Collective weak pinning model of vortex dissipation in SRF cavities

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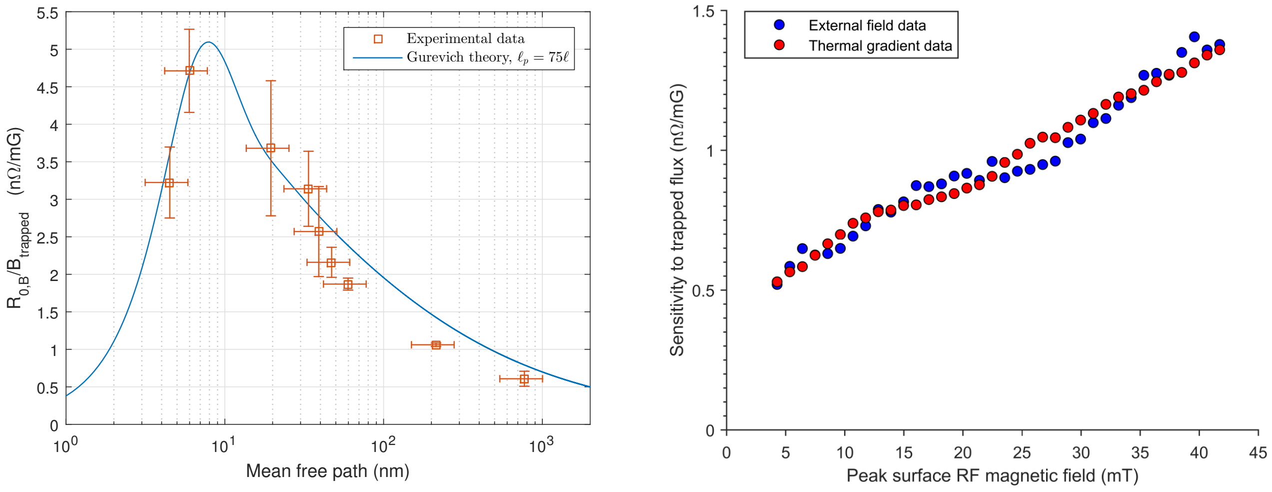
Center for BRIGHT BEAMS
A National Science Foundation Science & Technology Center

TTC Topical Workshop - RF Superconductivity: Pushing Cavity Performance Limits

Outline

a) Introduction
b) Collective weak pinning
c) Results
d) Final remarks

a)



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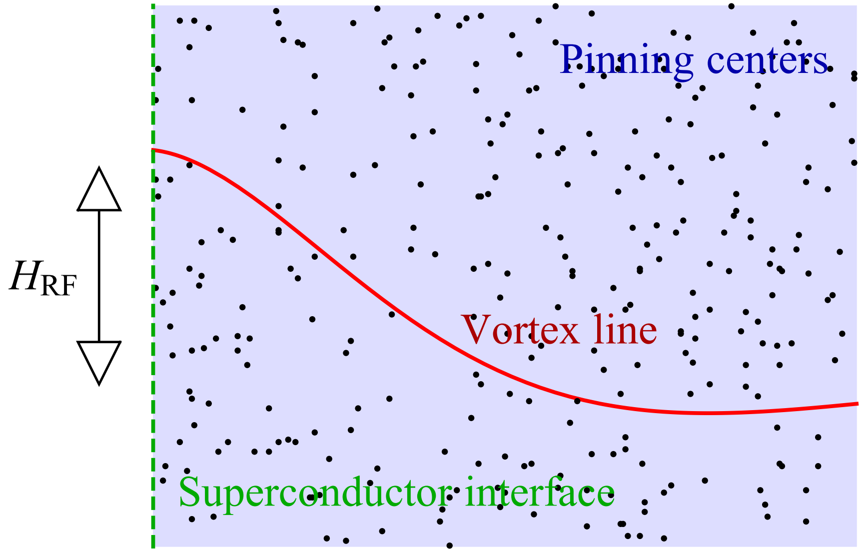

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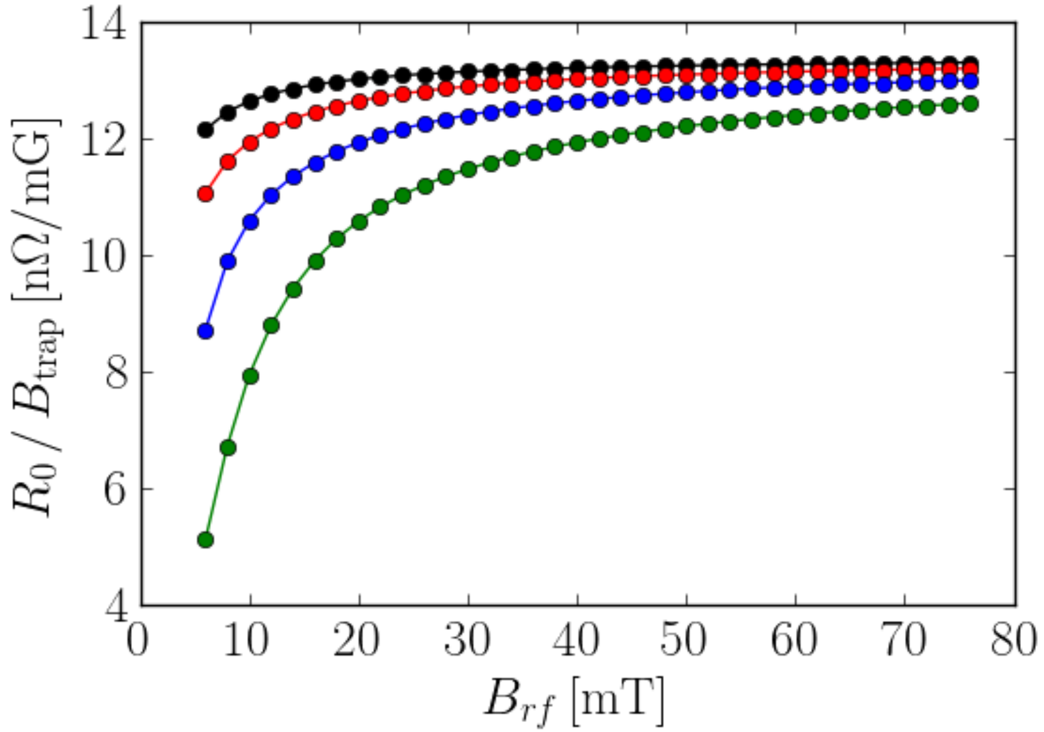

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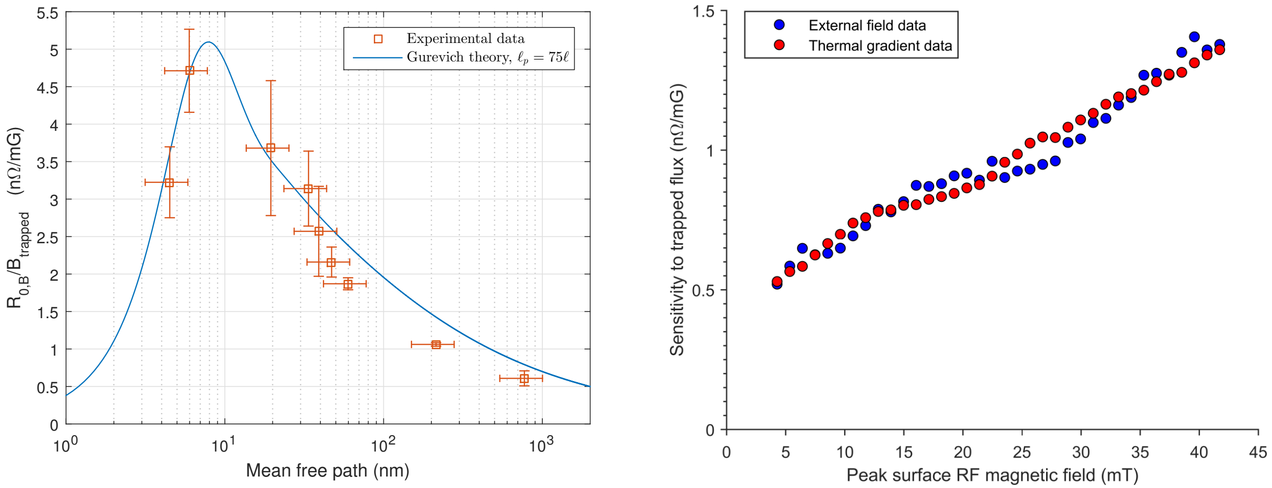

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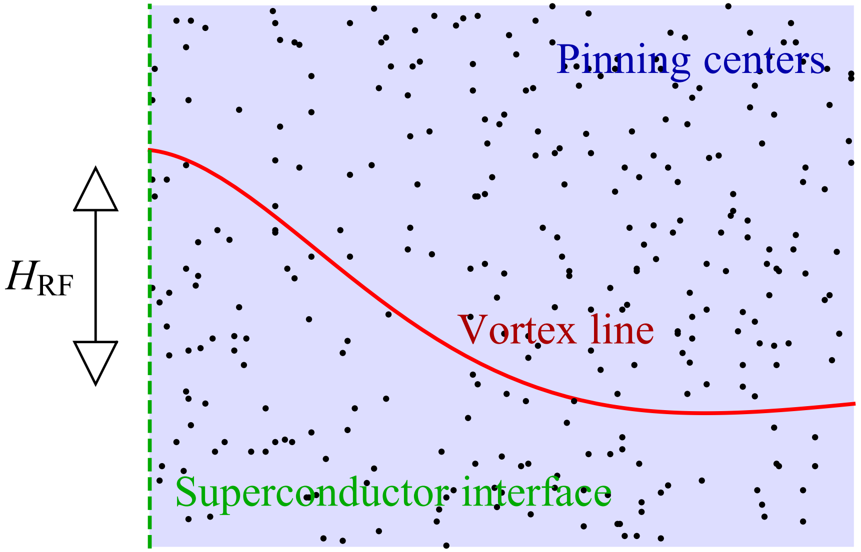

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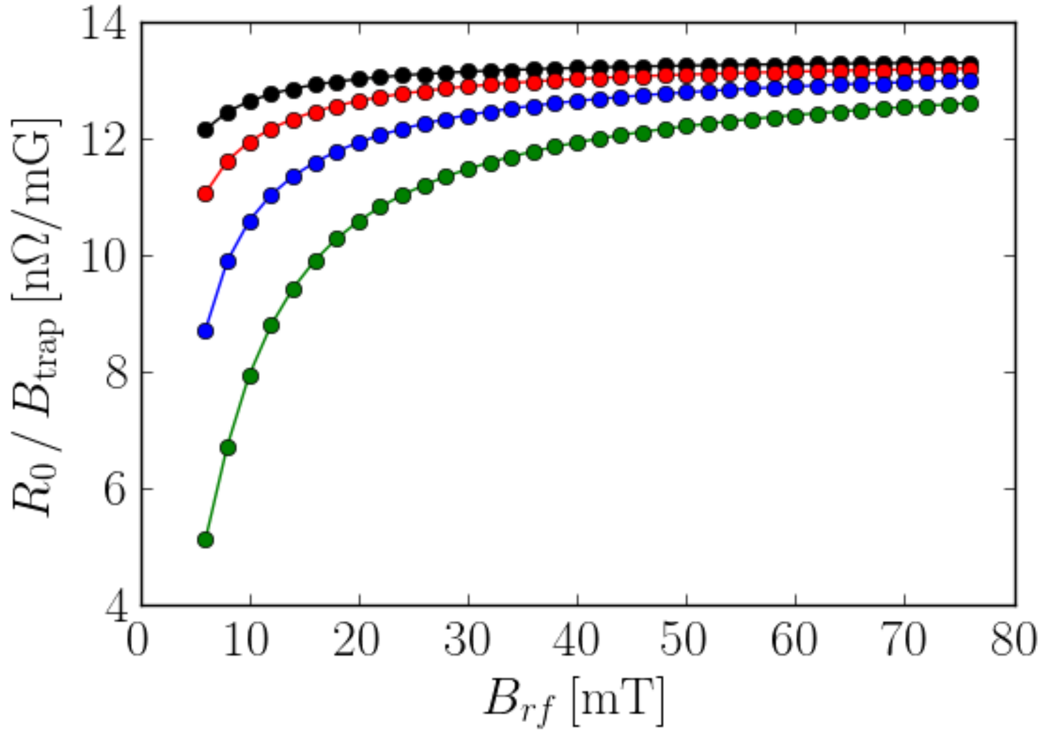
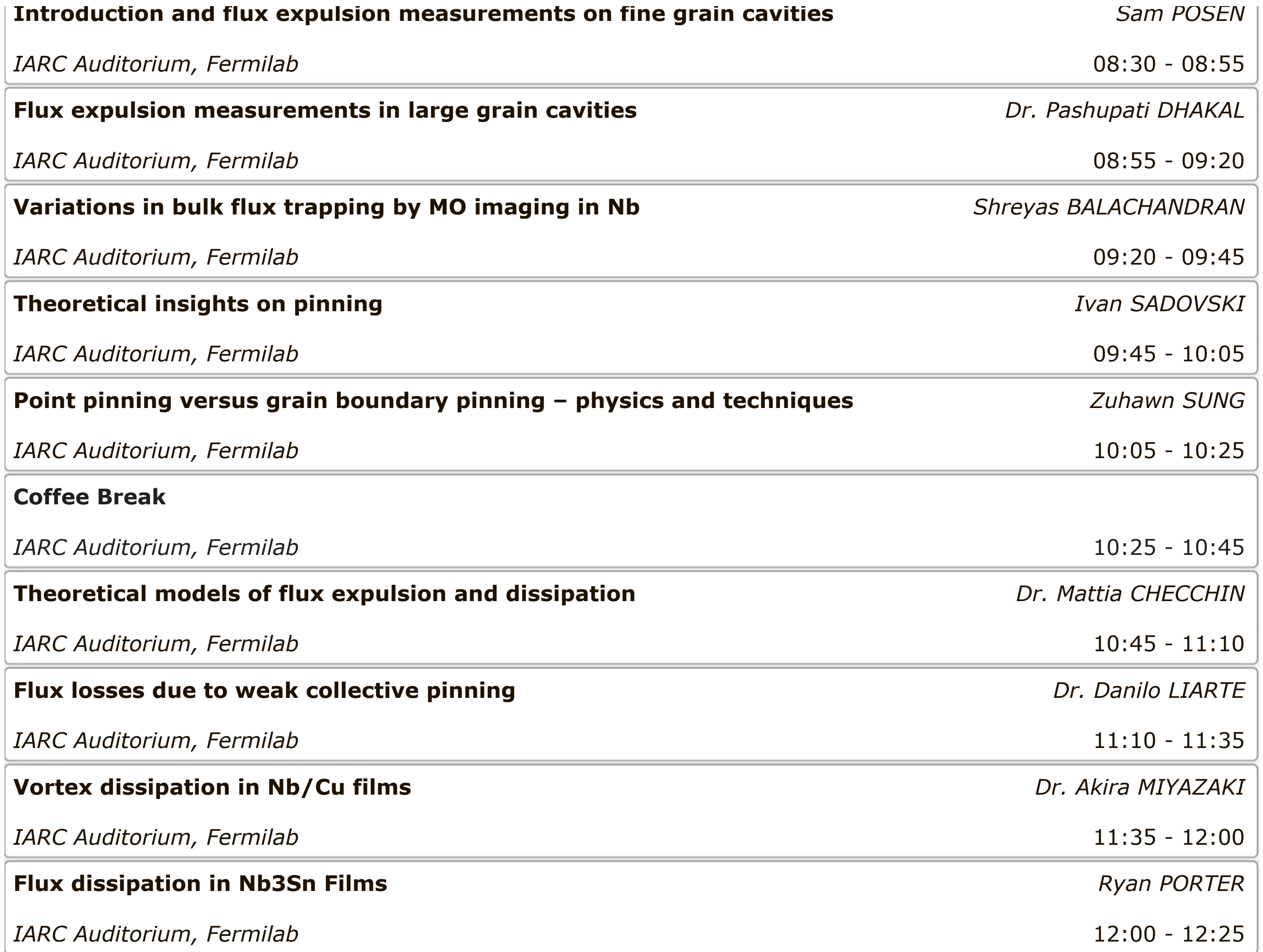
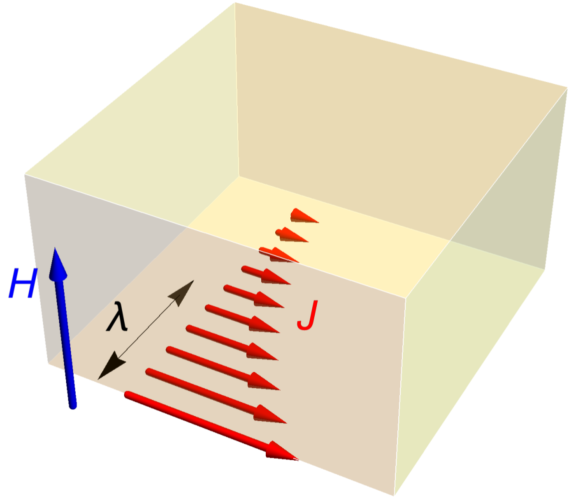


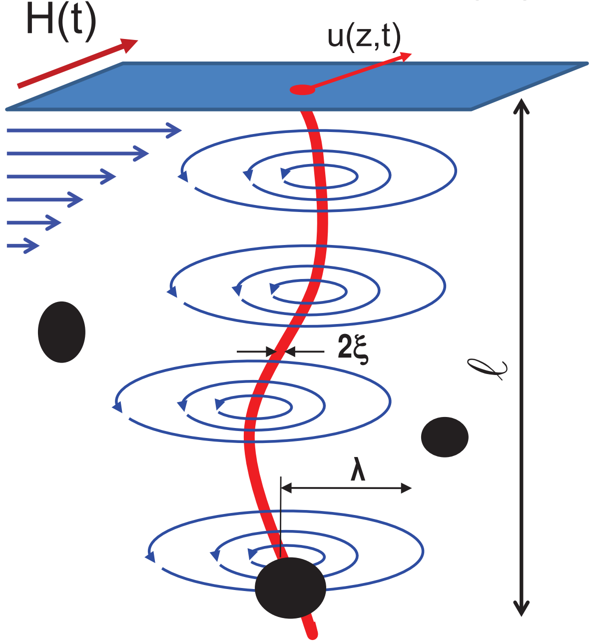
Gaining insight on trapped flux



Introduction and flux expulsion measurements on fine grain cavities	*Sam POSEN*	*IARC Auditorium, Fermilab*	08:30 - 08:55
Flux expulsion measurements in large grain cavities	*Dr. Pashupati DHAKAL*	*IARC Auditorium, Fermilab*	08:55 - 09:20
Variations in bulk flux trapping by MO imaging in Nb	*Shreyas BALACHANDRAN*	*IARC Auditorium, Fermilab*	09:20 - 09:45
Theoretical insights on pinning	*Ivan SADOVSKI*	*IARC Auditorium, Fermilab*	09:45 - 10:05
Point pinning versus grain boundary pinning – physics and techniques	*Zuhawn SUNG*	*IARC Auditorium, Fermilab*	10:05 - 10:25
Coffee Break		*IARC Auditorium, Fermilab*	10:25 - 10:45
Theoretical models of flux expulsion and dissipation	*Dr. Mattia CHECCHIN*	*IARC Auditorium, Fermilab*	10:45 - 11:10
Flux losses due to weak collective pinning	*Dr. Danilo LIARTE*	*IARC Auditorium, Fermilab*	11:10 - 11:35
Vortex dissipation in Nb/Cu films	*Dr. Akira MIYAZAKI*	*IARC Auditorium, Fermilab*	11:35 - 12:00
Flux dissipation in Nb3Sn Films	*Ryan PORTER*	*IARC Auditorium, Fermilab*	12:00 - 12:25

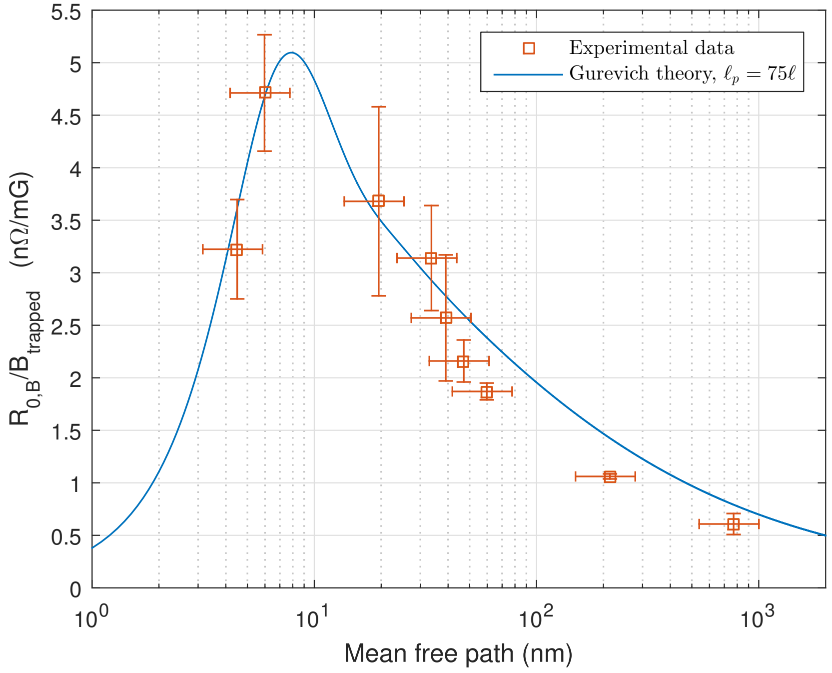


Gurevich & Ciovati
PRB 2013



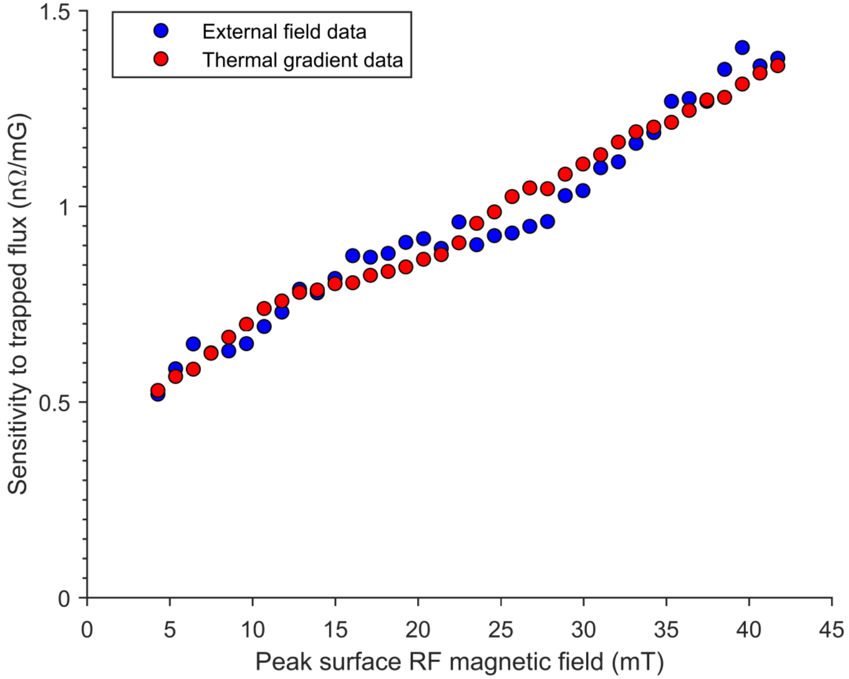
Sensitivity of R_0 to trapped flux

Dependence on MFP (Gonnella)



Gonnella et al. J. Appl. Phys. 2016

Dependence on RF field (Hall)



Hall et al. IPAC 2017

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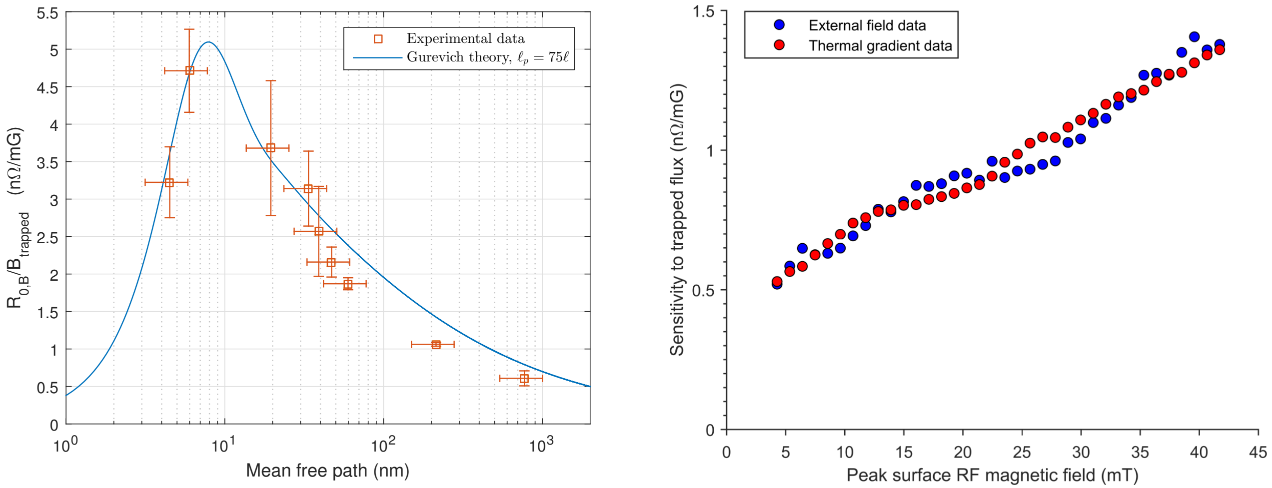

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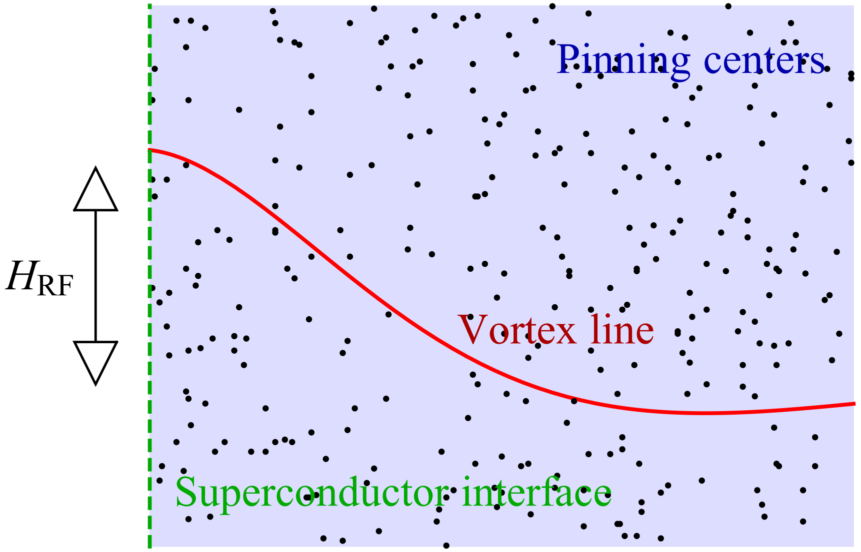

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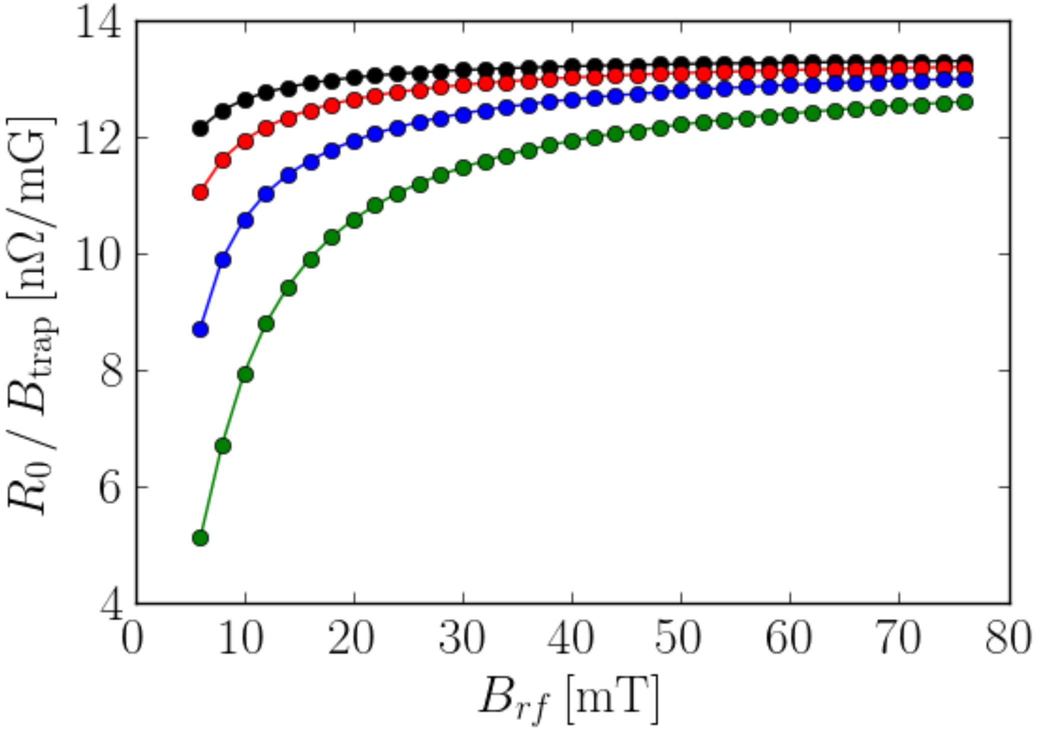


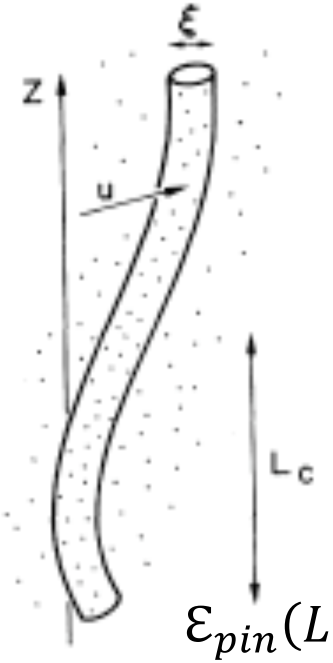
$\sqrt{N}$ fluctuations and collective weak pinning

$$\underset{\text{inertial}}{M\ddot{u}} = \overset{\text{viscous}}{f_v} + \underset{\text{Lorentz}}{f_L} + \overset{\text{pinning}}{f_p} + \underset{\text{elastic}}{f_e} + \overset{\text{Magnus}}{f_M}$$

Pinning forces add up randomly; only fluctuations can pin the line.

$$F_{pin} \cong \sqrt{{f_{pin}}^2\, n\, \xi^2\, L}$$ accumulated pinning force

For $L > L_c$, a vortex can bend to find a favorable position in the pinning potential, cutting off the square-root growth of F_{pin}.



$$\mathcal{E}_{pin}(L_c) = \mathcal{E}_{elastic}(L_c)$$

FIG. 4. Single vortex line pinned by the collective action of many weak pointlike pinning centers. Only fluctuations in the pin density are able to pin the vortex. In order to accommodate optimally to the pinning potential, the vortex line deforms by ξ (the minimal transverse length scale the vortex core is able to resolve equals the scale of the pinning potential) on a longitudinal length scale L_c, the collective pinning length.

$\sqrt{N}$ fluctuations and collective weak pinning

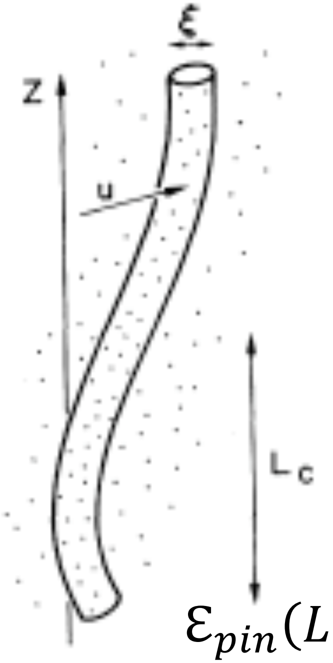
$$\underset{\text{inertial}}{M\ddot{u}} = \overset{\text{viscous}}{f_v} + \underset{\text{Lorentz}}{f_L} + \overset{\text{pinning}}{f_p} + \underset{\text{elastic}}{f_e} + \overset{\text{Magnus}}{f_M}$$

A vortex breaks up into segments of size L_c;
each will compete with the Lorentz force.

Pinning force & depinning current…

$$F_{pin}(L_c) = F_{Lorentz}(j_d, L_c)$$

$$\frac{F_{pin}}{L_c} = {H_c}^2 \xi \, \frac{j_d}{j_o}$$ (cgs units)



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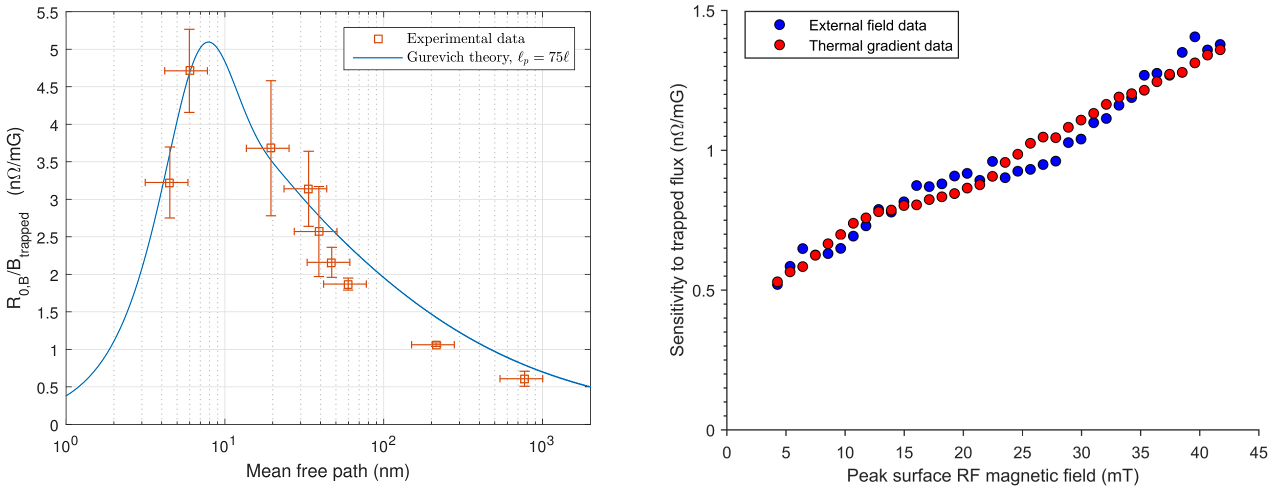

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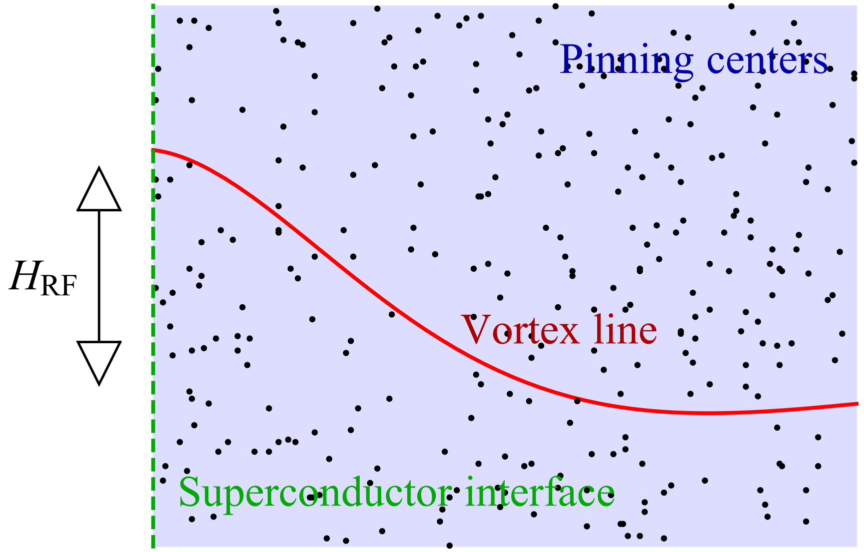

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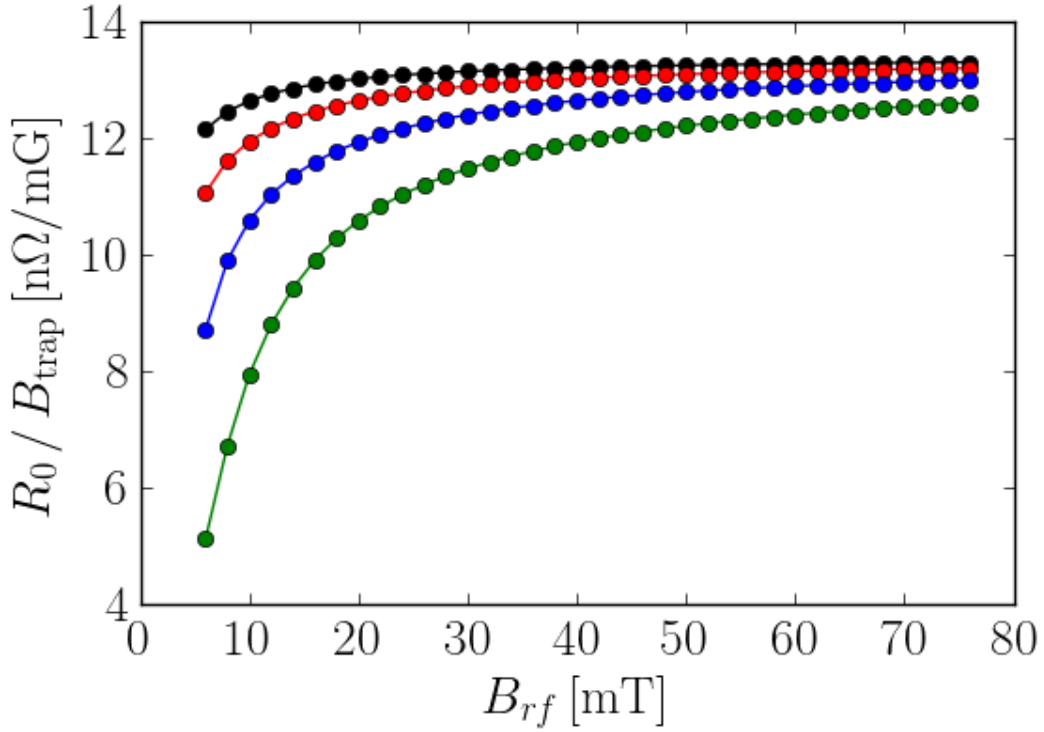


Collective weak pinning at low frequency

Derivation and analytical solution

MF pinning

$$0 = f_L + f_p^{MF} + f_e$$

Lorentz elastic

$$y'' = \alpha - \beta \sin(t)\,\delta(z)$$

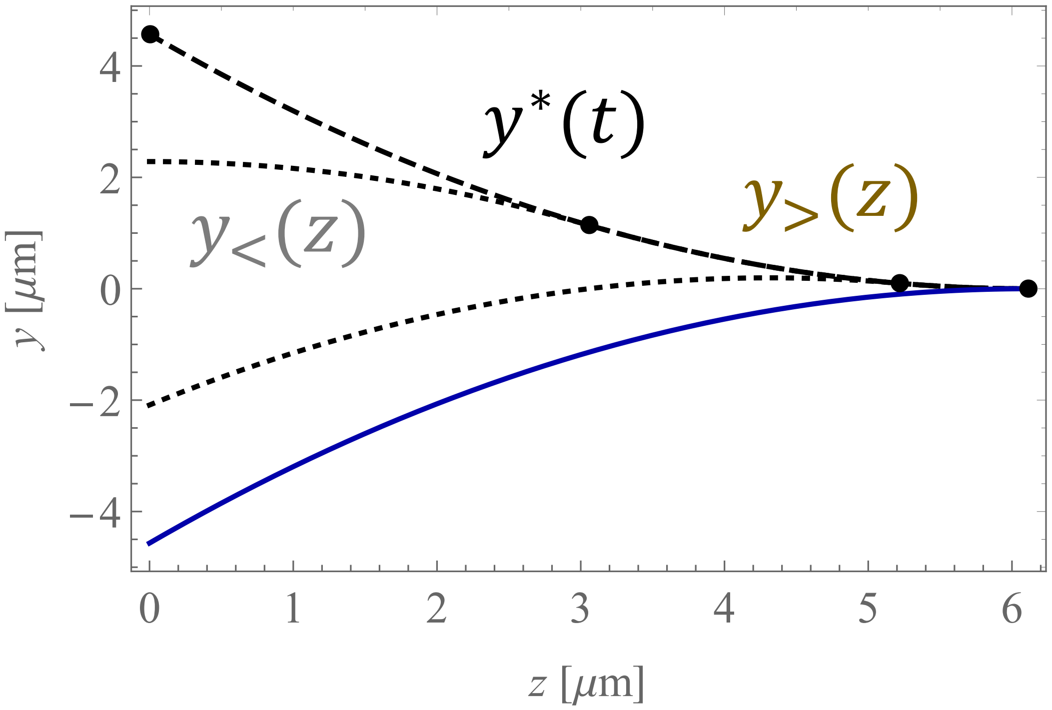
$$y_<(z) = a(t) - \beta \sin(t)\, z - \frac{|\alpha|}{2} z^2$$

for $y < y^*$

$$y_>(z) = \frac{|\alpha|}{2}\left(z - {}^{\beta}/_{|\alpha|}\right)^2$$

for $y > y^*$

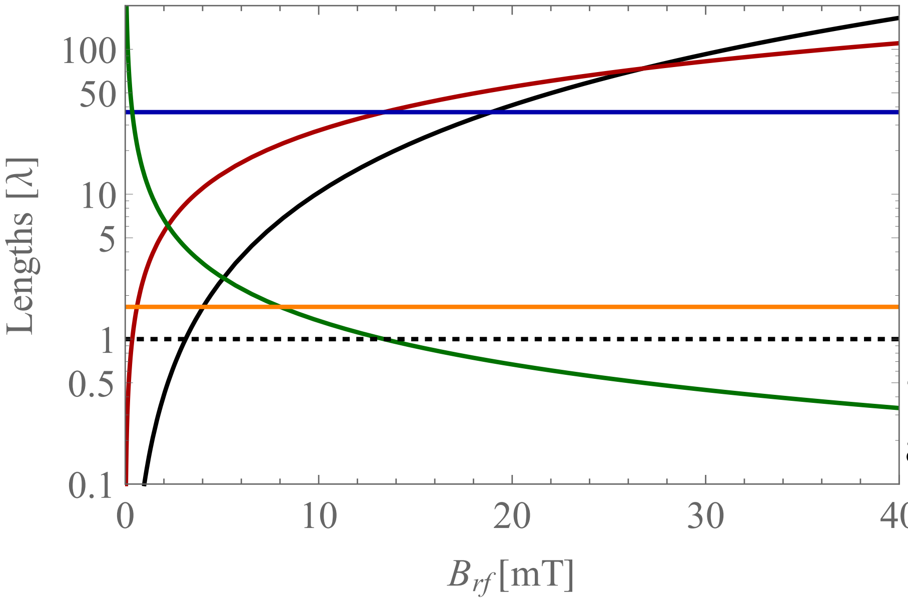
Solution for Nb_3Sn at 20mT RF field and 1mA/μm² depinning current



Collective weak pinning at low frequency

‘Sanity’ tests

Point-like force, collective weak pinning, BC…



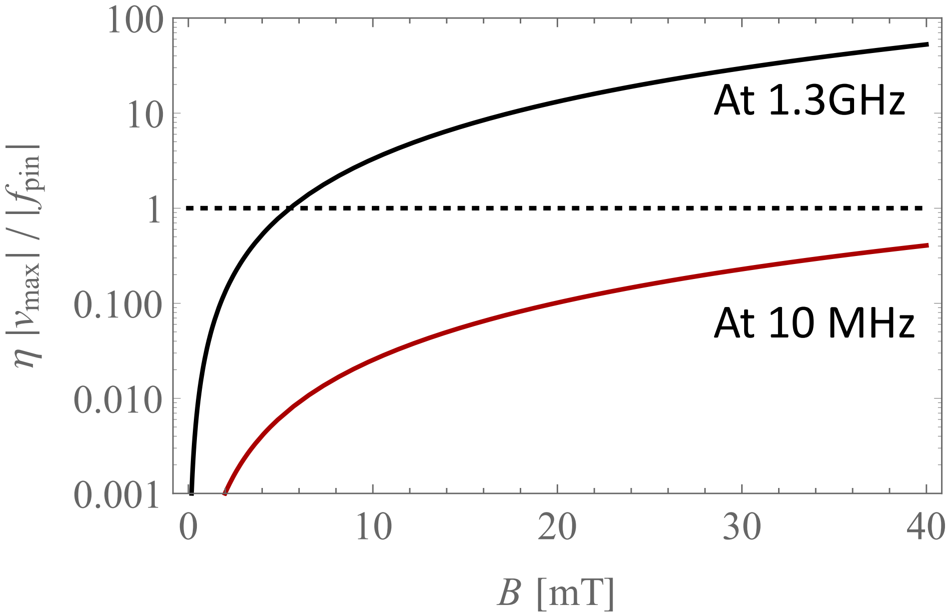
Amplitudes in y and z

Curvature radius at the surface

Pinning length

‘Curvature radius’ at the surface

Viscous dissipation term



Dependence on RF field

$$\frac{R_0}{B_{trapped}} = a\,B_{rf} + b$$

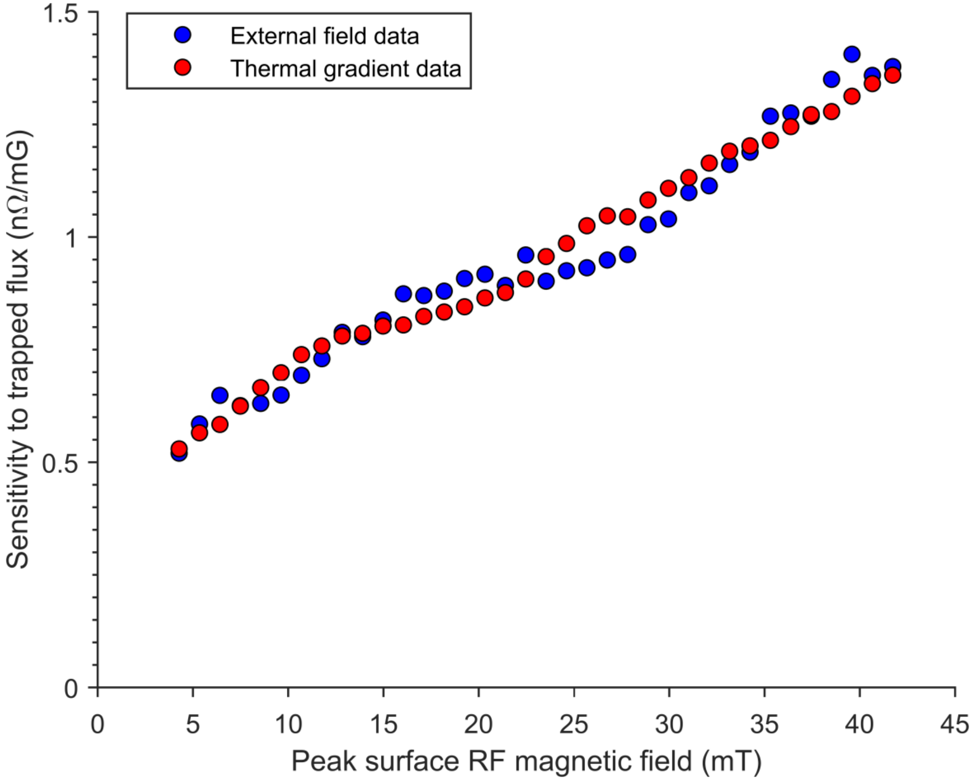
- The linear behavior is consistent with collective weak pinning (but not accurate)

Using $j_d{\sim}3\times10^{-3}j_o$

- There is a factor of 100 off in comparison with the experimental results. Viscous dissipation is needed.

$$a = \frac{4}{3}\frac{f\lambda^2\mu_0}{B_c^2\xi}\frac{j_o}{j_d};$$

$$b = 0$$



DBL, Hall, Liepe, Sethna, in progress

Collective weak pinning at low frequency

‘Sanity’ tests

Estimate density of impurities assuming experimental value for $j_d \sim 3 \times 10^{-3} j_o$

$$F_{pin} \cong \sqrt{{f_{pin}}^2\, n\, \xi^2\, L_c}$$

$$\frac{F_{pin}}{L_c} = {H_c}^2 \xi\, \frac{j_d}{j_o}$$

Atomic scale

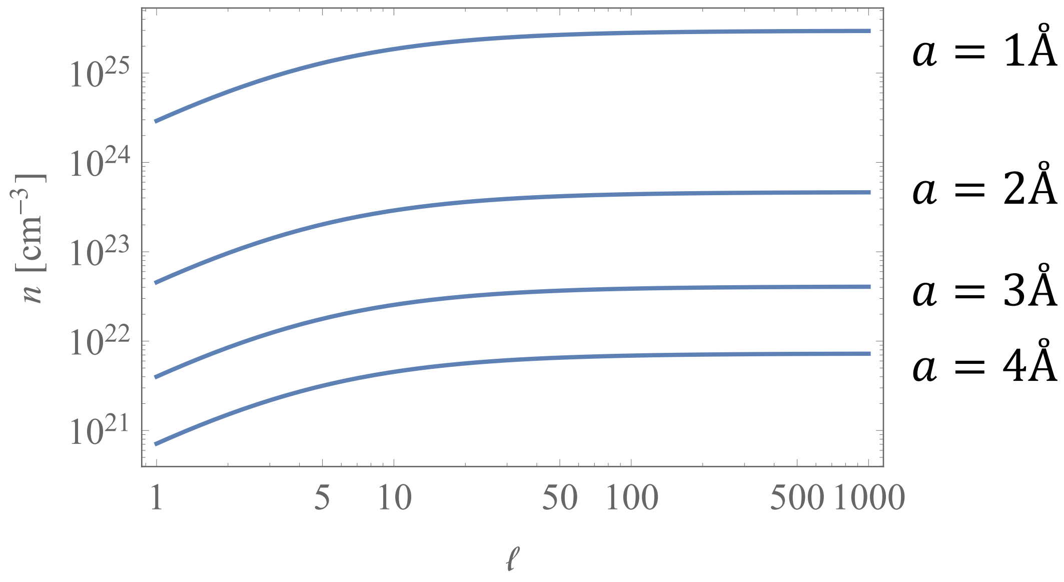
$$f_{pin} \cong \xi^{-1}\, \frac{a^3 {H_c}^2}{8\pi}$$

Individual pinning force

$$\left(\frac{j_o}{j_d}\right)^3 = 256\, \pi^4\, \frac{\xi^6}{n^{12} a^{12}}$$

$$n^{1/3} = {}^{\xi(l)}\!\big/_{10\, a^2}$$

$$\xi(l) = {}^{0.738\, \xi_0}\!\Big/\!\sqrt{1 + 0.882\, {}^{\xi_0}\!/_{l}}$$



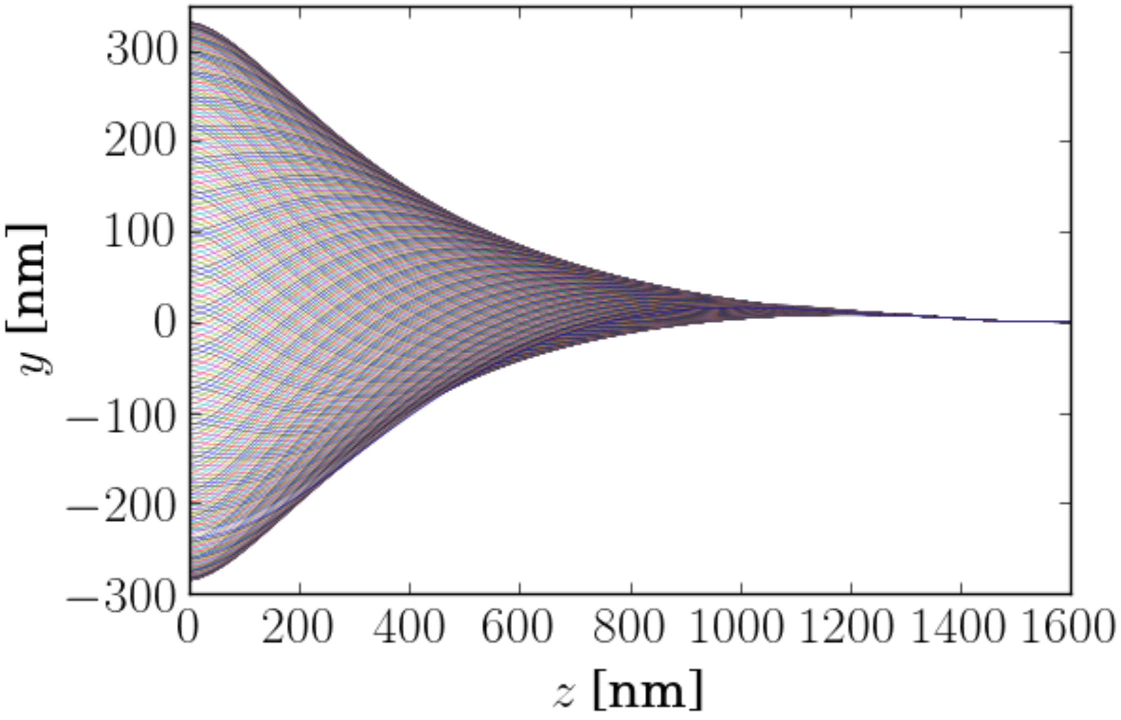
Collective weak pinning at high frequency

Simulated solution

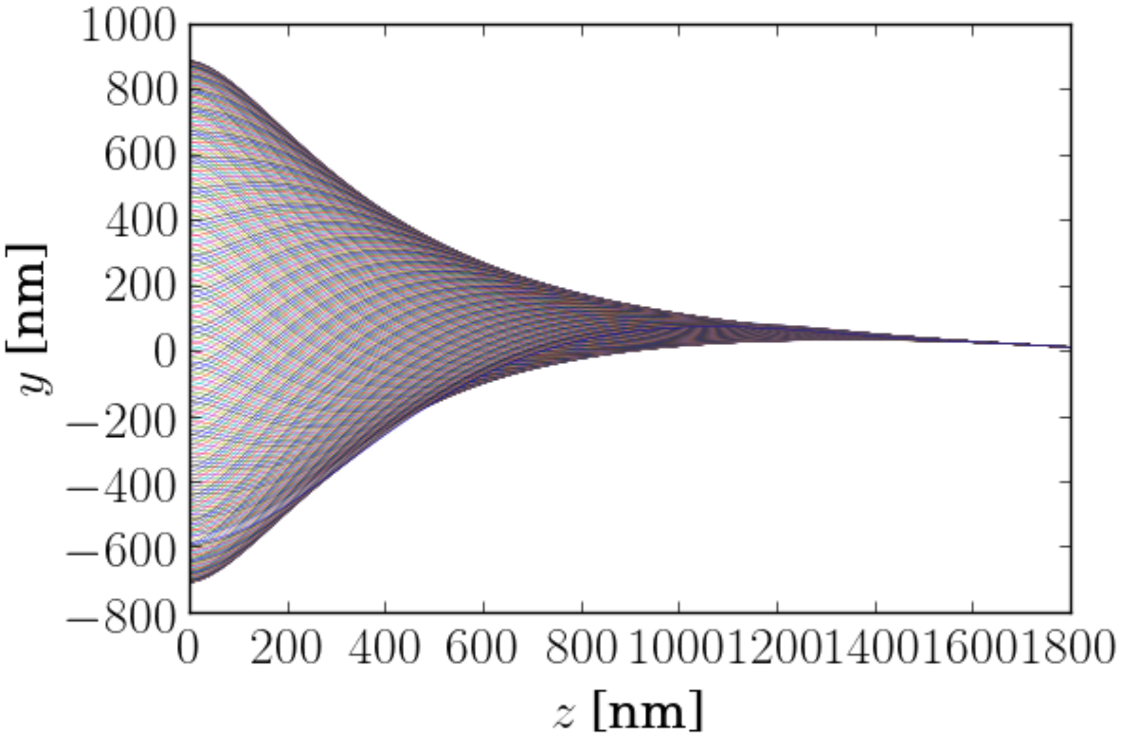
viscous, MF pinning

$$0 = f_v + f_L + f_p^{MF} + f_e$$

Lorentz, elastic



At $B_{rf} = 20\text{mT}$

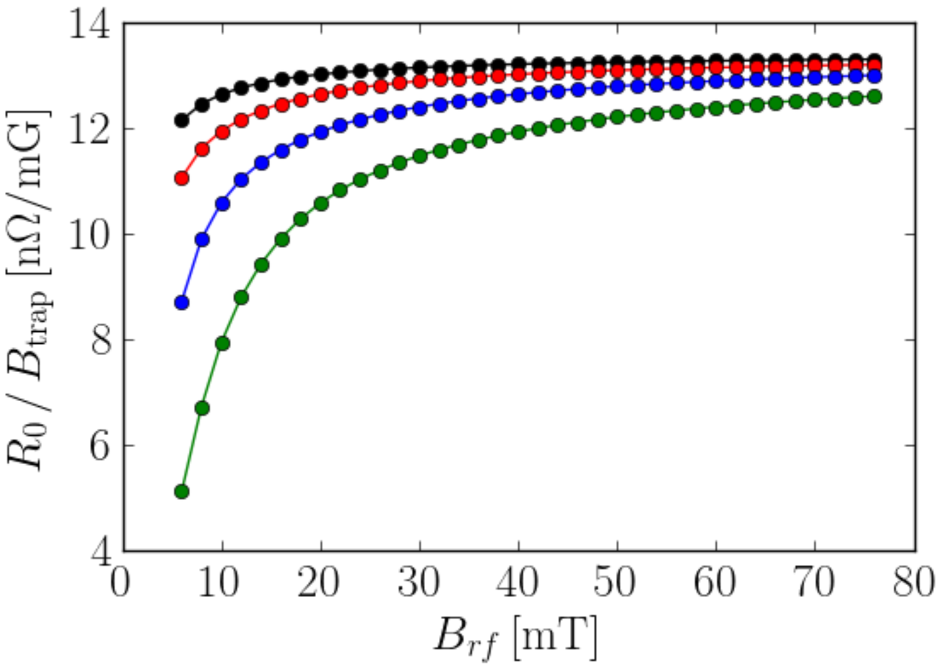


At $B_{rf} = 50\text{mT}$

Sensitivity of R_0 to trapped flux

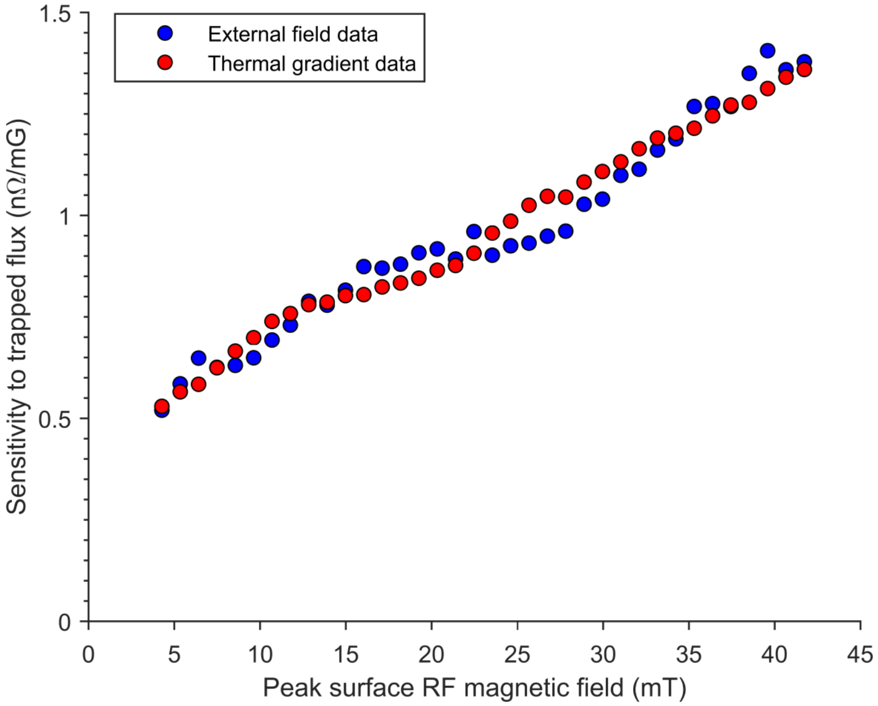
as a function of the RF field

- Simulation



At $j_d = 1$ (black), 2 (red), 3 (blue), and 4mA/µm² (green)

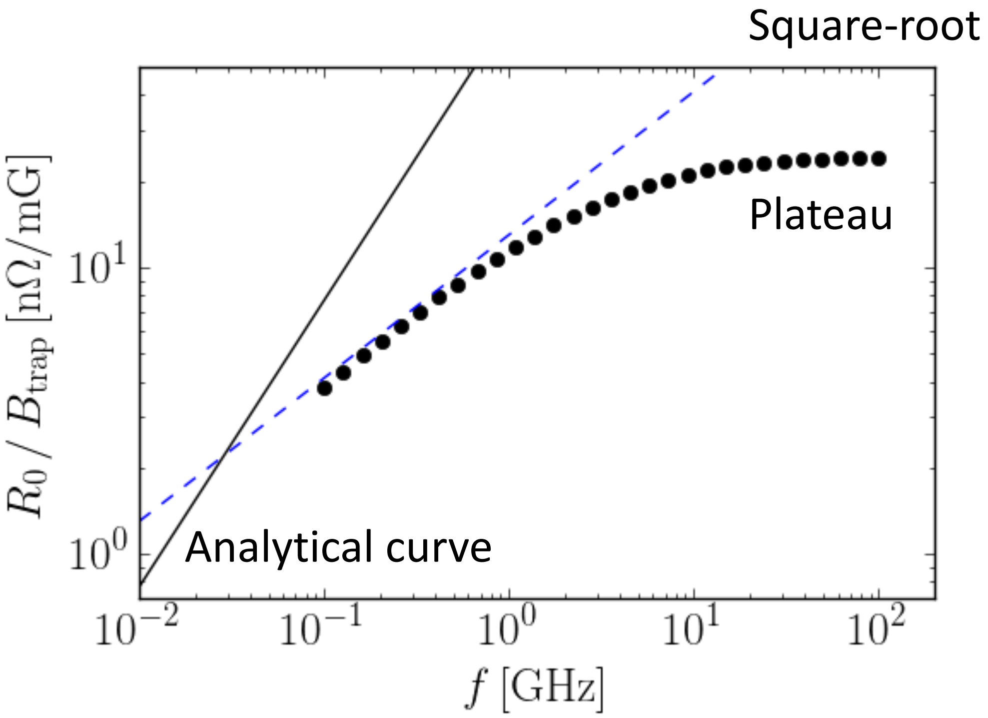
- Experiment (Hall)



Sensitivity of R_0 to trapped flux

as a function of frequency

- Simulation



- Experiment (Oseroff)

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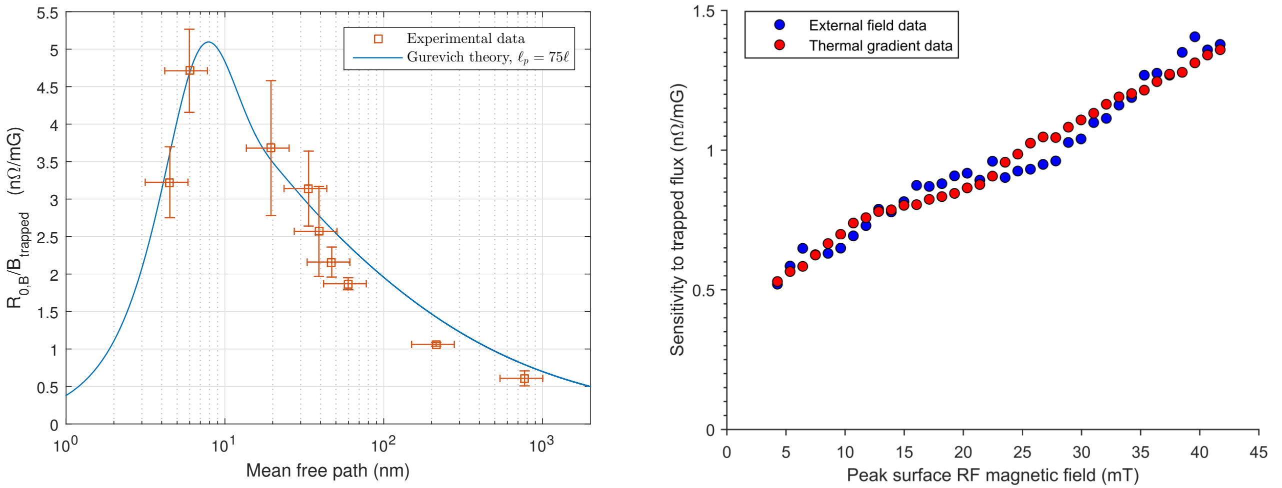

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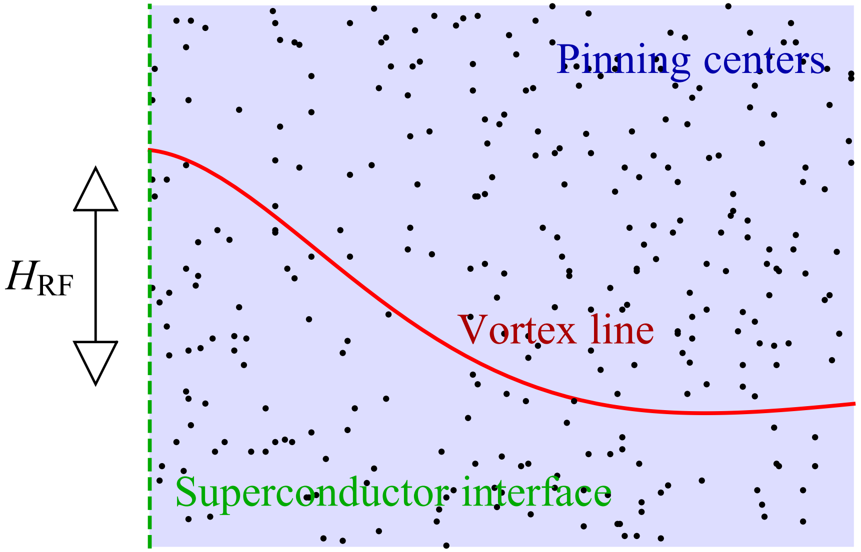

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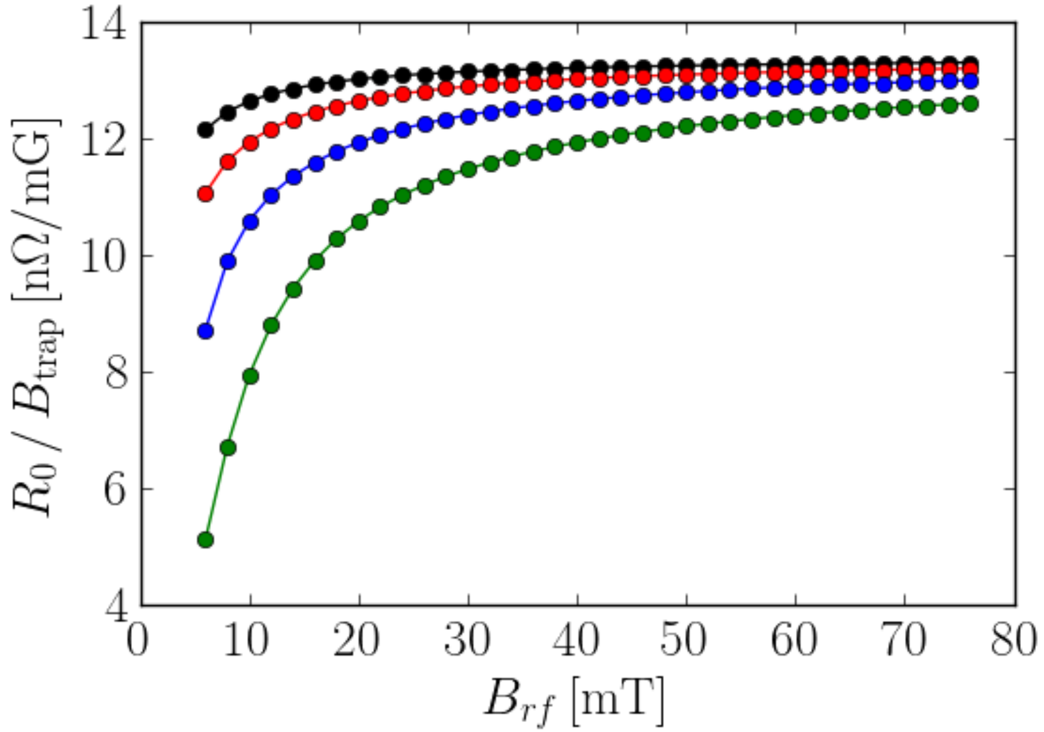


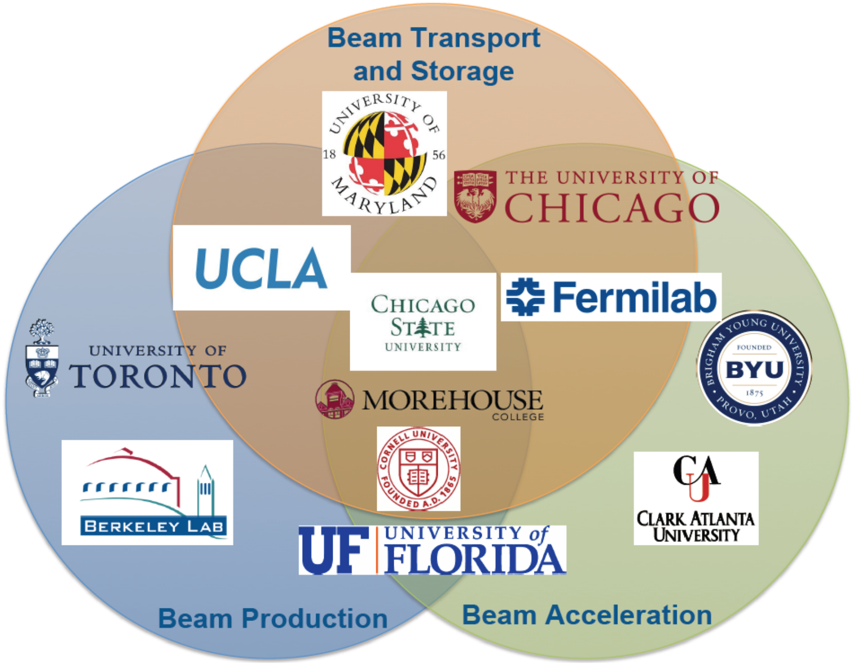
Conclusions and future work

- Hysteretic losses might explain the dependence of the residual resistance sensitivity to trapped flux on the RF field.
- Our approximations are consistent, though we predict dissipations larger than the experimental ones by a factor of about eight.
- The collective weak-pinning model predicts three distinct regimes for the dissipation as a function of frequency: linear, square-root, and a plateau.

- Simulations with explicit inclusion of impurities.
- Large amplitudes, grain boundaries, and mixed (strong and weak pinning) scenarios.
- Experimental check: do most trapped vortices lie perpendicular to the interface? (We have assumed vortices that are normally aligned with respect to the interface.)

Acknowledgments

- TTC Topical Workshop Committee, for the invitation.
- The Sethna and the Liepe groups in Cornell.
- The Center for Bright Beams SRF team.
- Prof. Alex Gurevich, for useful consultation.
- Financial support from the Center for Bright Beams.



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