Superheating fields in impurity diffusion layers

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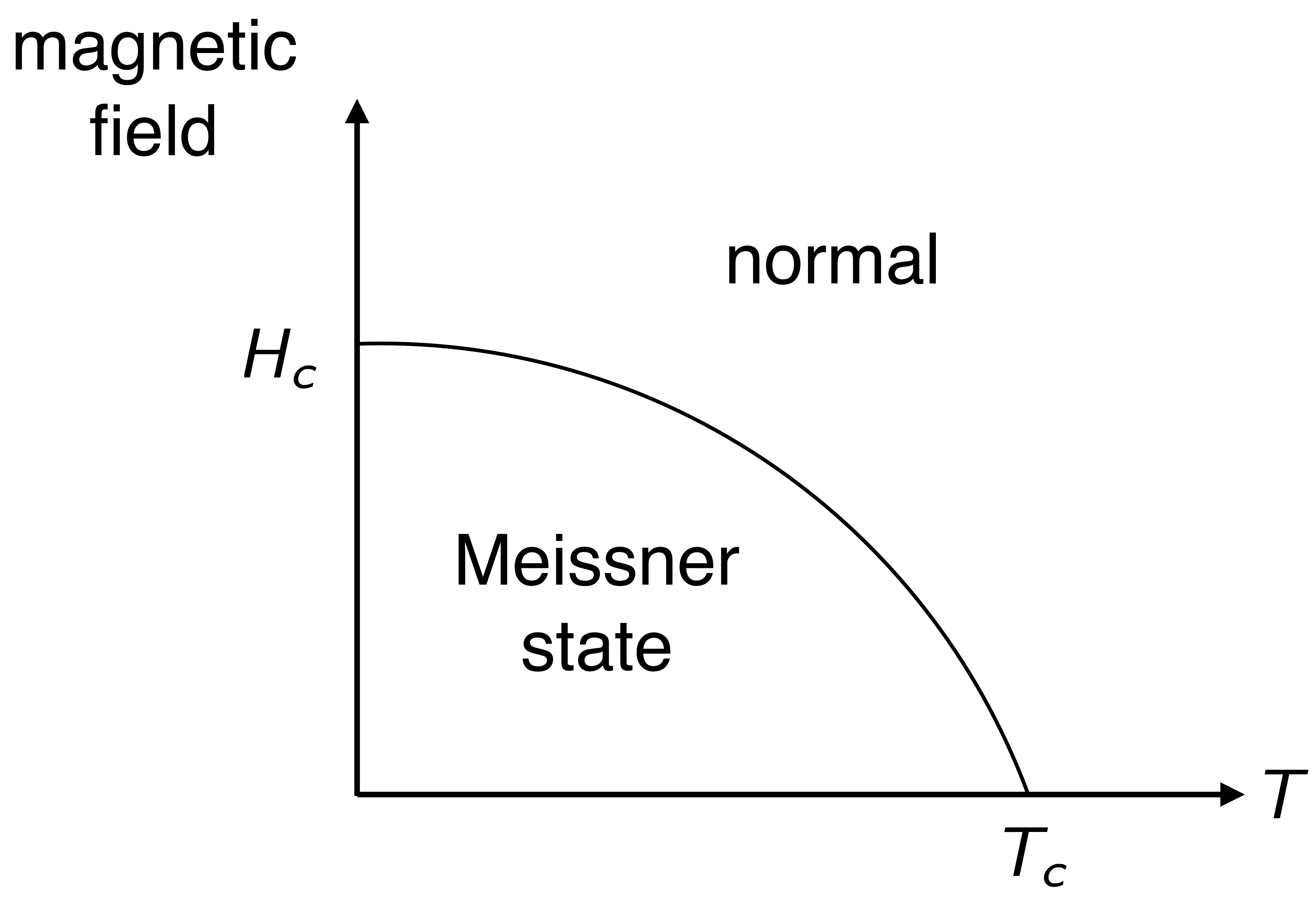
Recent advances in SRF cavity performance pose theoretical challenges

- ✦ Recent advances in surface treatments have increased both Q-factor and the quench field of SRF cavities
  - Polishing modifies surface roughness
  - Doping introduces impurities in a semi-controlled way
- ✦ What is the dominant mechanism which enhances SRF performance?
  - Theoretically this is a challenging problem which may involve more than one mechanism in the non-linear regime (strong fields)
- ✦ We study the effect of inhomogeneous impurity density in an impurity diffusion layer

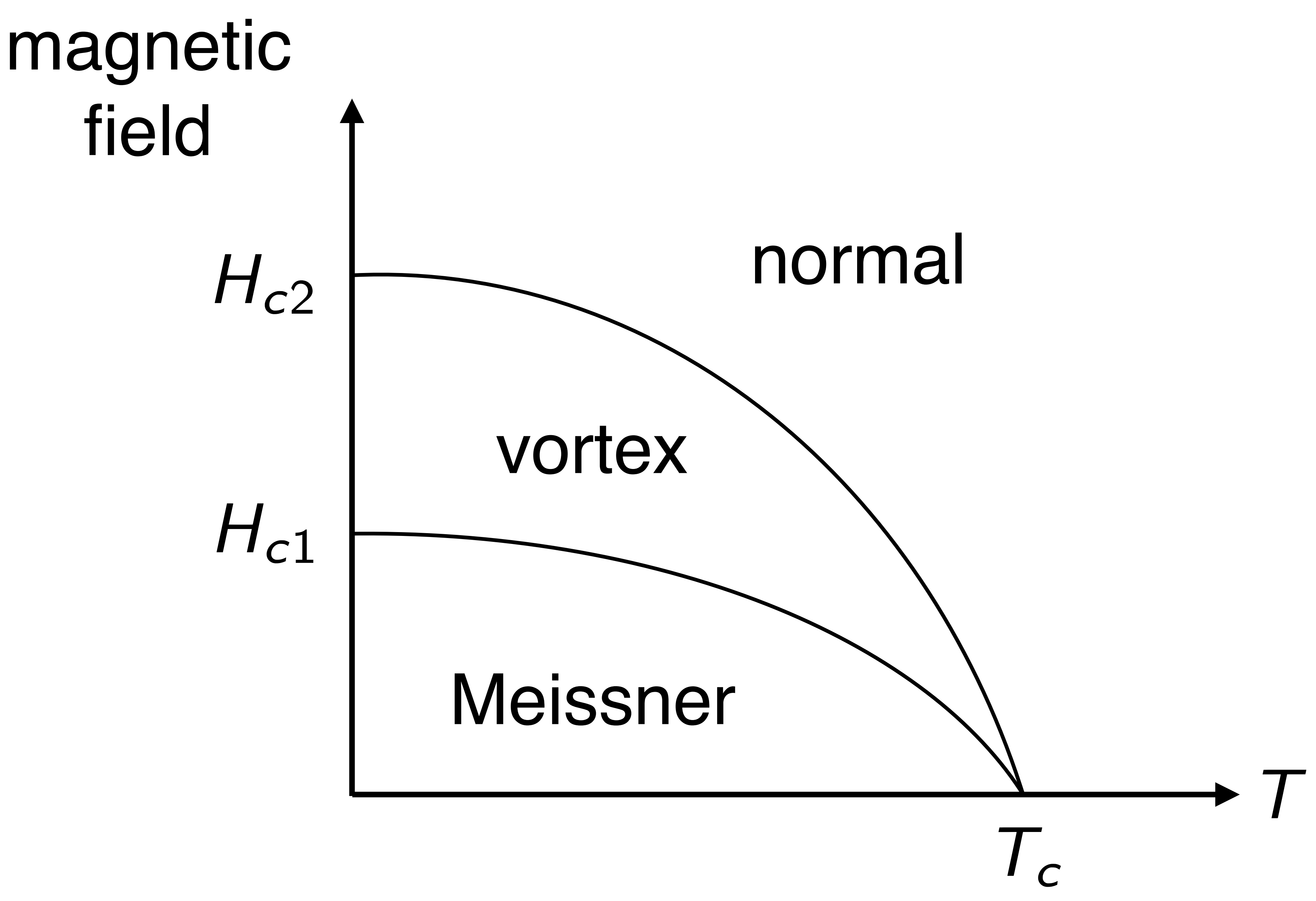
see, e.g., Gurevich, Supercond. Sci. Technol. (2017)

Type-I and Type-II superconductors
Meissner: weak dissipation; Vortex: strong dissipation



Type-I

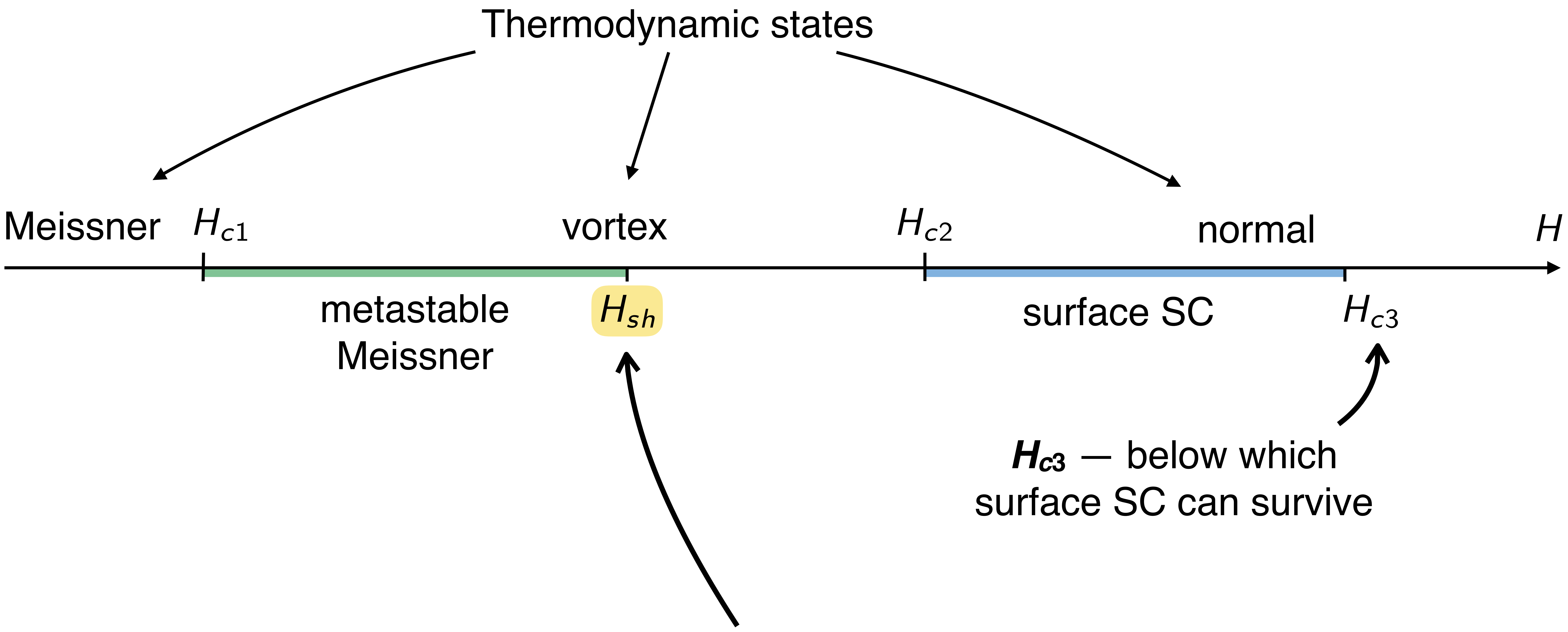
most elemental SC
low critical temperature/field



Type-II

most alloys and Nb

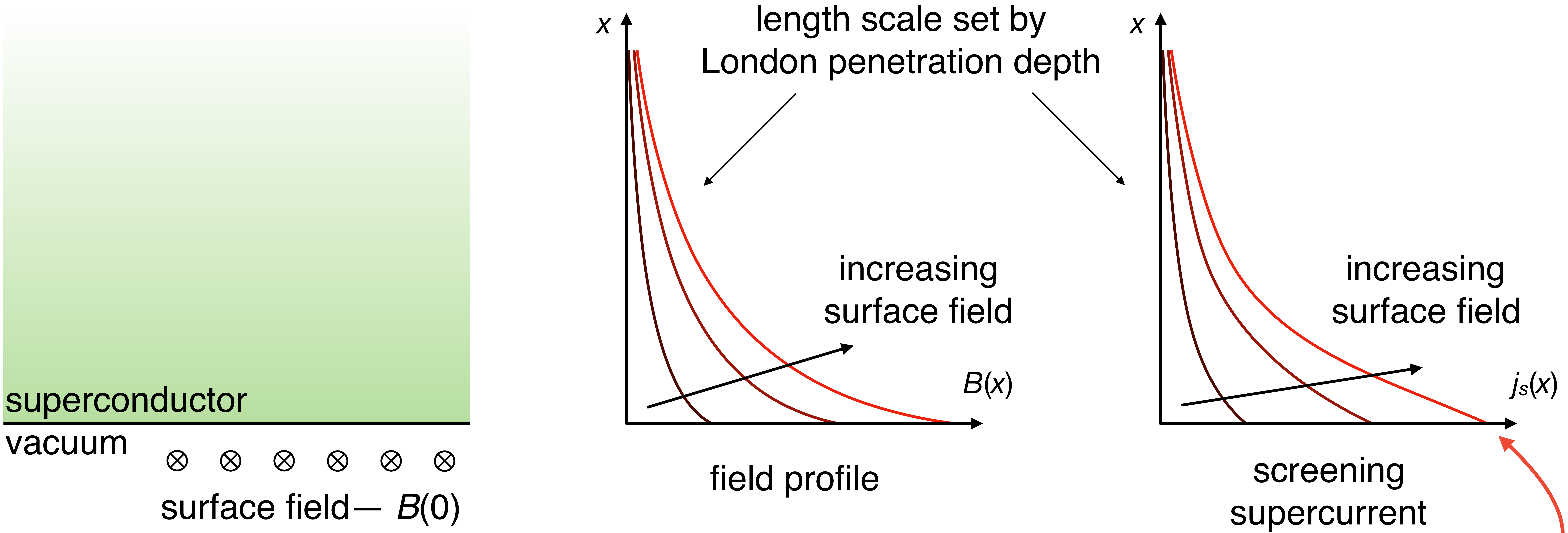
H_{c1}, H_{c2}, H_{c3} and superheating field H_{sh}



Superheating field

above which Meissner state (low-dissipation) is unstable

H_{sh} in a theorist's cavity



Superheating field reached when current at surface is equal to critical current— $j_s(0) = j_c$

For recent work, see
Catelani & Sethna, PRB (2008)
Lin & Gurevich, PRB (2012)

What we assume

- *Zero temperature* — SRF cavities are cooled to 2K (0.20-0.25T_c)
- *Static field* — RF field in cavities is1-10GHz, still much smaller than the gap frequency (100 times larger)
- *Born scattering* — Weak scattering from point-like impurities
- *Local limit (extreme Type-II limit)* — This simplifies the calculations but Nb is *not* extreme Type-II

Quasiclassical transport equations

Spectral channels encode excitation spectrum

$$\left[\varepsilon\hat{\tau}_3 - \hat{\sigma}_{\text{ext}} - \hat{\sigma}^{R,A}, \hat{g}^{R,A}\right]_\circ + i\vec{v}_f \cdot \vec{\nabla}\hat{g}^{R,A} = 0$$

Keldysh channel encodes spectrum and occupation

$$\left(\varepsilon\hat{\tau}_3 - \hat{\sigma}_{\text{ext}} - \hat{\sigma}^R\right) \circ \hat{g}^K - \hat{g}^K \circ \left(\varepsilon\hat{\tau}_3 - \hat{\sigma}_{\text{ext}} - \hat{\sigma}^A\right)$$
$$+\hat{g}^R \circ \hat{\sigma}^K - \hat{\sigma}^K \circ \hat{g}^A + i\vec{v}_f \cdot \vec{\nabla}\hat{g}^K = 0$$

Normalization conditions

$$\hat{g}^{R,A} \circ \hat{g}^{R,A} = -\pi^2\hat{1}, \qquad \hat{g}^R \circ \hat{g}^K - \hat{g}^K \circ \hat{g}^A = 0$$

4x4 matrix propagators depend on Fermi vector, energy, position and time

$$\hat{g}^{R,A,K} = \hat{g}^{R,A,K}(\vec{p}_f, \varepsilon, \vec{R}, t)$$

Self-energies encode interactions
Fermionic pairing and impurity scattering

Spectral channels encode excitation spectrum

$$\left[\varepsilon\hat{\tau}_3 - \hat{\sigma}_{\text{ext}} - \hat{\sigma}^{R,A}, \hat{g}^{R,A}\right]_\circ + i\vec{v}_f \cdot \vec{\nabla}\hat{g}^{R,A} = 0$$

Keldysh channel encodes spectrum and occupation

$$\left(\varepsilon\hat{\tau}_3 - \hat{\sigma}_{\text{ext}} - \hat{\sigma}^{R}\right) \circ \hat{g}^{K} - \hat{g}^{K} \circ \left(\varepsilon\hat{\tau}_3 - \hat{\sigma}_{\text{ext}} - \hat{\sigma}^{A}\right) + \hat{g}^{R} \circ \hat{\sigma}^{K} - \hat{\sigma}^{K} \circ \hat{g}^{A} + i\vec{v}_f \cdot \vec{\nabla}\hat{g}^{K} = 0$$

Self-energies encode interactions

$$\hat{\sigma}^{R,A} = \hat{\Delta}_{\text{mf}} + \hat{\sigma}^{R,A}_{\text{imp}}, \qquad \hat{\sigma}^{K} = \hat{\sigma}^{K}_{\text{imp}}$$

s-wave gap equation

$$\Delta_{\text{mf}} = V_0 \int \frac{d\varepsilon}{4\pi i} \left\langle f^K(\vec{p}_f;\varepsilon)\right\rangle_{\vec{p}_f}$$

Off-diagonal Keldysh propagator (occupied pair spectral func.)

Impurity self-energy

$$\hat{\sigma}^{R,A,K}_{\text{imp}}(\vec{p}_f;\varepsilon) = n_{\text{imp}}\hat{t}^{R,A,K}(\vec{p}_f,\vec{p}_f;\varepsilon)$$

Single-impurity *t*-matrix

Coupling to electromagnetic fields

Spectral channels encode excitation spectrum

$$\left[\varepsilon\hat{\tau}_3 - \hat{\sigma}_{\text{ext}} - \hat{\sigma}^{R,A}, \hat{g}^{R,A}\right]_\circ + i\vec{v}_f \cdot \vec{\nabla}\hat{g}^{R,A} = 0$$

Keldysh channel encodes spectrum and occupation

$$\left(\varepsilon\hat{\tau}_3 - \hat{\sigma}_{\text{ext}} - \hat{\sigma}^R\right) \circ \hat{g}^K - \hat{g}^K \circ \left(\varepsilon\hat{\tau}_3 - \hat{\sigma}_{\text{ext}} - \hat{\sigma}^A\right)$$
$$+\hat{g}^R \circ \hat{\sigma}^K - \hat{\sigma}^K \circ \hat{g}^A + i\vec{v}_f \cdot \vec{\nabla}\hat{g}^K = 0$$

EM coupling in terms of vector potential **A** *or* SF momentum $\mathbf{p}_s$

$$\hat{\sigma}_{\text{ext}} = -\frac{e}{c}\vec{v}_f \cdot \vec{A}\hat{\tau}_3 \equiv \vec{v}_f \cdot \vec{p}_s\hat{\tau}_3$$

Maxwell's equation for vector potential (div**A** = 0 & *no charging*)

$$\left(-\nabla^2 + \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\vec{A}(\vec{R},t) = \frac{4\pi}{c}\vec{j}(\vec{R},t)$$

Current computed from diagonal Keldysh propagator (occupied spectral function)

$$\vec{j}(\vec{R},t) = 2N_f \int \frac{d\varepsilon}{4\pi i} \left\langle e\vec{v}_{f,\vec{p}_f}\, g^K(\vec{p}_f,\varepsilon,\vec{R},t)\right\rangle_{\vec{p}_f}$$

Coupling to electromagnetic fields

Spectral channels encode excitation spectrum

$$\left[\varepsilon\hat\tau_3 - \hat\sigma_{\text{ext}} - \hat\sigma^{R,A}, \hat g^{R,A}\right]_\circ + i\vec v_f \cdot \vec\nabla \hat g^{R,A} = 0$$

Keldysh channel encodes spectrum and occupation

$$\left(\varepsilon\hat\tau_3 - \hat\sigma_{\text{ext}} - \hat\sigma^R\right) \circ \hat g^K - \hat g^K \circ \left(\varepsilon\hat\tau_3 - \hat\sigma_{\text{ext}} - \hat\sigma^A\right)$$
$$+ \hat\sigma^R \circ \hat g^K - \hat g^K \circ \hat\sigma^A + i\vec v_f \cdot \vec\nabla \hat g^K = 0$$

EM coupling in terms of vector potential **A** *or* SF momentu

Maxwell's equation for vector pote (div**A** = 0 & *no charging*)

Current computed from diagonal Keldysh propagator (occupied spectral function)

Transport equations & Maxwell's equation must be solved simultaneously!

$$\left(-\nabla^2 + \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\vec A(\vec R, t) = \frac{4\pi}{c}\vec j(\vec R, t)$$

$$\vec j(\vec R, t) = 2N_f \int \frac{d\varepsilon}{4\pi i}\left\langle e\vec v_{f,\vec p_f}\, g^K(\vec p_f, \varepsilon, \vec R, t)\right\rangle_{\vec p_f}$$

Extreme type-II SC under static surface field

Extreme type-II — $\xi \ll \lambda$

ξ — coherence length scale (propagators variation length scale)

λ — London penetration depth (B-field variation length scale)

Propagators do not 'see' B-field variation — $i\vec{v}_f \cdot \vec{\nabla}\hat{g}^{R,A} = 0$

Static B field

Occupation function is the equilibrium Fermi function

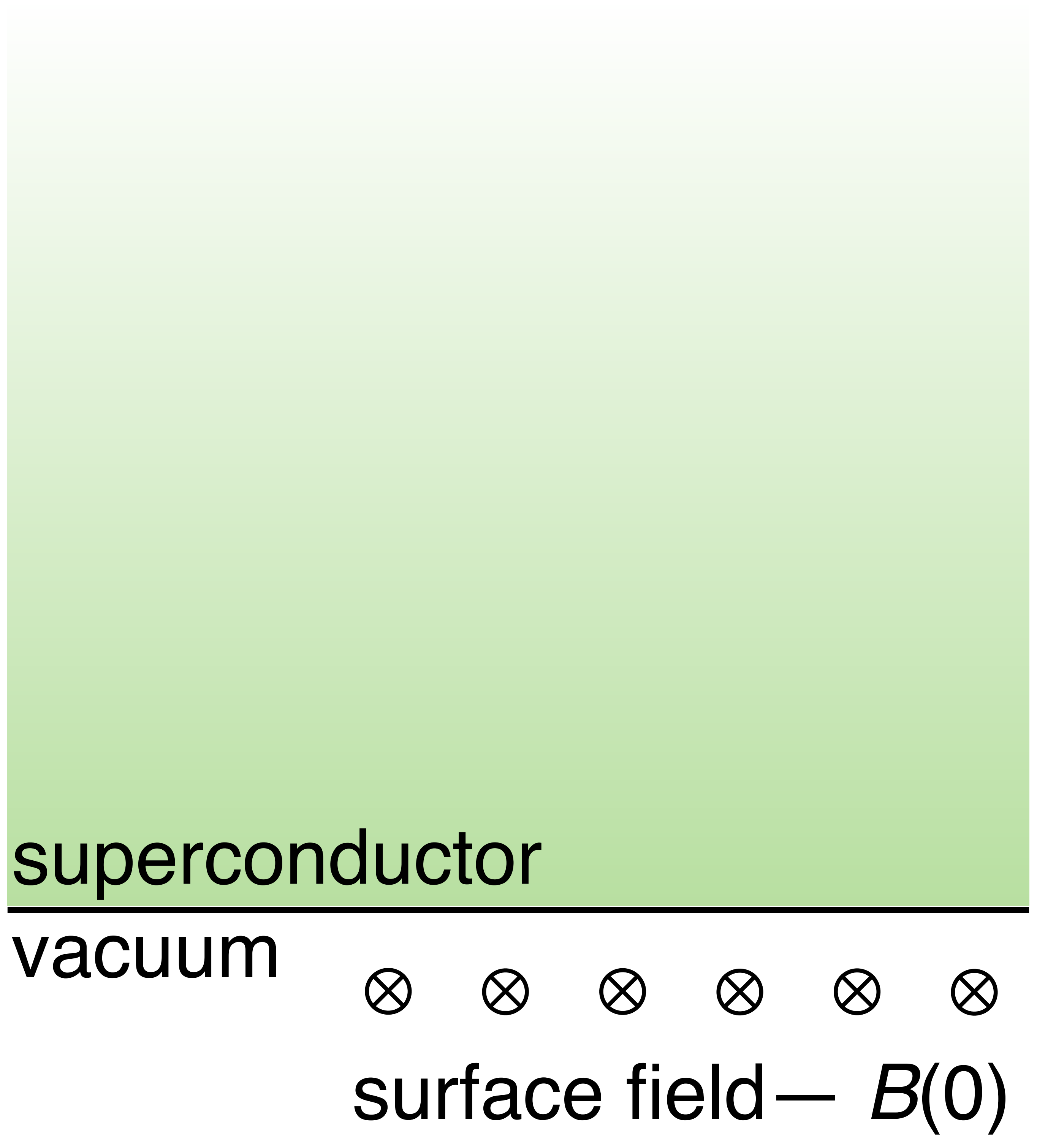
Keldysh propagator $\hat{g}^K = (\hat{g}^R - \hat{g}^A)\tanh\frac{\varepsilon}{2T}$

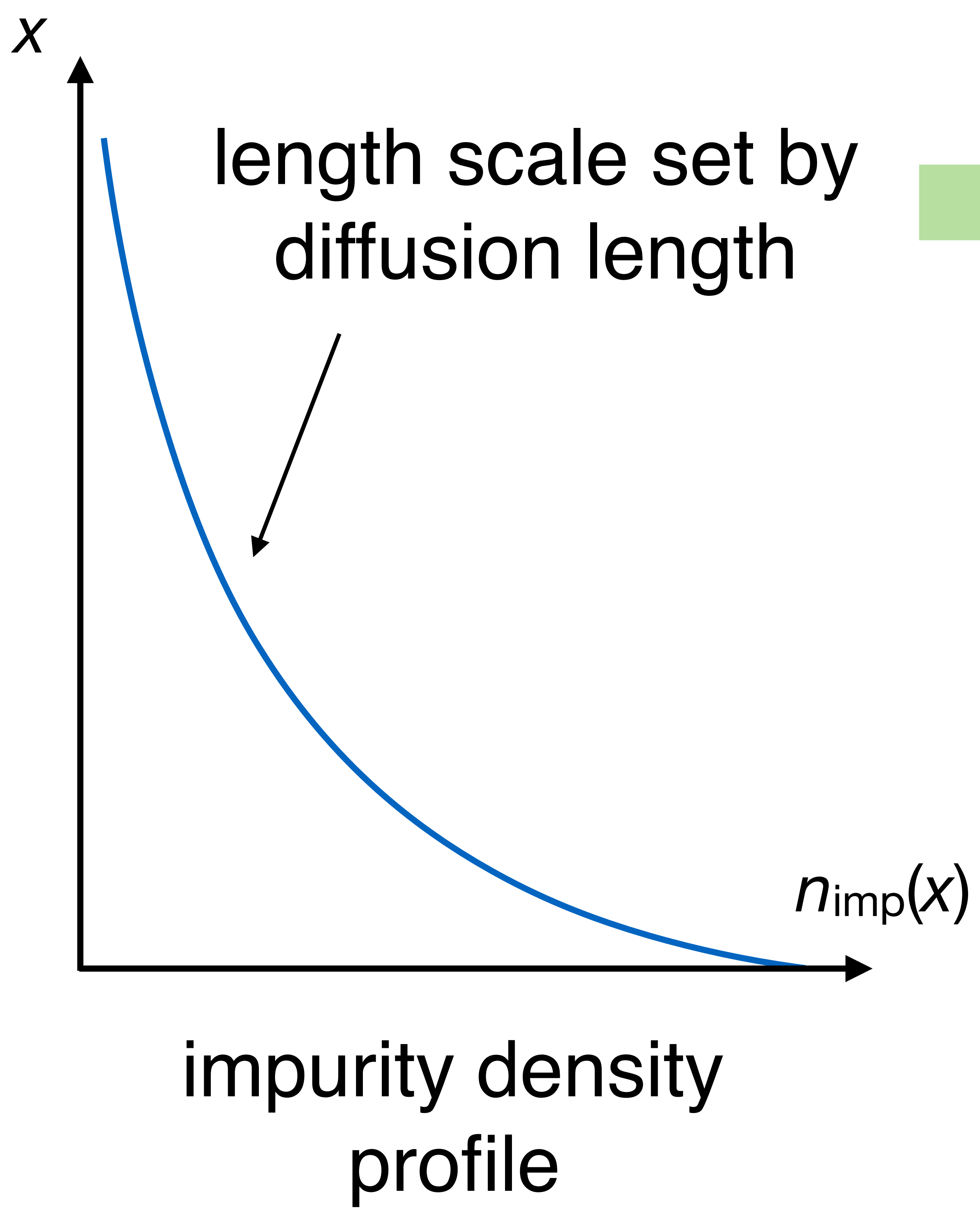
Spectral propagators computed from

$$\left[(\varepsilon - \vec{v}_f \cdot \vec{p}_s)\hat{\tau}_3 - \hat{\sigma}^{R,A}, \hat{g}^{R,A}\right] + \cancel{i\vec{v}_f \cdot \vec{\nabla}\hat{g}^{R,A}} = 0$$

Impurity infusion results in diffusion layer

What is the effect of diffusion layer on superheating field?





Varying scattering rate:

$$\gamma(x) = \gamma_0 e^{-x/\zeta} \propto n_{imp}(x),$$

ζ denotes the diffusion length

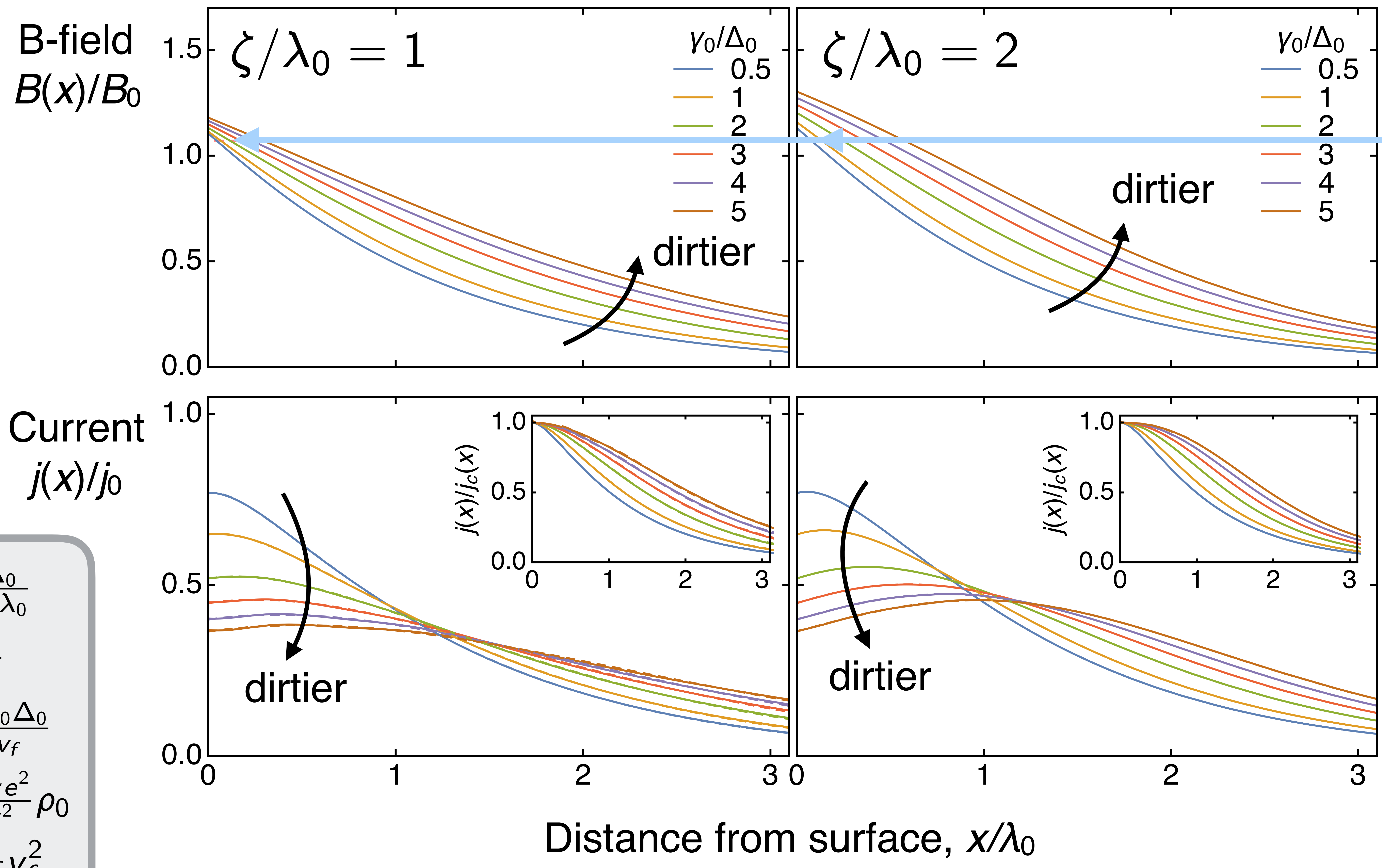
Maxwell's equation

ODE for field/current profile

$$p_s''(x) = \frac{4\pi e}{c^2} j\left(p_s(x), \gamma(x)\right)$$

SF momentum (vector potential) — supercurrent — scattering rate

Effect of impurity density at surface



Maximum B_{sh}
for constant
homogeneous
scattering rate
Lin & Gurevich (2012)

$$B_0 = \frac{e}{c}\frac{\Delta_0}{v_f \lambda_0}$$

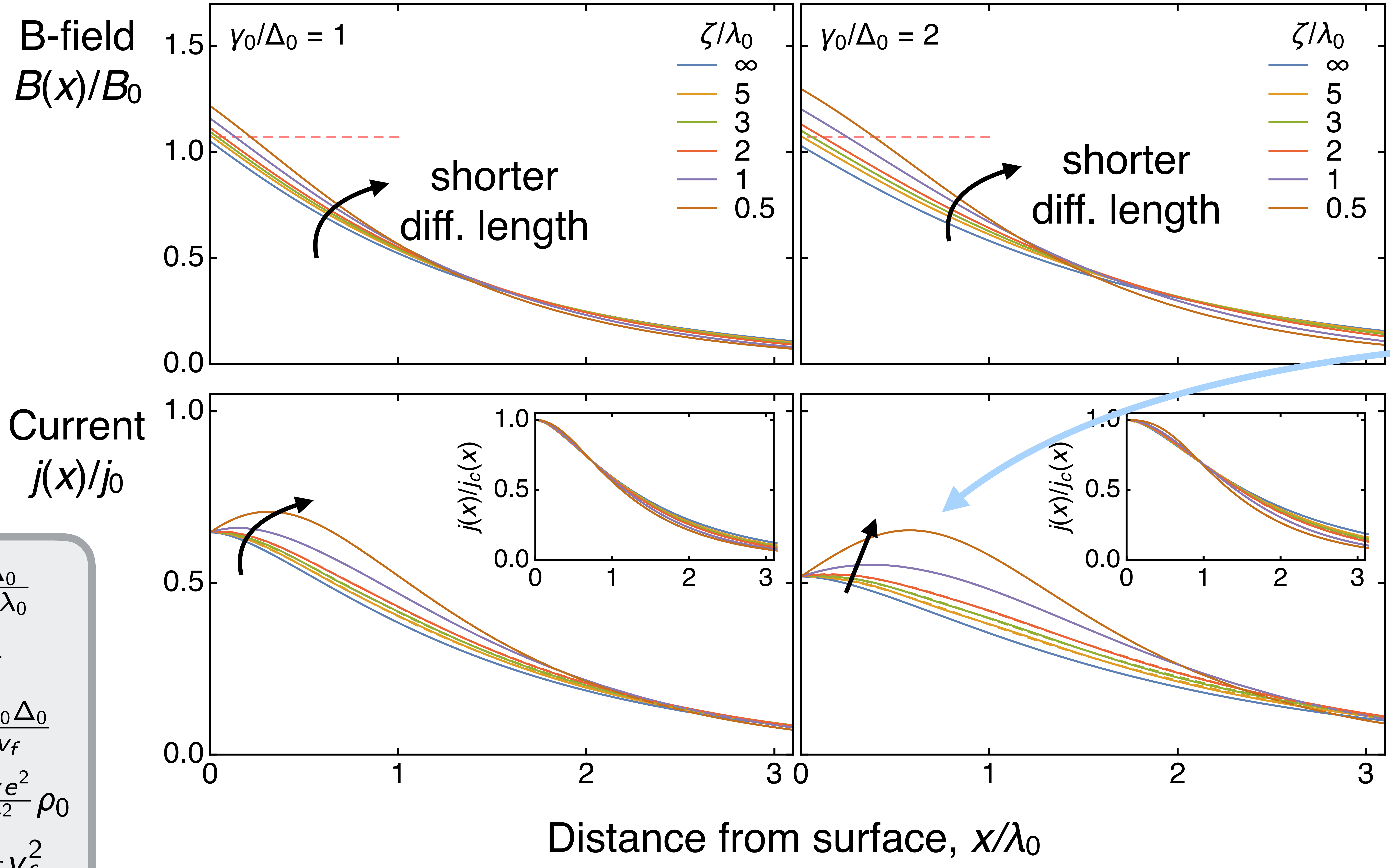
$$\sim \frac{\phi_0}{\lambda_0 \xi_0}$$

$$j_0 = \frac{1}{3}\frac{e \rho_0 \Delta_0}{v_f}$$

$$\lambda_0^{-2} = \frac{1}{3}\frac{4\pi e^2}{c^2}\rho_0$$

$$\rho_0 = 2 N_f v_f^2$$

Effect of diffusion length



Maximum current away from surface

$$B_0 = \frac{e}{c}\frac{\Delta_0}{v_f \lambda_0}$$
$$\sim \frac{\phi_0}{\lambda_0 \xi_0}$$
$$j_0 = \frac{1}{3}\frac{e \rho_0 \Delta_0}{v_f}$$
$$\lambda_0^{-2} = \frac{1}{3}\frac{4\pi e^2}{c^2}\rho_0$$
$$\rho_0 = 2 N_f v_f^2$$

Conclusions & outlooks

- Inhomogeneous impurity profiles can lead to superheating fields that are higher than homogeneous profiles
- Diffusion layers are not only an experimental fact — they could play an important role in understanding SRF cavity performance
- We only explore the relatively easy limit of extreme type-II SC — we expect richer physics in the more realistic non-local, dynamic case

Thank you!

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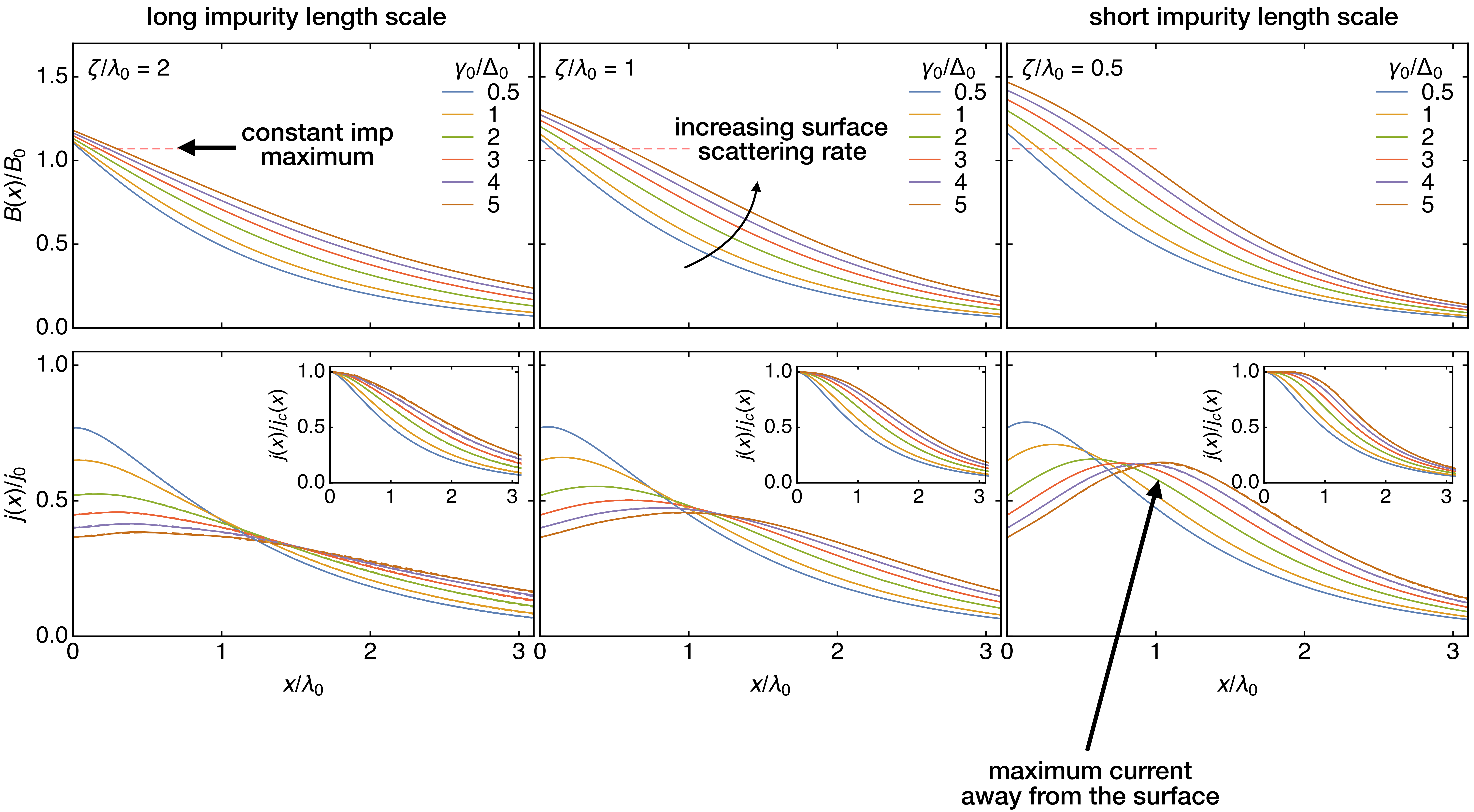
Anna Grassellino
Alex Romanenko
Mattia Checchin
Martina Martinello

Lin & Gurevich, PRB (2012)

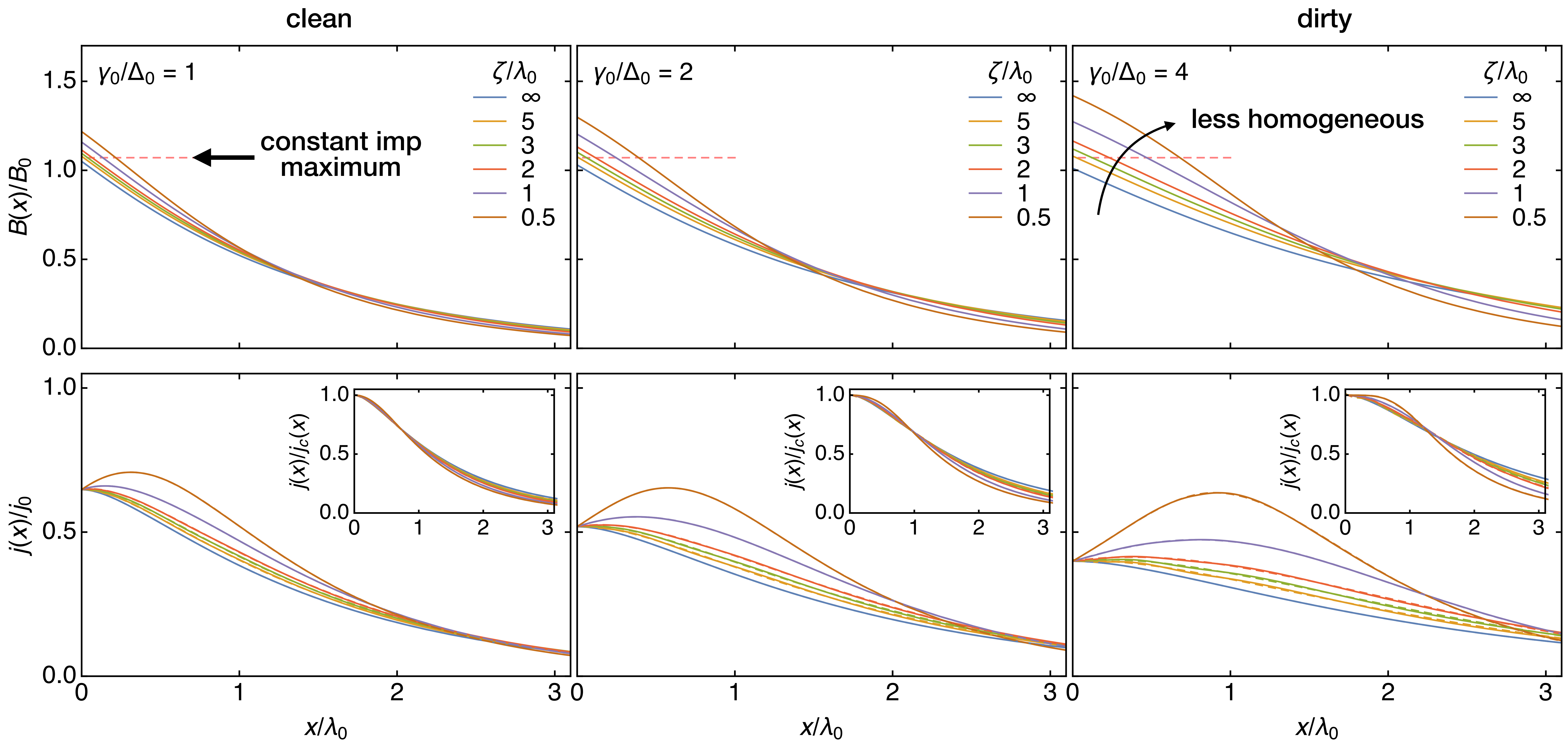
effects of impurity density at surface



scattering rate

$\gamma(x) = \gamma_0 e^{-x/\zeta}$

effects of diffusion length



scattering rate

$\gamma(x) = \gamma_0 e^{-x/\zeta}$