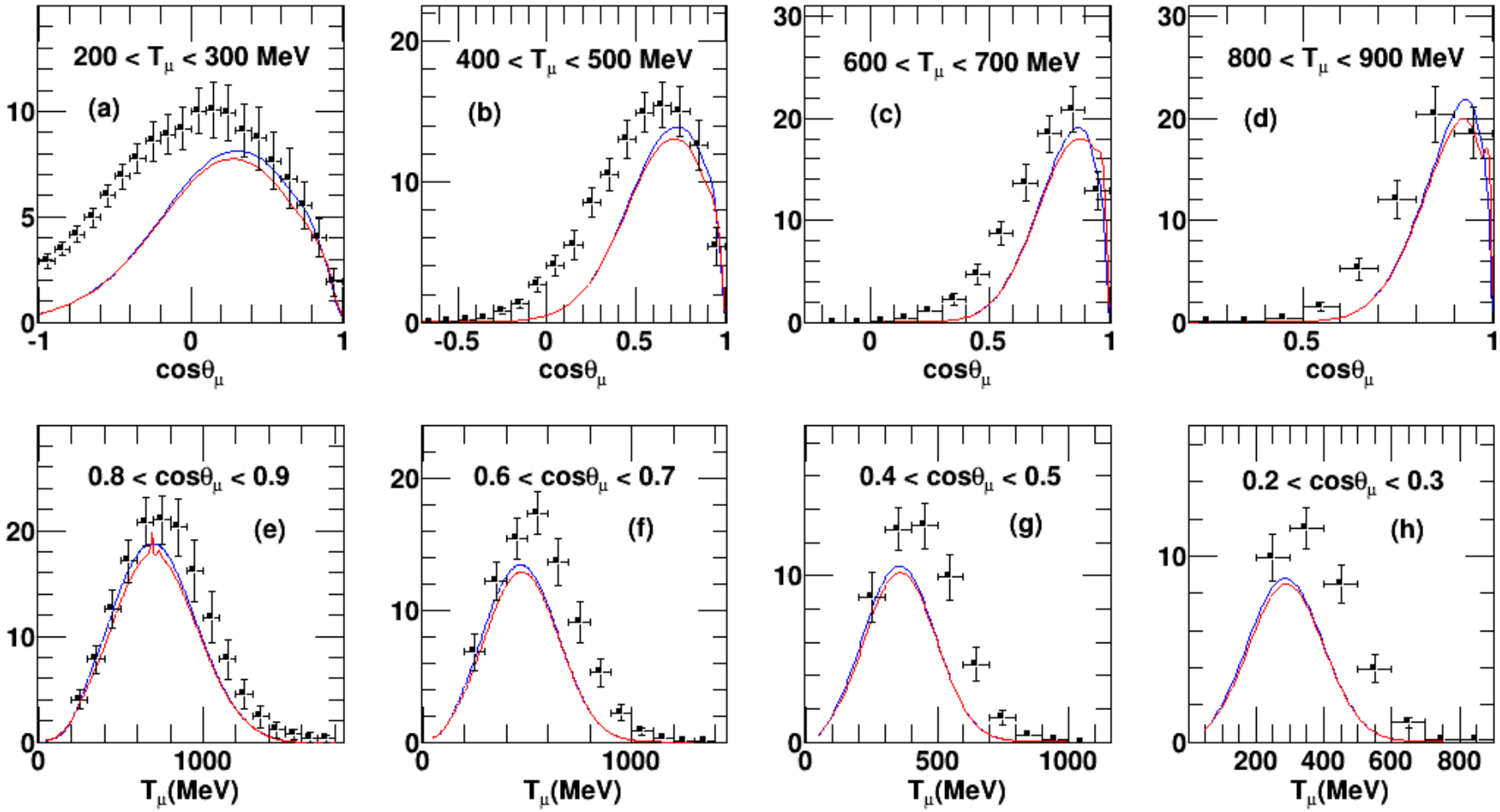
- Good overall agreement with e-scattering data

V. Pandey, N. Jachowicz, M. Martini, *et al.* Phys. Rev. C94, 054609 (2016)

P. Barreau et al., Nucl. Phys. A402, 515 (1983), J. S. O'Connell et al., Phys. Rev. C35, 1063 (1987), R. M. Sealock et al., Phys. Rev. Lett.62, 1350 (1989).,D. S. Bagdasaryan et al., YERPHI-1077-40-88 (1988),D. B. Day et al., Phys. Rev. C 48, 1849 (1993).,D. Zeller, DESY-F23-73-2 (1973).

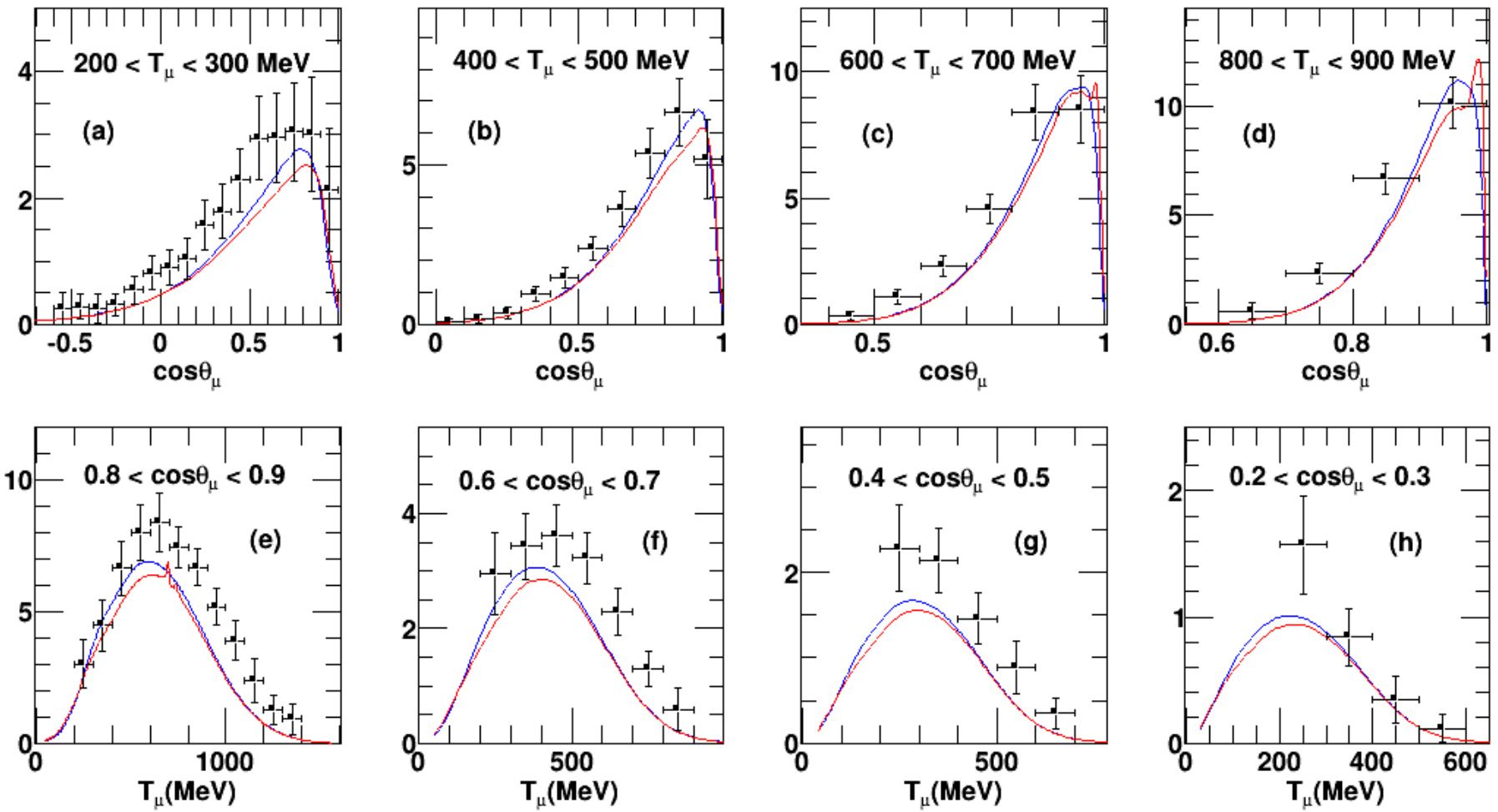
MiniBooNe ν_μ

- Satisfactory general agreement
- Good agreement for forward scattering
- Missing strength for low T_μ , backward scattering



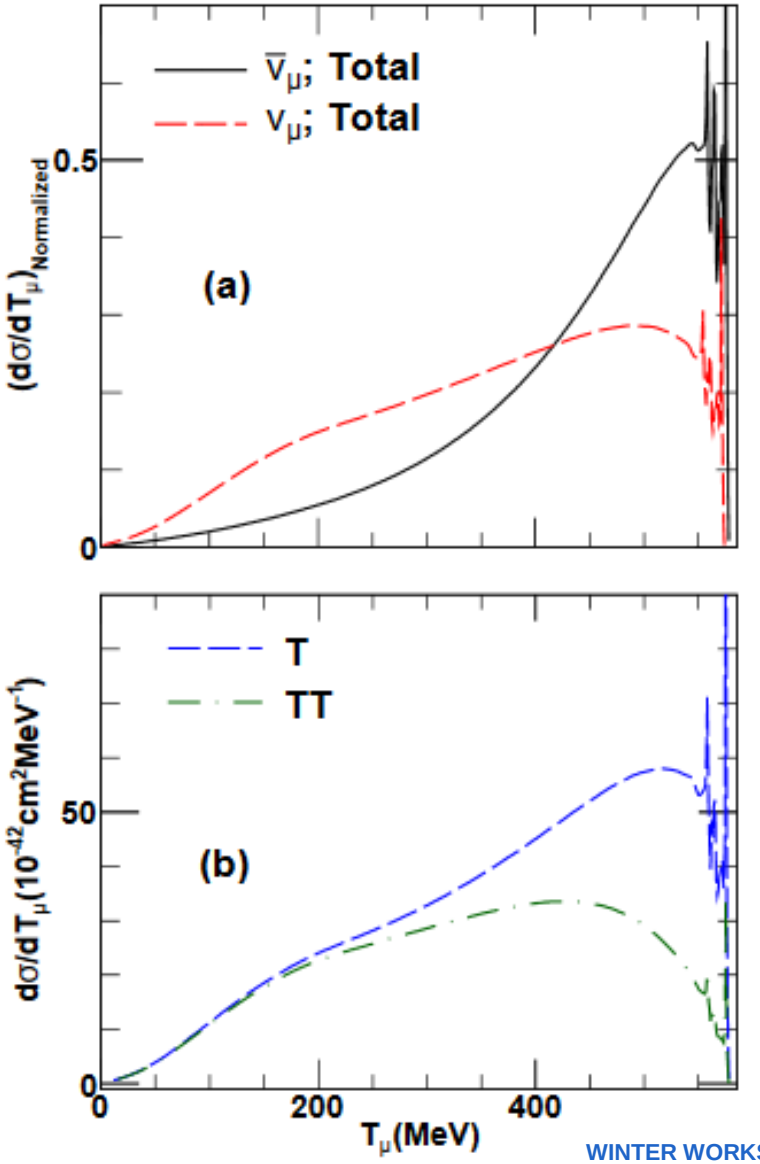
MiniBooNe $\bar{\nu}_\mu$

- Good general agreement
- Good agreement for forward scattering
- Missing strength for high T_μ , backward scattering
- Better agreement with data than neutrino cross sections

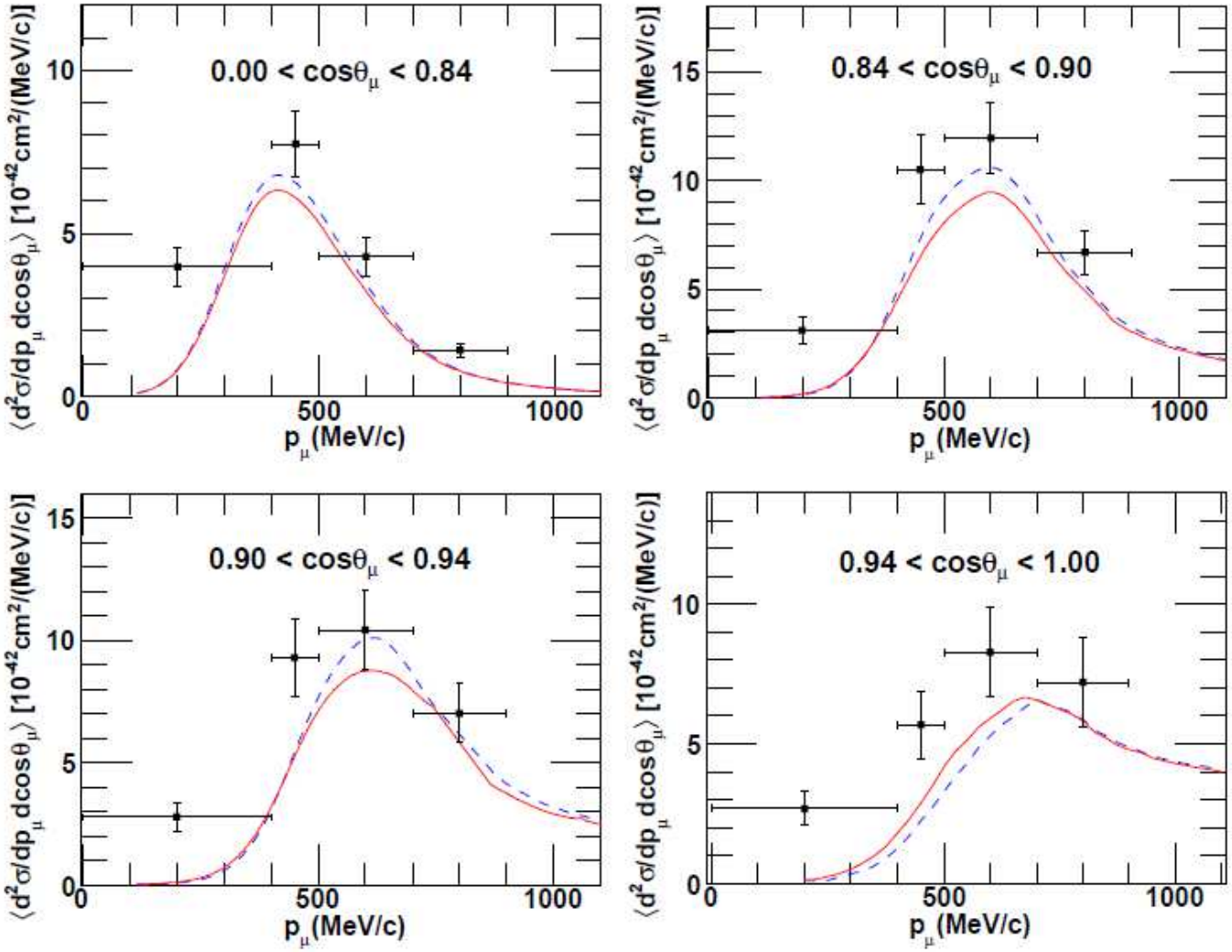


neutrino vs anti neutrinos

E=700 MeV



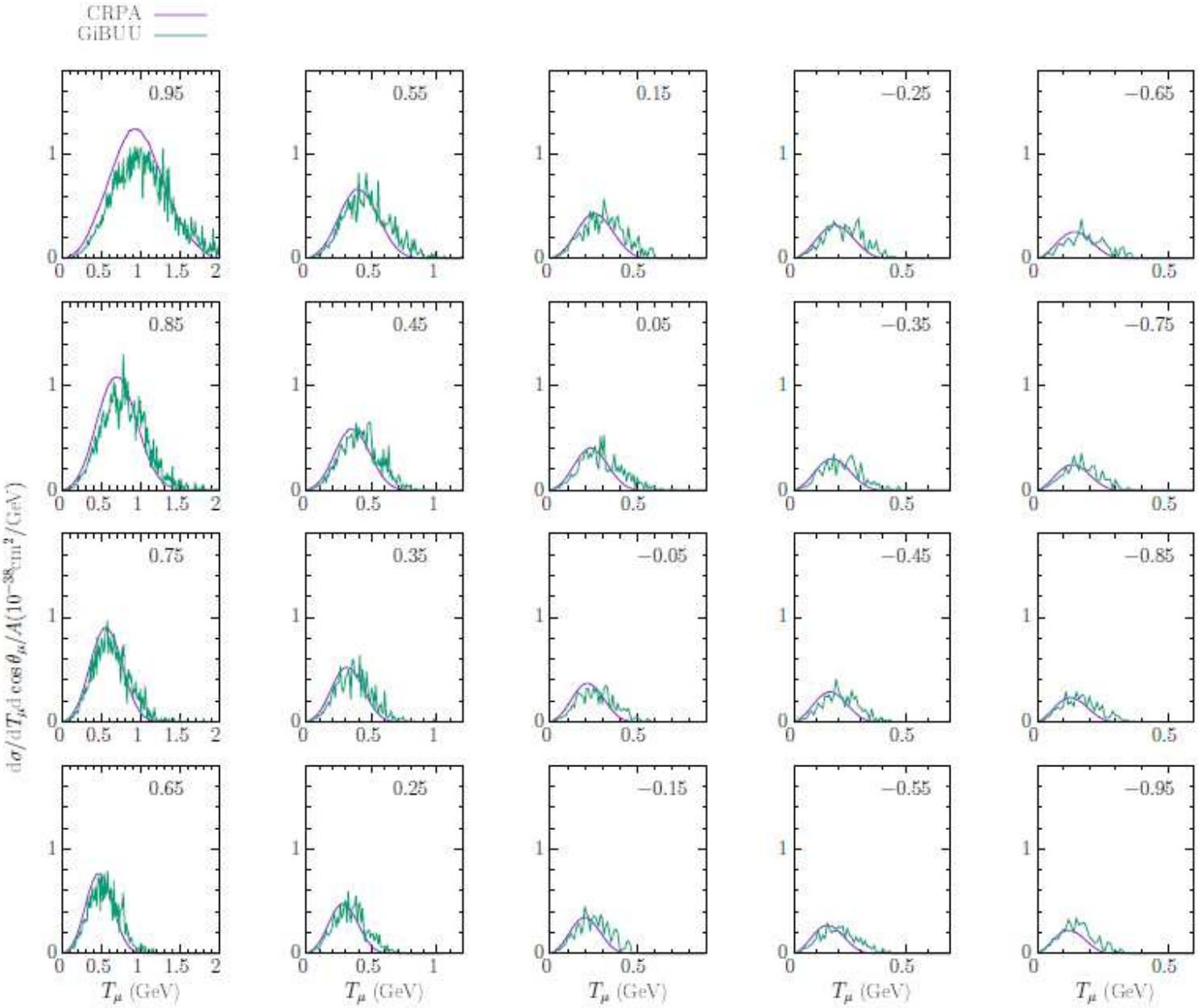
V. Pandey et al, Phys. Rev. C 92, 024606 (2015)



T2K ν_μ

- General agreement quite good
- Missing strength for low p_μ

^{40}Ar



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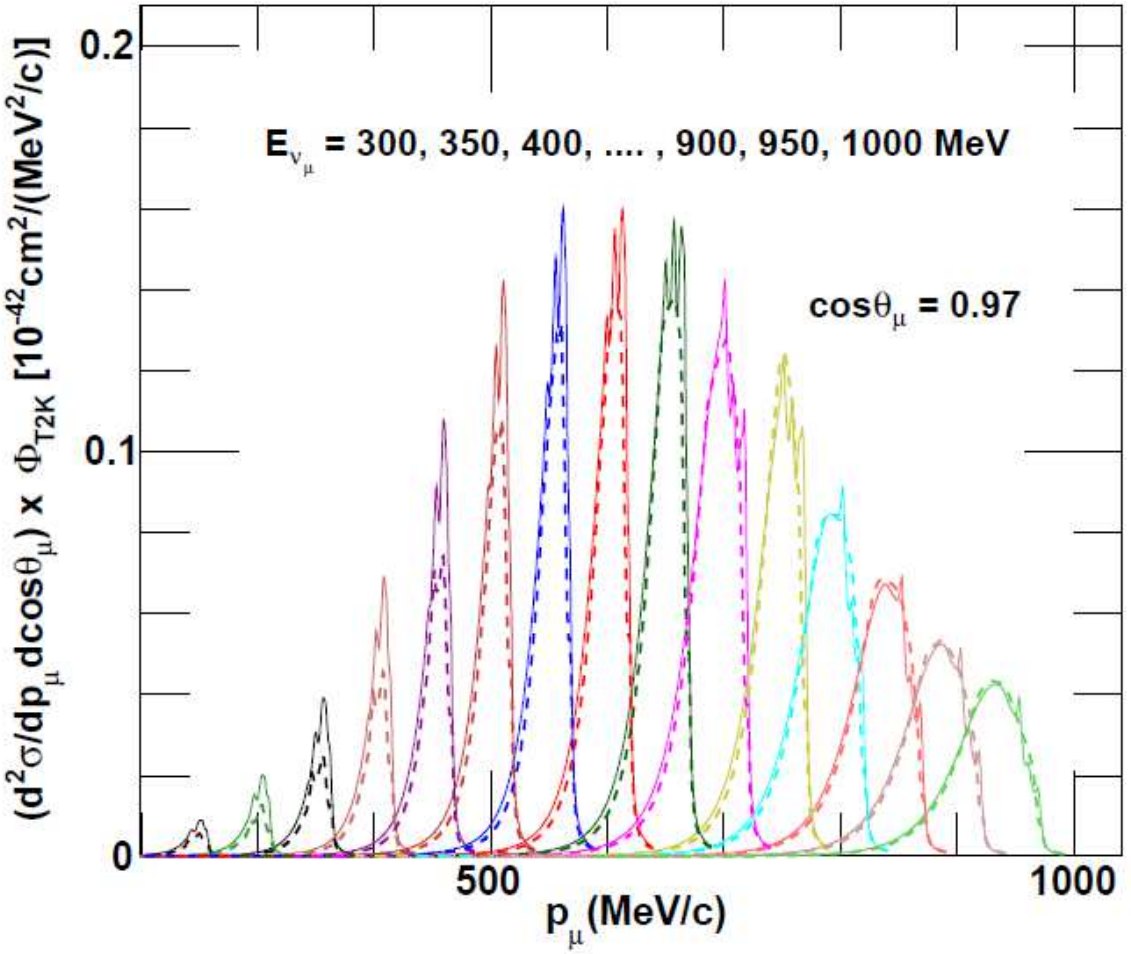
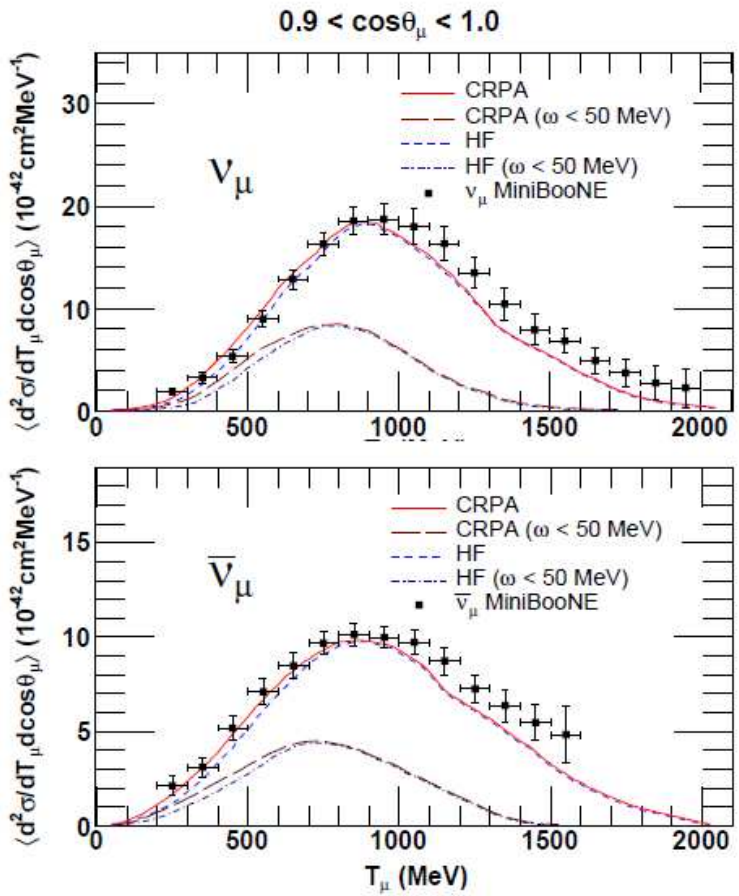
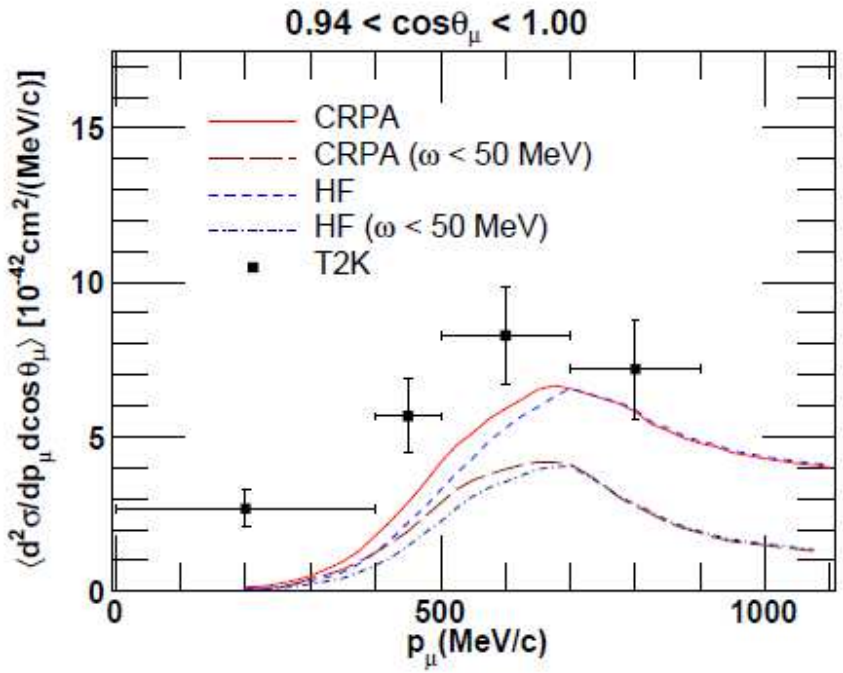
Forward scattering



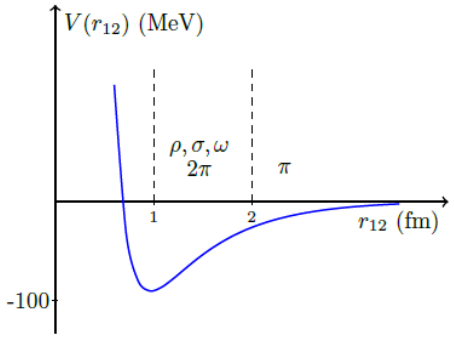
MiniBooNe



T2K

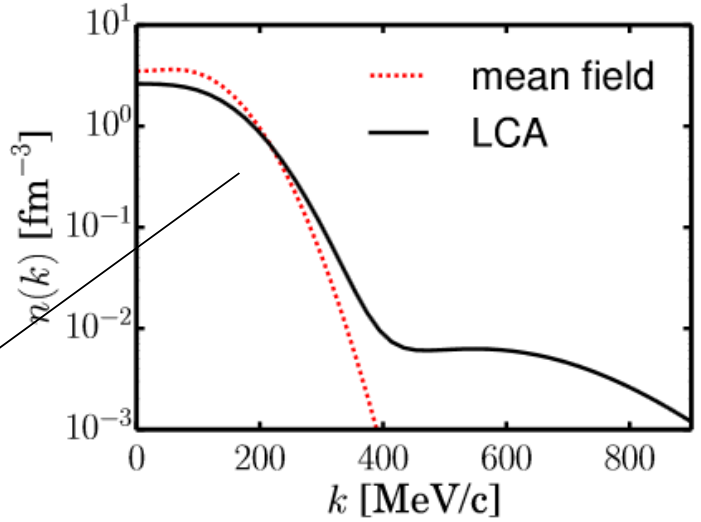


Short-range correlations



- The short-range repulsive character of the nuclear force, which correlates with the Pauli exclusion principle, results in a large mean free path of the nucleons with respect to the size of the nucleus
- In an independent particle model nucleons move independently from each other in a mean field
- This approach fails to capture short-range features of nucleon-nucleon correlations

- SRC : short-range repulsive, tensor component of the nuclear force
- Individual nucleons receive large momenta compared to the Fermi momentum



IPM single-particle orbitals are depleted by SRC and higher energy levels are populated

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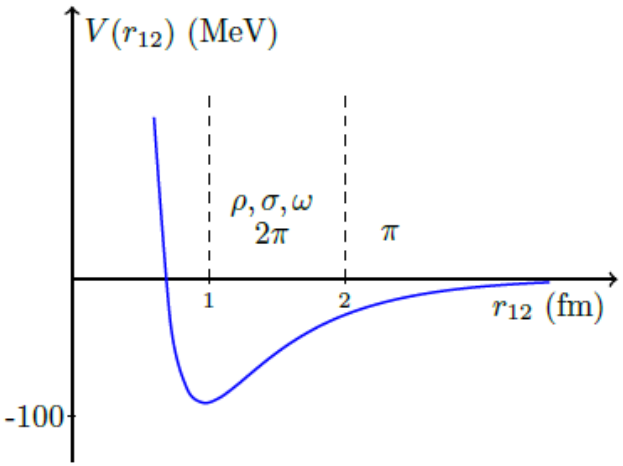
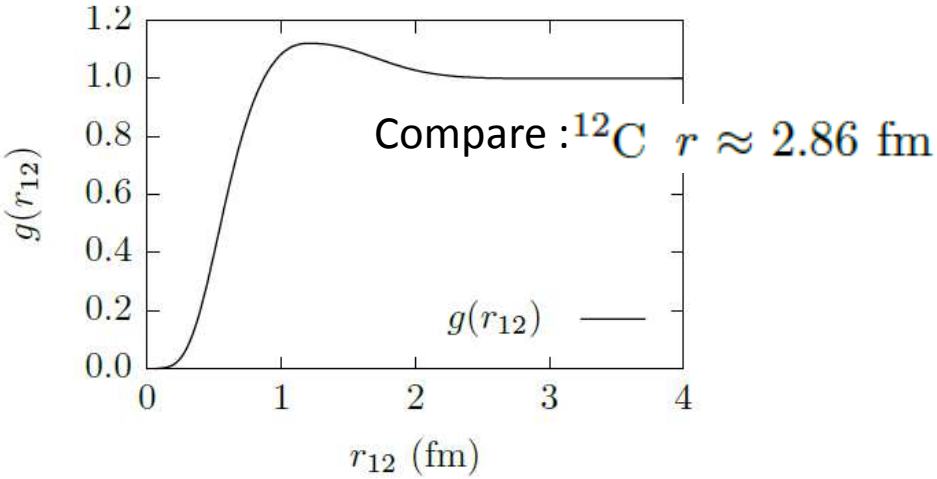
Two-body density :

$$\rho^{[2]}(\boldsymbol{r}_1, \boldsymbol{r}_2) = \rho^{[1]}(\boldsymbol{r}_1)\rho^{[1]}(\boldsymbol{r}_2)g(r_{12})$$

Independent particle model

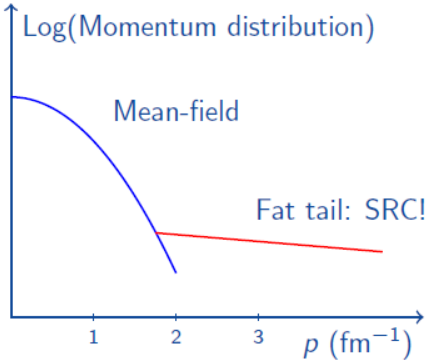
Correlation function





Compare : $^{12}\mathrm{C}$ $r \approx 2.86$ fm

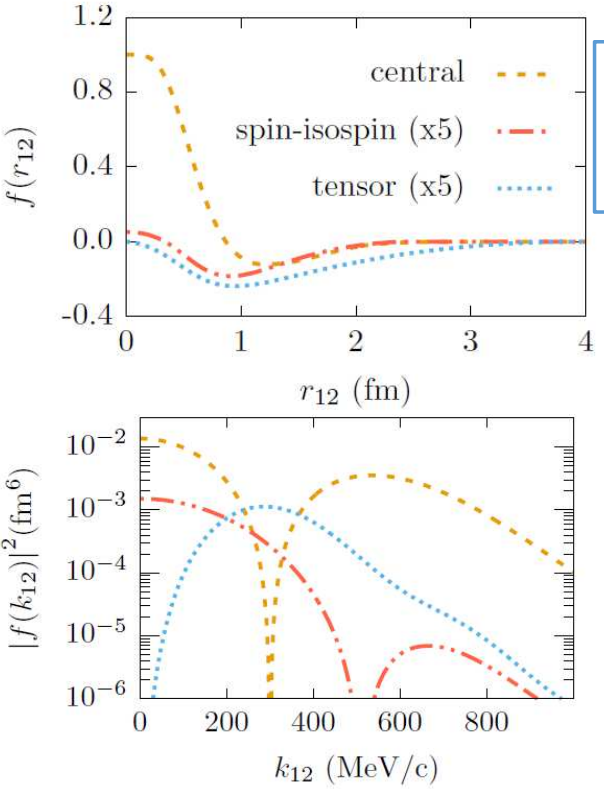
Short-range correlations



$$|\Psi\rangle = \frac{1}{\sqrt{\mathcal{N}}}\hat{\mathcal{G}}|\Phi\rangle \qquad \text{with} \qquad \hat{\mathcal{G}} \approx \hat{\mathcal{S}}\left(\prod_{i<j}^{A}\left[1+\hat{l}(i,j)\right]\right)$$

$$\hat{l}(i,j) = -\,g_c(r_{ij}) + f_{\sigma\tau}(r_{ij})\,(\vec{\sigma}_i\cdot\vec{\sigma}_j)\,(\vec{\tau}_i\cdot\vec{\tau}_j) + f_{t\tau}(r_{ij})\hat{S}_{ij}\,(\vec{\tau}_i\cdot\vec{\tau}_j)\,,$$

$$\hat{S}_{ij} = \frac{3}{r_{ij}^2}(\boldsymbol{\sigma}_i\cdot\boldsymbol{r}_{ij})(\boldsymbol{\sigma}_j\cdot\boldsymbol{r}_{ij}) - (\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j)$$



Gearheart (1994) Pieper (1992)

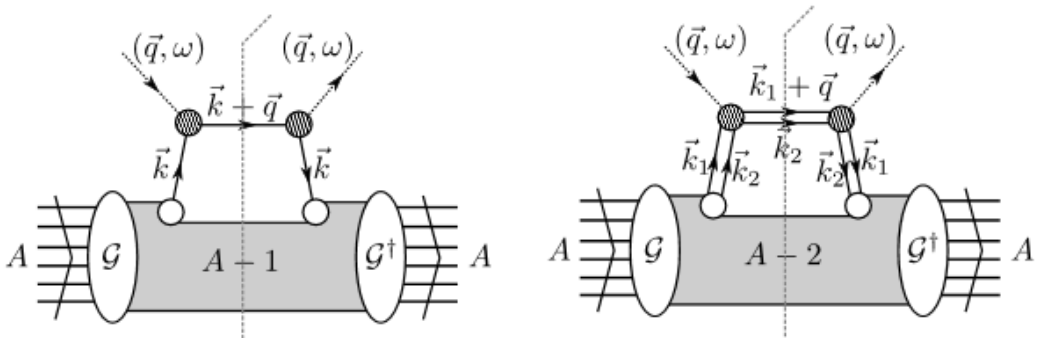
Shifting the complexity induced by correlations from the wave functions to the operators

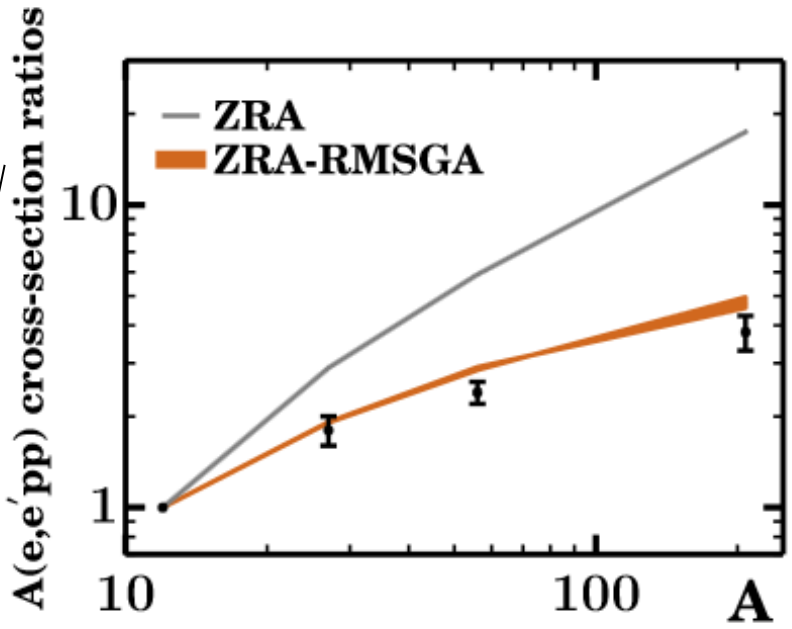
$$\langle\Psi_\mathrm{f}|\hat{J}_\mu^{\mathrm{nucl}}|\Psi_\mathrm{i}\rangle = \frac{1}{\sqrt{\mathcal{N}_\mathrm{i}\mathcal{N}_\mathrm{f}}}\langle\Phi_\mathrm{f}|\hat{J}_\mu^{\mathrm{eff}}|\Phi_\mathrm{i}\rangle$$

$$\hat{J}_\mu^{\mathrm{eff}} \approx \sum_{i=1}^{A}\hat{J}_\mu^{[1]}(i) + \sum_{i<j}^{A}\hat{J}_\mu^{[1],\mathrm{in}}(i,j) + \left[\sum_{i<j}^{A}\hat{J}_\mu^{[1],\mathrm{in}}(i,j)\right]^\dagger$$

$$\hat{J}_\mu^{[1],\mathrm{in}}(i,j) = \left[J_\mu^{\lfloor 1\rfloor}(i) + J_\mu^{\lfloor 1\rfloor}(j)\right] l(i,j)$$

- Short-range correlations affect 1nucleon and 2-nucleon knockout processes
- The SRC-prone nucleon pairs are predominantly in a back-to-back configuration with a small center-of-mass and high relative momentum
- Final-state interactions for the outgoing nucleons affect the experimental observations





To ^{12}C

Data : CLAS A(e,e',pN) data

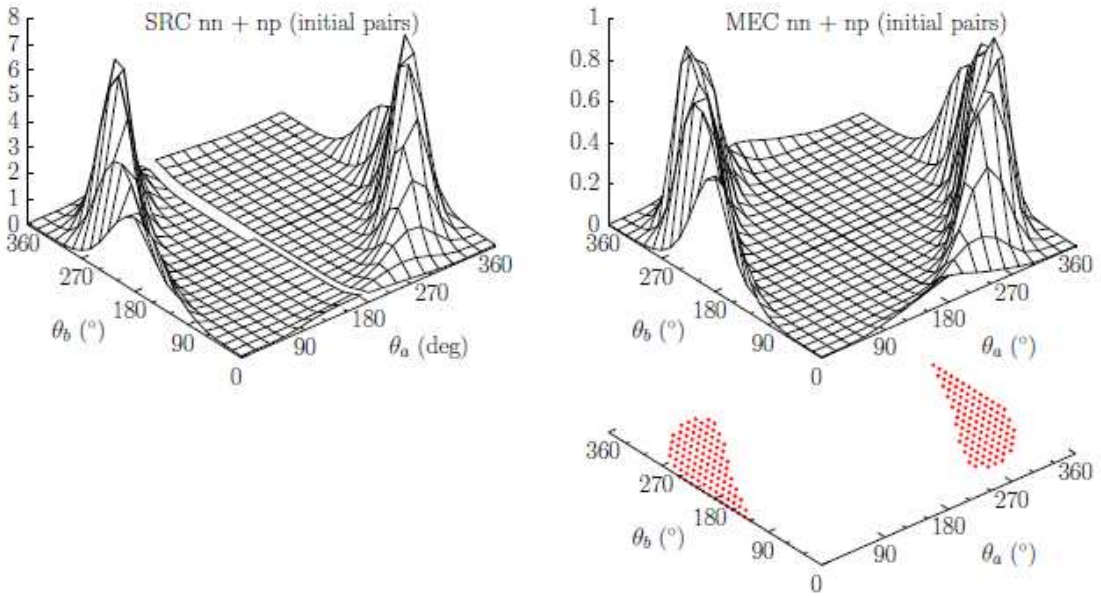
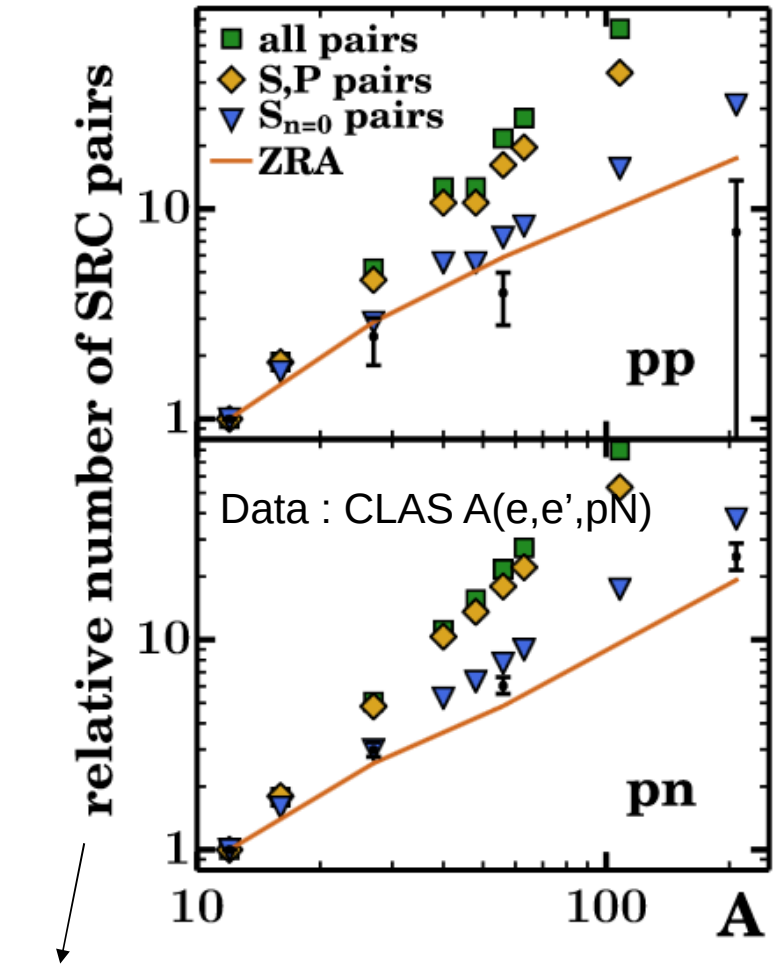


Figure 4.5: The $^{12}C(\nu_\mu, \mu^- N_a N_b)$ cross section (N_a = p, N_b = p′, n) at ϵ_{ν_μ} = 750 MeV, ϵ_μ = 550 MeV, $\theta_\mu = 15°$ and T_p = 50 MeV for in-plane kinematics. Left with SRCs, right with MECs, the bottom plot shows the (θ_a, θ_b) regions with $P_{12} < 300$ MeV/c.

- Strength residing in restricted part of phase space
- $\boldsymbol{p}_b \approx \boldsymbol{p}_b^{ave}$
- Quasi-deuteron kinematics

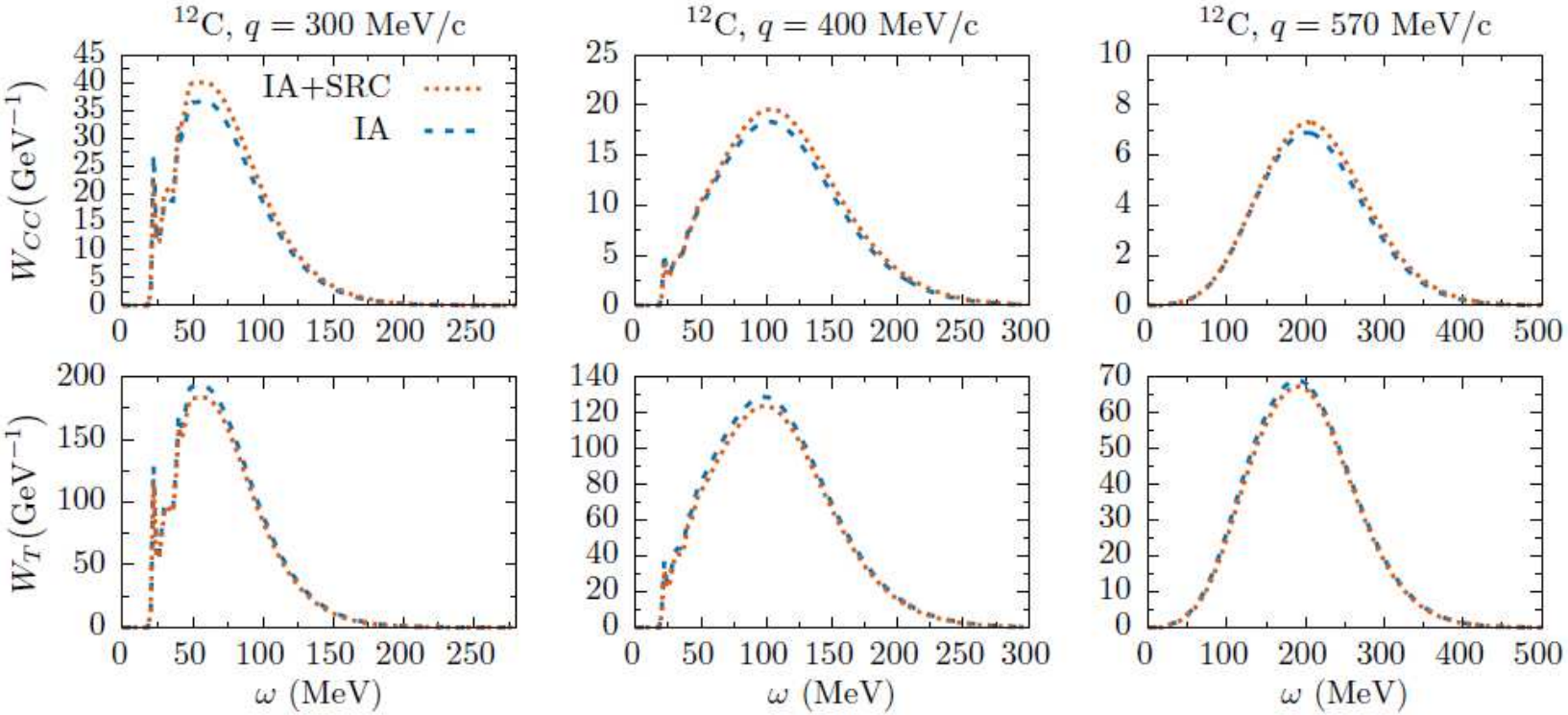


To ^{12}C

- mass dependence ~ $A^{1.12}$:soft !
- Predominantly pn, s-pairs
- Universal over mass tabel
- Tensor force dominates at short distances

C. Colle, W. Cosyn, and J. Ryckebusch, Phys. Rev. C **93**, 034608 (2016)
C. Colle, W. Cosyn, J. Ryckebusch, and M. Vanhalst, Phys. Rev. C **89**, 024603 (2014)
C. Colle, O. Hen, W. Cosyn, I. Korover, E. Piasetzky, J. Ryckebusch, and L. B. Weinstein, Phys. Rev. C **92**, 024604 (2015).

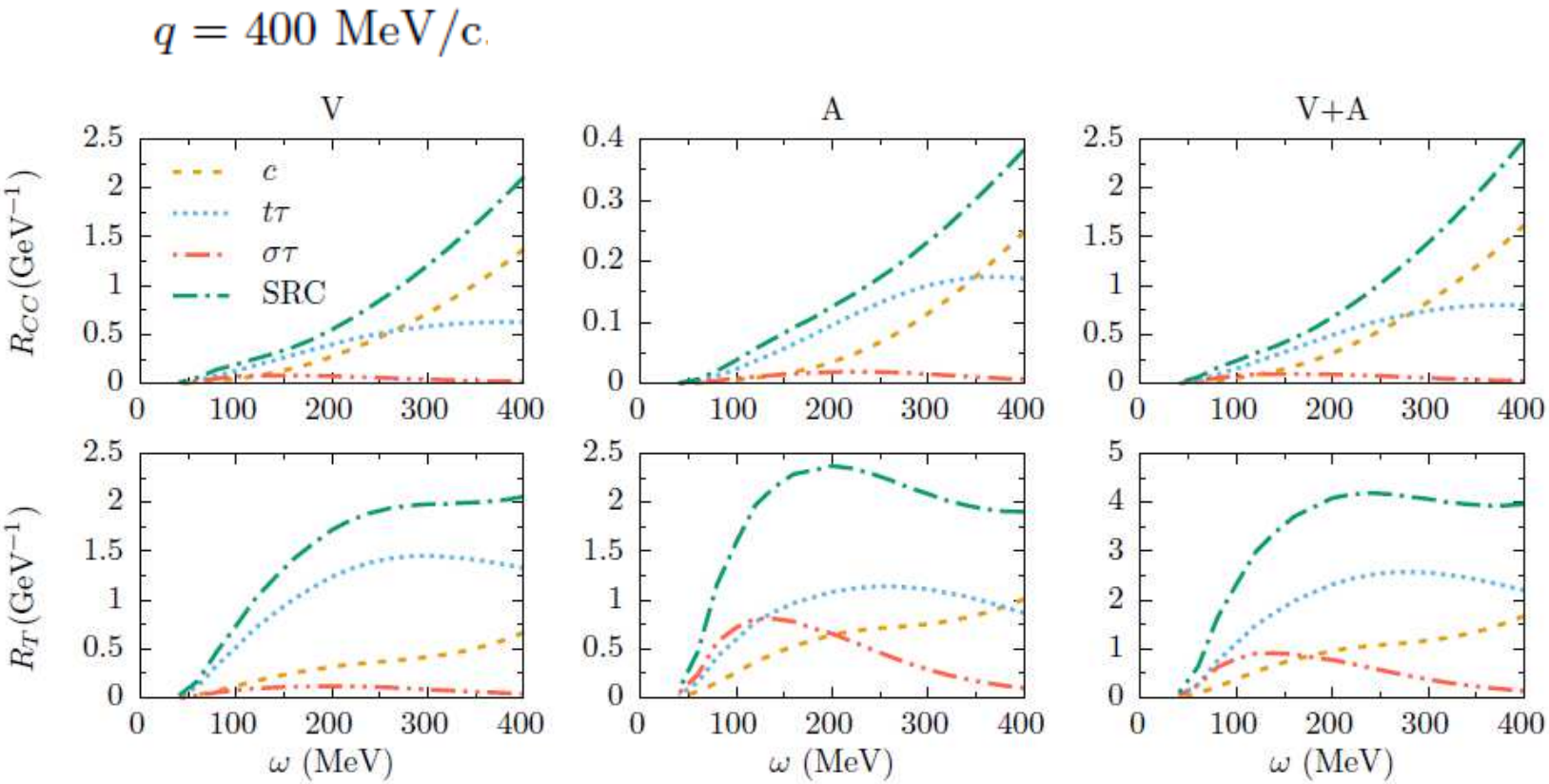
SRC neutrinos 1p1h



- Reduction of transverse response
- Enhancement of Coulomb-longitudinal

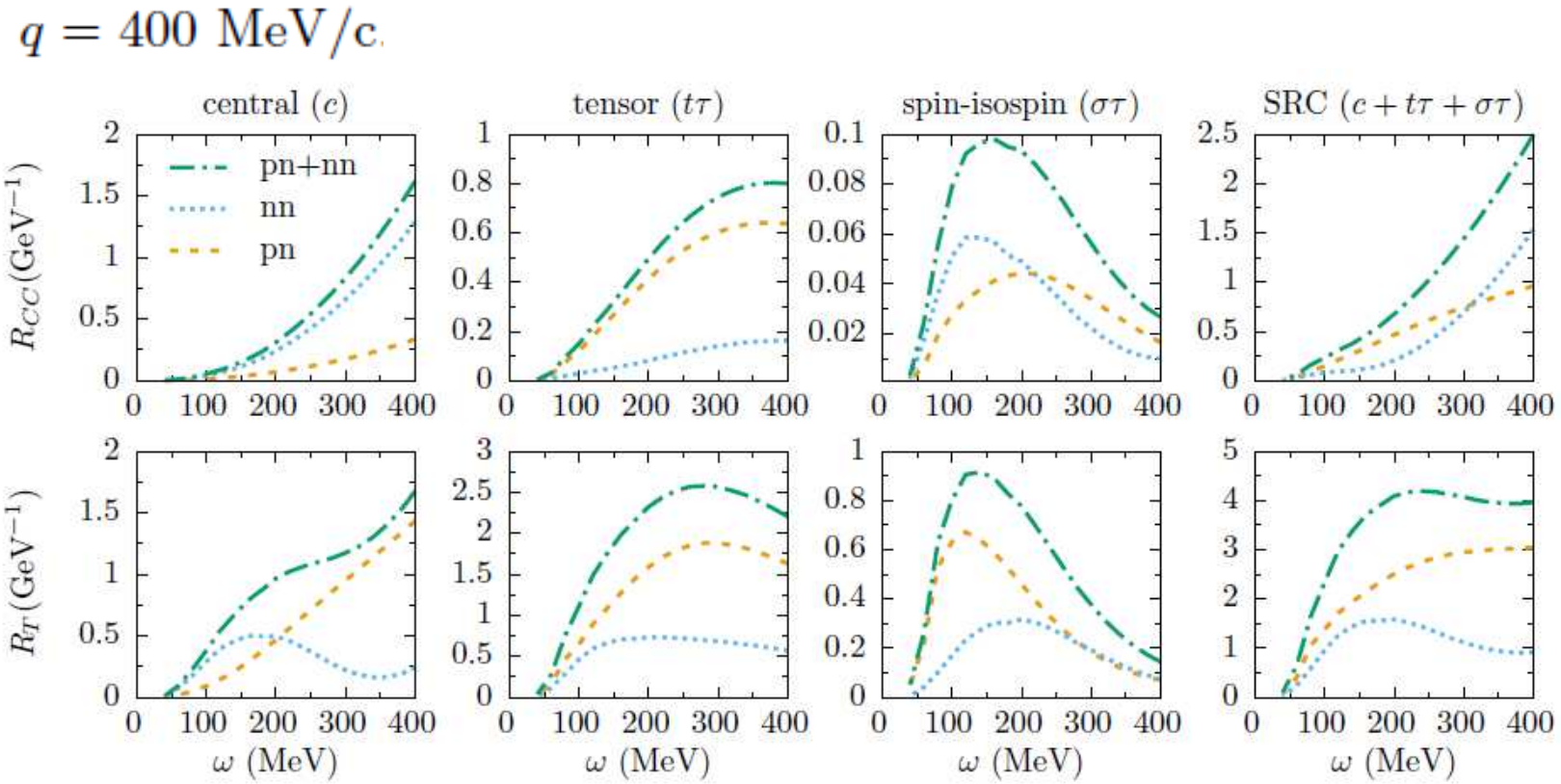
T. Van Cuyck, N. Jachowicz, R. González–Jiménez, *et al.* Phys. Rev. **C94**, 024611 (2016).

SRC neutrinos 2p2h



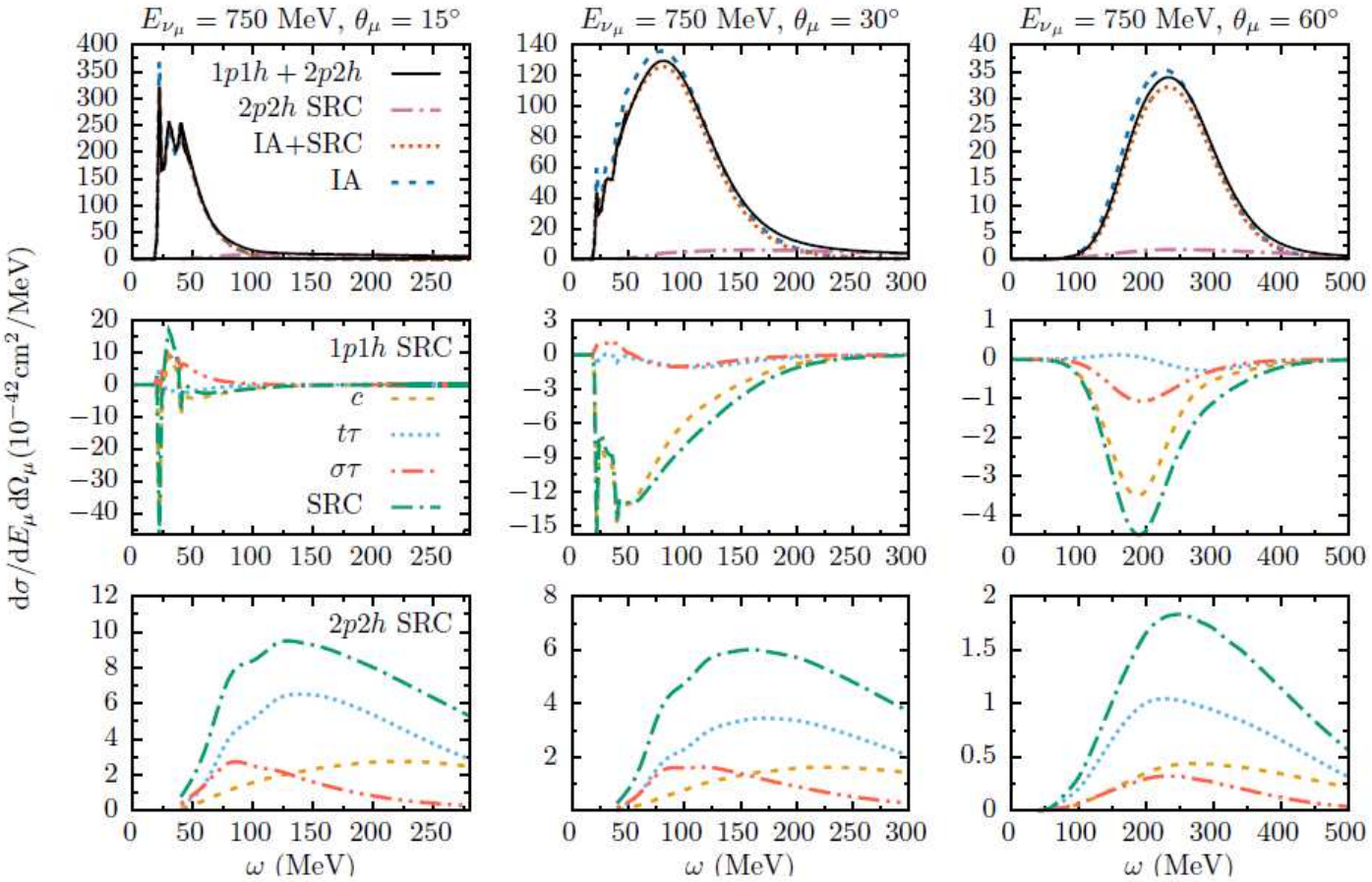
- Vector and axial contributions have comparable strength
- Tensor often dominates, but not for all kinematics

SRC neutrinos 2p2h



- Vector and axial contributions have comparable strength
- Tensor often dominates, but not for all kinematics
- pn pairs dominate

SRC neutrinos 1p1h+2p2h



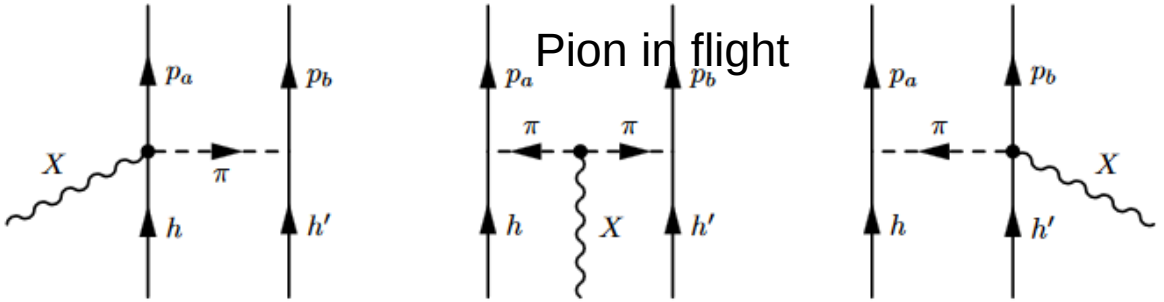
T. Van Cuyck et al
Phys. Rev. C 94,
024611 (2016)

Meson-exchange currents

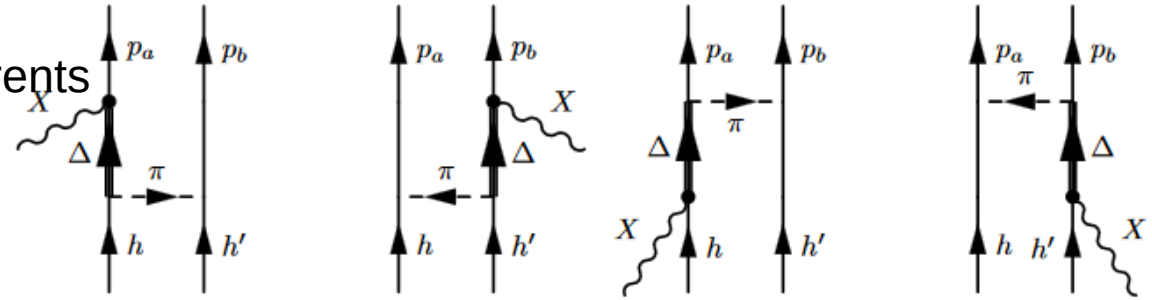
When an electroweak boson interacts with a pair of nucleons which are correlated through the exchange of a meson, this will cause the knockout of one or both of the particles from the nucleus. The boson was interacting with a current consisting of two nucleons, a two-body current, called a MEC

Seagull

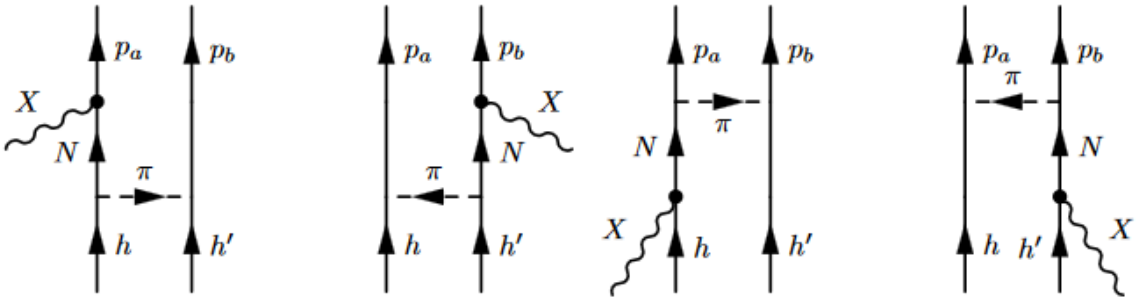
Pion in flight



Delta currents



Correlation currents

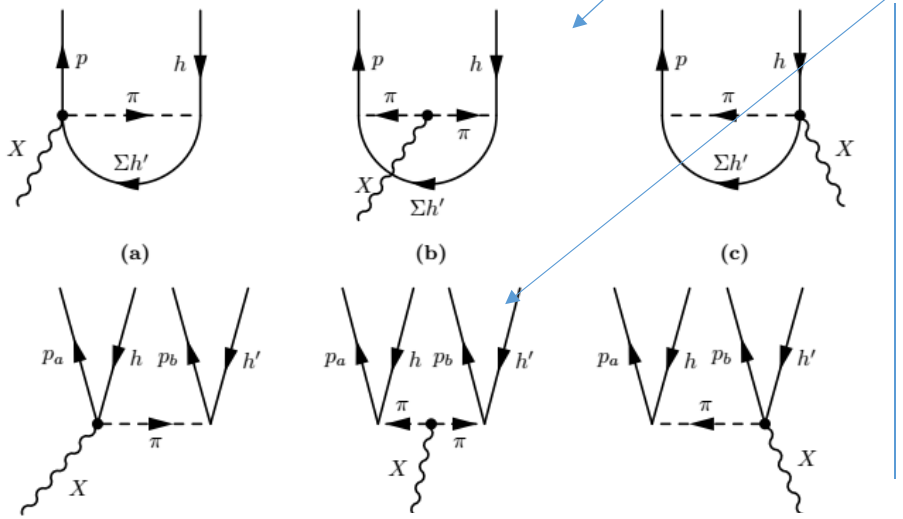


Already included in mean field models !

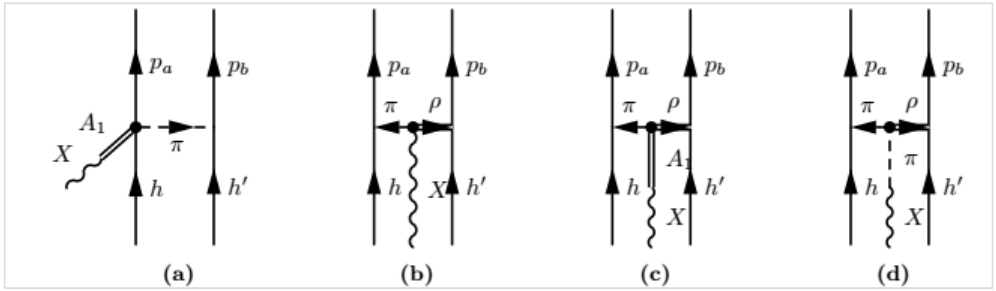
Contributions of heavier mesons :

$m_\pi \approx 135\text{MeV}$, $m_\rho \approx 775\text{MeV}$, $m_\omega \approx 782\text{MeV}$

III. MEC in 1p1h and 2p2h



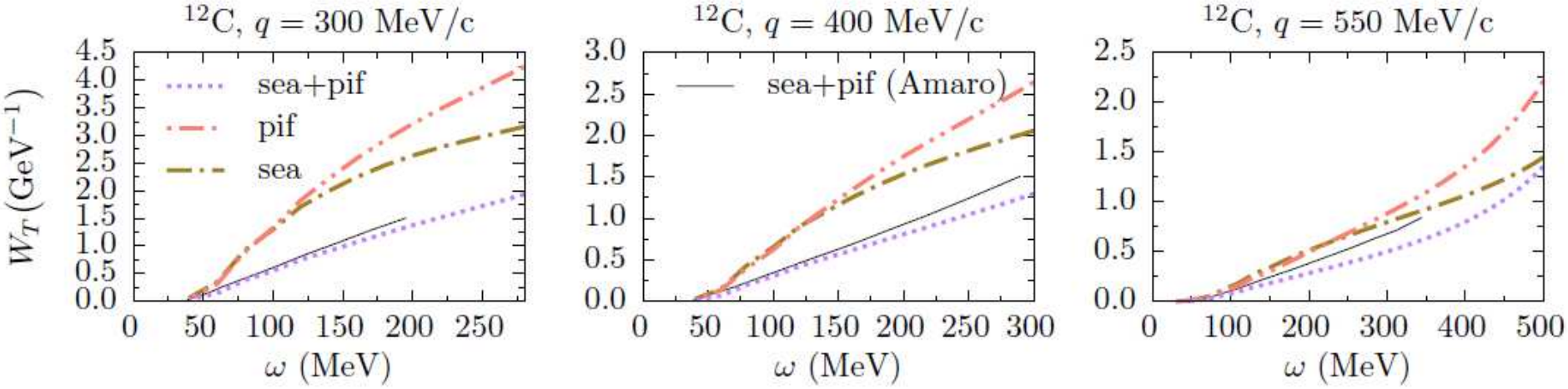
Axial contributions :



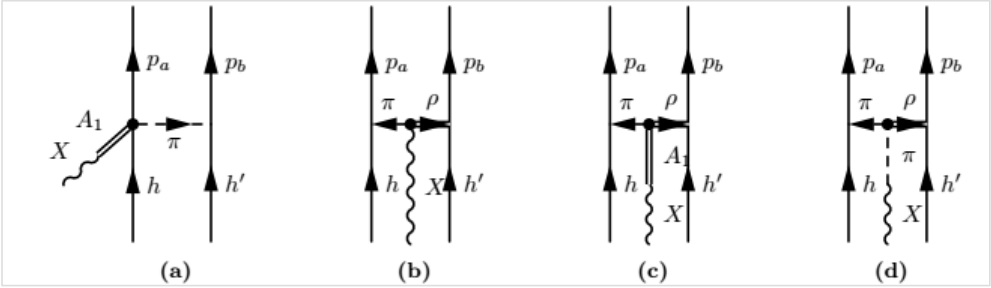
$$\hat{\rho}_A^{[2],\mathrm{axi}}(\boldsymbol{q}) = \frac{i}{g_A}\left(\frac{f_{\pi NN}}{m_\pi}\right)^2 (\boldsymbol{I}_V)\left(F_\pi(q_2^2)\,\Gamma_\pi^2(q_2^2)\frac{\boldsymbol{\sigma}_2\cdot\boldsymbol{q}_2}{q_2^2+m_\pi^2} - F_\pi(q_1^2)\,\Gamma_\pi^2(q_1^2)\frac{\boldsymbol{\sigma}_1\cdot\boldsymbol{q}_1}{q_1^2+m_\pi^2}\right)$$

I. Towner, Nucl. Phys.A542, 631 (1992)

$^{12}C(e, e')$

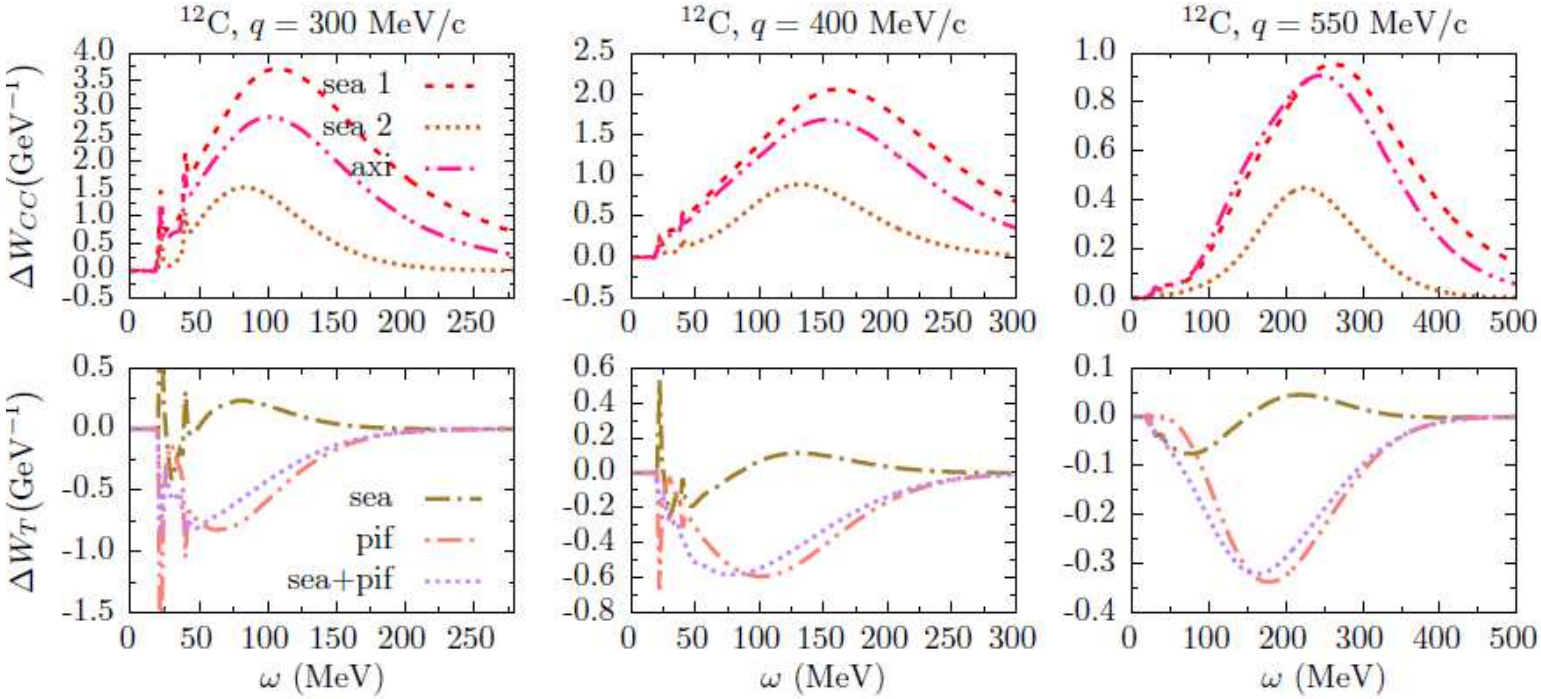


- Only seagull have axial counterpart
- timelike
- Partially constrained by PCAC
- Non-relativistic reduction not unambiguous

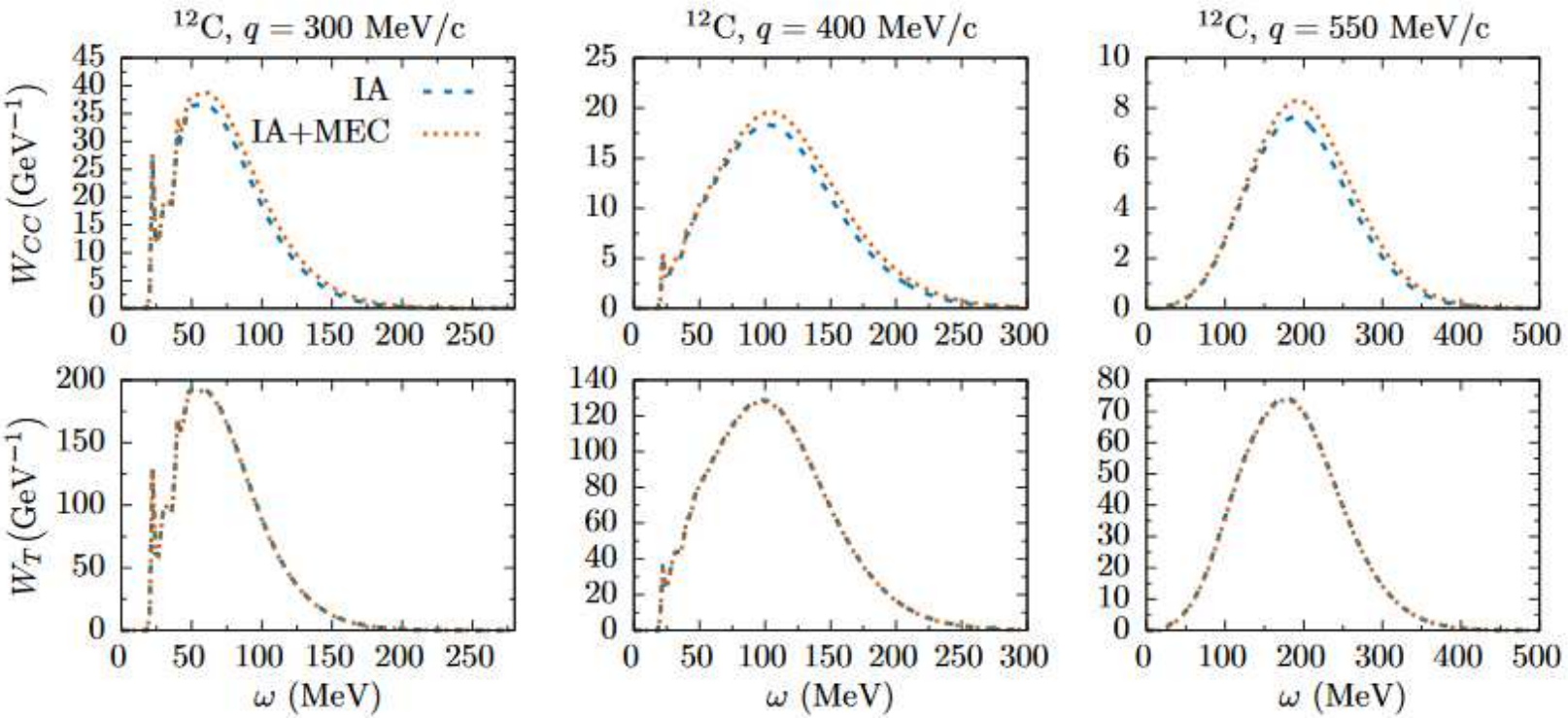


$$\widehat{\rho}_A^{[2],\mathrm{axi}}(\boldsymbol{q}) = \frac{i}{g_A}\left(\frac{f_{\pi NN}}{m_\pi}\right)^2 (\boldsymbol{I}_V)\left(F_\pi(\boldsymbol{q}_2^2)\,\Gamma_\pi^2(\boldsymbol{q}_2^2)\frac{\boldsymbol{\sigma}_2\cdot\boldsymbol{q}_2}{\boldsymbol{q}_2^2+m_\pi^2} - F_\pi(\boldsymbol{q}_1^2)\,\Gamma_\pi^2(\boldsymbol{q}_1^2)\frac{\boldsymbol{\sigma}_1\cdot\boldsymbol{q}_1}{\boldsymbol{q}_1^2+m_\pi^2}\right)$$

I. Towner, Nucl. Phys.A542, 631 (1992)

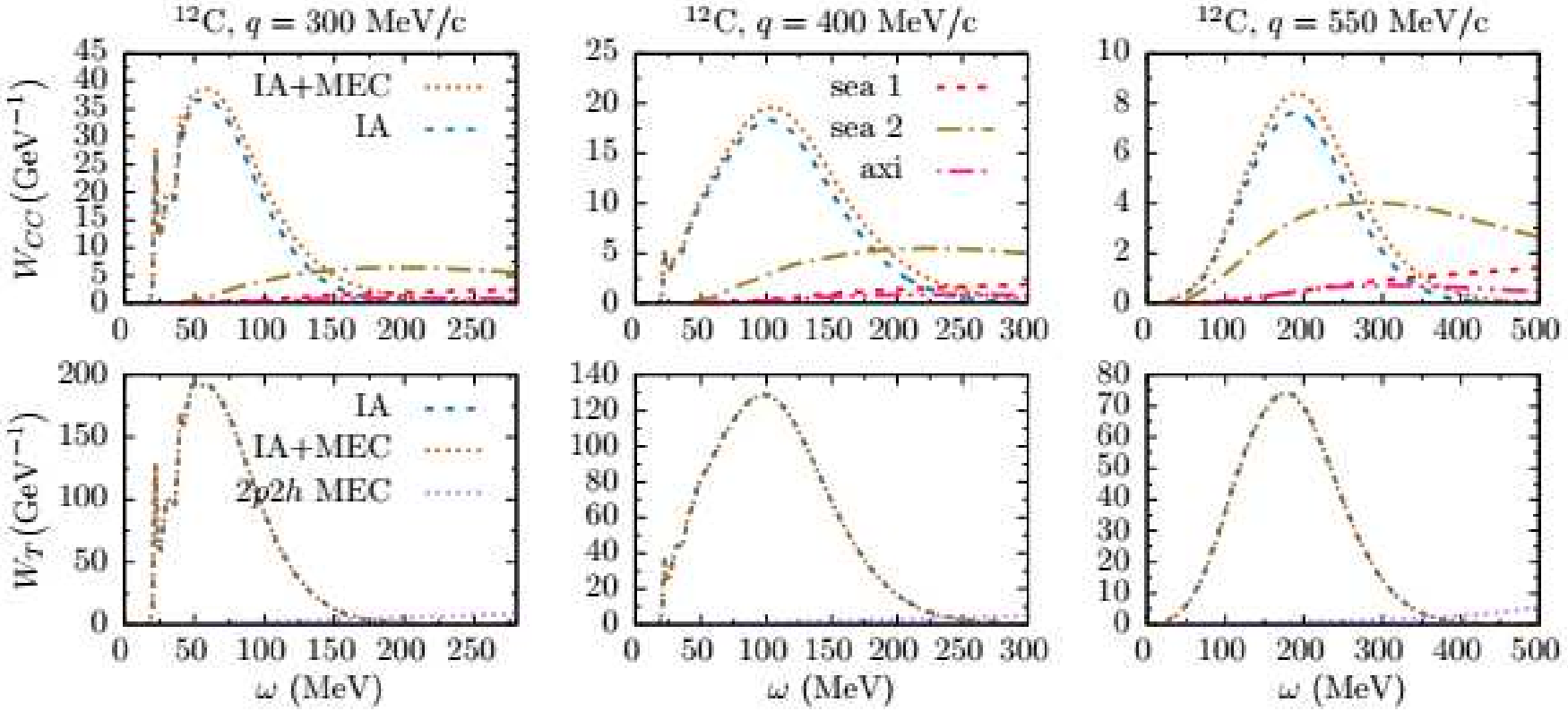


Seagull and PIF in neutrino 1p1h

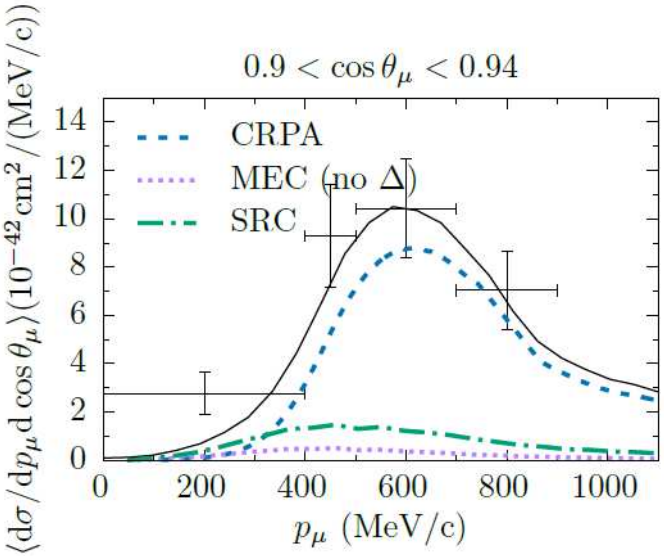


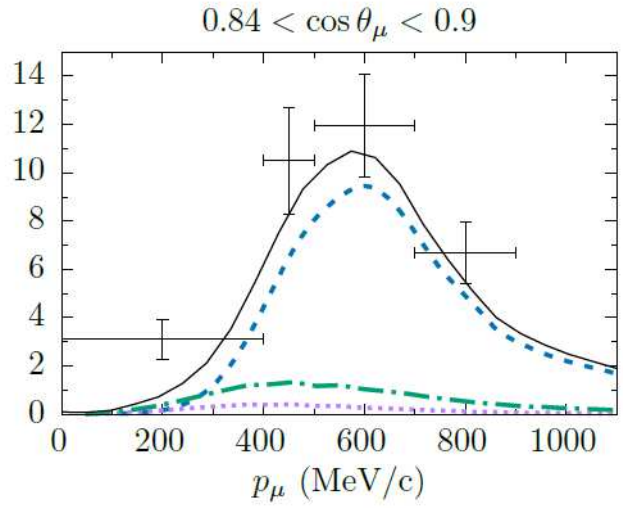
T. Van Cuyck, N. Jachowicz, R. González–Jiménez, *et al.* PRC95, 054611 (2017)

Seagull and PIF in neutrino 2p2h

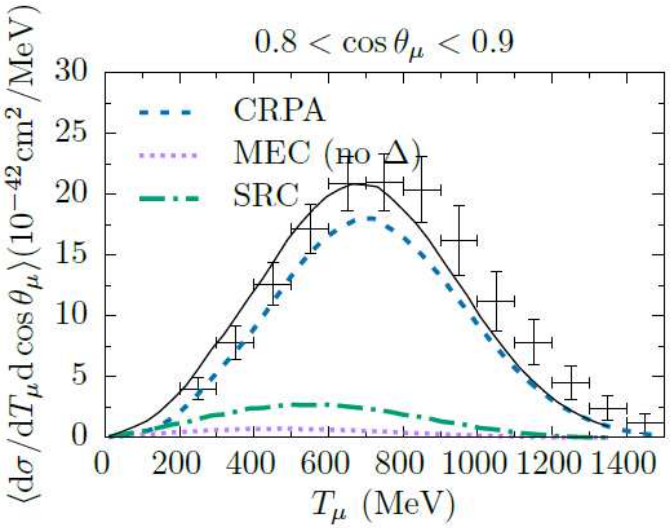


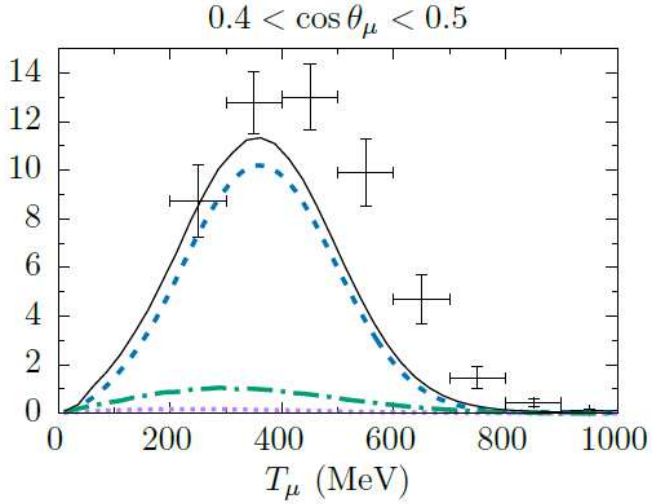
T2K





MiniBooNe





Summary

- Long- and short-range correlations in QE-like cross sections
- CRPA calculations provide extra strength for forward scattering arising from low-energy excitations
- This might affect CCQE neutrino cross sections as measured by MiniBooNe, T2K, …
- SRC and MEC affect 1- and 2-nucleon knockout processes

- Scheme-dependent separation
- Double counting