Quantum Simulation of Abelian and non-Abelian Gauge Theories

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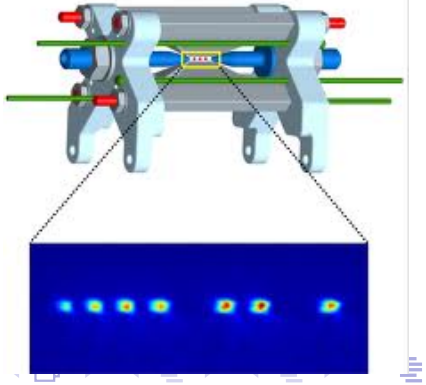
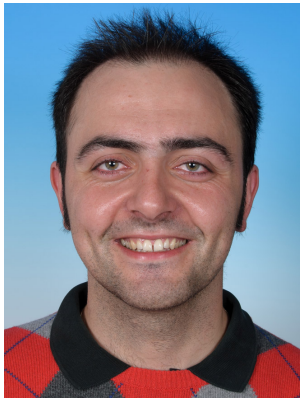
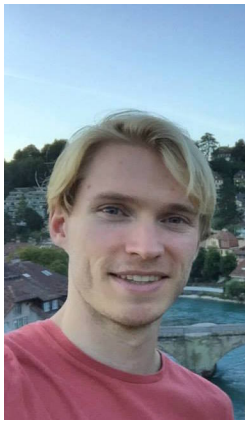
Near-Term Applications
of Quantum Computation
Fermilab
December 6-7, 2017



Bern-Innsbruck Particle Physics AMO Collaboration



Trieste: Marcello Dalmonte
Innsbruck: Catherine Laflamme, Peter Zoller
Bilbao: Enrique Rico Ortega



Bern: Wynne Evans, Florian Hebenstreit, Philippe Widmer
DESY Zeuthen: Debasish Banerjee

Outline

A Brief History of Computing

Pioneers of Quantum Simulation

The Nature of the Sign Problem

From Wilson's Lattice QCD to Quantum Link Models

Quantum Simulators for Abelian Lattice Gauge Theories

Quantum Simulators for non-Abelian Gauge Theories

How to Reach the Continuum Limit in a Toy Model for QCD

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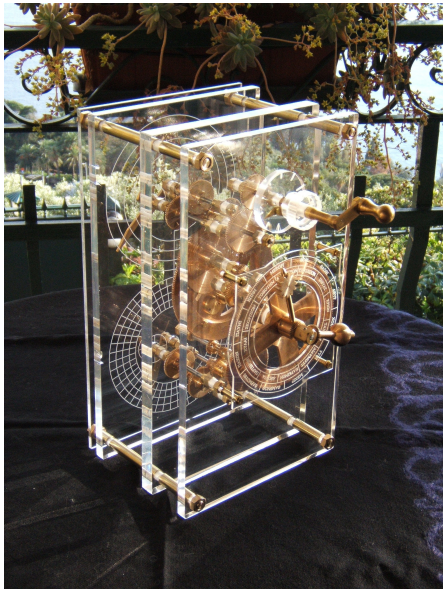
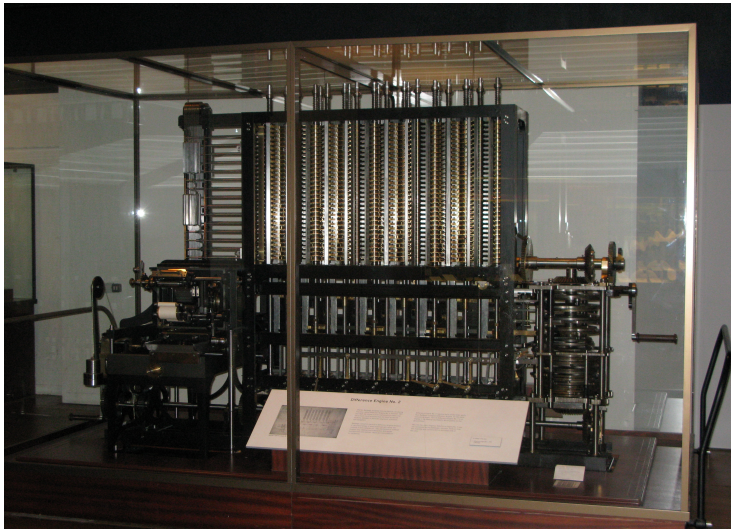
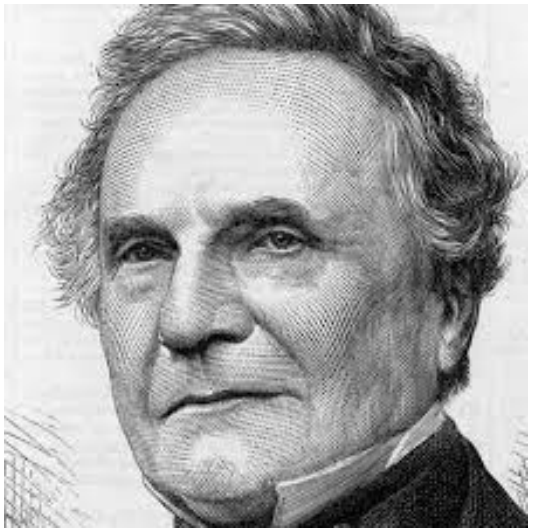
The first "digital computer" in Babylonia about 2400 b.c.

The first "analog computer": Antikythera for determining the position of celestial bodies, Crete, about 100 b.c.



The first programmable computer: Charles Babbagge's (1791-1871) "difference engine" was realized by his son.





The first software developer: Ada Lovelace (1815-1852).

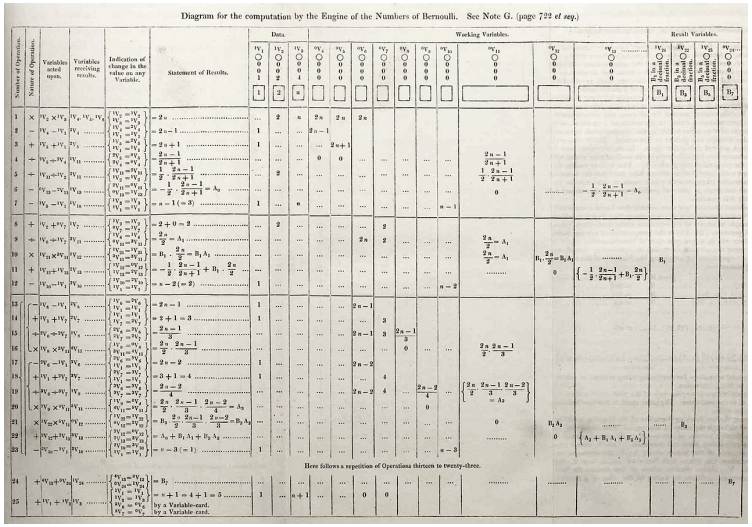
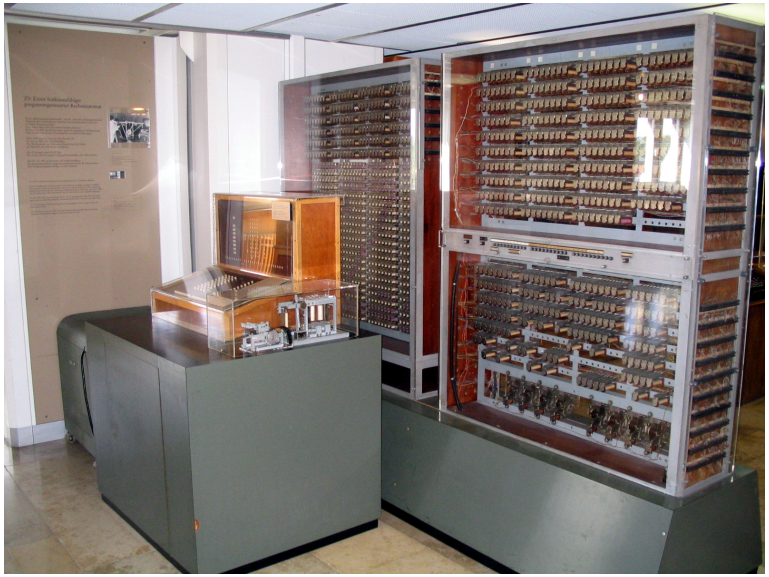


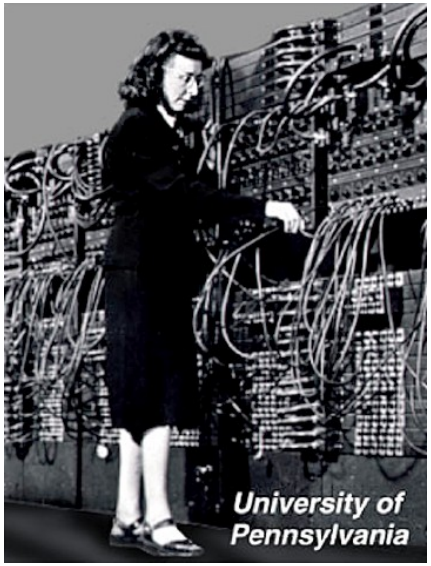
Diagram for the computation by the Engine of the Numbers of Bernoulli. See Note G. (page 722 *et seq.*)

Number of Operation.	Nature of Operation.	Variables acted upon.	Variables receiving results.	Indication of change in the value on any Variable.	Statement of Results.	Data.			Working Variables.											Result Variables.			
						$^{1}V_1$ 1	$^{1}V_2$ 2	$^{1}V_3$ n	$^{2}V_1$	$^{2}V_2$	$^{2}V_3$	$^{2}V_4$	$^{2}V_5$	$^{2}V_6$	$^{2}V_7$	$^{2}V_8$	$^{2}V_{11}$	$^{2}V_{12}$	$^{2}V_{13}$	$^{3}V_{11}$ B_1	$^{3}V_{12}$ B_3	$^{3}V_{13}$ B_5	$^{3}V_{14}$ B_7
1	×	$^{1}V_2 \times {}^{1}V_3$	$^{1}V_4$	$^{1}V_2 \cdot {}^{1}V_3$	$\left\{\begin{array}{l} {}^{2}V_2 = {}^{1}V_2 \\ {}^{2}V_3 = {}^{1}V_3 \end{array}\right\} = 2n$	...	2	n	2n	2n	2n												
2	−	$^{1}V_4 - {}^{1}V_1$	$^{1}V_4$		$\left\{\begin{array}{l} {}^{2}V_4 = {}^{1}V_4 \\ {}^{2}V_5 = {}^{1}V_1 \end{array}\right\} = 2n-1$	1	...	...	2n−1														
3	+	$^{1}V_5 + {}^{1}V_1$	$^{1}V_5$		$\left\{\begin{array}{l} {}^{2}V_6 = {}^{1}V_5 \\ {}^{2}V_7 = {}^{1}V_1 \end{array}\right\} = 2n+1$	1	...	...	...	2n+1													
4	÷	$^{2}V_6 \div {}^{2}V_4$	$^{2}V_{11}$		$\left\{\begin{array}{l} {}^{3}V_6 = {}^{2}V_6 \\ {}^{3}V_7 = {}^{2}V_4 \end{array}\right\} = \frac{2n+1}{2n-1}$	...	...	...	0	0	...	...	...	...	...		$\frac{2n-1}{2n+1}$						
5	×	$^{2}V_{11} \times {}^{2}V_2$	$^{2}V_{11}$		$\left\{\begin{array}{l} {}^{3}V_{11} = {}^{2}V_{11} \\ {}^{3}V_{12} = {}^{2}V_2 \end{array}\right\} = \frac{1}{2} \cdot \frac{2n-1}{2n+1}$	...	2	...	...	...	...	...	...	...	...		$\frac{1}{2} \cdot \frac{2n-1}{2n+1}$						
6	−	$^{3}V_{13} - {}^{3}V_{11}$	$^{3}V_{13}$		$\left\{\begin{array}{l} {}^{4}V_{13} = {}^{3}V_{13} \\ {}^{4}V_{14} = {}^{3}V_{11} \end{array}\right\} = \frac{1}{2} \cdot \frac{2n-1}{2n+1} + A_3$	...	...	...	...	...	...	...	...	...	...		0		.....	$-\frac{1}{2} \cdot \frac{2n-1}{2n+1} - A_3$			
7	−	$^{1}V_8 - {}^{1}V_1$	$^{1}V_{13}$		$\left\{\begin{array}{l} {}^{2}V_{13} = {}^{1}V_8 \\ {}^{2}V_{14} = {}^{1}V_1 \end{array}\right\} = n-1\ (=3)$	1	...	n	...	...	...	...	...	...	...	n−1							
8	+	$^{1}V_2 + {}^{2}V_7$	$^{2}V_7$		$\left\{\begin{array}{l} {}^{2}V_7 = {}^{1}V_2 \\ {}^{2}V_8 = {}^{2}V_7 \end{array}\right\} = 2+0=2$	...	2	...	...	...	...	2											
9	÷	$^{2}V_8 \div {}^{2}V_2$	$^{2}V_{11}$		$\left\{\begin{array}{l} {}^{2}V_{11} = {}^{2}V_8 \\ {}^{2}V_{12} = {}^{2}V_2 \end{array}\right\} = \frac{2n}{2} = A_1$	...	...	...	...	2n	2	...	...	...			$\frac{2n}{2} = A_1$						
10	×	$^{2}V_{12} \times {}^{2}V_{11}$	$^{2}V_{12}$		$\left\{\begin{array}{l} {}^{3}V_{11} = {}^{2}V_{11} \\ {}^{3}V_{12} = {}^{2}V_{12} \end{array}\right\} = B_1 \cdot \frac{2n}{2} = B_1 A_1$	...	...	...	...	...	...	...	...	...	...		$\frac{2n}{2} = A_1$	$B_1 \cdot \frac{2n}{2} = B_1 A_1$	.....		B_1		
11	+	$^{3}V_{12} + {}^{3}V_{13}$	$^{3}V_{13}$		$\left\{\begin{array}{l} {}^{4}V_{13} = {}^{3}V_{12} \\ {}^{4}V_{14} = {}^{3}V_{13} \end{array}\right\} = -\frac{1}{2} \cdot \frac{2n-1}{2n+1} + B_1 \cdot \frac{2n}{2}$	...	...	...	...	...	...	...	...	...	...		.....		0	$\left\{-\frac{1}{2} \cdot \frac{2n-1}{2n+1} + B_1 \frac{2n}{2}\right\}$			
12	−	$^{1}V_{10} - {}^{1}V_1$	$^{1}V_{10}$		$\left\{\begin{array}{l} {}^{2}V_{10} = {}^{1}V_{10} \\ {}^{2}V_{11} = {}^{1}V_1 \end{array}\right\} = n-2\ (=2)$	1	...	...	...	...	...	...	...	...	...	n−2							
13	−	$^{2}V_4 - {}^{2}V_1$	$^{2}V_4$		$\left\{\begin{array}{l} {}^{3}V_4 = {}^{2}V_4 \\ {}^{3}V_5 = {}^{2}V_1 \end{array}\right\} = 2n-1$	1	...	...	...	...	2n−1												
14	+	$^{3}V_5 + {}^{3}V_1$	$^{3}V_5$		$\left\{\begin{array}{l} {}^{4}V_5 = {}^{3}V_5 \\ {}^{4}V_6 = {}^{3}V_1 \end{array}\right\} = 2+1=3$	1	...	...	...	...	...	3											
15	÷	$^{3}V_4 \div {}^{3}V_5$	$^{3}V_4$		$\left\{\begin{array}{l} {}^{4}V_4 = {}^{3}V_4 \\ {}^{4}V_5 = {}^{3}V_5 \end{array}\right\} = \frac{2n-1}{3}$	...	...	...	...	...	2n−1	3	$\frac{2n-1}{3}$										
16	×	$^{4}V_5 \times {}^{4}V_{11}$	$^{4}V_{11}$		$\left\{\begin{array}{l} {}^{5}V_{11} = {}^{4}V_5 \\ {}^{5}V_{12} = {}^{4}V_{11} \end{array}\right\} = \frac{2n}{2} \cdot \frac{2n-1}{3}$	...	...	...	...	...	...	...	0	...	...		$\frac{2n}{2} \cdot \frac{2n-1}{3}$						
17	−	$^{2}V_4 - {}^{4}V_1$	$^{4}V_4$		$\left\{\begin{array}{l} {}^{5}V_4 = {}^{4}V_4 \\ {}^{5}V_5 = {}^{4}V_1 \end{array}\right\} = 2n-2$	1	...	...	...	...	2n−2												
18	+	$^{5}V_5 + {}^{5}V_1$	$^{5}V_5$		$\left\{\begin{array}{l} {}^{6}V_5 = {}^{5}V_5 \\ {}^{6}V_6 = {}^{5}V_1 \end{array}\right\} = 3+1=4$	1	...	...	...	...	...	4											
19	÷	$^{5}V_4 \div {}^{5}V_5$	$^{5}V_6$		$\left\{\begin{array}{l} {}^{6}V_4 = {}^{5}V_4 \\ {}^{6}V_5 = {}^{5}V_5 \end{array}\right\} = \frac{2n-2}{4}$	...	...	...	...	...	2n−2	4	...	$\frac{2n-2}{4}$	...	$\left\{\frac{2n}{2} \cdot \frac{2n-1}{3} \cdot \frac{2n-2}{3}\right\} = A_3$							
20	×	$^{6}V_5 \times {}^{6}V_{11}$	$^{6}V_{11}$		$\left\{\begin{array}{l} {}^{7}V_{11} = {}^{6}V_5 \\ {}^{7}V_{12} = {}^{6}V_{11} \end{array}\right\} = \frac{2n}{2} \cdot \frac{2n-1}{3} \cdot \frac{2n-2}{4} = A_3$	...	...	...	...	...	...	...	...	0	...								
21	×	$^{7}V_{12} \times {}^{7}V_{11}$	$^{7}V_{12}$		$\left\{\begin{array}{l} {}^{8}V_{11} = {}^{7}V_{11} \\ {}^{8}V_{12} = {}^{7}V_{12} \end{array}\right\} = B_1 \cdot \frac{2n}{2} \cdot \frac{2n-1}{3} \cdot \frac{2n-2}{3} = B_1 A_3$	...	...	...	...	...	...	...	...	...	...		0	$B_1 A_3$	.....		B_3		
22	+	$^{8}V_{12} + {}^{3}V_{12}$	$^{8}V_{12}$		$\left\{\begin{array}{l} {}^{9}V_{12} = {}^{8}V_{12} \\ {}^{9}V_{13} = {}^{3}V_{12} \end{array}\right\} = A_3 + B_1 A_1 + B_1 A_3$	...	...	...	...	...	...	...	...	...	...		.....		0	$\left\{A_3 + B_1 A_1 + B_1 A_3\right\}$			
23	−	$^{3}V_{13} - {}^{1}V_1$	$^{3}V_{13}$		$\left\{\begin{array}{l} {}^{4}V_{13} = {}^{3}V_{13} \\ {}^{4}V_{14} = {}^{1}V_1 \end{array}\right\} = n-3\ (=1)$	1	...	...	...	...	...	...	...	...	...	n−3							
Here follows a repetition of Operations thirteen to twenty-three.																							
24	+	$^{4}V_{10} + {}^{4}V_{11}$	$^{4}V_{10}$		$\left\{\begin{array}{l} {}^{5}V_{10} = {}^{4}V_{10} \\ {}^{5}V_{11} = {}^{4}V_{11} \end{array}\right\} = B_7$	...	...	...	...	...	...	...	...	...	...		.....		.....		.....	.....	.....	B_7
25	+	$^{1}V_1 + {}^{4}V_8$	$^{4}V_8$		$\left\{\begin{array}{l} {}^{5}V_8 = {}^{4}V_8 \\ {}^{5}V_9 = {}^{4}V_9 \end{array}\right\} = n+1 = 4+1 = 5$ by a Variable-card. by a Variable-card.	1	...	n+1	...	...	0	0											

Konrad Zuse's (1910-1992) relay-driven computer Z3



From the vacuum-tube ENIAC to the IBM Blue Gene



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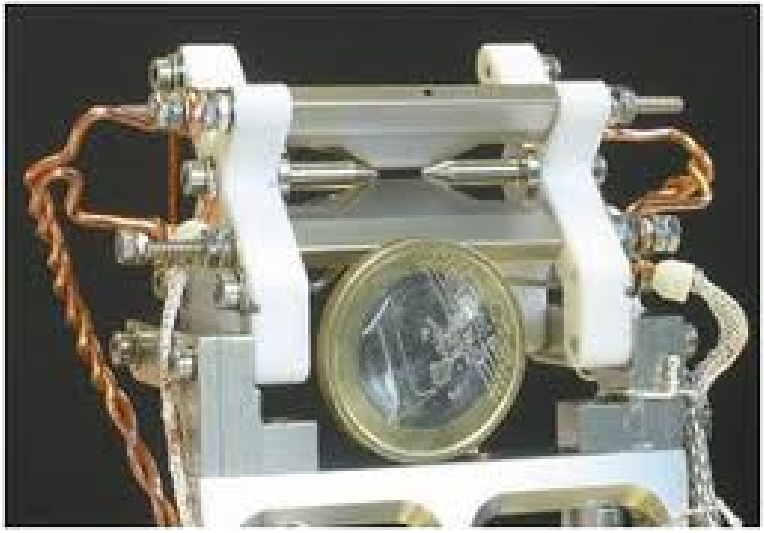
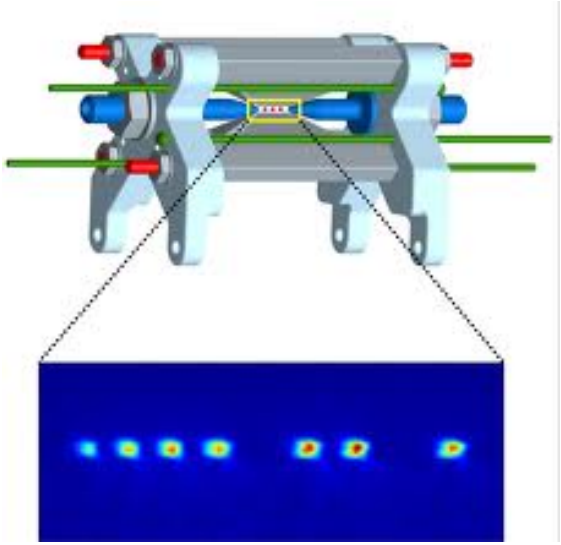
Richard Feynman's vision of 1982

"I'm not happy with all the analyses that go with just the classical theory, because nature isn't classical, dammit, and if you want to make a simulation of nature, you'd better make it quantum mechanical, and by golly it's a wonderful problem, because it doesn't look so easy."

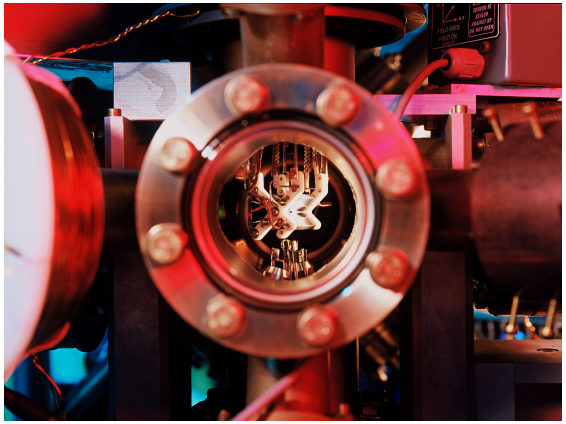
Ion traps as a digital quantum computer?

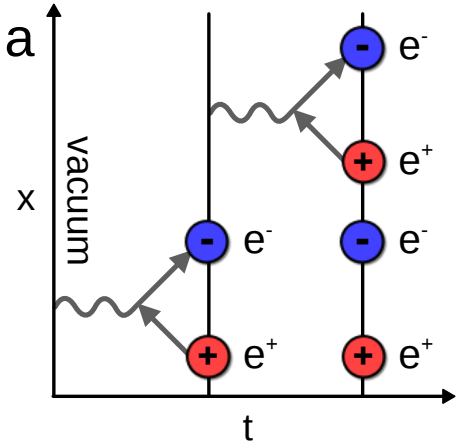


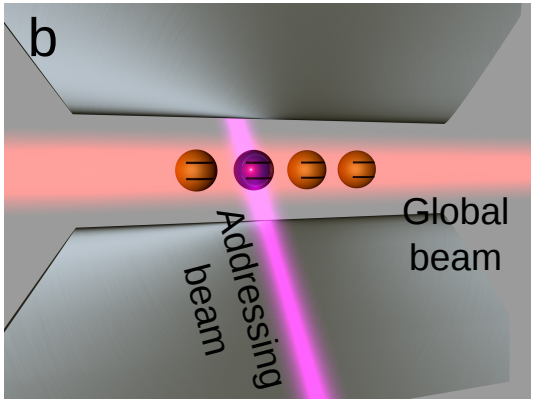


Franklin Medal 2010: I. Cirac, D. Wineland, P. Zoller

Digital 4-qubit ion-trap quantum computation of pair creation



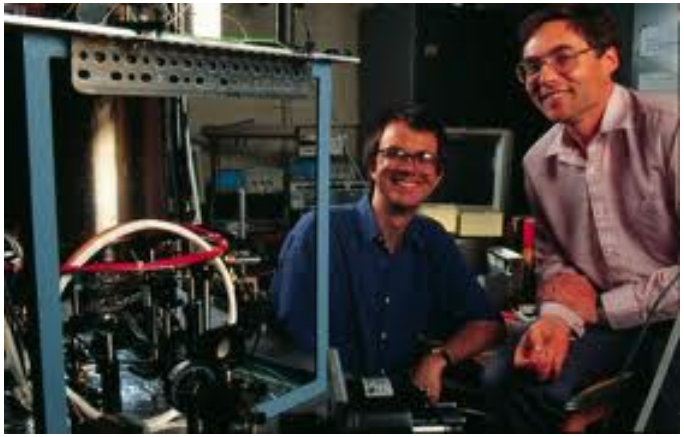




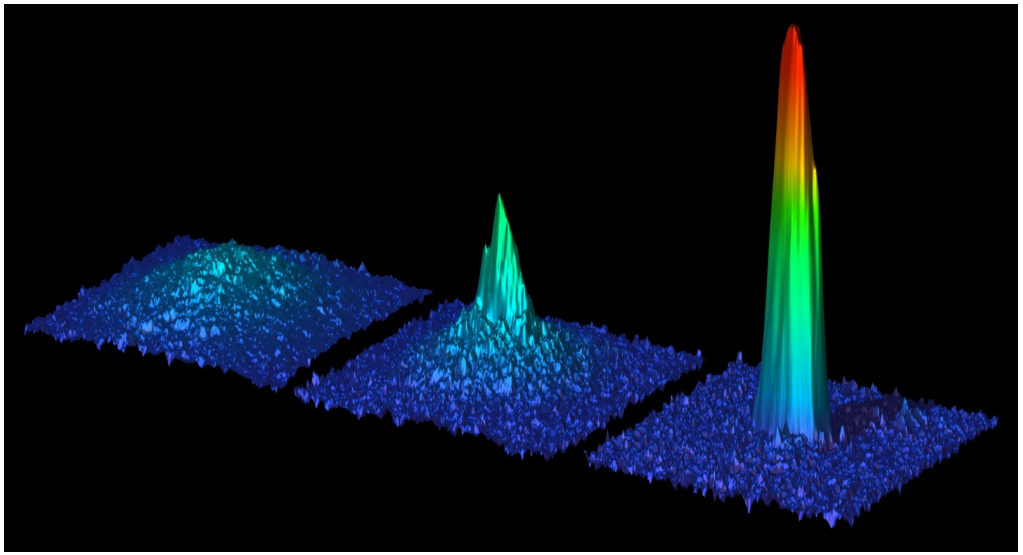
• Quantum computer consisting of four trapped Ca ions that act as four qubits, which are manipulated by external laser beams.
• Precisely controllable many-body quantum device, executing a prescribed sequence of quantum gate operations.
• State of simulated system is encoded as quantum information.
• Dynamics is represented by a sequence of quantum gates, following a stroboscopic Trotter decomposition.

E. A. Martinez, C. A. Muschik, P. Schindler, D. Nigg, A. Erhard, M. Heyl, P. Hauke, M. Dalmonte, T. Monz, P. Zoller, R. Blatt, Nature 534 (2016) 516.

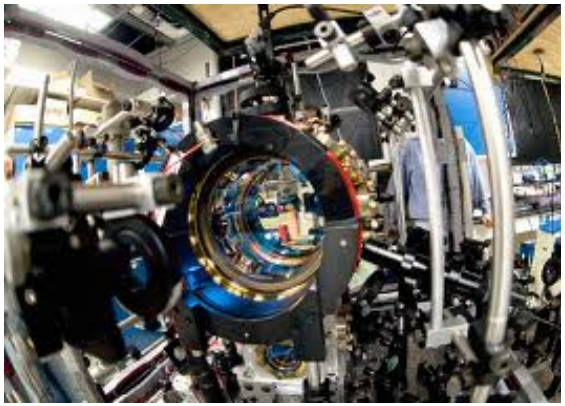
Bose-Einstein condensation in ultra-cold atomic gases

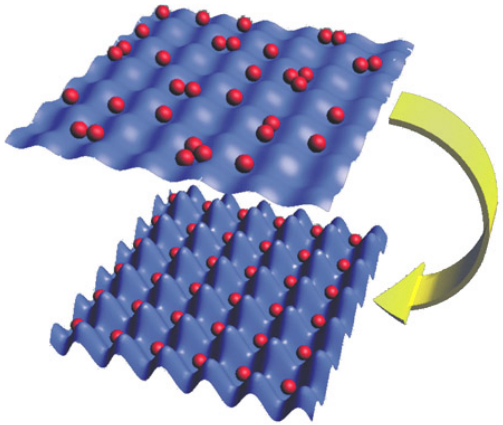


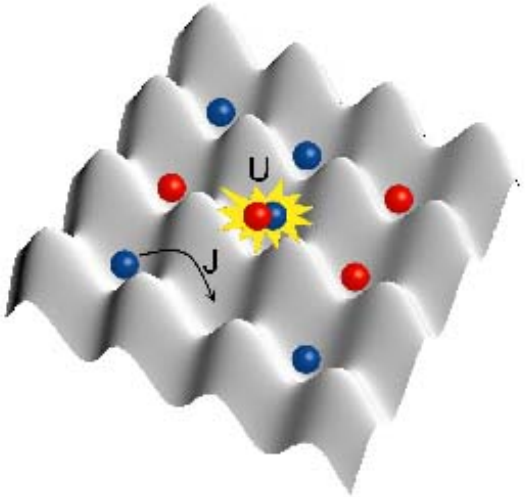
Eric Cornell, Carl Wieman, Wolfgang Ketterle, 1995



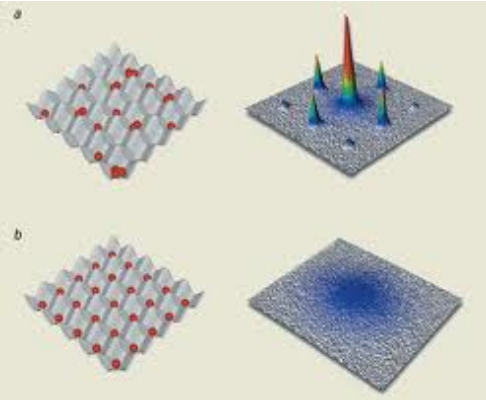
Ultra-cold atoms in optical lattices as analog quantum simulators







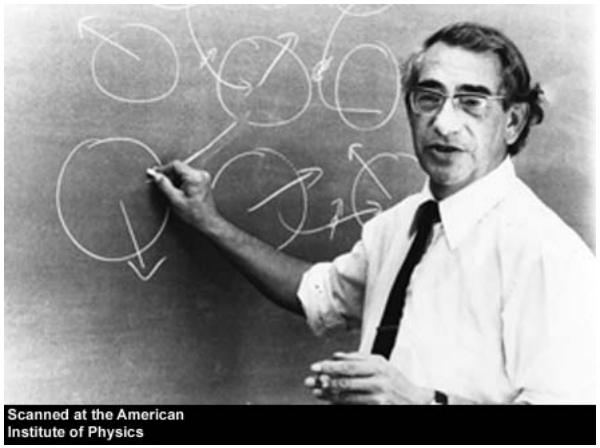
Transition from a superfluid to a Mott insulator

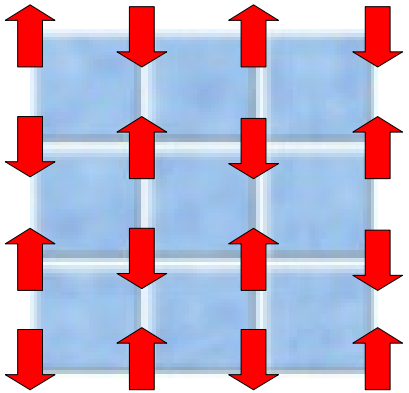


Theodor Hänsch Immanuel Bloch

Can one understand high-T_c superconductivity in this way?

The Hubbard Model for doped antiferromagnets





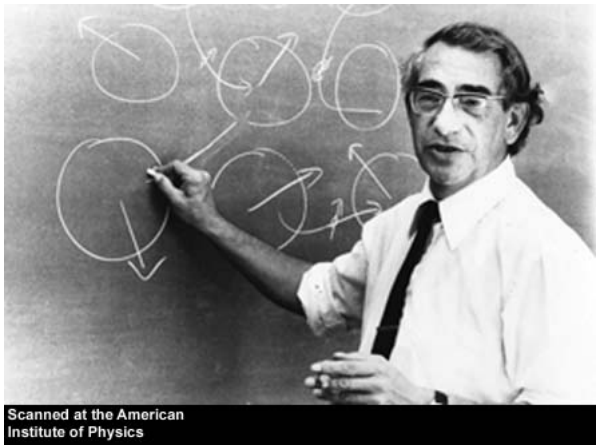
$$H = -t \sum_{\langle xy \rangle} (c_x^\dagger c_y + c_y^\dagger c_x) + U \sum_x (c_x^\dagger c_x - 1)^2, \quad c_x = \begin{pmatrix} c_{x\uparrow} \\ c_{x\downarrow} \end{pmatrix}$$

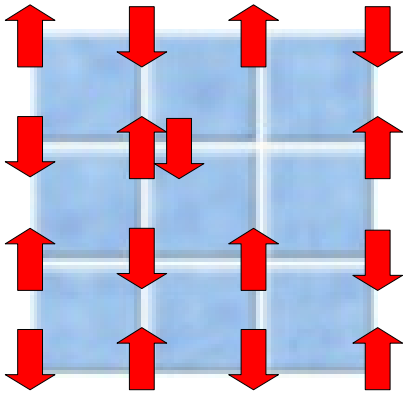
reduces to the Heisenberg model at half-filling for $U \gg t$

$$H = J \sum_{\langle xy \rangle} \vec{S}_x \cdot \vec{S}_y$$

Important open question:
Does the Hubbard model explain high-T_c superconductivity?

The Hubbard Model for doped antiferromagnets





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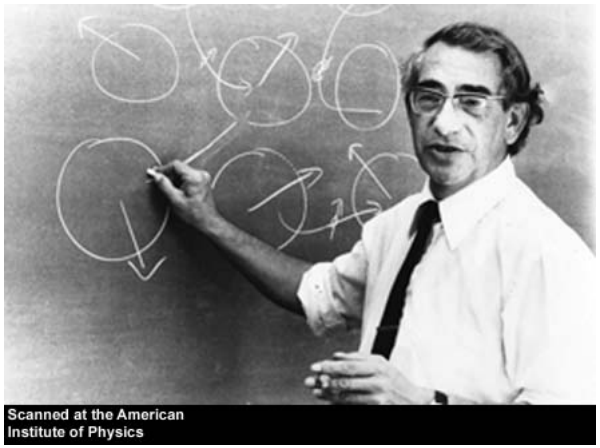
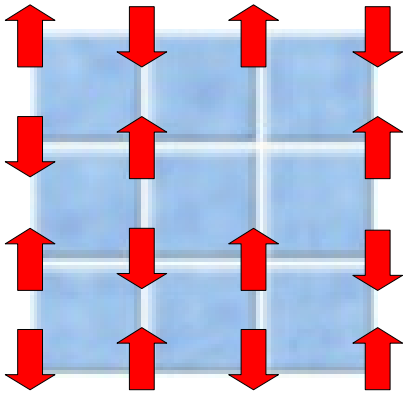




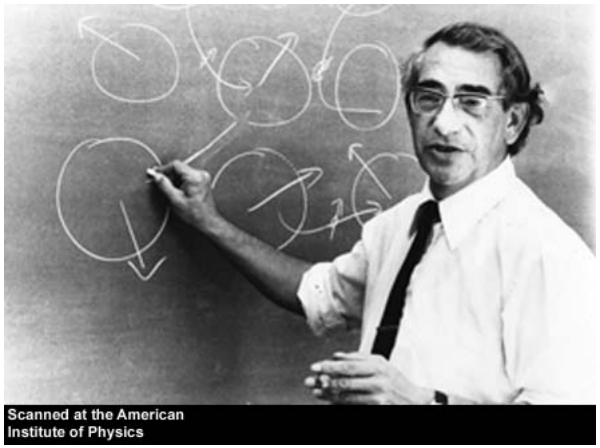
$$H = -t \sum_{\langle xy \rangle} (c_x^\dagger c_y + c_y^\dagger c_x) + U \sum_x (c_x^\dagger c_x - 1)^2, \quad c_x = \begin{pmatrix} c_{x\uparrow} \\ c_{x\downarrow} \end{pmatrix}$$

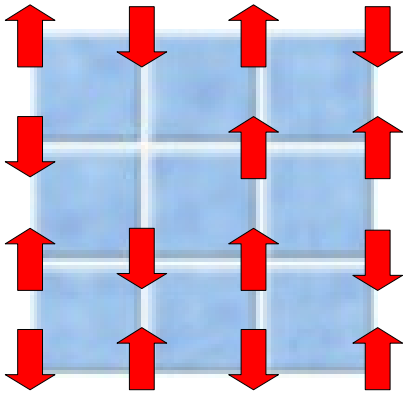
reduces to the Heisenberg model at half-filling for $U \gg t$

$$H = J \sum_{\langle xy \rangle} \vec{S}_x \cdot \vec{S}_y$$

Important open question:
Does the Hubbard model explain high-T_c superconductivity?

The Hubbard Model for doped antiferromagnets





$$H = -t \sum_{\langle xy \rangle} (c_x^\dagger c_y + c_y^\dagger c_x) + U \sum_x (c_x^\dagger c_x - 1)^2, \quad c_x = \begin{pmatrix} c_{x\uparrow} \\ c_{x\downarrow} \end{pmatrix}$$

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$$H = J \sum_{\langle xy \rangle} \vec{S}_x \cdot \vec{S}_y$$

Important open question:
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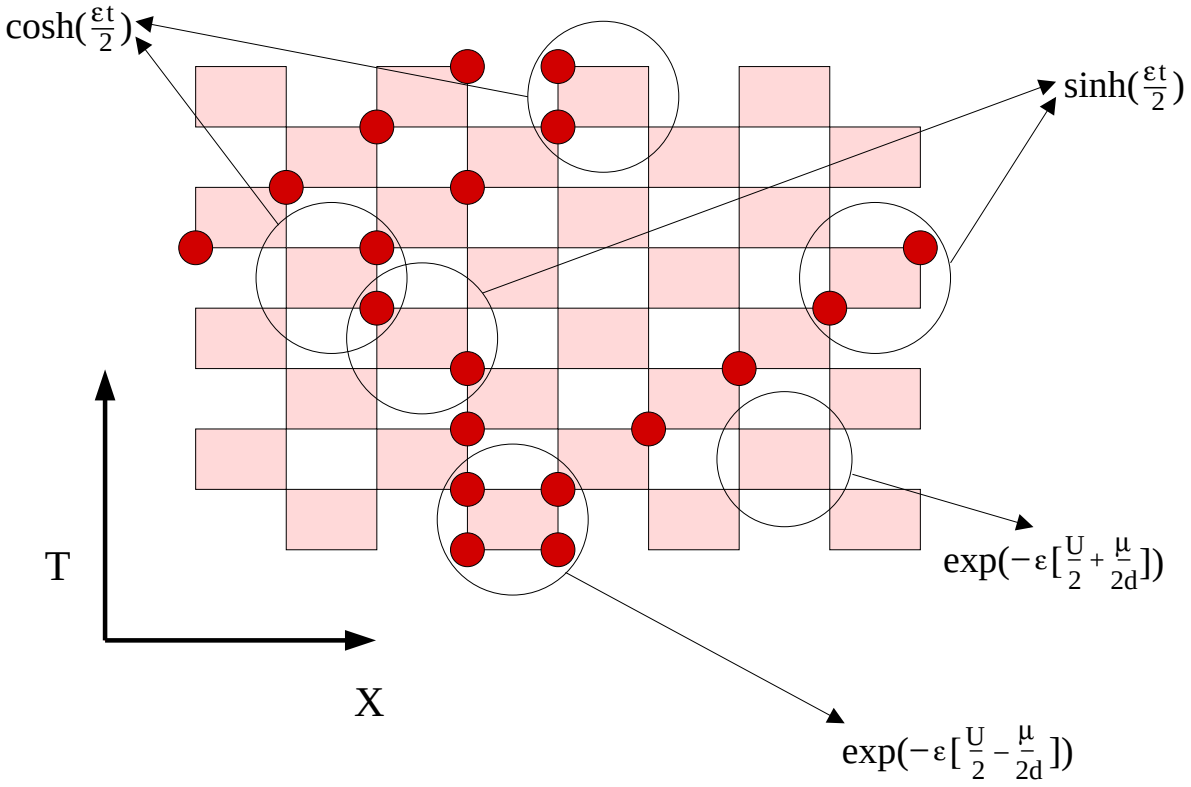
Path integral

$$
\begin{aligned}
Z_f &= \mathrm{Tr}[\exp(-\varepsilon H_1)\exp(-\varepsilon H_2)\ldots\exp(-\varepsilon H_M)]^N \\
&= \sum_{[n]} \mathrm{Sign}[n]\exp(-S[n])
\end{aligned}
$$



Sign problem of fermionic path integrals

$$Z_f = \mathrm{Tr}\exp(-\beta H) = \sum_{[n]} \mathrm{Sign}[n]\exp(-S[n]) \ , \quad \mathrm{Sign}[n] = \pm 1$$

Average sign is exponentially small

$$\langle \mathrm{Sign} \rangle = \frac{\sum_{[n]} \mathrm{Sign}[n]\exp(-S[n])}{\sum_{[n]} \exp(-S[n])} = \frac{Z_f}{Z_b} = \exp(-\beta V \Delta f)$$

The statistical error is exponentially large

$$\frac{\sigma_{\mathrm{Sign}}}{\langle \mathrm{Sign} \rangle} = \frac{\sqrt{\langle \mathrm{Sign}^2 \rangle - \langle \mathrm{Sign} \rangle^2}}{\sqrt{N}\langle \mathrm{Sign} \rangle} = \frac{\exp(\beta V \Delta f)}{\sqrt{N}} \ .$$

Some very hard sign problems are NP complete

M. Troyer, UJW, Phys. Rev. Lett. 94 (2005) 170201.

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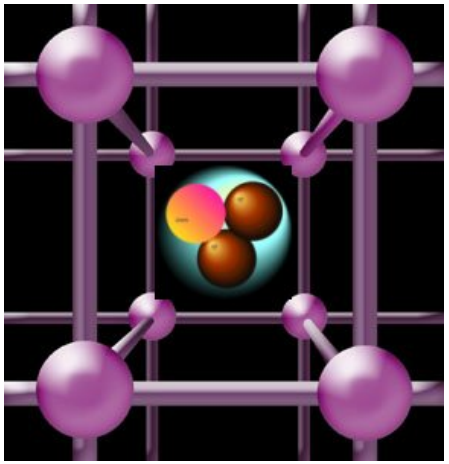
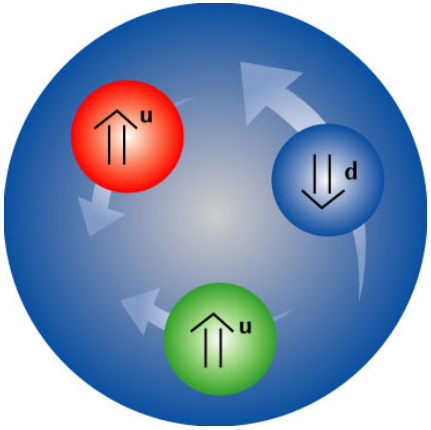
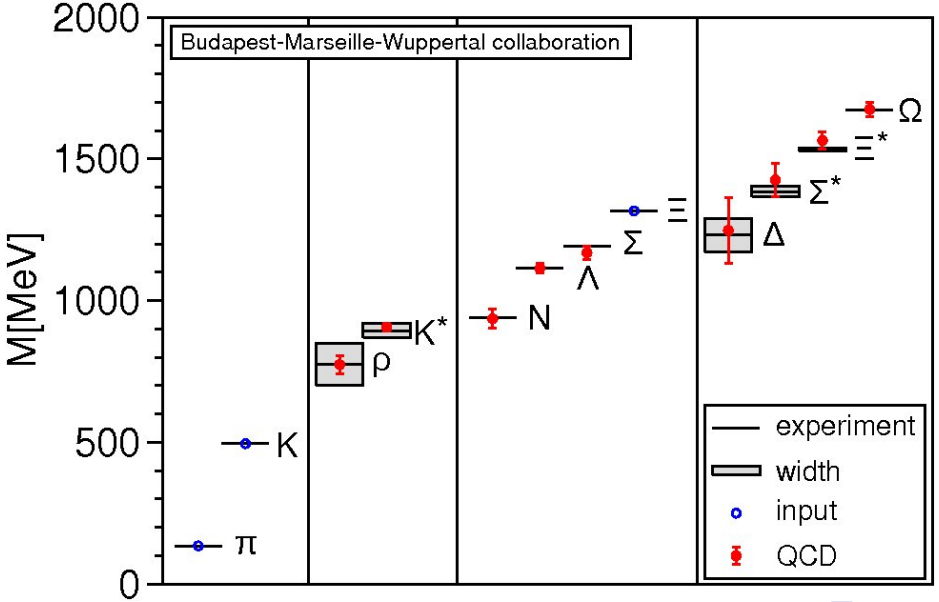
Kenneth Wilson's lattice QCD describes confinement of quarks and gluons inside protons und neutrons

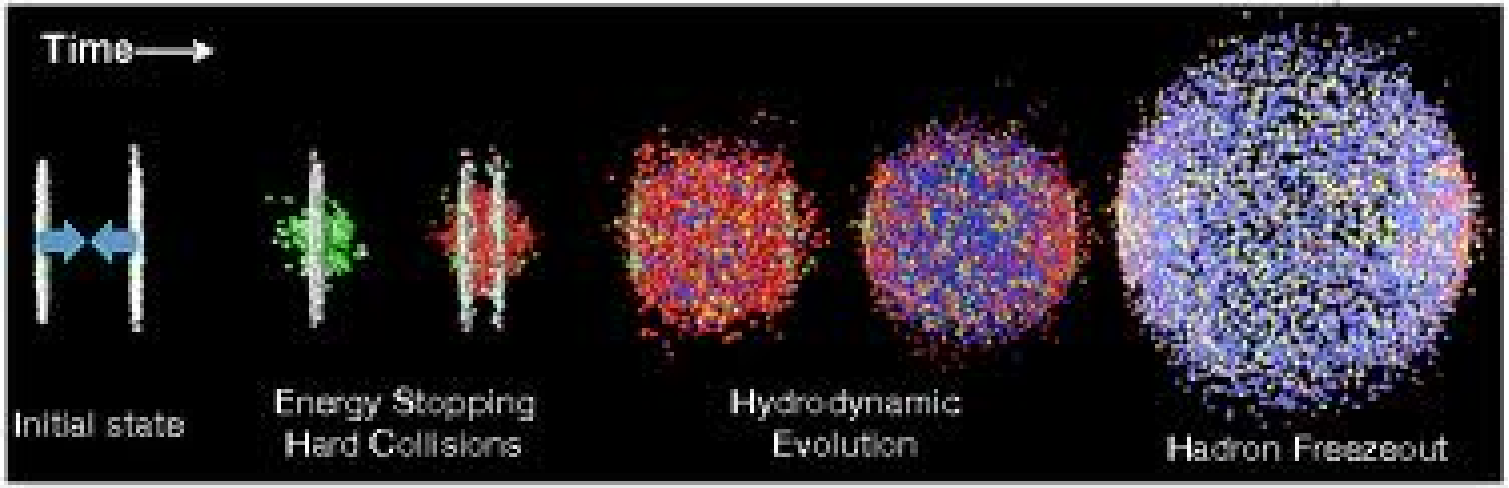


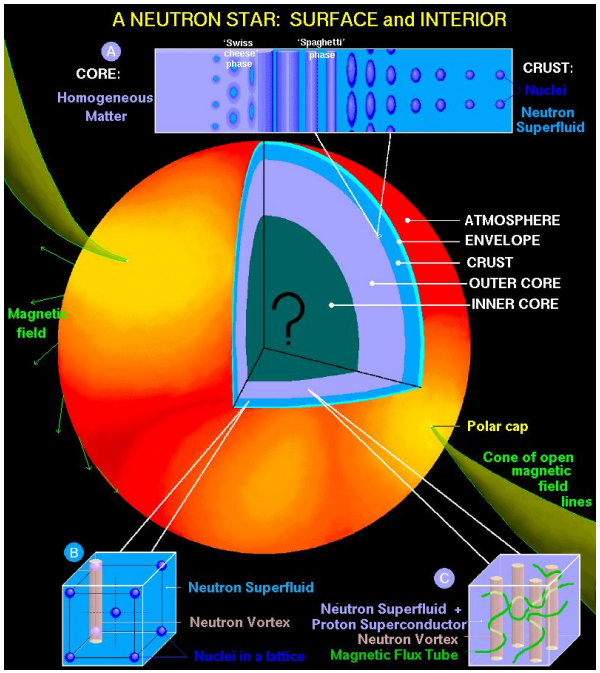
and confirms the experimentally measured mass spectrum

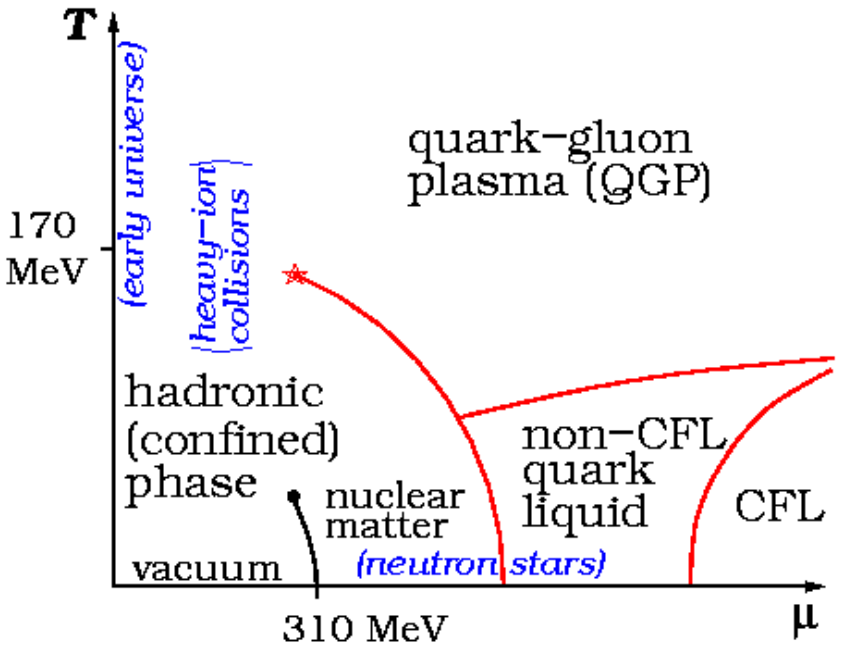




Can heavy-ion collision physics or nuclear astrophysics benefit from quantum simulations in the long run?







Different descriptions of dynamical Abelian gauge fields:

Maxwell's classical electromagnetic gauge fields

$$\vec{\nabla}\cdot\vec{E}(\vec{x},t)=\rho(\vec{x},t),\quad \vec{\nabla}\cdot\vec{B}(\vec{x},t)=0,\quad \vec{B}(\vec{x},t)=\vec{\nabla}\times\vec{A}(\vec{x},t)$$

Quantum Electrodynamics (QED) for perturbative treatment

$$E_i(\vec{x},t)=-i\frac{\partial}{\partial A_i(\vec{x},t)},\quad [E_i,A_j]=i\delta_{ij},\quad \left[\vec{\nabla}\cdot\vec{E}-\rho\right]|\Psi[A]\rangle=0$$

Wilson's $U(1)$ lattice gauge theory for classical simulation

$$U_{xy}=\exp\left(ie\int_x^y d\vec{l}\cdot\vec{A}\right)=\exp(i\varphi_{xy})\in U(1),\quad E_{xy}=-i\frac{\partial}{\partial\varphi_{xy}},$$

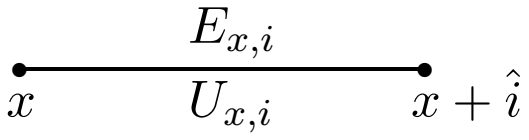
$$[E_{xy},U_{xy}]=U_{xy},\quad \left[\sum_i(E_{x,x+\hat{i}}-E_{x-\hat{i},x})-\rho\right]|\Psi[U]\rangle=0$$

$U(1)$ quantum link models for quantum simulation

$$U_{xy}=S^+_{xy},\quad U^\dagger_{xy}=S^-_{xy},\quad E_{xy}=S^3_{xy},$$

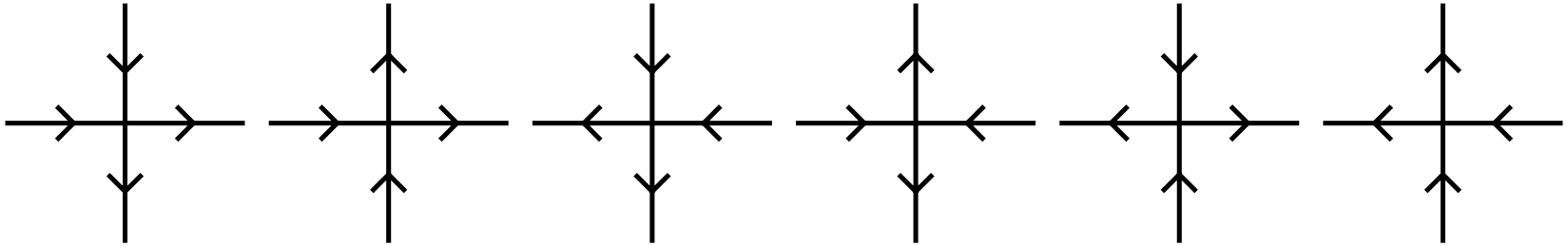
$$[E_{xy},U_{xy}]=U_{xy},\quad [E_{xy},U^\dagger_{xy}]=-U^\dagger_{xy},\quad [U_{xy},U^\dagger_{xy}]=2E^\dagger_{xy}$$

$U(1)$ gauge fields from spins $\frac{1}{2}$

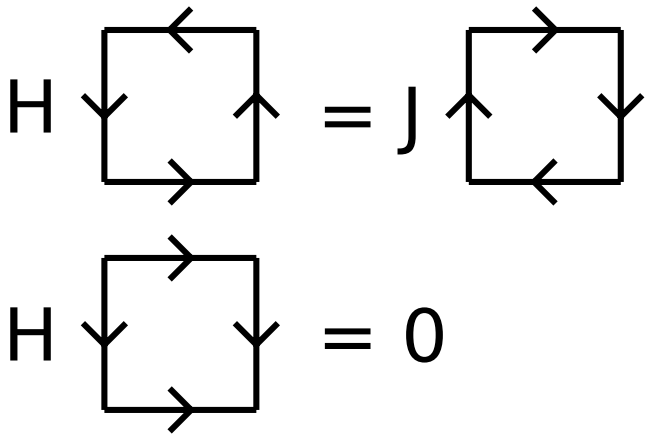


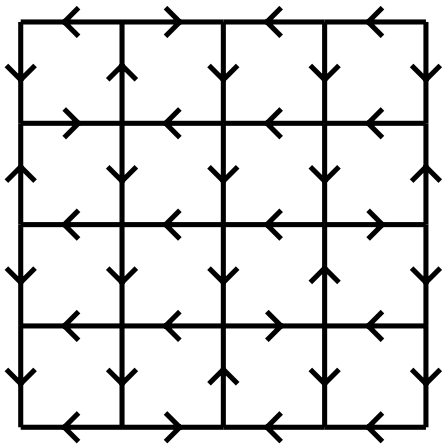
$$U = S^+, \; U^\dagger = S^-, \; E = S^3.$$

Gauss law



Ring-exchange plaquette Hamiltonian



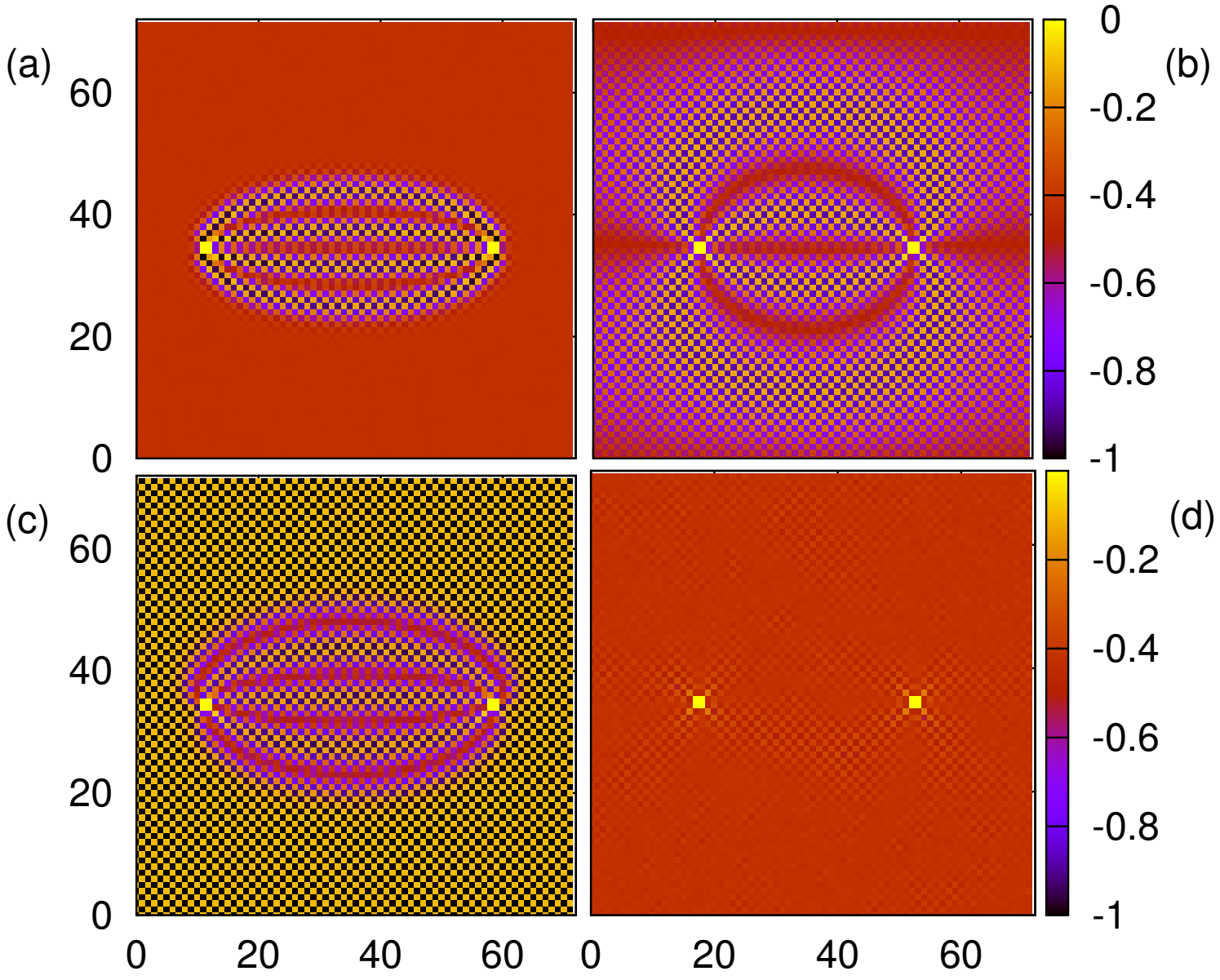


D. Horn, Phys. Lett. B100 (1981) 149

P. Orland, D. Rohrlich, Nucl. Phys. B338 (1990) 647

S. Chandrasekharan, UJW, Nucl. Phys. B492 (1997) 455

Energy density of charge-anti-charge pair $Q = \pm 2$



D. Banerjee, F.-J. Jiang, P. Widmer, UJW, JSTAT (2013) P12010.

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Hamiltonian for staggered fermions and $U(1)$ quantum links

$$H = -t \sum_x \left[\psi_x^\dagger U_{x,x+1} \psi_{x+1} + \text{h.c.} \right] + m \sum_x (-1)^x \psi_x^\dagger \psi_x + \frac{g^2}{2} \sum_x E_{x,x+1}^2$$

Bosonic rishon representation of the quantum links

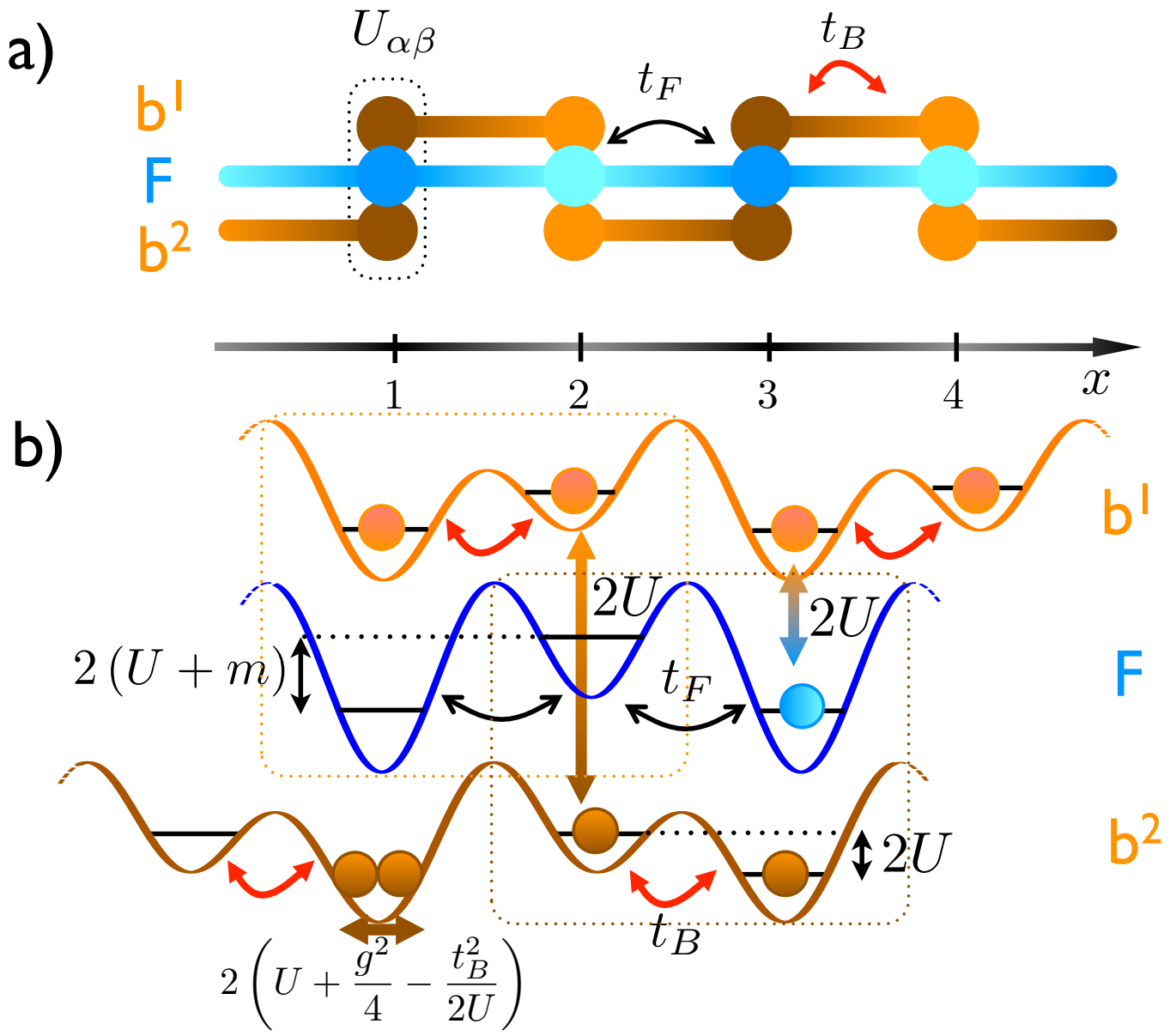
$$U_{x,x+1} = b_x b_{x+1}^\dagger, \; E_{x,x+1} = \frac{1}{2} \left(b_{x+1}^\dagger b_{x+1} - b_x^\dagger b_x \right)$$

Microscopic Hubbard model Hamiltonian

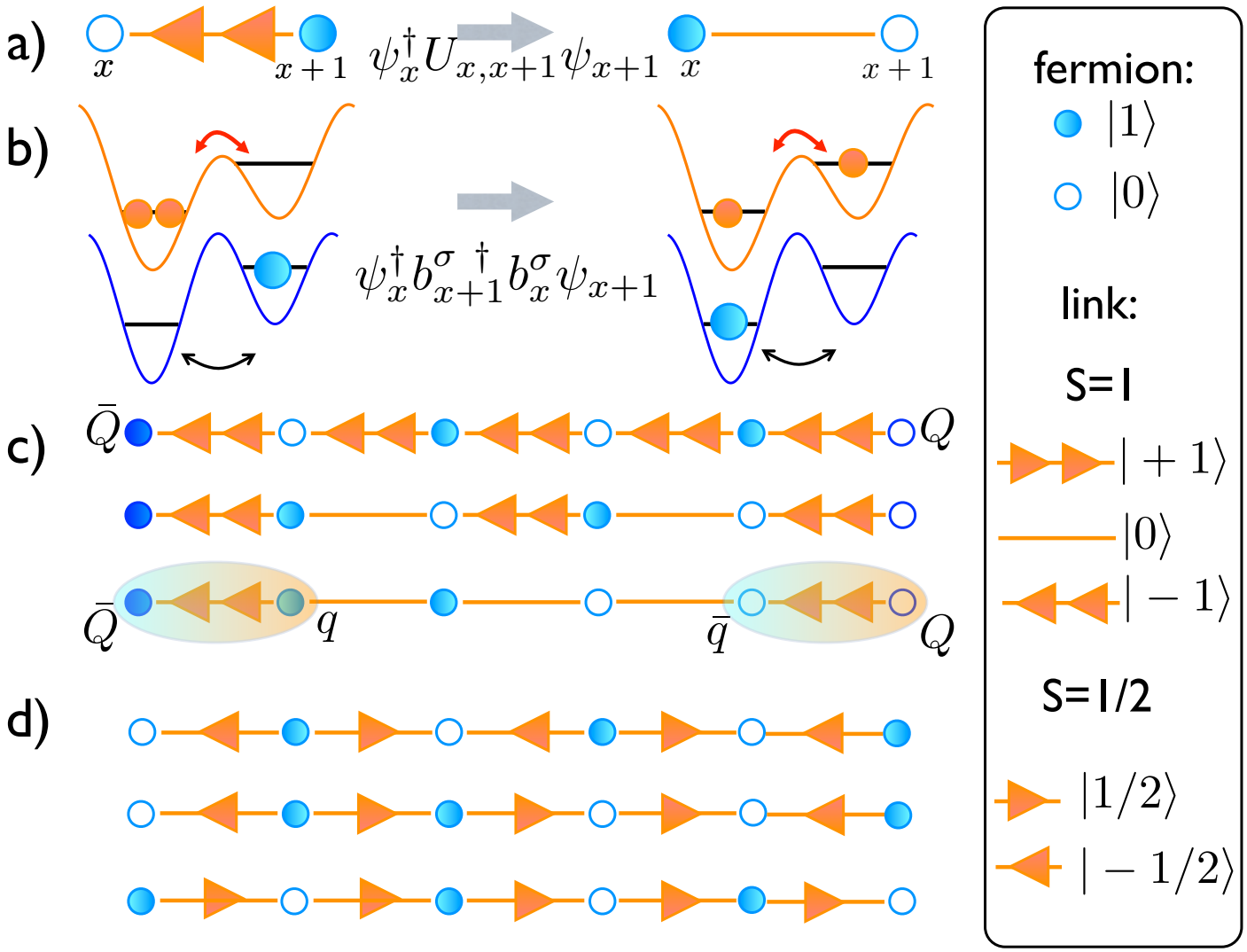
$$\begin{aligned} \widetilde{H} &= \sum_x h_{x,x+1}^B + \sum_x h_{x,x+1}^F + m \sum_x (-1)^x n_x^F + U \sum_x \widetilde{G}_x^2 \\ &= -t_B \sum_{x \text{ odd}} b_x^{1\dagger} b_{x+1}^1 - t_B \sum_{x \text{ even}} b_x^{2\dagger} b_{x+1}^2 - t_F \sum_x \psi_x^\dagger \psi_{x+1} + \text{h.c.} \\ &+ \sum_{x,\alpha,\beta} n_x^\alpha U_{\alpha\beta} n_x^\beta + \sum_{x,\alpha} (-1)^x U_\alpha n_x^\alpha \end{aligned}$$

D. Banerjee, M. Dalmonte, M. Müller, E. Rico, P. Stebler, UJW, P. Zoller, Phys. Rev. Lett. 109 (2012) 175302.

Optical lattice with Bose-Fermi mixture of ultra-cold atoms



From string breaking to false vacuum decay



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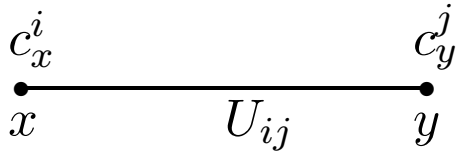
Fermionic rishons at the two ends of a link

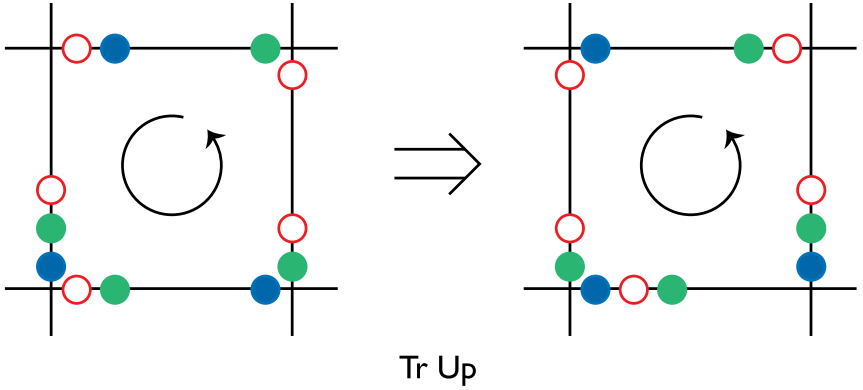
$$\{c_x^i, c_y^{j\dagger}\} = \delta_{xy}\delta_{ij}, \ \{c_x^i, c_y^j\} = \{c_x^{i\dagger}, c_y^{j\dagger}\} = 0$$

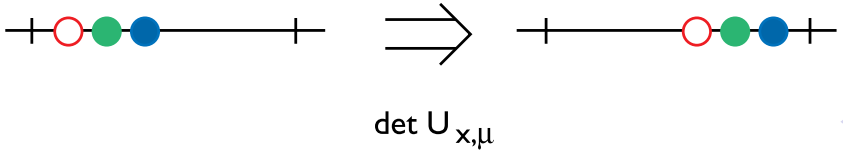
Rishon representation of link algebra



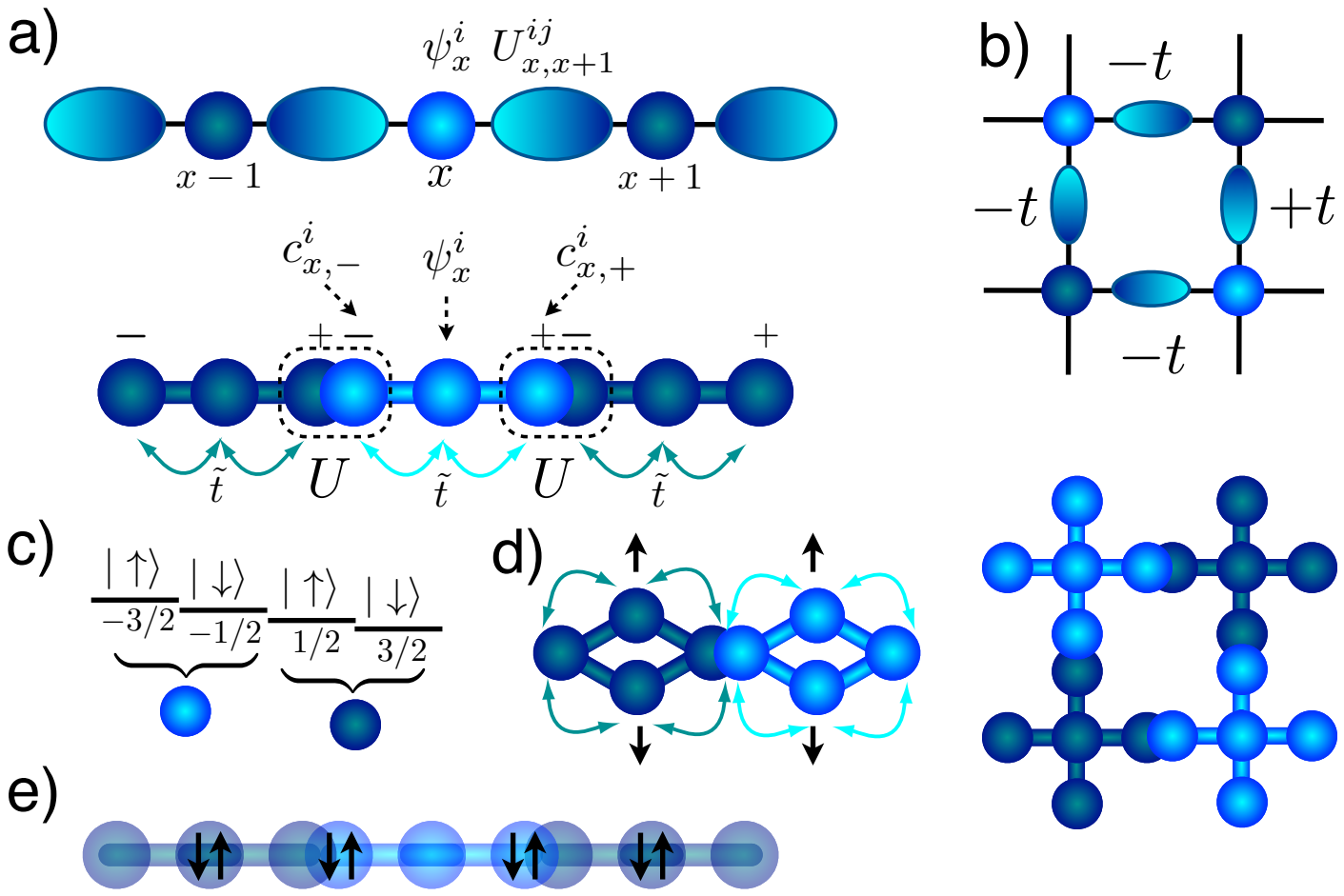
$$U_{xy}^{ij} = c_x^i c_y^{j\dagger}, \ L_{xy}^a = c_x^{i\dagger}\lambda_{ij}^a c_x^j, \ R_{xy}^a = c_y^{i\dagger}\lambda_{ij}^a c_y^j, \ E_{xy} = \frac{1}{2}(c_y^{i\dagger}c_y^i - c_x^{i\dagger}c_x^i)$$

Can a "rishon abacus" implemented with ultra-cold atoms be used as a quantum simulator?



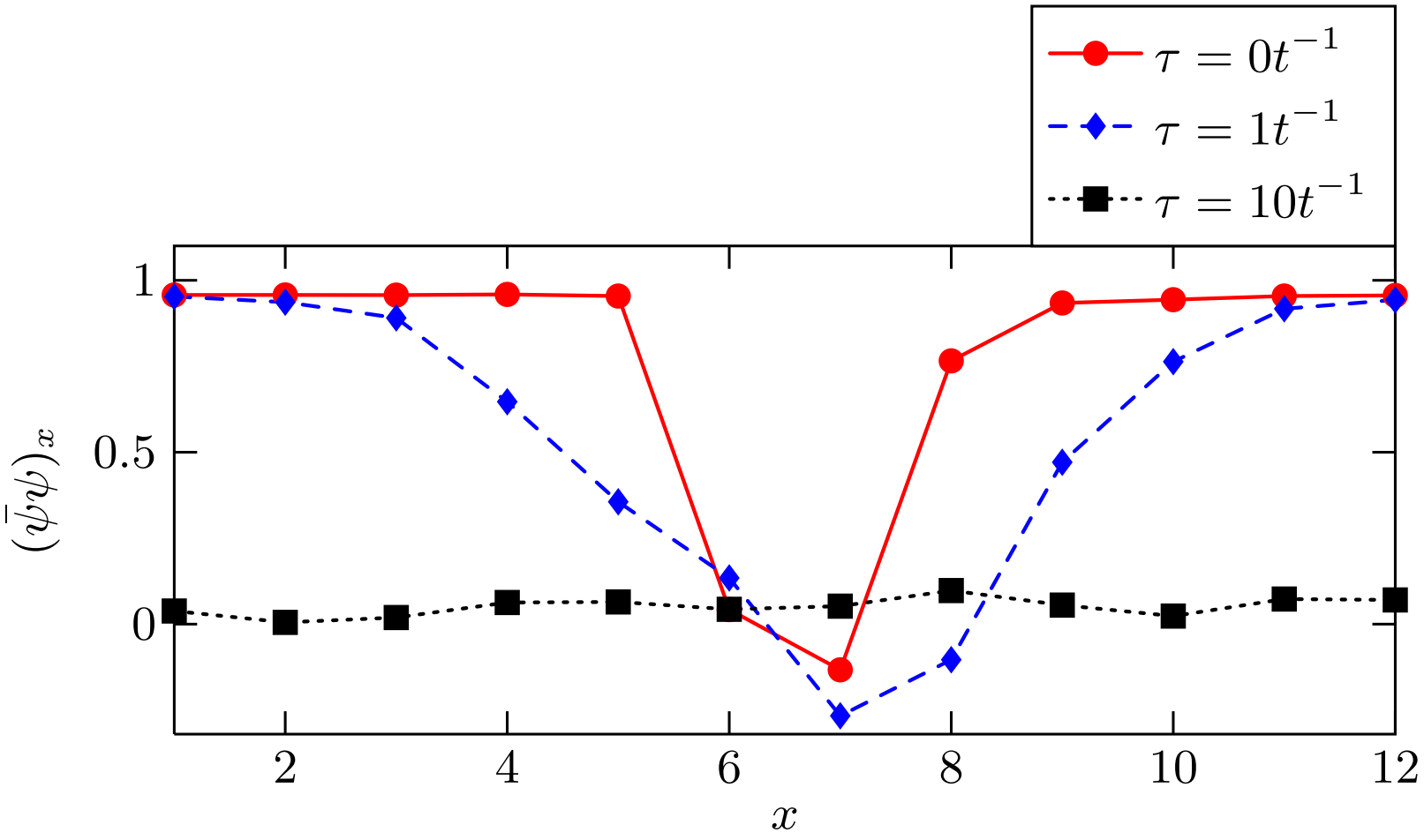


Optical lattice with ultra-cold alkaline-earth atoms (^{87}Sr or ^{173}Yb) with color encoded in nuclear spin



D. Banerjee, M. Bögli, M. Dalmonte, E. Rico, P. Stebler, UJW, P. Zoller, Phys. Rev. Lett. 110 (2013) 125303

Expansion of a "fireball" mimicking a hot quark-gluon plasma

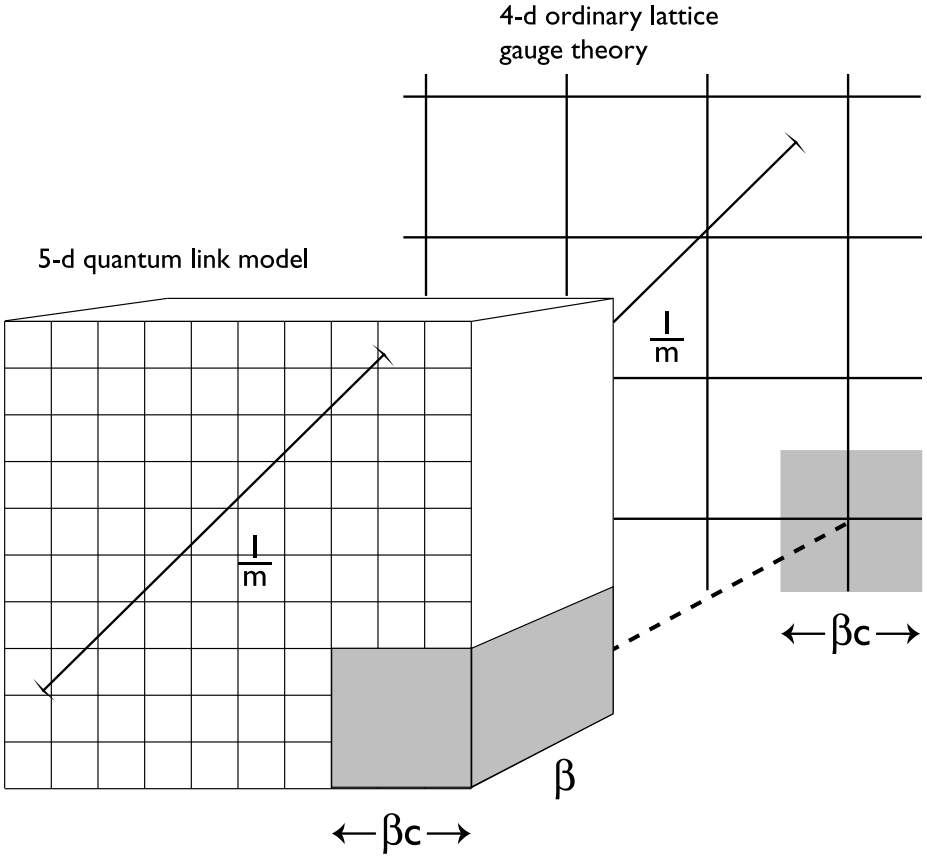


Low-energy effective action of a quantum link model

$$S[G_\mu] = \int_0^\beta dx_5 \int d^4x \, \frac{1}{2e^2} \left(\mathrm{Tr}\ G_{\mu\nu} G_{\mu\nu} + \frac{1}{c^2} \mathrm{Tr}\ \partial_5 G_\mu \partial_5 G_\mu \right), \ G_5 = 0$$

undergoes dimensional reduction from $4+1$ to 4 dimensions

$$S[G_\mu] \to \int d^4x \, \frac{1}{2g^2} \mathrm{Tr}\ G_{\mu\nu} G_{\mu\nu}, \ \frac{1}{g^2} = \frac{\beta}{e^2}, \ \frac{1}{m} \sim \exp\left(\frac{24\pi^2 \beta}{11 N e^2} \right)$$



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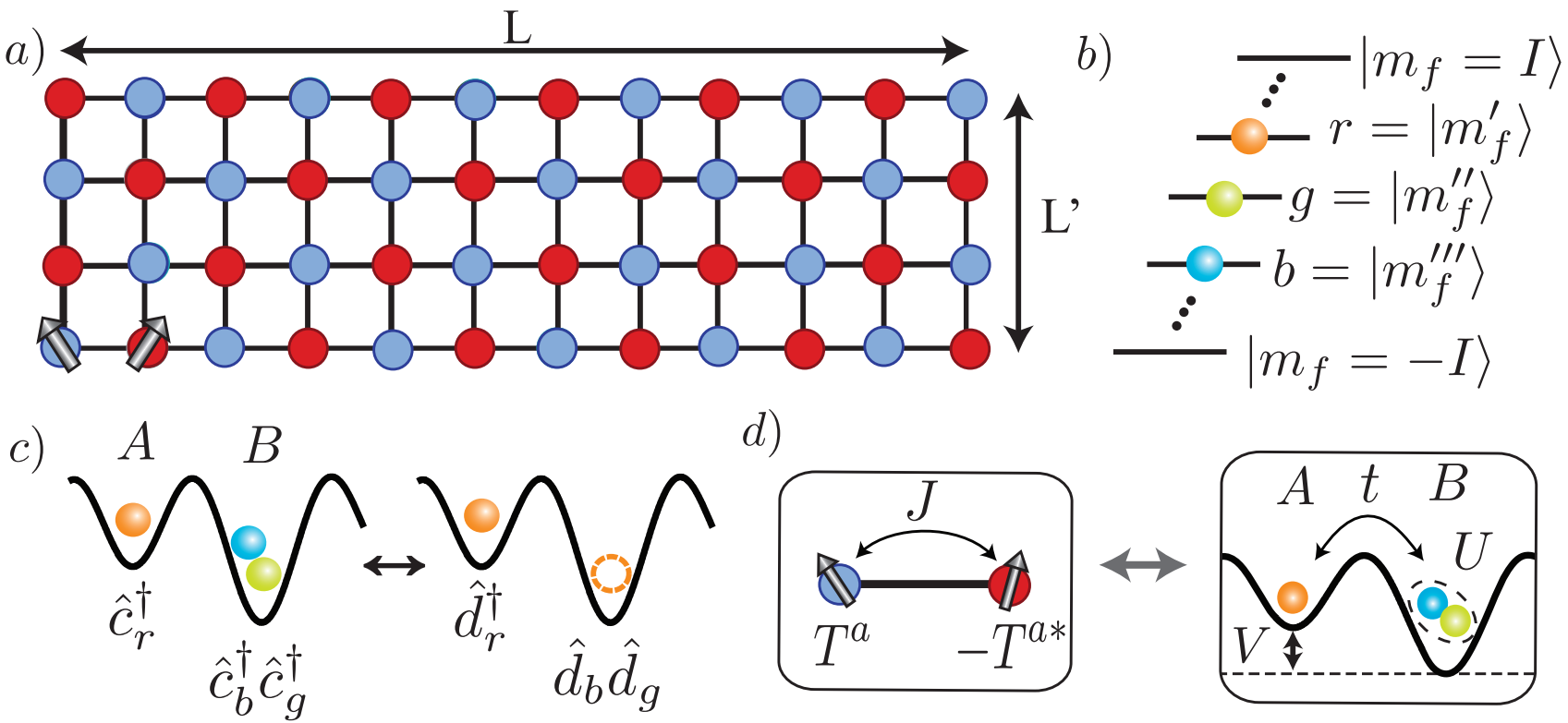
How to reach the continuum limit in $\mathbb{C}P(N-1)$ Models?

Ladder of $SU(N)$ quantum spins embodied with alkaline-earth atoms.

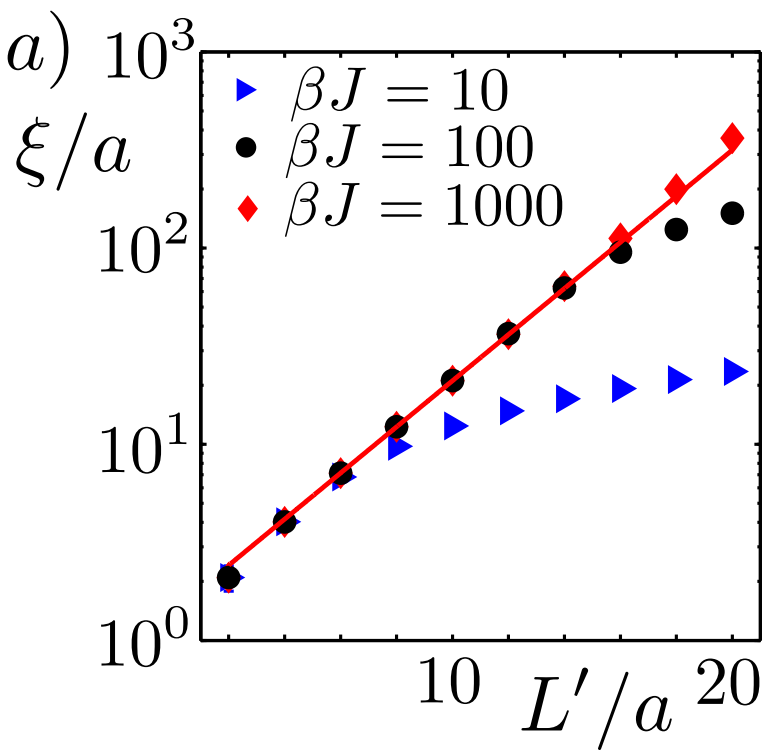
$$H = -J \sum_{\langle xy \rangle} T_x^a T_y^{a*}, \quad [T_x^a, T_y^b] = i\delta_{xy} f_{abc} T_x^c$$

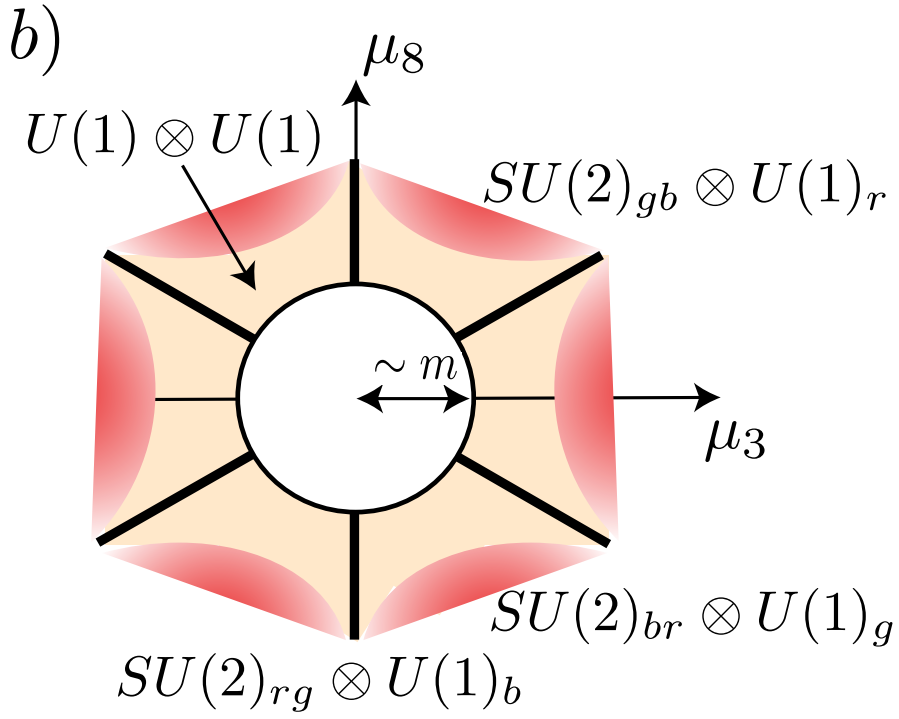


C. Laflamme, W. Evans, M. Dalmonte, U. Gerber, H. Meja-Daz, W. Bietenholz, UJW, and P. Zoller, Annals Phys. 360 (2016) 117.

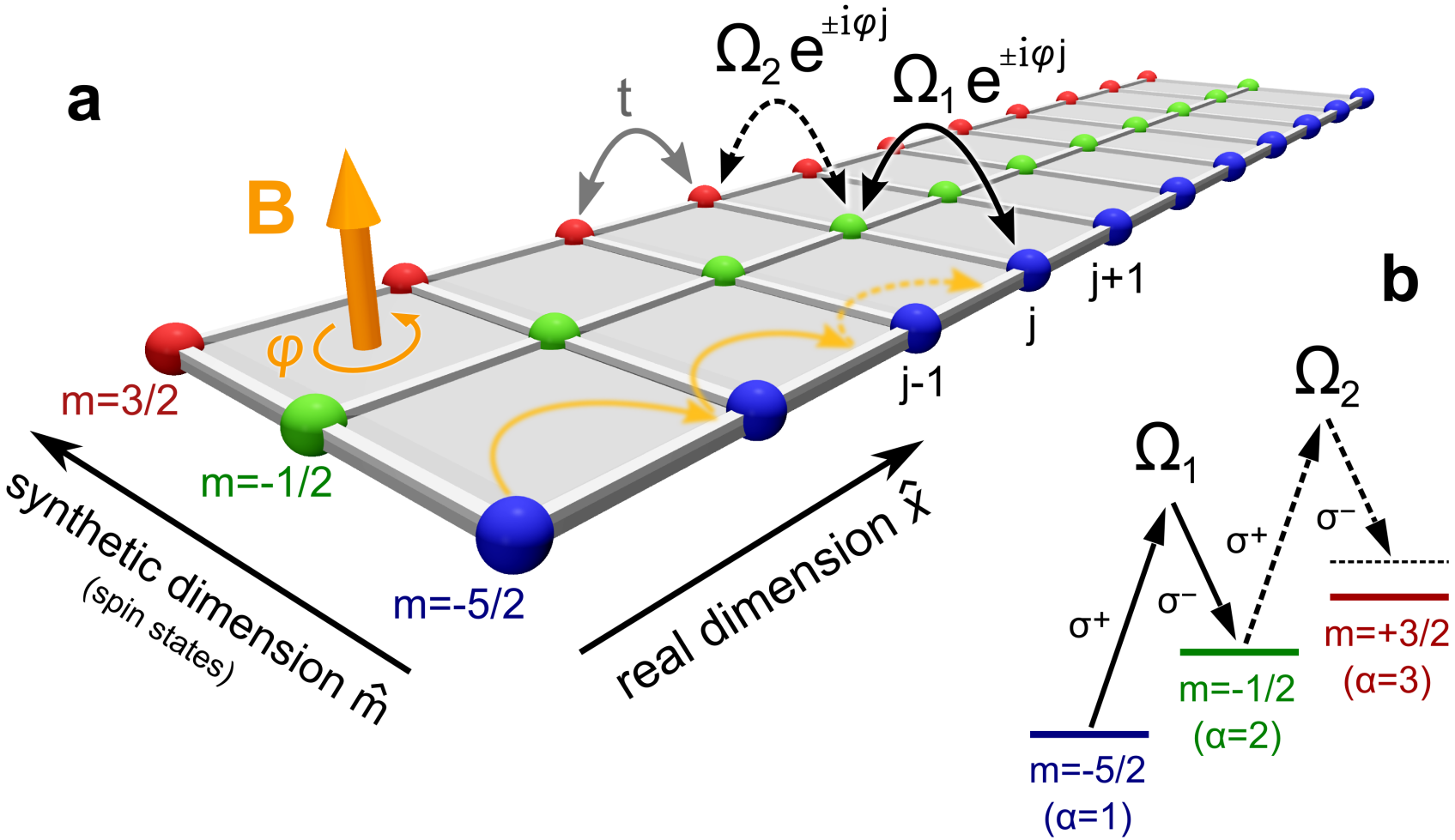
Very large correlation length $\xi \propto \exp(4\pi L' \rho_s / cN) \gg L'$.
Reduction to the $(1+1)$-d $\mathbb{C}P(N-1)$ model at $\theta = n\pi$.

$$S[P] = \int_0^\beta dt \int_0^L dx \, \mathrm{Tr} \left\{ \frac{1}{g^2} \left[\partial_x P \partial_x P + \frac{1}{c^2} \partial_t P \partial_t P \right] - n P \partial_x P \partial_t P \right\}$$





Engineering an Extra Dimension with Alkaline-Earth Atoms



A. Celi, P. Massignan, J. Ruseckas, N. Goldman, I. B. Spielman, G. Juzeliunas, M. Lewenstein, Phys. Rev. Lett. 112 (2014) 043001.
M. Mancini, G. Pagano, G. Cappellini, L. Livi, M. Rider, J. Catani, C. Sias, P. Zoller, M. Inguscio, M. Dalmonte, L. Fallani, arXiv:1502.02495.

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Analog quantum simulator proposals

H. P. Büchler, M. Hermele, S. D. Huber, M. P. A. Fisher, P. Zoller, Phys. Rev. Lett. 95 (2005) 040402.

E. Zohar, B. Reznik, Phys. Rev. Lett. 107 (2011) 275301.

E. Zohar, J. I. Cirac, B. Reznik, Phys. Rev. Lett. 110 (2013) 125304.

D. Banerjee, M. Dalmonte, M. Müller, E. Rico, P. Stebler, UJW, P. Zoller, Phys. Rev. Lett. 109 (2012) 175302.

D. Banerjee, M. Bögli, M. Dalmonte, E. Rico, P. Stebler, UJW, P. Zoller, Phys. Rev. Lett. 110 (2013) 125303.

V. Kasper, F. Hebenstreit, M. Oberthaler, J. Berges, PLB 760 (2016).

V. Kasper, F. Hebenstreit, F. Jendrzejewski, M. Oberthaler, J. Berges, New J. Phys. 19 (2017) 023030.

Digital quantum simulator proposals

M. Müller, I. Lesanovsky, H. Weimer, H. P. Büchler, P. Zoller, Phys. Rev. Lett. 102 (2009) 170502; Nat. Phys. 6 (2010) 382.

L. Tagliacozzo, A. Celi, P. Orland, M. Lewenstein, Nature Communications 4 (2013) 2615.

Review on quantum simulators for lattice gauge theories

UJW, Annalen der Physik 525 (2013) 777, arXiv:1305.1602.

Conclusions

• The computational treatment of large quantum systems is severely restricted by sign and complex action problems, whose solution would benefit greatly from a universal quantum computer or from special purpose digital or analog quantum simulators.

• Promising platforms for quantum computers or quantum simulators are, for example, ion traps or ultracold atoms in optical lattices.

• Quantum link models provide an alternative formulation of lattice gauge theory with a finite-dimensional Hilbert space per link, which allows implementations with ultracold atoms in optical lattices.

• Quantum simulator constructions have already been presented for the $U(1)$ quantum link model as well as for $U(N)$ and $SU(N)$ quantum link models with fermionic matter, using ultracold Bose-Fermi mixtures or alkaline-earth atoms.

• This allows the quantum simulation of the real-time evolution of string breaking as well as the quantum simulation of dense "quark" matter, at least in toy models for QCD.

• The path towards quantum simulation of QCD will be a long one. However, with a lot of interesting physics along the way.